SCMx1

1. Supply Chain Management & Logistics

1.1. Classical Logistics

1.1.1. Logistics as "organization of a complex operation"
<empty citation>

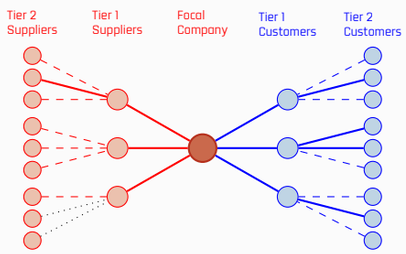
1.1.2. Logistics in Manufacturing
<empty citation>

1.1.3. Logistics in Services
<empty citation>

1.2. Supply Chain Management

1.2.1. What is a Supply Chain?
<empty citation>

1.2.2. Supply Networks
<empty citation>



In a manufacturing context, a supply chain can be seen as a network of suppliers, manufacturers, distributors, and retailers.

1.2.3. SCM vs. Logistics
<empty citation>

1.2.4. SCM Cycles
<empty citation>

1.2.5. SCM Processes
<empty citation>

1.2.6. SCOR Model
<empty citation>

1.2.7. Supply Chains as Systems
<empty citation>

2. Flow & Capacity

2.1. Flows

2.1.1. Types of Flows in a Supply Chain
<empty citation>

2.2. Capacity

2.2.1. Buffers
<empty citation>

2.2.2. Matching Supply with Demand
<empty citation>

3. Push-Pull Systems & Segmentation

3.1. Push-Pull Systems

3.1.1. Push and Pull Processes
<empty citation>

3.1.2. Product-Process Matrix
<empty citation>

3.2. Postponement & Mass Customization

3.2.1. Customer Order Decoupling Point
<empty citation>

3.3. Product Segmentation

3.3.1. Criteria for Segmentation
<empty citation>

- Product Segmentation

- Fast Moving Items
- FMCG
- Slow Moving Items, etc.

3.3.2. Power "Law"
<empty citation>

3.3.3. ABC Analysis
<empty citation>

3.3.4. Multicriteria ABC Analysis
<empty citation>

3.3.5. AI/ML techniques for Segmentation
<empty citation>

3.4. Supply Chain Segmentation

3.4.1. Supply Chain Portfolios
<empty citation>

4. Accounting POV for Inventory

4.1. Capital and Financial Statements

4.1.1. Sources of capital
1,2

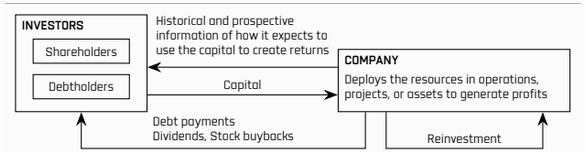
Management seeks capital to finance operations from two main sources:

Shareholders: Individuals or entities that purchase and hold shares of a company's stock, thereby owning a portion of the company's equity. They expect (and are entitled to receive) returns through investment appreciation and/or dividends, and also have the right to vote on certain company decisions. Unlike debtholders, shareholders have an ownership stake in the company, which carries both potential financial rewards and risks, as their investment value can fluctuate with the company's performance.

Debtholders: Individuals or entities that lend capital to a company, usually in the form of loans or bonds, with the expectation of being repaid the principal amount along with interest over time. Unlike shareholders, debtholders do not obtain ownership stakes in the company, but hold a financial claim that is prioritized in the event of liquidation.

The blended result of these contributions is called the **capital structure**.

4.1.2. Flow of capital
1,2



4.1.3. Fundamental Business Activities
1,2

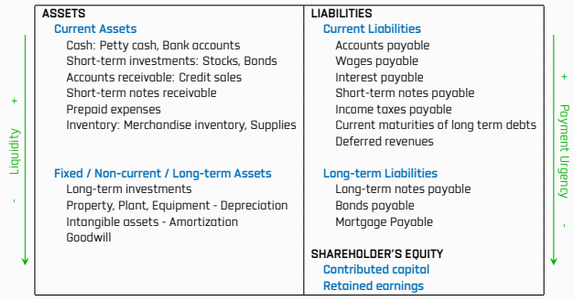
Operating Activities:
- Form the core of a business through the management of operating assets for the production and/or sale of goods and services.
- Encompass everyday functions to maintain business continuity.
- Ideally, these activities ensure smooth operations for profit generation.

