

1. Supply Chain Management & Logistics

1.1. Classical Logistics

1.1.1. Logistics as “organization of a complex operation”	<empty citation>
1.1.2. Logistics in Manufacturing	<empty citation>
1.1.3. Logistics in Services	<empty citation>

1.2. Supply Chain Management

1.2.1. What is a Supply Chain?	<empty citation>
--------------------------------	------------------

1.2.2. Supply Networks

In a manufacturing context, a supply chain can be seen as a network of suppliers, manufacturers, distributors, and retailers.

1.2.3. SCM vs. Logistics	<empty citation>
1.2.4. SCM Cycles	<empty citation>
1.2.5. SCM Processes	<empty citation>
1.2.6. SCOR Model	<empty citation>
1.2.7. Supply Chains as Systems	<empty citation>

2. Flow & Capacity

2.1. Flows

2.1.1. Types of Flows in a Supply Chain	<empty citation>
---	------------------

2.2. Capacity

2.2.1. Buffers	<empty citation>
2.2.2. Matching Supply with Demand	<empty citation>

3. Push-Pull Systems & Segmentation

3.1. Push-Pull Systems

3.1.1. Push and Pull Processes	<empty citation>
3.1.2. Product-Process Matrix	<empty citation>

3.2. Postponement & Mass Customization

3.2.1. Customer Order Decoupling Point	<empty citation>
--	------------------

3.3. Product Segmentation

3.3.1. Criteria for Segmentation	<empty citation>
3.3.2. Power “Law”	<empty citation>
3.3.3. ABC Analysis	<empty citation>
3.3.4. Multicriteria ABC Analysis	<empty citation>
3.3.5. AI/ML techniques for Segmentation	<empty citation>

3.4. Supply Chain Segmentation

3.4.1. Supply Chain Portfolios	<empty citation>
--------------------------------	------------------

4. Accounting POV for Inventory

4.1. Capital and Financial Statements

4.1.1. Sources of capital

Management seeks capital to finance operations from two main sources:

Shareholders: Individuals or entities that purchase and hold shares of a company's stock, thereby owning a portion of the company's equity. They expect (and are entitled to receive) returns through investment appreciation and/or dividends, and also have the right to vote on certain company decisions. Unlike debtholders, shareholders have an ownership stake in the company, which carries both potential financial rewards and risks, as their investment value can fluctuate with the company's performance.

Debtholders: Individuals or entities that lend capital to a company, usually in the form of loans or bonds, with the expectation of being repaid the principal amount along with interest over time. Unlike shareholders, debtholders do not obtain ownership stakes in the company, but hold a financial claim that is prioritized in the event of liquidation.

The blended result of these contributions is called the **capital structure**.

4.1.2. Flow of capital

4.1.3. Fundamental Business Activities

Operating Activities:

- Form the core of a business through the management of operating assets for the production and/or sale of goods and services.
- Encompass everyday functions to maintain business continuity.
- Ideally, these activities ensure smooth operations for profit generation.

Investing Activities:

- Acquisition, replacement, and disposition of operating assets like inventory, buildings, and equipment.
- Investments in intangible assets like know-how or Research and Development.
- Investments in digital assets such as platforms and software.
- Full or partial acquisition of other companies.
- Planning and control of cash inflows to ensure rational and timely, opportune amounts.

Financial Activities:

- Focused on capital management, raising funds from shareholders and/or debtholders.
- Selling financial assets or securities such as shares of stock and bonds.
- Managing debt and dividend payments, or engaging in stock buybacks.
- Evaluating various debt and equity financing options, designing a sound capital structure.

4.1.4. The Balance Sheet: A Statement of Financial Position

Provides a snapshot of a company's financial position at a specific point in time, often at the end of a fiscal year, showcasing assets, liabilities, and shareholders' equity.

The amount of highly liquid assets indicates ability to meet debt payments as they come due.

4.1.5. The Income Statement Visualized

Let's examine a scenario where better demand-supply alignment results in a 5% sales increase, while also fairly accounting for a rise in costs and expenses.

4.1.6. Elements of the Income Statement

Revenues indicate inflow of assets or reduction in liabilities, primarily from sales of inventories or services.

COGS or Cost of Sales reflects the original cost of inventory sold, either its purchase price or its manufacturing cost. By subtracting this from Revenues, we arrive at the **Gross Margin**.

R&D Expenses or Research & Development Expenses cover costs like product innovation or supply chain optimizations. Whereas **SGA Expenses** or Selling, General, and Administrative expenses, encompass costs that aren't directly tied to producing an item. This includes expenses such as salaries, rent, utilities, marketing, distribution costs, customer service as well as administrative costs like office supplies, legal fees, etc.

By subtracting the aforementioned expenses, we derive **EBITDA**, which stands for Earnings Before Interest, Taxes, Depreciation, and Amortization. Further adjustments, primarily subtracting depreciation and amortization from EBITDA, yield the **Operating Income**, also known as **EBIT** (Earnings Before Interest and Taxes).

Other revenues (or expenses) represent minor cash inflows or outflows not related to core operations. After accounting for these, we determine the **Net Income**, also referred to as **Profit**.

4.1.7. Profitable operations as a source of capital

Retained earnings represent the cumulative profits a company has generated and chosen to reinvest in the business rather than distribute as dividends. They don't pinpoint a specific tangible asset or cash pool. Instead, they indicate the portion of the assets listed on the balance sheet that stems from profitable operations.

These earnings highlight the capital sourced directly from profitable operations, distinguishing it from capital derived from borrowings or owner contributions.

4.2. Financial Performance

4.2.1. Measuring Performance	<empty citation>
Foo	

4.2.2. The Operating Cycle

The operating cycle refers to the duration it takes for a company to convert its cash outflows into cash inflows through its core operations.

