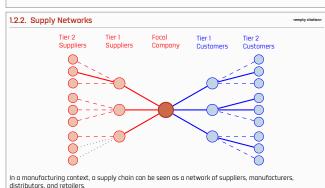
1. Supply Chain Management & Logistics

1.1. Classical Logistics

1.2.1. What is a Supply Chain?

111 Glassical Logistics	
1.1.1. Logistics as "organization of a complex operation"	<empty citation=""></empty>
1.1.2. Logistics in Manufacturing	<empty citation=""></empty>
1.1.3. Logistics in Services	<empty citation=""></empty>

1.2. Supply Chain Management



distributors, and retailers.		
1.2.3. SCM vs. Logistics		<emp< th=""></emp<>

1.2.4. SCM Cycles	<empty citation=""></empty>
1.2.5. SCM Processes	<empty citation=""></empty>

1.0	.2.6. SUUR MODEI	<empty citor<="" th=""></empty>
1.3	.2.7. Supply Chains as Systems	<empty cita<="" td=""></empty>

2. Flow & Capacity

2.1. Flows

2.1.1. Types of Flows in a Supply Chain	<empty citations<="" td=""></empty>

2.2. Capacity

2.2.1. Buffers	<empty citation=""></empty>
2.2.2. Matching Supply with Demand	<empty citation=""></empty>

3. Push-Pull Systems & Segmentation

3.1. Push-Pull Systems

3.1.1. Push and Pull Processes	<empty citation=""></empty>
3.1.2. Product-Process Matrix	<empty citation=""></empty>

3.2. Postponement & Mass Customization

3.2.1. Customer Order Decoupling Point <empty citation

3.3. Product Segmentation

3.3.1. Criteria for Segmentation



3.4. Supply Chain Segmentation

3.3.5. AI/ML techniques for Segmentation

3.4.1. Supply Chain Portfolios

4. Accounting POV for Inventory

4.1. Capital and Financial Statements

Management seeks capital to finance operations from two main sources:

Sources of capital	
	Sources of capital

Shareholders: Individuals or entities that purchase and hold shares of a company's stock, thereby owning a portion of the company's equity. They expect (and are entitled to receive) returns through investment appreciation and/or dividends, and also have the right to vote on certain company decisions. Unlike debtholders, shareholders have an ownership stake in the company, which carries both potential financial rewards and risks, as their investment value can fluctuate with the company's performance.

Debtholders: Individuals or entities that lend capital to a company, usually in the form of loans or bonds, with the expectation of being repaid the principal amount along with interest over time. Unlike shareholders, debtholders do not obtain ownership stakes in the company, but hold a financial claim that is prioritized in the event of liquidation

The blended result of these contributions is called the capital structure

4.1.2. Flow of capital



4.1.3. Fundamental Business Activities

Operating Activities:

- Form the core of a business through the management of operating assets for the production and/or sale of goods and services.
- → Encompass everyday functions to maintain business continuity
- → Ideally, these activities ensure smooth operations for profit generation

Investing Activities

- Acquisition, replacement, and disposition of operating assets like inventory, buildings, and equipment.
 Investments in intanaible assets like know-how or Research and Development.
- Investments in digital assets such as platforms and software.
- → Full or partial acquisition of other companies.
- → Planning and control of cash inflows to ensure rational and timely, opportune amounts.

Financial Activities:

- → Focused on capital management, raising funds from shareholders and/or debtholders.
- → Selling financial assets or securities such as shares of stock and bonds.
- → Managing debt and dividend payments, or engaging in stock buybacks. → Evaluating various debt and equity financing options, designing a sound capital structure

4.1.4. The Balance Sheet: A Statement of Financial Position

Provides a snapshot of a company's financial position at a specific point in time, often at the end of a fiscal year, showcasing assets, liabilities, and shareholders' equity.



The amount of highly liquid assets indicates ability to meet debt payments as they come due

4.1.5. Elements of the Income Statement

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Revenues indicate inflow of assets or reduction in liabilities, primarily from sales of inventories or services.