Investing Activities:
- Acquisition, replacement, and disposition of operating assets like inventory, buildings, and equipment.
- Investments in intangible assets like know-how or Research and Development.
- Investments in digital assets such as platforms and software.
- Full or partial acquisition of other companies.
- Planning and control of cash inflows to ensure rational and timely, opportune amounts.

Financial Activities:
- Focused on capital management, raising funds from shareholders and/or debtholders.
- Selling financial assets or securities such as shares of stock and bonds.
- Managing debt and dividend payments, or engaging in stock buybacks.
- Evaluating various debt and equity financing options, designing a sound capital structure.

4.1.4. The Balance Sheet: A Statement of Financial Position
1,2

Provides a snapshot of a company's financial position at a specific point in time, often at the end of a fiscal year, showcasing assets, liabilities, and shareholders' equity.



The amount of highly liquid assets indicates ability to meet debt payments as they come due.

4.1.5. Elements of the Income Statement
1

Revenues indicate inflow of assets or reduction in liabilities, primarily from sales of inventories or services.

COGS or **Cost of Sales** reflects the original cost of inventory sold, either its purchase price or its manufacturing cost. By subtracting this from Revenues, we arrive at the **Gross Margin**.

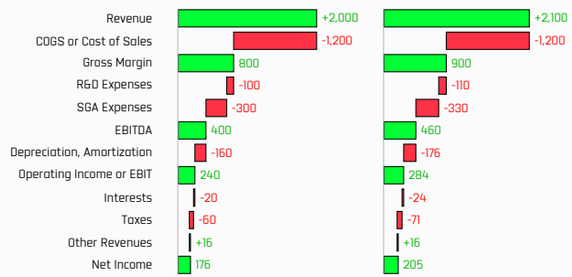
R&D Expenses or Research & Development Expenses cover costs like product innovation or supply chain optimizations. Whereas **SGA Expenses** or Selling, General, and Administrative expenses, encompass costs that aren't directly tied to producing an item. This includes expenses such as salaries, rent, utilities, marketing, distribution costs, customer service as well as administrative costs like office supplies, legal fees, etc.

By subtracting the aforementioned expenses, we derive **EBITDA**, which stands for Earnings Before Interest, Taxes, Depreciation, and Amortization. Further adjustments, primarily subtracting depreciation and amortization from EBITDA, yield the **Operating Income**, also known as **EBIT** (Earnings Before Interest and Taxes).

Other revenues (or expenses) represent minor cash inflows or outflows not related to core operations. After accounting for these, we determine the **Net Income**, also referred to as **Profit**.

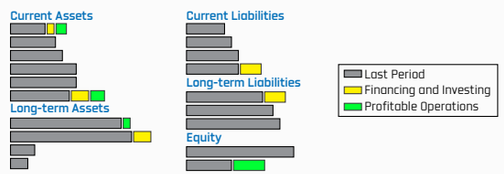
4.1.6. The Income Statement Visualized
1,2

Let's examine a scenario where better demand-supply alignment results in a 5% sales increase, while also fairly accounting for a rise in costs and expenses.



4.1.7. Profitable operations as a source of capital
1

Retained earnings represent the cumulative profits a company has generated and chosen to reinvest in the business rather than distribute as dividends. They don't pinpoint a specific tangible asset or cash pool. Instead, they indicate the portion of the assets listed on the balance sheet that stems from profitable operations.



These earnings highlight the capital sourced directly from profitable operations, distinguishing it from capital derived from borrowings or owner contributions.

4.2. Accounting for Inventory

4.2.1. The Inventory Accounting Flow/Cycle
1

Create a flowchart (rather than a cycle figure) for this:
- Acquisition: Purchase or Manufacture
- Carrying Inventory: Periodic or Perpetual Methods
- Selling Invetory: FIFO, LIFO and register COGS for Income Statement
- or goes to —> Ending Inventory

4.2.2. Accounting categories of Inventory
1,3

Inventory plays a central role in accounting, reflecting a company's financial well-being and operational stance. It represents a major portion of a firm's assets, with its management directly affecting profitability and liquidity. Therefore, precise record-keeping is required to offer stakeholders a concise financial perspective crucial for investment decisions.

Shifting our lens to manufacturing, we can delineate these specific inventory categories:
- **Raw Materials:** Fundamental inputs of a manufacturing process
- **Work in Progress (WIP):** Inventory undergoing transformation from raw materials to final products.
- **Components:** Individual parts, sourced or produced, essential for final product assembly.
- **Finished Goods:** Fully processed products ready for sale.