The duration of this cycle can vary based on the company type, its policies, industry norms, market conditions, and the nature of its transactions.

Current assets are those that can be converted into cash within a company's operating cycle or within a year, whichever is longer.

Factors	Example 1	Example 2
Company Type	Local craft brewery	High-end furniture manufacturer
Company Policies	Offers extended credit terms to distributors to promote their brand	Has a strict return policy, allowing only exchanges within a short window
Industry Norms	Typically, the alcoholic beverage industry sees seasonal spikes in sales, especially during holidays	In luxury furniture, customers expect customization options, leading to longer production times
Market Conditions	Due to a recent health trend, there's a surge in demand for craft beverages with natural ingredients	The economy is in a downturn, and fewer consumers are investing in luxury goods
Nature of its Transactions	Primarily engages in B2B transactions with retailers and restaurants, which often involve negotiated rates and bulk deals	Engages mainly in B2C transactions through their showroom and online store, with occasional bespoke orders from corporate clients

4.2.3. Working Capital	<empty citation>
Foo	
4.2.4. Ratios	<empty citation>
Foo	
4.2.5. Inventory Turnover Revisited	<empty citation>
Foo	

4.3. Cost of Capital

4.3.1. Cost of Capital

Foo

4.3.2. WACC

The weighted average cost of capital uses the following formula:

$$WACC = \frac{E}{E + D} (R_E + \beta \cdot MRP) + \frac{D}{E + D} R_D (1 - t)$$

Let's analyze this concept. First, notice the terms $\frac{E}{E + D}$ and $\frac{D}{E + D}$; they create a weighted measure of the individual contributions of $(R_f + \beta \cdot MRP)$ and $R_D (1 - t)$.

4.4. Economic and Investment Analysis

4.4.1. Time Value of Money	<empty citation>
Foo	
4.4.2. Present Value	<empty citation>
Foo	
4.4.3. Combine with Pratt 6?	<empty citation>
Check if Pratt chapter 6 has something to do with Cash Flow Analysis (it seems, but haven't done a diligent search).	

4.5. Pratt: Chapters 6, 7)

4.5.1. Why 6?	<empty citation>
It seems traditional ratio analysis is out of date, it isn't very useful. Future cash flows is a more realistic analysis. Must include The Statement of Cash Flows	
4.5.2. Why 7?	<empty citation>
Inventories → But this will be include in the next section.	

5. Accounting POV for Inventory

5.1. Accounting for Inventory

5.1.1. Accounting categories of Inventory

Inventory plays a central role in accounting, reflecting a company's financial well-being and operational stance. It represents a major portion of a firm's assets, with its management directly affecting profitability and liquidity. Therefore, precise record-keeping is required to offer stakeholders a concise financial perspective crucial for investment decisions.

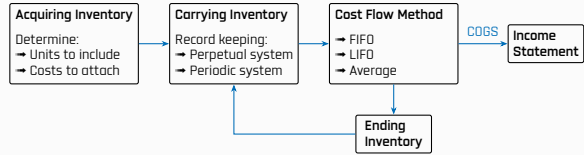
Shifting our lens to manufacturing, we can delineate these specific inventory categories:

- **Raw Materials:** Fundamental inputs of a manufacturing process
- **Work in Progress (WIP):** inventory undergoing transformation from raw materials to final products.
- **Components:** individual parts, sourced or produced, essential for final product assembly.
- **Finished Goods:** Fully processed products ready for sale.

In a broader operational context, beyond pure manufacturing, we also consider:

- **Merchandise Inventory:** Ready-to-sell products acquired for resale without additional modification.
- **Supplies:** Operational items not for sale, such as office materials.
- **MRO Items:** Resources for maintenance, repair, and operations, distinct from final product materials.

5.1.2. The Inventory Accounting Flow/Cycle



5.1.3. Capitalization of Inventory Costs [Cost Deduction]

Inventories are acquired at a cost and don't generate revenues until they're sold; thus, their cost is capitalized. Per the matching principle, the sale revenue is matched with the inventory's cost at the time of sale. To determine the capitalized cost, first identify the number of inventory items, then assign a cost to each item.

To understand the policy conversation about LIFO and FIFO, one must understand two main philosophies of calculating income: the pure income approach and the cash flow approach. The income approach focuses on matching deductions for costs with the revenues they generate. For example, if a farm invests in a new tractor that it will use for 10 years, it should spread the deductions for that tractor out over the next 10 years. When applying this principle to inventories, companies should deduct the cost of a unit of inventory when it is sold. The cash flow approach suggests companies should deduct their costs right when those costs are incurred. In the case of the farm investing in a new combine, it should deduct the full cost of the combine immediately. When applying this principle to inventories, companies should deduct the cost of a unit of inventory when it is acquired.

For accountants and financial analysts, the income (or book income) approach makes sense for determining the financial health of a company—this approach shows that, for instance, a decline in cash flow thanks to major investment decisions does not mean that the company is “losing money,” as those investment costs can get spread out. Meanwhile, the cash flow-based approach makes the most sense from a tax economics perspective because deducting investment costs when they are incurred means a company can deduct the full real cost, without inflation and the time value of money eating away at the deduction's value [which occurs when deductions are spread over several years]. For some issues, companies can use one set of rules to calculate financial income and another set of rules to calculate taxable income—which also makes sense, as they measure different things. But in the case of LIFO and FIFO, both systems are, at least on paper, based on the book income approach. Both systems have companies deduct the cost of a unit of inventory when it is sold, not when it is acquired. Additionally, companies must use the same system for both financial and taxable income.