COGS or Cost of Sales reflects the original cost of inventory sold, either its purchase price or its manufacturing cost. By subtracting this from Revenues, we arrive at the Gross Margin

RSD Expenses or Research & Development Expenses cover costs like product innovation or supply chain optimizations. Whereas SGA Expenses or Selling, General, and Administrative expenses, encompass costs that aren't directly tied to producing an item. This includes expenses such as salaries, rent, utilities. marketing, distribution costs, customer service as well as administrative costs like office supplies, legal

By subtracting the aforementioned expenses, we derive EBITDA, which stands for Earnings Before Interest, Taxes, Depreciation, and Amortization. Further adjustments, primarily subtracting depreciation and amortization from EBITDA, yield the Operating Income, also known as EBIT (Earnings Before Interest and

Other revenues (or expenses) represent minor cash inflows or outflows not related to core operations. After accounting for these, we determine the Net Income, also referred to as Profit.

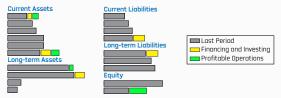
4.1.6. The Income Statement Visualized

Let's examine a scenario where better demand-supply alignment results in a 5% sales increase, while also fairly accounting for a rise in costs and expenses



4.1.7. Profitable operations as a source of capital

Retained earnings represent the cumulative profits a company has generated and chosen to reinvest in the business rather than distribute as dividends. They don't pinpoint a specific tangible asset or cash pool. Instead, they indicate the portion of the assets listed on the balance sheet that stems from profitable operations



These earnings highlight the capital sourced directly from profitable operations, distinguishing it from capital derived from borrowings or owner contributions

4.2. Accounting for Inventory

4.2.1. Accounting categories of Inventory

Inventory plays a central role in accounting, reflecting a company's financial well-being and operational stance. It represents a major portion of a firm's assets, with its management directly affecting profitability and liquidity. Therefore, precise record-keeping is required to offer stakeholders a concise financial perspective crucial for investment decisions.

Shifting our lens to manufacturing, we can delineate these specific inventory categories:

- → Raw Materials: Fundamental inputs of a manufacturing process
- → Work in Progress (WIP): Inventory undergoing transformation from raw materials to final products.
- Components: Individual parts, sourced or produced, essential for final product assembly.
- → Finished Goods: Fully processed products ready for sale

In a broader operational context, beyond pure manufacturing, we also consider

- → Merchandise Inventory: Ready-to-sell products acquired for resale without additional modification.
- → Supplies: Operational items not for sale, such as office materials
- → MRO Items: Resources for maintenance, repair, and operations, distinct from final product materials.

4.2.2. The Inventory Accounting Flow/Cycle Selling Inventory COGS Income LIFO Statement Acquiring Inventory Carrying Inventory Average Units to include erpetual method Costs to attach Periodic method Ending Inventory Something

4.2.3. Capitalization of Inventory Costs & Units to include

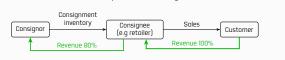
Inventories are acquired at a cost and don't generate revenues until they're sold; thus, their cost is capitalized. Per the matching principle, the sale revenue is matched with the inventory's cost at the time of sale. To determine the capitalized cost, first identify the number of inventory items, then assign a cost to

General Rule: Items intended for manufacturing, sale, or consumption should be included in a company's inventory only if the company has full ownership of them, meaning that it begrs all associated risks and benefits. Usually, ownership implies possession of the items, and in such cases, the units to be included in the inventory can be straightforwardly counted.

However, there are situations where ownership doesn't necessarily mean direct possession. Two of these notable exceptions are: Consignments and Goods in Transit.

4.2.4. Consignments

In a consignment arrangement, the consignor transfers inventory to a consignee, such as a retailer, who physically holds and sells the items. While the consignor retains full ownership, the consignee, after selling keens a service fee and remits the rest of the proceeds to the consignor

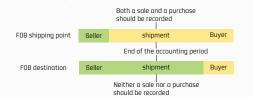


Inventory should only be disclosed in the consignor's balance sheet

4.2.5. Goods in Transit

Theoretically, both a seller and a buyer should record a transaction simultaneously. However, in practice, most sales are recorded when goods are shipped, while purchases are typically recorded upon receipt of the goods. This method is generally acceptable, unless there are goods in transit at the end of an accounting period

To properly account for such transactions, it's essential to determine the ownership of the goods while they are in transit. Freight shipping terms like FOB (free on board) serve this purpose. This term is commonly used in domestic shinning within the U.S., and should not be confused with the FOB term from the International Commercial Terms, INCOTERMS @ 2020.



FOB shipping point: The seller is responsible for the goods only to the point from which they are shipped. FOB destination: The seller is responsible for the goods all the way to their destination.