In a broader operational context, beyond pure manufacturing, we also consider:
- **Merchandise Inventory:** Ready-to-sell products acquired for resale without additional modification.
- **Supplies:** Operational items not for sale, such as office materials.
- **MRO Items:** Resources for maintenance, repair, and operations, distinct from final product materials.

4.2.3. Units to include
<empty citation>

General Rule:

Consignments:

Goods in Transit:

FOB:

4.2.4. Costs to attach
<empty citation>

Foo

4.2.5. Record of carried inventory: Periodic and Perpetual Methods
<empty citation>

Foo

4.2.6. COGS: FIFO vs. LIFO
<empty citation>

Foo

4.2.7. COGS Computation: Example
<empty citation>

Foo

4.2.8. Ending Inventory
<empty citation>

Foo

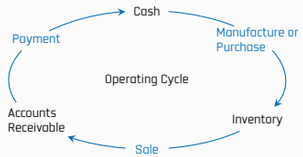
4.3. Financial Performance

4.3.1. Measuring Performance
<empty citation>

Foo

4.3.2. The Operating Cycle

The operating cycle refers to the duration it takes for a company to convert its cash outflows into cash inflows through its core operations.



The duration of this cycle can vary based on the company type, its policies, industry norms, market conditions, and the nature of its transactions.

Current assets are those that can be converted into cash within a company's operating cycle or within a year, whichever is longer.

Factors	Example 1	Example 2
Company Type	Local craft brewery	High-end furniture manufacturer
Company Policies	Offers extended credit terms to distributors to promote their brand	Has a strict return policy, allowing only exchanges within a short window
Industry Norms	Typically, the alcoholic beverage industry sees seasonal spikes in sales, especially during holidays	In luxury furniture, customers expect customization options, leading to longer production times
Market Conditions	Due to a recent health trend, there's a surge in demand for craft beverages with natural ingredients	The economy is in a downturn, and fewer consumers are investing in luxury goods
Nature of its Transactions	Primarily engages in B2B transactions with retailers and restaurants, which often involve negotiated rates and bulk deals	Engages mainly in B2C transactions through their showroom and online store, with occasional bespoke orders from corporate clients

4.3.3. Working Capital

<empty citation>

Foo

4.3.4. Ratios

<empty citation>

Foo

4.3.5. Inventory Turnover Revisited

<empty citation>

Foo

4.4. Cost of Capital

4.4.1. Cost of Capital

<empty citation>

Foo

4.4.2. WACC

[4]

The weighted average cost of capital uses the following formula:

$$\text{WACC} = \frac{E}{E+D}(R_E + \beta \cdot \text{MRP}) + \frac{D}{E+D}R_D(1-t)$$

Let's analyze this concept. First, notice the terms $\frac{E}{D+E}$ and $\frac{D}{E+D}$; they create a weighted measure of the individual contributions of $(R_f + \beta \cdot \text{MRP})$ and $R_b(1-t)$.

4.5. Pratt: Chapters 6, 7)

4.5.1. Why 6?

<empty citation>

It seems traditional ratio analysis is out of date, it isn't very useful.
Future cash flows is a more realisitc analysis
Must include **The Statement of Cash Flows**

4.5.2. Why 7?

<empty citation>

Inventories —> But this will be include in the next section.

5. Inventory I: Deterministic Models

5.1. What is Inventory and why does it matter?

5.1.1. Accounting POV vs. Logistics/SCM POV

<empty citation>

5.1.2. Logistics/SCM types of inventory

<empty citation>

5.1.3. Why hold inventory?

<empty citation>

- Cover process time
- Decouple process
- ...

5.1.4. Inventory decisions

<empty citation>

5.2. Inventory Models

5.2.1. Models

<empty citation>

Trade-offs between complexity and ease of understanding/communication/implementation.

5.2.2. Models for Inventory Management

<empty citation>

- Focus on costs
- Focus on service level
- ...

5.3. Inventory Costs

5.3.1. Unit Cost: $c \longrightarrow \$/\text{unit}$

[5,6,7,8]

The cost of obtaining one unit of a SKU, either through procurement or production.

For merchants: It's the sum of the purchase price paid to the supplier, combined with additional costs necessary for preparing the product for sale, such as packaging and labeling. Typically, it also incorporates per-unit costs related to freight transportation and material handling, like loading and unloading.