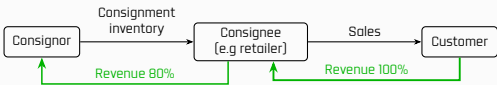
5.1.4. Units to include

General Rule: Items intended for manufacturing, sale, or consumption should be included in a company's inventory only if the company has full ownership of them, meaning that it bears all associated risks and benefits. Usually, ownership implies possession of the items, and in such cases, the units to be included in the inventory can be straightforwardly counted.

However, there are situations where ownership doesn't necessarily mean direct possession. Two of these notable exceptions are: Consignments and Goods in Transit.

5.1.5. Consignments

In a consignment arrangement, the *consignor* transfers inventory to a *consignee*, such as a retailer, who physically holds and sells the items. While the consignor retains full ownership, the consignee, after selling, keeps a service fee and remits the rest of the proceeds to the consignor.

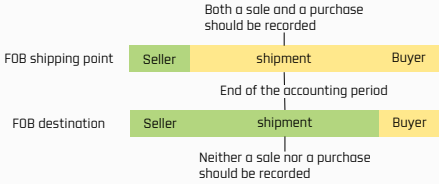


Inventory should only be disclosed in the consignor's balance sheet.

5.1.6. Goods in Transit

Theoretically, both a seller and a buyer should record a transaction simultaneously. However, in practice, most sales are recorded when goods are shipped, while purchases are typically recorded upon receipt of the goods. This method is generally acceptable, unless there are *goods in transit* at the end of an accounting period.

To properly account for such transactions, it's essential to determine the ownership of the goods while they are in transit. Freight shipping terms like FOB (free on board) serve this purpose. This term is commonly used in domestic shipping within the U.S., and should not be confused with the FOB term from the International Commercial Terms, INCOTERMS © 2020.



FOB shipping point: The seller is responsible for the goods only to the point from which they are shipped.
FOB destination: The seller is responsible for the goods all the way to their destination.

5.1.7. Costs to Attach

General Rule: Attach all the costs required to bring inventory to a saleable condition:

- Acquisition (purchased or manufactured)
 - Shipping in (or inbound transportation)
 - Storage
 - Packaging
- Distribution costs → These are considered as “selling expenses”, not inventory costs.

5.1.8. Types of Discounts

Sales-boosting Discounts:

Designed to increase sales volume, either by attracting more customers or incentivizing larger purchases. Promotions, Introductory Offers, Trade Discounts, Volume or Quantity Discounts

Inventory Management Discounts:

Discounts used to manage stock levels, clear old inventory, or promote specific products. Seasonal Discounts, Markdowns, Two-for-One (and related), Overstock Discounts, Closeout Discounts

Liquidity-Improving Discounts:

Discounts offered to accelerate payment, enhancing the seller's (or even the buyer's) cash flow. Cash Discounts (Early Payment Discounts), Deferred Payment / Simple Trade Credit.

Cost and Service Discounts:

Discounts that either reduce ancillary costs or add value through supplementary services. Free Shipping, Loyalty Discounts, Pick-up Incentives, Tiered Service Discounts.

5.1.9. Cash Discounts [Early Payment Discounts]

They're incentives for buyers to pay invoices early, improving the seller's cash flow. For instance, “2/10 net 30” provides a 2% discount if payment is made within 10 days; otherwise, the full amount is due by day 30.



Based on the provided terms, buyers can either capitalize on a 2% discount by paying on day 10, often via a loan, or leverage the extended trade credit for 20 more days by settling on day 30. From a financial perspective, it's essential to balance the associated debt cost against the opportunity cost of missing the discount or paying the extra amount.

Simple interest of the opportunity cost:

$$\left(\frac{0.02}{0.98}\right)\left(\frac{365}{20}\right) \approx 0.37$$

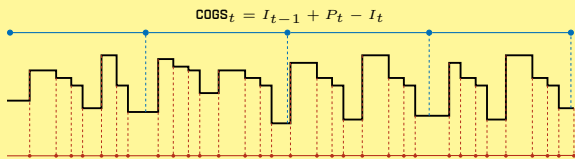
Annualization helps to compare the costs and evaluate alternatives, factoring in either simple or compounded interest for the lost funds, based on how the business handles its money.

Compound interest of the opportunity cost:

$$\left(\frac{1}{0.98}\right)^{(365/20)} - 1 \approx 0.45$$

5.1.10. Inventory record keeping: Periodic and Perpetual Systems

Periodic System: Inventory records are updated only at specific intervals, often at the end of a month or quarter. For instance, *COGS* would be calculated considering the *beginning inventory* plus the *purchases* during the period minus the *ending inventory*. This approach may be chosen due to technological constraints or specific company policies.



Perpetual System: Inventory is continuously updated with each transaction. Real-time tracking is often facilitated by Point-of-Sale Systems or technologies such as RFID, QR codes, barcodes, and ERP systems. This system provides enhanced tracking precision, though it may come at a significantly higher cost

5.1.11. Inventory Valuation Methods and COGS Allocation

While theoretically possible, allocating inventory costs to specific items often proves impractical for most firms due to the need for extensive physical counting or advanced tracking systems. This challenge leads companies to adopt standardized valuation methods, which, beyond simplifying the process, can also strategically influence financial reporting and taxation.