4.2.6. Costs to Attach

General Rule: Attach all the costs required to bring inventory to a saleable condition:

- → Acquisition (purchased or manufactured) → Shipping in (or inbound transportation)
- → Storage → Packaging

Distribution costs → These are considered as "selling expenses", not inventory costs.

4.2.7. Types of Discounts

Sales-boosting Discounts:

Designed to increase sales volume, either by attracting more customers or incentivizing larger purchases. Promotions, Introductory Offers, Trade Discounts, Volume or Quantity Discounts

Discounts used to manage stock levels, clear old inventory, or promote specific products. Seasonal Discounts, Markdowns, Two-for-One (and related). Overstock Discounts, Claseout Discounts

Liquidity-Improving Discounts:

Discounts offered to accelerate payment, enhancing the seller's (or even the buyer's) cash flow. Cash Discounts (Early Payment Discounts), Deferred Payment / Simple Trade Credit.

Cost and Service Discounts:

Discounts that either reduce ancillary costs or add value through supplementary services. Free Shipping, Loyalty Discounts, Pick-up Incentives, Tiered Service Discounts.

4.2.8. Cash Discounts [Early Payment Discounts]

Incentives for buyers to pay invoices early, improving the seller's cash flow. For instance, "2/10 net 30" provides a 2% discount if payment is made within 10 days; otherwise, the full amount is due by day 30.



Based on the provided terms, buyers can either capitalize on a 2% discount by paying on day 10, often via a loan, or leverage the extended trade credit for 20 more days by settling on day 30. From a financial perspective, it's essential to balance the associated debt cost against the opportunity cost of missing the discount or paying the extra amount

Simple interest of the opportunity cost:

Compound interest of the opportunity cost:

$$\left(\frac{0.02}{0.98}\right)\left(\frac{365}{20}\right) \approx 0.37$$

$$\left(\frac{1}{0.08}\right)^{(365/20)} - 1 \approx 0.45$$

Annualization helps to compare the costs and evaluate alternatives, factoring in either simple or compounded interest for the lost funds, based on how the business handles its money.

4.2.9. Inventory record keeping: Periodic and Perpetual Systems

Periodic System: Inventory records are updated only at specific intervals, often at the end of a month or quarter. This approach may be chosen due to technological constraints or specific company policies.

Perpetual System: Inventory is continuously updated with each transaction. Real-time tracking is often facilitated by Point-of-Sale Systems or technologies such as RFID, OR codes, barcodes, and ERP systems. Additionally, advanced innovations like the Internet of Thinas, Blockchain, Al. and Automation can further enhance tracking precision

4.2.10. COGS: FIFO, LIFO, Average Method	<empty citation=""></empty>
Foo	
4.2.11 COGS Computation: Example	<empty citation=""></empty>



4.2.12. Ending Inventory	<empty citation=""></empty>
F00	

4.3. Financial Performance

4.3.1. Measuring Performance	<empty citation=""></empty>	
Fon		

4.3.2. The Operating Cycle

The operating cycle refers to the duration it takes for a company to convert its cash outflows into cash inflows through its core operations.



The duration of this cycle can vary based on the company type, its nolicies, industry norms, market conditions, and the nature of its transactions

Current assets are those that can be converted into cash within a company's operating cycle or within a year, whichever is longer.

Factors	Example 1	Example 2
Company Type	Local craft brewery	High-end furniture manufacturer
Company Policies	Offers extended credit terms to distributors to promote their brand	Has a strict return policy, allowing only exchanges within a short window
Industry Norms	Typically, the alcoholic beverage industry sees seasonal spikes in sales, especially during holidays	In luxury furniture, customers expect customization options, leading to longer production times
Market Conditions	Due to a recent health trend, there's a surge in demand for craft beverages with natural ingredients	The economy is in a downturn, and fewer consumers are investing in luxury goods
Nature of its Transactions	Primarily engages in B2B transactions with retailers and restaurants, which often involve negotiated rates and bulk deals	Engages mainly in B2C transactions through their showroom and online store, with occasional bespoke orders from corporate clients

	4.3.3. Working Capital	<empty citation=""></empty>
L	/ L	

4.3.4. Ratios	<empty citation=""></empty>
Enn	

4.3.5. Inventory Turnover Revisited	<empty citation=""></empty>
Foo	

4.4. Cost of Capital

4.4.1. Cost of Capital	<empty citation=""></empty>
Foo	

4.4.2. WACC

The weighted average cost of capital uses the following formula:

$$\text{WACC} = \frac{E}{E+D}(R_E + \beta \cdot \text{MRP}) + \frac{D}{E+D}R_D(1-t)$$

Let's analyze this concept. First, notice the terms $\frac{E}{D+E}$ and $\frac{D}{E+D}$; they create a weighted measure of the individual contributions of $(R_f + \beta \cdot MRP)$ and $R_h(1-t)$.