For producers: It's the total unitary production cost. Similarly, it can also include material handling and freight transportation costs incurred from production-related activities. Determining the unit value in manufacturing can be more challenging due to its intricate nature.

In basic inventory models, the unit cost is typically considered lot-size independent for simplicity. However, some models account for economies of scale by incorporating discounts related to the volume of items purchased or produced, recognizing that unit costs can vary with lot size.

Typically, the unit value is derived from the company's internal accounting system, representing its "book value", therefore, it may differ from what SCM/logistics specialists might consider. Ideally, the unit value should be determined collaboratively, taking into account the actual money spent on that specific SKU to prepare it for either internal or external use.

5.3.2. Ordering/Setup Cost: $c_t \longrightarrow \$/\text{order}$

<empty citation>

5.3.3. Cost components of holding inventory

[6,9,4,10]

They vary across companies and SKUs, but, in general, they include the following major components, which must be incremental in nature, otherwise, they would have been incorporated as part of the fixed ordering cost:

Cost of Capital: Capital is allocated to either purchase or produce inventory units, so less inventory means more available capital for alternative investments, each with their respective rates of return. Given that capital can be sourced from either equity or debt, the Weighted Average Cost of Capital [WACC] is often used here, as it's a blended measure for both sources of inventory financing.

Incremental Costs of Storage: Warehouse space often represents a significant expenditure, especially in prime locations. Handling inventory -i.e. moving, organizing within the storage space—adds to the costs. Periodic counting or inventory audits are essential for accuracy, but require time and resources. Some inventory items might also necessitate special storage conditions, such as refrigeration or specific humidity levels, leading to additional expenses.

Costs of Depreciation: Inventory value can diminish over time due to several reasons. Perishable items may degrade, rendering them unsellable. As new products are introduced, older items may become obsolete, especially in industries with rapid innovation cycles. Moreover, shrinkage, resulting from items being lost, stolen, or damaged, further erodes the inventory's value.

5.3.4. Holding Cost: $c_e \longrightarrow \$/(\text{unit} \times \text{period})$

[5,6]

Encapsulates all costs incurred from carrying a unit of inventory for a designated period. We can model it as:

$$c_e = rc$$

where the *holding rate* r denotes a percentage of the unit value c per period of demand [e.g. for 1 year]. A multi-SKU company may opt for $c_{e_i} = r_i c_i$ for each SKU i or, to alleviate the complexity of individual analysis, apply a uniform holding rate r across all SKUs. Accordingly, c_e has the following dimensions:

$$\frac{\$}{\text{unit} \times \text{period}}$$

By modeling it this way, we can evaluate the cost of keeping inventory proportionally to the amount held.

However, in certain scenarios, the cost of storing an item remains consistent, regardless of its value. When we employ a singular, aggregated rate r, we inadvertently allow the storage component to escalate in proportion to the item's unit value. A more nuanced approach would be to utilize

$$c_e = rc + h$$

Within this framework, h stands as a constant unitary storage fee, while r is solely representative of the cost of capital and depreciation associated with the item.

Furthermore, consider the scenario where storage capacity is limited; if Q exceeds this threshold, an additional warehouse is required, incurring a fixed cost. This scenario can be modeled using a piecewise function, for instance:

$$\text{Total Holding Cost} = \begin{cases} Q(rc+h) & \text{, for } Q \leq \text{threshold} \\ Q(rc+h) + \text{Fixed Cost} & \text{, for } Q > \text{threshold} \end{cases}$$

Given the complexity of the holding cost, it's advisable to model it in collaboration with Finance/Accounting specialists.

5.3.5. Stockout/Shortage Cost: c_s

<empty citation>

Can be modeled using stockout event or units short

5.3.6. Coordinated Cost Estimation: Finance and SCM/Logistics

<empty citation>

Foo

5.3.7. Total Cost & Total Relevant Cost

<empty citation>

$$\text{TC} = \text{Purchase Cost} + \text{Ordering Cost} + \text{Holding Cost} + \text{Shortage Cost}$$

$$TC = cD + c_t\frac{D}{Q} + c_e\frac{Q}{2} + c_s E[\text{Units Short}]$$

Procurement activities have influence on the Purchase Cost, while Inventory Management activities have influence on the other costs.