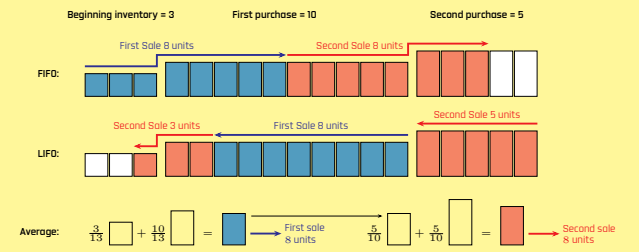
Specific Identification: Often used for high-value items and increasingly with common items through barcode and RFID technology, ensures the highest accuracy in inventory acquisition cost allocation and ending inventory valuation by tracking individual items. However, it allows for potential manipulation of net income, as the cost of goods sold can be selectively influenced by choosing specific items for sale.

The **FIFO [First-in-First-Out]** method calculates COGS using the oldest inventory costs, while the ending inventory is valued at the most recent costs. During times of rising prices, this can result in COGS being reported at lower, outdated costs, potentially understating the actual expense and inflating profits in inflationary periods.

The **LIFO [Last-in-First-Out]** method determines COGS using the most recent inventory costs and ending inventory is valued at the oldest costs. In inflationary times, this can lead to COGS reflecting higher, more current prices, possibly overstating expenses and reducing reported profits, but it may also offer tax advantages due to lower taxable income.

Weighted Average: This method calculates COGS and ending inventory value using a periodically updated weighted average cost, reflecting not just a simple average but a proportionate cost of goods, thus ensuring a balanced approach to inventory valuation.

5.1.12. FIFO, LIFO and the Weighted Average Method



5.1.13. LIFO Liquidation

This occurs when older, lower-cost LIFO inventory layers are sold, often due to cash flow issues, like shortages or the need to smooth earnings variability¹, demand spikes, supply shortages, or the disposal of outdated stock. Such liquidations can significantly distort COGS, impacting taxable income and reported profits.

			price
greatest revenue			
smallest COGS layer			

Notice that LIFO causes some harm...we need mitigation

1 Firms may use LIFO liquidation during certain periods as part of *earnings management*, aiming to smooth earnings and present a stable, predictable financial performance.

5.1.14. LIFO: Implications for Financial Reporting and Taxation

In periods of rising prices, inventory acquired later is often more expensive, and so the LIFO method allows for the deduction of higher inventory costs, leading to decreased reported earnings and lower tax burdens. Conversely, during times of declining prices, LIFO can result in lower deductions, overstating both earnings and tax liabilities. The strategic implications of these scenarios under LIFO require careful considerations.

Inventory Turnover: Determined by COGS divided by Average Inventory, serves as a benchmark for comparing efficiency across similar industries. However, in times of inflation, LIFO's elevated COGS can lead to seemingly higher Inventory Turnover ratios, which may overstate a company's effectiveness in managing its inventory.

The **LIFO Reserve**, representing the difference between FIFO and LIFO inventory valuations, facilitates comparisons between companies with different accounting methods. It indicates inventory undervaluation on the balance sheet and the cumulative reduction in taxable income for the firm.

5.1.15. LIFO and Just-in-Time Inventory Management

[11] observed the following behaviors related to Just-in-Time inventory management, LIFO reserve dynamics, and financial decision-making in firms during the 1980s and 1990s, a period in which JIT gained popularity:

Large LIFO Reserve firms: Firms with significant LIFO reserves often see important short-term earnings increases after JIT adoption, as the liquidation of lower-cost LIFO inventories reduces COGS and boosts profits.

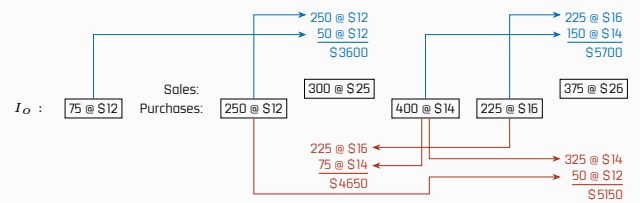
FIFO and Minimal LIFO Reserve firms: Companies operating under FIFO or with small LIFO reserves may find that the initial costs of implementing JIT can offset or even negate short-term productivity or efficiency gains.

JIT adoption in High-Leverage Firms: High-debt firms with large LIFO reserves tend to adopt JIT, suggesting they use JIT-induced LIFO liquidations to manage debt-related constraints.

Tax rates influencing JIT adoption: In high-tax scenarios, the likelihood of JIT adoption diminishes for firms with large LIFO reserves, as JIT's low inventory levels limit the tax benefits of LIFO reserves, and their liquidation can lead to unfavorable tax consequences.

5.1.16. COGS Computation Example

Consider the following scenario with beginning inventory I_O , purchases and sales during a period. The **perpetual** system computes COGS in each transaction and the **periodic** system computes COGS at the end of the whole period. Cost allocation is **LIFO**.



The **Perpetual LIFO COGS is \$9300** whereas the **Periodic LIFO COGS is \$9800**.

6. Inventory I: Deterministic Models

6.1. What is Inventory and why does it matter?

6.1.1. Accounting POV vs. Logistics/SCM POV

6.1.2. Logistics/SCM types of inventory

6.1.3. Why hold inventory?

- Cover process time
- Decouple process
- ...

6.1.4. Inventory decisions

6.2. Inventory Models

6.2.1. Models

Trade-offs between complexity and ease of understanding/communication/implementation.

6.2.2. Models for Inventory Management

- Focus on costs
- Focus on service level
- ...

6.3. Inventory Costs

6.3.1. Unit Cost: $c \rightarrow \$/\text{unit}$

The cost of obtaining one unit of a SKU, either through procurement or production.

For merchants: It's the sum of the purchase price paid to the supplier, combined with additional costs necessary for preparing the product for sale, such as packaging and labeling. Typically, it also incorporates per-unit costs related to freight transportation and material handling, like loading and unloading.

For producers: It's the total unitary production cost. Similarly, it can also include material handling and freight transportation costs incurred from production-related activities. Determining the unit value in manufacturing can be more challenging due to its intricate nature.