4.5. Pratt: Chapters 6. 7)

Must include The Statement of Cash Flows

 Cover process time Decouple process

4.5.1. Why 6?	<empty citation=""></empty>
It seems traditional ratio analysis is out of date, it isn't very useful.	
Future cash flows is a more realisite analysis	

Inventories --> But this will be include in the next section.

5. Inventory I: Deterministic Models

5.1. What is Inventory and why does it matter?

5.1.1. Accounting POV vs. Logistics/SCM POV	<empty citation=""></empty>
5.1.2. Logistics/SCM types of inventory	<empty citation=""></empty>
5.1.3. Why hold inventory?	<empty citation=""></empty>

5.1.4. Inventory decisions

5.2. Inventory Models

5.2.1. Models

Trade-offs between complexity and ease of understanding/communication/implementation.

5.2.2. Models for Inventory Management

- Focus on costs
- Focus on service level

5.3. Inventory Costs

5.3.1. Unit Cost:
$$c \rightarrow \$$$
/unit

The cost of obtaining one unit of a SKU, either through procurement or production.

For merchants: It's the sum of the purchase price paid to the supplier, combined with additional costs necessary for preparing the product for sale, such as packaging and labeling. Typically, it also incorporates per-unit costs related to freight transportation and material handling, like loading and unloading.

For producers: It's the total unitary production cost. Similarly, it can also include material handling and freight transportation costs incurred from production-related activities. Determining the unit value in manufacturing can be more challenging due to its intricate nature

In basic inventory models, the unit cost is typically considered lot-size independent for simplicity. However, some models account for economies of scale by incorporating discounts related to the volume of items purchased or produced, recognizing that unit costs can vary with lot size.

Typically, the unit value is derived from the company's internal accounting system, representing its "book value", therefore, it may differ from what SCM/logistics specialists might consider. Ideally, the unit value should be determined collaboratively, taking into account the actual money spent on that specific SKU to prepare it for either internal or external use

5.3.2. Ordering/Setup Cost:
$$c_t
ightarrow \$/$$
order

5.3.3. Cost components of holding inventory

They vary across companies and SKUs, but, in general, they include the following major components, which must be incremental in nature, otherwise, they would have been incorporated as part of the fixed ordering

Cost of Capital: Capital is allocated to either purchase or produce inventory units, so less inventory means more available capital for alternative investments, each with their respective rates of return. Given that capital can be sourced from either equity or debt, the Weighted Average Cost of Capital [WACC] is often used here, as it's a blended measure for both sources of inventory financing.

Incremental Costs of Storage: Warehouse space often represents a significant expenditure, especially in prime locations. Handling inventory -i.e. moving, organizing within the storage space—adds to the costs. Periodic counting or inventory guidits are essential for accuracy, but require time and resources. Some inventory items might also necessitate special storage conditions, such as refrigeration or specific humidity levels, leading to additional expenses.

Costs of Depreciation: Inventory value can diminish over time due to several reasons. Perishable items may degrade, rendering them unsellable. As new products are introduced, older items may become obsolete, especially in industries with rapid innovation cycles. Moreover, shrinkage, resulting from items being lost, stolen, or damaged, further erodes the inventory's value.

5.3.4. Holding Cost: $c_{m{e}} ightarrow \$/(\mathsf{unit} imes \mathsf{period})$

Encapsulates all costs incurred from carrying a unit of inventory for a designated period. We can model it as:

$$c_e = rc$$

where the holding rate r denotes a percentage of the unit value c per period of demand (e.g. for 1 year). A multi-SKU company may opt for $c_{e_i}=r_ic_i$ for each SKU i or, to alleviate the complexity of individual analysis, apply a uniform holding rate r across all SKUs. Accordingly, c_e has the following dimensions:

By modeling it this way, we can evaluate the cost of keeping inventory proportionally to the amount held.