5.4. EOQ: Economic Order Quantity

5.4.1. EOQ model assumptions

<empty citation>

- ➞ Known demand —> Constant
- ➞ Zero or Constant Lead Time
- ➞ Something else

Checkar papers review sobre EOQ

Checkar variaciones en el modelado de costos (i.e. variable holding cost, setup cost, etc.) en Silver, Chopra, Nahmias, etc...Hay muchas variaciones, pero incluir las mas frecuentes en los libros)

5.4.2. EOQ formula derivation

<empty citation>

Since demand is deterministic, we can get rid of the Stockout Cost concept for now. So,

$$TRC(Q) = c_t\frac{D}{Q} + c_e\frac{Q}{2}$$

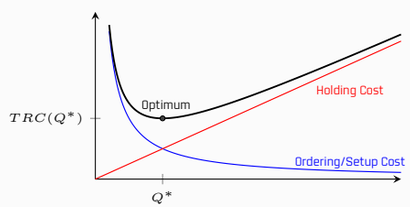
From the first-order optimal condition (first derivative equals zero), we have

$$0 = \frac{\mathrm{d}}{\mathrm{d}Q}\left(\frac{c_t D}{Q}\right) + \frac{\mathrm{d}}{\mathrm{d}Q}\left(\frac{c_e Q}{2}\right)$$

$$0 = -\frac{c_t D}{Q^2} + \frac{c_e}{2}$$

$$\boxed{Q^* = \sqrt{\frac{2c_t D}{c_e}}}$$

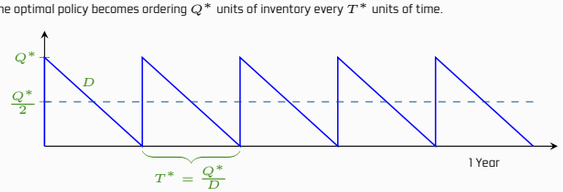
The EOQ or Q^* gives the minimum TRC under deterministic conditions:



5.4.3. EOQ Sawtooth Plot

<empty citation>

The optimal policy becomes ordering Q^* units of inventory every T^* units of time.



Notice that the total consumption of the last order may take place after the 1 year (unit time) period.

5.4.4. Sensitivity Analysis for the EOQ model

<empty citation>

Resaltar que, pese a que algunos parametros se asumen alegremente como deterministicos, el modelo es lo suficientemente robusto como para compensar variaciones en los mismos (e.g. demanda, costos, etc.)
Usar los 5 libros en ...Analisis y logistica de la produccion + otros complementos