In basic inventory models, the unit cost is typically considered lot-size independent for simplicity. However, some models account for **economies of scale** by incorporating discounts related to the volume of items purchased or produced, recognizing that unit costs can vary with lot size.

Typically, the unit value is derived from the company's internal accounting system, representing its “**book value**”, therefore, it may differ from what SCM/logistics specialists might consider. Ideally, the unit value should be determined collaboratively, taking into account the actual money spent on that specific SKU to prepare it for either internal or external use.

6.3.2. Ordering/Setup Cost: $c_t \rightarrow \$/\text{order}$

6.3.3. Cost components of holding inventory	15,18,3,19
They vary across companies and SKUs, but, in general, they include the following major components, which must be incremental in nature, otherwise, they would have been incorporated as part of the fixed ordering cost:	
Cost of Capital: Capital is allocated to either purchase or produce inventory units, so less inventory means more available capital for alternative investments, each with their respective rates of return. Given that capital can be sourced from either equity or debt, the Weighted Average Cost of Capital (WACC) is often used here, as it's a blended measure for both sources of inventory financing.	
Incremental Costs of Storage: Warehouse space often represents a significant expenditure, especially in prime locations. Handling inventory -i.e. moving, organizing within the storage space--adds to the costs. Periodic counting or inventory audits are essential for accuracy, but require time and resources. Some inventory items might also necessitate special storage conditions, such as refrigeration or specific humidity levels, leading to additional expenses.	
Costs of Depreciation: Inventory value can diminish over time due to several reasons. Perishable Items may degrade, rendering them unsellable. As new products are introduced, older items may become obsolete, especially in industries with rapid innovation cycles. Moreover, shrinkage, resulting from items being lost, stolen, or damaged, further erodes the inventory's value.	

6.3.4. Holding Cost: $c_e \rightarrow \text{\\$/ (unit} \times \text{period)}$	14,15
Encapsulates all costs incurred from carrying a unit of inventory for a designated period. We can model it as:	
$c_e = r c$	
where the <i>holding rate</i> r denotes a percentage of the unit value c per period of demand [e.g. for 1 year]. A multi-SKU company may opt for $c_{e_i} = r_i c_i$ for each SKU i or, to alleviate the complexity of individual analysis, apply a uniform holding rate r across all SKUs. Accordingly, c_e has the following dimensions:	
$\frac{\text{\$}}{\text{unit} \times \text{period}}$	
By modeling it this way, we can evaluate the cost of keeping inventory proportionally to the amount held.	
However, in certain scenarios, the cost of storing an item remains consistent, regardless of its value. When we employ a singular, aggregated rate r , we inadvertently allow the storage component to escalate in proportion to the item's unit value. A more nuanced approach would be to utilize	
$c_e = r c + h$	
Within this framework, h stands as a constant unitary storage fee, while r is solely representative of the cost of capital and depreciation associated with the item.	
Furthermore, consider the scenario where storage capacity is limited; if Q exceeds this threshold, an additional warehouse is required, incurring a fixed cost. This scenario can be modeled using a piecewise function, for instance:	
$\text{Total Holding Cost} = \begin{cases} Q(r c + h) & , \text{ for } Q \leq \text{threshold} \\ Q(r c + h) + \text{Fixed Cost} & , \text{ for } Q > \text{threshold} \end{cases}$	
Given the complexity of the holding cost, it's advisable to model it in collaboration with Finance/Accounting specialists.	

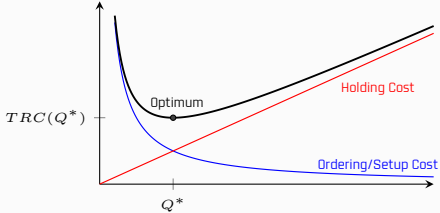
6.3.5. Stockout/Shortage Cost: c_s	<empty citation>
Can be modeled using stockout event or units short	

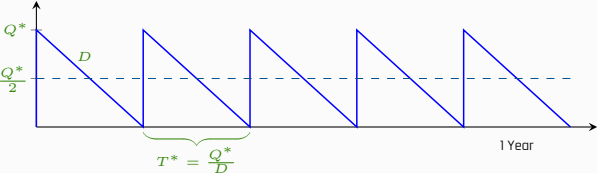
6.3.6. Coordinated Cost Estimation: Finance and SCM/Logistics	<empty citation>
Foo	

6.3.7. Total Cost & Total Relevant Cost	<empty citation>
$TC = \text{Purchase Cost} + \text{Ordering Cost} + \text{Holding Cost} + \text{Shortage Cost}$	
$TC = cD + c_t \frac{D}{Q} + c_e \frac{Q}{2} + c_s E[\text{Units Short}]$	
Procurement activities have influence on the Purchase Cost, while Inventory Management activities have influence on the other costs.	

6.4. EOQ: Economic Order Quantity

6.4.1. EOQ model assumptions	<empty citation>
<div> <div>→ Known demand → Constant</div> <div>→ Zero or Constant Lead Time</div> <div>→ Something else</div> </div> <div> <div>Checkar papers review sobre EOQ</div> </div>	
<div> <div>Checkar variaciones en el modelado de costos (i.e. variable holding cost, setup cost, etc.) en Silver, Chopra, Nahmias, etc...Hay muchas variaciones, pero incluir las mas frecuentes en los libros)</div> </div>	

6.4.2. EOQ formula derivation	<empty citation>
Since demand is deterministic, we can get rid of the Stockout Cost concept for now. So,	
$TRC(Q) = c_t \frac{D}{Q} + c_e \frac{Q}{2}$	
From the first-order optimal condition (first derivative equals zero), we have	
$0 = \frac{d}{dQ} \left(\frac{c_t D}{Q} \right) + \frac{d}{dQ} \left(\frac{c_e Q}{2} \right)$	
$0 = -\frac{c_t D}{Q^2} + \frac{c_e}{2}$	
$Q^* = \sqrt{\frac{2c_t D}{c_e}}$	
The EOQ or Q^* gives the minimum TRC under deterministic conditions:	
	

6.4.3. EOQ Sawtooth Plot	<empty citation>
The optimal policy becomes ordering Q^* units of inventory every T^* units of time.	
	