However, in certain scenarios, the cost of storing an item remains consistent, regardless of its value. When we employ a singular, aggregated rate r, we inadvertently allow the storage component to escalate in proportion to the item's unit value. A more nuanced approach would be to utilize

$$c_e = rc + h$$

Within this framework, h stands as a constant unitary storage fee, while r is solely representative of the cost of capital and depreciation associated with the item

Furthermore, consider the scenario where storage capacity is limited; if Q exceeds this threshold, an additional warehouse is required, incurring a fixed cost. This scenario can be modeled using a piecewise

$$\text{Total Holding Cost} = \begin{cases} Q(rc+h) & \text{, for } Q \leq \text{threshold} \\ Q(rc+h) + \text{Fixed Cost} & \text{, for } Q > \text{threshold} \end{cases}$$

Given the complexity of the holding cost, it's advisable to model it in collaboration with Finance/Accounting specialists



Can be modeled using stockout event or units short

5.3.7. Total Cost & Total Relevant Cost

5.3.6. Coordinated Cost Estimation: Finance and SCM/Logistics

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7.10.5.11

TC = Purchase Cost + Ordering Cost + Holding Cost + Shortage Cost

$$IU = Purchase Cost + Urdering Cost + Holding Cost + Shortage Cost$$

$$TC = cD + c_t \frac{D}{Q} + c_e \frac{Q}{2} + c_s E[\text{Units Short}] \label{eq:TC}$$

Procurement activities have influence on the Purchase Cost, while Inventory Management activities have influence on the other costs.

5.4. EOQ: Economic Order Quantity

5.4.1. EOQ model assumptions

→ Known demand → Constant

- → Zero or Constant Lead Time
- → Something else

Checkar papers review sobre EOQ

Checkar variaciones en el modelado de costos (i.e. variable holding cost, setup cost, etc.) en Silver, Chopra, Nahmias, etc...Hay muchas variaciones, pero incluir las mas frecuentes en los libros)

5.4.2. EOQ formula derivation

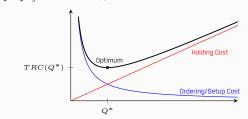
Since demand is deterministic, we can get rid of the Stockout Cost concept for now. So,

$$TRC(Q) = c_t \frac{D}{Q} + c_e \frac{Q}{2}$$

From the first-order optimal condition (first derivative equals zero), we have

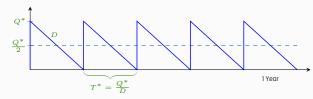
$$0 = \frac{\mathrm{d}}{\mathrm{d}Q} \left(\frac{c_t D}{Q} \right) + \frac{\mathrm{d}}{\mathrm{d}Q} \left(\frac{c_e Q}{2} \right)$$
$$0 = -\frac{c_t D}{Q^2} + \frac{c_e}{2}$$
$$Q^* = \sqrt{\frac{2c_t D}{Q}}$$

The EQQ or Q^* gives the minimum TRC under deterministic conditions:



5.4.3. EOO Sawtooth Plot

The optimal policy becomes ordering Q^* units of inventory every T^* units of time.



Notice that the total consumption of the last order may take place after the 1 year (unit time) period.

5.4.4. Sensitivity Analysis for the EOQ model

Resaltar que, pese a que algunos parametros se asumen alegremente como deterministicos, el modelo es lo suficientemente robusto como para compensar variaciones en los mismos (e.a. demanda, costos, etc.) Usar los 5 libros en ... Analisis y logistica de la producción + otros complementos