5.4.5. Powers of Two Policies

<empty citation>

5.5. EOQ Extensions

5.5.1. Lead Time > 0

<empty citation>

5.5.2. Discounts: All units

<empty citation>

5.5.3. Discounts: Incremental

<empty citation>

5.5.4. Discounts: One-time

<empty citation>

5.5.5. Backorders

<empty citation>

5.5.6. EPQ: Economic Production Quantity

<empty citation>

5.5.7. Perishability

<empty citation>

5.5.8. Trade Credit

<empty citation>

6. Forecasting I

6.1. Demand Planning

6.1.1. Demand Planning

<empty citation>

6.1.2. Demand Forecasting

<empty citation>

6.2. Data Collection

6.2.1. Obtaining data

<empty citation>

6.2.2. Aggregated data, Aggregated forecasts

<empty citation>

6.3. Time Series

6.3.1. Time Series Components

<empty citation>

6.3.2. Decomposition

<empty citation>

6.3.3. Cummulative & Naive Forecasting

<empty citation>

6.3.4. Moving Averages Forecasting

<empty citation>

6.4. Forecasting Metrics

6.4.1. Accuracy & Bias

<empty citation>

6.4.2. Error Metrics

<empty citation>

6.5. Exponential Smoothing

6.5.1. Simple Exponential Smoothing

<empty citation>

6.5.2. Damped Trend

<empty citation>

7. Forecasting II

7.1. Exponential Smoothing with Seasonality

7.1.1. Seasonality Patterns

<empty citation>

7.1.2. Double Exponential Smoothing

<empty citation>

7.1.3. Holt-Winter Model <empty citation>

7.1.4. Initialization of Parameters <empty citation>

7.1.5. Comments and Comparison of Models <empty citation>

7.2. Intermittent Demand Forecasting

7.2.1. Intermittent demand patterns and examples <empty citation>

7.2.2. Approaches <empty citation>

7.2.3. Croston's Method <empty citation>

7.3. Regression & Causal Analysis

7.3.1. Explaining causes of demand phenomena <empty citation>

7.3.2. Correlation and Causation <empty citation>

7.3.3. Simple Linear Regression <empty citation>

7.3.4. Multiple Linear Regression <empty citation>

7.4. Product Development, Marketing & Forecasting

7.4.1. New Products Introduction <empty citation>

7.4.2. Forecasting techniques & Product Life Cycle <empty citation>

7.5. AI/ML techniques for Forecasting

7.5.1. Clustering <empty citation>

8. Inventory II: Stochastic Models

8.1. Stochastic Demand

8.1.1. Demand distribution <empty citation>

8.1.2. Expected Demand <empty citation>

8.1.3. Expected Units Short <empty citation>

8.1.4. Expected Units Sold <empty citation>

8.2. Demand Modelling

8.2.1. Empirical Distribution <empty citation>

8.2.2. Discrete Uniform <empty citation>

8.2.3. Poisson <empty citation>

8.2.4. Continuous Uniform <empty citation>

8.2.5. Normal <empty citation>

8.2.6. Triangle <empty citation>

8.2.7. Chi-Square Test <empty citation>

8.3. SPIM: Single Period Inventory Models

8.3.1. SPIM: Problem introduction <empty citation>

8.3.2. Data Table <empty citation>

8.3.3. Marginal Analysis <empty citation>

8.3.4. Salvage Value <empty citation>

8.3.5. Penalty Value <empty citation>

8.3.6. Critical Ratio <empty citation>

8.3.7. Expected Profits <empty citation>

8.4. The Newsvendor Problem

8.4.1. Newsvendor Problem: Introduction <empty citation>

NFL Jersey Problem in the MicroMasters

8.4.2. Unit Normal Loss Function <empty citation>

8.4.3. Newsvendor Problem: Solution <empty citation>

8.5. The Newsvendor Problem Extensions

8.5.1. Foo <empty citation>

9. Inventory III: Multiple Period Inventory Models

9.1. Introductory Models

9.1.1. Rescaling of Parameters <empty citation>

9.1.2. Base Stock Model <empty citation>

9.2. Continuous Review Models

9.2.1. (s, Q) model <empty citation>

9.2.2. (s, S) model <empty citation>

9.3. Safety Stock: Service Cost and Metrics

9.3.1. Cycle Service Level <empty citation>

9.3.2. Cost per Stockout Event <empty citation>

9.3.3. Item Fill Rate <empty citation>

9.3.4. Cost per Item Short <empty citation>

9.3.5. Inputted and Implied Metrics <empty citation>

9.4. Periodic Review Models

9.4.1. (R, S) model <empty citation>

9.4.2. $(\ldots)$ model <empty citation>

10. Inventory IV: Multiple Dimension Models

10.1. Multiple Items

10.1.1. Grouping <empty citation>

10.1.2. Grouping: Powers of Two <empty citation>

10.1.3. Grouping: Exchange Curves <empty citation>

10.2. Multiple Locations

10.2.1. Location Pooling <empty citation>

10.3. Multiple Classes

10.3.1. Segmentation Revisited <empty citation>

- Fast moving items
- Slow moving items
...

10.3.2. A Items <empty citation>

10.3.3. B Items <empty citation>

10.3.4. C Items <empty citation>

10.4. Multiple Echelons

10.4.1. Multiple Echelons <empty citation>

11. Transportation I: Freight Transportation

11.1. Freight Transportation

11.1.1. Time-Space Diagram <empty citation>

11.1.2. Packaging <empty citation>

- Cases
- Pallets
- Containers
...

11.1.3. Transportation Modes and Routes <empty citation>

?