Notice that the total consumption of the last order may take place after the 1 year (unit time) period.	

6.4.4. Sensitivity Analysis for the EOQ model	<empty citation>
Resaltar que, pese a que algunos parametros se asumen alegremente como determinísticos, el modelo es lo suficientemente robusto como para compensar variaciones en los mismos (e.g. demanda, costos, etc.) Usar los 5 libros en ...Análisis y logística de la producción + otros complementos	

6.4.5. Powers of Two Policies	<empty citation>
--------------------------------------	------------------

6.5. EOQ Extensions

6.5.1. Lead Time > 0	<empty citation>
6.5.2. Discounts: All units	<empty citation>
6.5.3. Discounts: Incremental	<empty citation>
6.5.4. Discounts: One-time	<empty citation>
6.5.5. Backorders	<empty citation>
6.5.6. EPQ: Economic Production Quantity	<empty citation>
6.5.7. Perishability	<empty citation>
6.5.8. Trade Credit	<empty citation>

7. Forecasting I

7.1. Demand Planning

7.1.1. Demand Planning	<empty citation>
7.1.2. Demand Forecasting	<empty citation>

7.2. Data Collection

7.2.1. Obtaining data	<empty citation>
7.2.2. Aggregated data, Aggregated forecasts	<empty citation>

7.3. Time Series

7.3.1. Time Series Components	<empty citation>
7.3.2. Decomposition	<empty citation>
7.3.3. Cummulative & Naive Forecasting	<empty citation>
7.3.4. Moving Averages Forecasting	<empty citation>

7.4. Forecasting Metrics

7.4.1. Accuracy & Bias	<empty citation>
7.4.2. Error Metrics	<empty citation>

7.5. Exponential Smoothing

7.5.1. Simple Exponential Smoothing	<empty citation>
7.5.2. Damped Trend	<empty citation>

8. Forecasting II

8.1. Exponential Smoothing with Seasonality

8.1.1. Seasonality Patterns	<empty citation>
8.1.2. Double Exponential Smoothing	<empty citation>
8.1.3. Holt-Winter Model	<empty citation>
8.1.4. Initialization of Parameters	<empty citation>
8.1.5. Comments and Comparison of Models	<empty citation>

8.2. Intermittent Demand Forecasting

8.2.1. Intermittent demand patterns and examples	<empty citation>
8.2.2. Approaches	<empty citation>
8.2.3. Craston's Method	<empty citation>

8.3. Regression & Causal Analysis

8.3.1. Explaining causes of demand phenomena	<empty citation>
---	------------------

8.3.2. Correlation and Causation	<empty citation>
8.3.3. Simple Linear Regression	<empty citation>
8.3.4. Multiple Linear Regression	<empty citation>

8.4. Product Development, Marketing & Forecasting

8.4.1. New Products Introduction	<empty citation>
8.4.2. Forecasting techniques & Product Life Cycle	<empty citation>

8.5. AI/ML techniques for Forecasting

8.5.1. Clustering	<empty citation>
--------------------------	------------------

9. Inventory II: Stochastic Models

9.1. Stochastic Demand

9.1.1. Demand distribution	<empty citation>
9.1.2. Expected Demand	<empty citation>
9.1.3. Expected Units Short	<empty citation>
9.1.4. Expected Units Sold	<empty citation>

9.2. Demand Modelling

9.2.1. Empirical Distribution	<empty citation>
9.2.2. Discrete Uniform	<empty citation>
9.2.3. Poisson	<empty citation>
9.2.4. Continuous Uniform	<empty citation>
9.2.5. Normal	<empty citation>
9.2.6. Triangle	<empty citation>
9.2.7. Chi-Square Test	<empty citation>

9.3. SPIM: Single Period Inventory Models

9.3.1. SPIM: Problem introduction	<empty citation>
9.3.2. Data Table	<empty citation>
9.3.3. Marginal Analysis	<empty citation>
9.3.4. Salvage Value	<empty citation>
9.3.5. Penalty Value	<empty citation>
9.3.6. Critical Ratio	<empty citation>

9.3.7. Expected Profits	<empty citation>
-------------------------	------------------

9.4. The Newsvendor Problem

9.4.1. Newsvendor Problem: Introduction	<empty citation>
NFL Jersey Problem in the MicroMasters	

9.4.2. Unit Normal Loss Function	<empty citation>
----------------------------------	------------------

9.4.3. Newsvendor Problem: Solution	<empty citation>
-------------------------------------	------------------

9.5. The Newsvendor Problem Extensions

9.5.1. Foo	<empty citation>
------------	------------------

10. Inventory III: Multiple Period Inventory Models

10.1. Introductory Models

10.1.1. Rescaling of Parameters	<empty citation>
---------------------------------	------------------

10.1.2. Base Stock Model	<empty citation>
--------------------------	------------------

10.2. Continuous Review Models

10.2.1. (s, Q) model	<empty citation>
------------------------	------------------

10.2.2. (s, S) model	<empty citation>
------------------------	------------------