5.4.5. Powers of Two Policies

<empty citation

5.5. EOQ Extensions	7.1.3. Holt-Winter Model	<empty citation=""></empty>	8.2.7. Chi-Square Test	<empty citation=""></empty>	9.4.2. () model <====	npty citation>
5.5.1. Lead Time > 0	7.1.4. Initialization of Parameters	<empty citation=""></empty>	8.3. SPIM: Single Period Inventory Models		10. Inventory IV: Multiple Dimension	
5.5.2. Discounts: All units	7.1.5. Comments and Comparison of Models	<empty citation=""></empty>	8.3.1. SPIM: Problem introduction	<empty citation=""></empty>	Models	
5.5.3. Discounts: Incremental	ottotion>		8.3.2. Data Table	<empty citation=""></empty>		
	7.2. Intermittent Demand Forecasting		6.3.2. Dutu Tubie	-Empty distant	10.1. Multiple Items 10.1. Grouping	onty ritations
5.5.4. Discounts: One-time	7.2.1. Intermittent demand patterns and examples	<empty citation=""></empty>	8.3.3. Marginal Analysis	<empty citation=""></empty>	IO.I.I. Grouping	
5.5.5. Backorders	7.2.2. Approaches	<empty citation=""></empty>	8.3.4. Salvage Value	<empty citation=""></empty>	10.1.2. Grouping: Powers of Two	npty citation>
5.5.6. EPQ: Economic Production Quantity	7.2.3. Croston's Method	<empty citation=""></empty>	8.3.5. Penalty Value	<empty citation=""></empty>	10.1.3. Grouping: Exchange Curves	npty citation>
5.5.7. Perishability			8.3.5. Periuity vuide		10.2. Multiple Locations	
5.5.7. Perisiduality	7.3. Regression & Causal Analysis		8.3.6. Critical Ratio	<empty citation=""></empty>		npty citation>
5.5.8. Trade Credit	7.3.1. Explaining causes of demand phenomena	<empty citation=""></empty>	8.3.7. Expected Profits	<empty citation=""></empty>		
6. Forecasting I	7.3.2. Correlation and Causation	<empty citation=""></empty>	8.4. The Newsvendor Problem		10.3. Multiple Classes	
	7.3.3. Simple Linear Regression	<empty citation=""></empty>	8.4.1. Newsyendor Problem: Introduction	<empty citation=""></empty>	10.3.1. Segmentation Revisited - Fost moving items	npty citation>
6.1. Demand Planning			NFL Jersey Problem in the MicroMasters		- rust moving items - Slow moving items	
6.1.1. Demand Planning	7.3.4. Multiple Linear Regression	<empty citation=""></empty>	8.4.2. Unit Normal Loss Function	<empty citation=""></empty>	10.3.2. A Items	npty citation>
6.1.2. Demand Forecasting	7.4. Product Development, Marketing &		8.4.3. Newsvendor Problem: Solution	<empty citation=""></empty>	10.3.3. B Items	npty citation>
6.2. Data Collection	Forecasting		8.5. The Newsvendor Problem Extensions			
6.2.1. Obtaining data	7.4.1. New Products Introduction	<empty citation=""></empty>	8.5.1. Foo	<empty citation=""></empty>	10.3.4. Citems	npty citation>
	7.4.2. Forecasting techniques & Product Life Cycle	<empty citation=""></empty>			10.4. Multiple Echelons	
6.2.2. Aggregated data, Aggregated forecasts	7.5. AI/ML techniques for Forecasting		9. Inventory III: Multiple Period Inve	ntory	10.4.1. Multiple Echelons	npty citation>
6.3. Time Series		<empty citation=""></empty>	Models		11 Transportation I. Freight	
6.3.1. Time Series Components	7.5.1. Clustering	veripty diddisin	9.1. Introductory Models		11. Transportation I: Freight Transportation	
6.3.2. Decomposition	8. Inventory II: Stochastic Models		9.1.1. Rescaling of Parameters	<empty citation=""></empty>	·	
6.3.3. Cummulative & Naive Forecasting	8.1. Stochastic Demand		9.1.2. Base Stock Model	<empty citation=""></empty>	11.1. Freight Transportation	
b.s.s. Cullinductive a Nuive Full clustering	8.1.1. Demand distribution	<empty citation=""></empty>	9.2. Continuous Review Models		11.1.1. Time-Space Diagram	npty citation>
6.3.4. Moving Averages Forecasting	8.1.2. Expected Demand	<empty citation=""></empty>	9.2.1. (s, Q) model	<empty citation=""></empty>	11.1.2. Packaging	npty citation>
6.4. Forecasting Metrics	S.i.z. Expected Demand				- Cases - Pallets	
6.4.1. Accuracy & Bias	8.1.3. Expected Units Short	<empty citation=""></empty>	9.2.2. (s,S) model	<empty citation=""></empty>	- Containers 	
5.4.2 Frrnr Matrirs	8.1.4. Expected Units Sold	<empty citation=""></empty>	9.3. Safety Stock: Service Cost and Metrics		11.1.3. Transportation Modes and Routes	npty citation>
G-FE Error records	8.2. Demand Modelling		9.3.1. Cycle Service Level	<empty citation=""></empty>	11.2. Transportation Networks	
6.5. Exponential Smoothing	821 Empirical Distribution	<empty citation=""></empty>	9.3.2. Cost per Stockout Event	<empty citation=""></empty>	•	npty citation>
6.5.1. Simple Exponential Smoothing	otation Carried State Carr					
6.5.2. Damped Trend	8.2.2. Discrete Uniform	<empty citation=""></empty>	9.3.3. Item Fill Rate	<empty citation=""></empty>	11.2.2. Operational Network	npty citation>
7. Favoration II	8.2.3. Poisson	<empty citation=""></empty>	9.3.4. Cost per Item Short	<empty citation=""></empty>	11.2.3. Strategic Network	npty citation>
7. Forecasting II	8.2.4. Continuous Uniform	<empty citation=""></empty>	9.3.5. Inputted and Implied Metrics	<empty citation=""></empty>	11.3. Transportation & Inventory	
7.1. Exponential Smoothing with Seasonality						npty citation>
7.1.1. Seasonality Patterns	8.2.5. Normal	<empty citation=""></empty>	9.4. Periodic Review Models	comply situation	-	
7.1.2. Double Exponential Smoothing	8.2.6. Triangle	<empty citation=""></empty>	9.4.1. (R, S) model	<empty citation=""></empty>	11.3.2. Total Inventory & Transportation Cost	npty citation>

11.3.3. Transit & Lead Time Variability	<empty citation=""></empty>	13.3.2. Put away	<empty citation=""></empty>
11.3.4. Random Sum of Random Variables	<empty citation=""></empty>	13.3.3. Store	<empty citation=""></empty>
11.4. Mode Selection		13.3.4. Pick	<empty citation=""></empty>
11.4.1. Foo	<empty citation=""></empty>	13.3.5. Check, Pack, Ship	<empty citation=""></empty>
12. Transportation II: Analysis		13.3.6. Return handling	<empty citation=""></empty>
12.1. The Transportation Product		13.3.7. Value-added services	<empty citation=""></empty>
12.1.1. Four Fundamental Operations	<empty citation=""></empty>	13.4. Layout design	
12.1.2. Loading & Unloading	<empty citation=""></empty>	13.4. Foo	<empty citation=""></empty>
12.1.3. Linehaul Moves	<empty citation=""></empty>	13.5. Cross-Docking	
12.1.4. Vehicle Routing	<empty citation=""></empty>	13.5.1. Foo	<empty citation=""></empty>
12.1.5. Facility Sorting	<empty citation=""></empty>	13.6. Segmentation & Benchmarking in Warehousing	
12.2. Transportation Economies		13.6.1. Foo	<empty citation=""></empty>
12.2.1. Economies of Scale	<empty citation=""></empty>		
12.2.2. Economies of Scope	<empty citation=""></empty>		
12.2.3. Economies of Density	<empty citation=""></empty>		
12.3. Transportation Economic Modes			
12.3.1. Direct Transportation	<empty citation=""></empty>		
12.3.2. Consolidated Transportation	<empty citation=""></empty>		
12.4. Transportation & Routing Problems	3		
12.4.1. 1 : 1	<empty citation=""></empty>		
12.4.2. 1 : ∞	<empty citation=""></empty>		
12.4.3. ∞ : 1	<empty citation=""></empty>		
12.4.4. ∞ : ∞	<empty citation=""></empty>		
13. Warehouse Management			
13.1. Warehousing			
13.1.1. Why warehouses?	<empty citation=""></empty>		
13.1.2. Types of warehouses	<empty citation=""></empty>		
13.2. Warehousing & Packaging			
13.2.1. Foo	<empty citation=""></empty>		

13.3. Core Operational Functions

13.3.1. Receive

13.7. Templates

13.7.1. Consequences of the Axioms

By set theory definitions we have:
$$A \cup A^c = \Omega$$
 and $A \cap A^c = \emptyset$

$$P(A) \leq 1$$

 $\begin{vmatrix} A \text{ and } A^c \text{ are disjoint} &\Rightarrow P(A \cup A^c) = 1 = P(A) + P(A^c) \Rightarrow P(A^c) = 1 - P(A), \\ \text{and by } \textit{nonnegativity} \text{ we get } P(A^c) \geq 0 \Rightarrow P(A) \leq 1 \ \blacksquare$

$$P(\emptyset) = 0$$

Let $A = \Omega \Rightarrow P(\Omega) + P(\Omega^c) = 1 \Rightarrow 1 + \emptyset = 1 \Rightarrow P(\emptyset) = 0$ Let Ω be a finite set and A_1,\ldots,A_n be disjoint events, then:

$$P\left(\bigcup_{i=1}^n A_i\right) = \sum_1^n P(A_i)$$

 $\begin{array}{c} P(A\cup B\cup C) = P\left[(A\cup B)\cup C\right]. \text{ From additivity, given that the events are disjoint, we have} \\ (P(A)+P(B)) + P(C). \text{ By induction we can extend this to } n \text{ disjoint sets } \blacksquare \\ \text{Let } \{\omega_1, ..., \omega_k\} \text{ be a discrete, finite set of sample points, then:} \end{array}$

$$\boxed{P\Big(\{\omega_1,...,\omega_k\}\Big) \Rightarrow P\left(\bigcup_{j=1}^k \{\omega_j\}\right) \Rightarrow \sum_{j=1}^k P\Big(\{\omega_j\}\Big)}$$

because $\{\omega_1,\ldots,\omega_k\}$, can be seen as the union of *unit sets*, and since they are disjoint, additivity

applies
$$\blacksquare$$
 . Although, a simpler, non rigorous notation can be used: $\sum_{j=1}^k P(\omega_j)$.

13.7.2. More Consequences of the Axioms

Consider the condition $P(A \cap B) > 0$, \Rightarrow The events could be joint, therefore, more generally:



$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Which can be generalized to the...

$$\boxed{P\left(\bigcup_{i=1}^{n}A_{i}\right) = -\sum_{k=1}^{n}(-1)^{k}\sum_{1\leq i_{1}<\ldots< i_{k}\leq n}P\left(\bigcap_{j=1}^{k}A_{i_{j}}\right)}$$

From the above, the *Union Bound* property follows: $P(A \cup B) \leq P(A) + P(B)$ Consider that A is included in B, then:



$$A \subset B \Rightarrow P(A) \leq P(B)$$

since $B = A \cup (B \cap A^c) \Rightarrow P(B) = P(A) + P(B \cap A^c) \ge P(A)$ Consider 3 sets not necessarily disjoint, e.g.:



$$P(A \cup B \cup C) = P(A) + P(A^c \cap B) + P(A^c \cap B^c \cap C)$$

Visually, we can check the boxed expression by the matching of the colors, and since the subsets are disjoint, additivity holds. Notice the expression also applies to disjoint sets.

13.7.3. Multiplication Rule

Notice that:

$$P(A \cap B) = P(B)P(A|B)$$
$$= P(A)P(B|A)$$

And for 3 events we have:

$$P[(A \cap B) \cap C] = P(A \cap B)P(C|A \cap B)$$
$$= P(A)P(B|A)P(C|A \cap B)$$

More generally:

$$\left| P\left(\bigcap_{i=1}^{n} A_i\right) = P(A_1) \prod_{i=2}^{n} P\left(A_i \left| \bigcap_{j=1}^{i-1} A_j\right.\right) \right|$$

A particular intersection of events would be represented as a full path in a probability tree.

REFERENCES:

- [1] Jamie Pratt. Financial Accounting in an Economic Context (8th ed.) John Wiley & Sons, 2011.
- [2] Franklin Allen, Richard A. Brealey, and Stewart C. Myers. Principles of Corporate Finance (12th ed.) McGraw-Hill, 2017.
- [3] Investopedia Financial Terms Dictionary. https://www.investopedia.com/. Accessed: 2023.
- [4] GPT-4. https://openai.com/gpt-4. Accessed: 2023.
- [5] Sunil Chopra and Peter Meindl. Supply Chain Management (6th ed.) Pearson, 2016.
- [6] Chris Caplice and Eva Ponce. MITx MicroMasters Program in SCM Key Concepts. 2023. URL: https://scx-static-assets.s3.amazonaws.com/SCx%20Key%20Concept%20Documents/MITx_MicroMasters_SCM_KeyConcepts.pdf.
- [7] Edward A. Silver, David F. Pyke, and Douglas J. Thomas. Inventory and Production Management in Supply Chains (4th ed.) CRC Press, 2016.
- [8] Gerard Cachon and Christian Terwiesch. Matching Supply with Demand (4th ed.) McGraw-Hill Publishing, 2018.
- [9] Paul Schonsleben. Handbook Integral Logistics Management (6th ed.) Springer, 2022.
- [10] Sven Axsater. *Inventory Control (3rd ed.)* Springer, 2015.
- [11] Paolo Brandimarte and Giulio Zotteri. Introduction to Distribution Logistics. John Wiley & Sons, 2000.