10.3. Safety Stock: Service Cost and Metrics

10.3.1. Cycle Service Level	<empty citation>
-----------------------------	------------------

10.3.2. Cost per Stockout Event	<empty citation>
---------------------------------	------------------

10.3.3. Item Fill Rate	<empty citation>
------------------------	------------------

10.3.4. Cost per Item Short	<empty citation>
-----------------------------	------------------

10.3.5. Inputted and Implied Metrics	<empty citation>
--------------------------------------	------------------

10.4. Periodic Review Models

10.4.1. (R, S) model	<empty citation>
------------------------	------------------

10.4.2. (...) model	<empty citation>
---------------------	------------------

11. Inventory IV: Multiple Dimension Models

11.1. Multiple Items

11.1.1. Grouping	<empty citation>
------------------	------------------

11.1.2. Grouping: Powers of Two	<empty citation>
---------------------------------	------------------

11.1.3. Grouping: Exchange Curves	<empty citation>
-----------------------------------	------------------

11.2. Multiple Locations

11.2.1. Location Pooling	<empty citation>
--------------------------	------------------

11.3. Multiple Classes

11.3.1. Segmentation Revisited	<empty citation>
- Fast moving items	
- Slow moving items	
...	

11.3.2. A Items	<empty citation>
-----------------	------------------

11.3.3. B Items	<empty citation>
-----------------	------------------

11.3.4. C Items	<empty citation>
-----------------	------------------

11.4. Multiple Echelons

11.4.1. Multiple Echelons	<empty citation>
---------------------------	------------------

12. Transportation I: Freight Transportation

12.1. Freight Transportation

12.1.1. Time-Space Diagram	<empty citation>
----------------------------	------------------

12.1.2. Packaging	<empty citation>
- Cases	
- Pallets	
- Containers	
...	

12.1.3. Transportation Modes and Routes	<empty citation>
?	

12.2. Transportation Networks

12.2.1. Physical Network	<empty citation>
--------------------------	------------------

12.2.2. Operational Network	<empty citation>
-----------------------------	------------------

12.2.3. Strategic Network	<empty citation>
---------------------------	------------------

12.3. Transportation & Inventory

12.3.1. Transportation Cost Functions	<empty citation>
---------------------------------------	------------------

12.3.2. Total Inventory & Transportation Cost	<empty citation>
---	------------------

12.3.3. Transit & Lead Time Variability	<empty citation>
---	------------------

12.3.4. Random Sum of Random Variables	<empty citation>
--	------------------

12.4. Mode Selection

12.4.1. Foo	<empty citation>
-------------	------------------

13. Transportation II: Analysis

13.1. The Transportation Product

13.1.1. Four Fundamental Operations	<empty citation>
-------------------------------------	------------------

13.1.2. Loading & Unloading	<empty citation>
-----------------------------	------------------

13.1.3. Linehaul Moves	<empty citation>
------------------------	------------------

13.1.4. Vehicle Routing	<empty citation>
-------------------------	------------------

13.1.5. Facility Sorting	<empty citation>
--------------------------	------------------

13.2. Transportation Economies

13.2.1. Economies of Scale	<empty citation>
----------------------------	------------------

13.2.2. Economies of Scope	<empty citation>
----------------------------	------------------

13.2.3. Economies of Density	<empty citation>
------------------------------	------------------

13.3. Transportation Economic Modes

13.3.1. Direct Transportation	<empty citation>
-------------------------------	------------------

13.3.2. Consolidated Transportation	<empty citation>
-------------------------------------	------------------

13.4. Transportation & Routing Problems

13.4.1. 1 : 1	<empty citation>
---------------	------------------

13.4.2. 1 : ∞	<empty citation>
----------------------	------------------

13.4.3. ∞ : 1	<empty citation>
----------------------	------------------

13.4.4. ∞ : ∞	<empty citation>
-----------------------------	------------------

14. Warehouse Management

14.1. Warehousing

14.1.1. Why warehouses?	<empty citation>
-------------------------	------------------

14.1.2. Types of warehouses	<empty citation>
-----------------------------	------------------

14.2. Warehousing & Packaging

14.2.1. Foo	<empty citation>
-------------	------------------

14.3. Core Operational Functions

14.3.1. Receive	<empty citation>
-----------------	------------------

14.3.2. Put away	<empty citation>
------------------	------------------

14.3.3. Store	<empty citation>
---------------	------------------

14.3.4. Pick	<empty citation>
--------------	------------------

14.3.5. Check, Pack, Ship	<empty citation>
---------------------------	------------------

14.3.6. Return handling	<empty citation>
-------------------------	------------------

14.3.7. Value-added services	<empty citation>
------------------------------	------------------


14.4. Layout design

14.4.1. Foo	<empty citation>
-------------	------------------

14.5. Cross-Docking

14.5.1. Foo	<empty citation>
-------------	------------------

14.6. Segmentation & Benchmarking in Warehousing

14.6.1. Foo	<empty citation>
	

14.7. Templates

14.7.1. Consequences of the Axioms

<empty citation>

By set theory definitions we have: $A \cup A^c = \Omega$ and $A \cap A^c = \emptyset$

$P(A) \leq 1$

A and A^c are disjoint $\Rightarrow P(A \cup A^c) = 1 = P(A) + P(A^c) \Rightarrow P(A^c) = 1 - P(A)$, and by *nonnegativity* we get $P(A^c) \geq 0 \Rightarrow P(A) \leq 1$ ■

$P(\emptyset) = 0$

Let $A = \Omega \Rightarrow P(\Omega) + P(\Omega^c) = 1 \Rightarrow 1 + \emptyset = 1 \Rightarrow P(\emptyset) = 0$ ■
Let Ω be a finite set and A_1, \dots, A_n be disjoint events, then:

$P\left(\bigcup_{i=1}^n A_i\right) = \sum_1^n P(A_i)$

$P(A \cup B \cup C) = P[(A \cup B) \cup C]$. From additivity, given that the events are disjoint, we have $(P(A) + P(B)) + P(C)$. By induction we can extend this to n disjoint sets ■
Let $\{\omega_1, \dots, \omega_k\}$ be a discrete, finite set of sample points, then:


$P\left(\{\omega_1, \dots, \omega_k\}\right) \Rightarrow P\left(\bigcup_{j=1}^k \{\omega_j\}\right) \Rightarrow \sum_{j=1}^k P\left(\{\omega_j\}\right)$

because $\{\omega_1, \dots, \omega_k\}$, can be seen as the union of *unit sets*, and since they are disjoint, additivity applies ■. Although, a simpler, non rigorous notation can be used: $\sum_{j=1}^k P(\omega_j)$.

14.7.2. More Consequences of the Axioms

<empty citation>

Consider the condition $P(A \cap B) \geq 0$, \Rightarrow The events could be joint, therefore, more generally:


 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Which can be generalized to the...:


$P\left(\bigcup_{i=1}^n A_i\right) = - \sum_{k=1}^n (-1)^k \sum_{1 \leq i_1 < \dots < i_k \leq n} P\left(\bigcap_{j=1}^k A_{i_j}\right)$

From the above, the *Union Bound* property follows: $P(A \cup B) \leq P(A) + P(B)$

Consider that A is included in B , then:

 $A \subset B \Rightarrow P(A) \leq P(B)$

since $B = A \cup (B \cap A^c) \Rightarrow P(B) = P(A) + P(B \cap A^c) \geq P(A)$ ■
Consider 3 sets not necessarily disjoint, e.g.:

 $P(A \cup B \cup C) = P(A) + P(A^c \cap B) + P(A^c \cap B^c \cap C)$

Visually, we can check the boxed expression by the matching of the colors, and since the subsets are disjoint, additivity holds. Notice the expression also applies to disjoint sets ■

14.7.3. Multiplication Rule

<empty citation>

Notice that:

$P(A \cap B) = P(B)P(A|B)$
 $= P(A)P(B|A)$

And for 3 events we have:

$P[(A \cap B) \cap C] = P(A \cap B)P(C|A \cap B)$
 $= P(A)P(B|A)P(C|A \cap B)$

More generally:

$P\left(\bigcap_{i=1}^n A_i\right) = P(A_1) \prod_{i=2}^n P\left(A_i \middle| \bigcap_{j=1}^{i-1} A_j\right)$

A particular intersection of events would be represented as a full path in a probability tree.

REFERENCES:

[1] Jamie Pratt. *Financial Accounting in an Economic Context (8th ed.)* John Wiley & Sons, 2011.

[2] Franklin Allen, Richard A. Brealey, and Stewart C. Myers. *Principles of Corporate Finance (12th ed.)* McGraw-Hill, 2017.

[3] Sunil Chopra and Peter Meindl. *Supply Chain Management (6th ed.)* Pearson, 2016.

[4] *Investopedia Financial Terms Dictionary*. <https://www.investopedia.com/>. Accessed: 2023.

[5] *Understanding the Tax Treatment of Inventory: The Role of LIFO*. <https://taxfoundation.org/research/all/federal/lifo-tax-treatment-inventory/>. Accessed: 2023.

[6] *GPT-4*. <https://openai.com/gpt-4>. Accessed: 2023.

[7] Janet Kiholm Smith. “Trade credit and informational asymmetry”. In: *The journal of finance* 42.4 (1987), pp. 863–872.

[8] *Repeal the “LIFO” and “Lower of Cost or Market” Inventory Accounting Methods*. <https://www.cbo.gov/budget-options/54812>. Accessed: 2023.

[9] *All About Amazon Costs of Goods Sold: Definition & Methods*. <https://trueprofit.io/blog/amazon-cost-of-goods-sold/>. Accessed: 2023.

[10] *LIFO Liquidation*. <https://corporatefinanceinstitute.com/resources/accounting/lifo-liquidation/>. Accessed: 2023.

[11] Michael R Kinney and William F Wempe. “JIT adoption: The effects of LIFO reserves and financial reporting and tax incentives”. In: *Contemporary Accounting Research* 21.3 (2004), pp. 603–638.

[12] Yibin Zhang et al. “Repealing the LIFO inventory accounting choice? A review of LIFO and inventory management”. In: *American Journal of Operations Research* 4.06 (2014), p. 351.

[13] *LIFO Reserve Meaning and How to Calculate It*. <https://www.investopedia.com/terms/l/lifo-reserve.asp>. Accessed: 2023.

[14] Chris Caplice and Eva Ponce. *MITx MicroMasters Program in SCM Key Concepts*. 2023. URL: https://scx-static-assets.s3.amazonaws.com/SCx%20Key%20Concept%20Documents/MITx_MicroMasters_SCM_KeyConcepts.pdf.

[15] Edward A. Silver, David F. Pyke, and Douglas J. Thomas. *Inventory and Production Management in Supply Chains (4th ed.)* CRC Press, 2016.

[16] Gerard Cachon and Christian Terwiesch. *Matching Supply with Demand (4th ed.)* McGraw-Hill Publishing, 2018.

[17] Paul Schonsleben. *Handbook Integral Logistics Management (6th ed.)* Springer, 2022.

[18] Sven Axäter. *Inventory Control (3rd ed.)* Springer, 2015.

[19] Paolo Brandimarte and Giulio Zotteri. *Introduction to Distribution Logistics*. John Wiley & Sons, 2000.