11.2. Transportation Networks

11.2.1. Physical Network <empty citation>

11.2.2. Operational Network <empty citation>

11.2.3. Strategic Network <empty citation>

11.3. Transportation & Inventory

11.3.1. Transportation Cost Functions <empty citation>

11.3.2. Total Inventory & Transportation Cost <empty citation>

11.3.3. Transit & Lead Time Variability <empty citation>

11.3.4. Random Sum of Random Variables <empty citation>

11.4. Mode Selection

11.4.1. Foo <empty citation>

12. Transportation II: Analysis

12.1. The Transportation Product

12.1.1. Four Fundamental Operations <empty citation>

12.1.2. Loading & Unloading <empty citation>

12.1.3. Linehaul Moves <empty citation>

12.1.4. Vehicle Routing <empty citation>

12.1.5. Facility Sorting <empty citation>

12.2. Transportation Economies

12.2.1. Economies of Scale <empty citation>

12.2.2. Economies of Scope <empty citation>

12.2.3. Economies of Density <empty citation>

12.3. Transportation Economic Modes

12.3.1. Direct Transportation <empty citation>

12.3.2. Consolidated Transportation <empty citation>

12.4. Transportation & Routing Problems

12.4.1. $1:1$ <empty citation>

12.4.2. $1:\infty$ <empty citation>

12.4.3. $\infty:1$ <empty citation>

12.4.4. $\infty:\infty$ <empty citation>

13. Warehouse Management

13.1. Warehousing

13.1.1. Why warehouses? <empty citation>

13.1.2. Types of warehouses <empty citation>

13.2. Warehousing & Packaging

13.2.1. Foo <empty citation>

13.3. Core Operational Functions

13.3.1. Receive <empty citation>

13.3.2. Put away

<empty citation>

13.3.3. Store

<empty citation>

13.3.4. Pick

<empty citation>

13.3.5. Check, Pack, Ship

<empty citation>

13.3.6. Return handling

<empty citation>

13.3.7. Value-added services

<empty citation>

13.4. Layout design

13.4.1. Foo

<empty citation>

13.5. Cross-Docking

13.5.1. Foo

<empty citation>

13.6. Segmentation & Benchmarking in Warehousing

13.6.1. Foo

<empty citation>

13.7. Templates

13.7.1. Consequences of the Axioms

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By set theory definitions we have: $\boxed{A \cup A^c = \Omega}$ and $\boxed{A \cap A^c = \emptyset}$

$$\boxed{P(A) \leq 1}$$

A and A^c are disjoint $\Rightarrow P(A \cup A^c) = 1 = P(A) + P(A^c) \Rightarrow P(A^c) = 1 - P(A)$, and by *nonnegativity* we get $P(A^c) \geq 0 \Rightarrow P(A) \leq 1$ ■

$$\boxed{P(\emptyset) = 0}$$

Let $A = \Omega \Rightarrow P(\Omega) + P(\Omega^c) = 1 \Rightarrow 1 + \emptyset = 1 \Rightarrow P(\emptyset) = 0$ ■
Let Ω be a finite set and $A_1, \ldots, A_n$ be disjoint events, then:

$$\boxed{P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{1}^{n} P(A_i)}$$

$P(A \cup B \cup C) = P\left[(A \cup B) \cup C\right]$. From additivity, given that the events are disjoint, we have $(P(A) + P(B)) + P(C)$. By induction we can extend this to n disjoint sets ■
Let $\{\omega_1, \ldots, \omega_k\}$ be a discrete, finite set of sample points, then:

$$\boxed{P\Big(\{\omega_1, \ldots, \omega_k\}\Big) \Rightarrow P\left(\bigcup_{j=1}^{k} \{\omega_j\}\right) \Rightarrow \sum_{j=1}^{k} P\Big(\{\omega_j\}\Big)}$$

because $\{\omega_1, \ldots, \omega_k\}$, can be seen as the union of *unit sets*, and since they are disjoint, additivity applies ■. Although, a simpler, non rigorous notation can be used: $\sum_{j=1}^{k} P(\omega_j)$.

13.7.2. More Consequences of the Axioms

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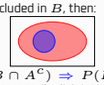
Consider the condition $P(A \cap B) \geq 0, \Rightarrow$ The events could be joint, therefore, more generally:

$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

Which can be generalized to the...:

$$\boxed{P\left(\bigcup_{i=1}^{n} A_i\right) = -\sum_{k=1}^{n} (-1)^k \sum_{1 \leq i_1 < \ldots < i_k \leq n} P\left(\bigcap_{j=1}^{k} A_{i_j}\right)}$$

From the above, the *Union Bound* property follows: $\boxed{P(A \cup B) \leq P(A) + P(B)}$
Consider that A is included in B, then:



$$\boxed{A \subset B \Rightarrow P(A) \leq P(B)}$$

since $B = A \cup (B \cap A^c) \Rightarrow P(B) = P(A) + P(B \cap A^c) \geq P(A)$ ■
Consider 3 sets not necessarily disjoint, e.g.:

$$\boxed{P(A \cup B \cup C) = P(A) + P(A^c \cap B) + P(A^c \cap B^c \cap C)}$$

Visually, we can check the boxed expression by the matching of the colors, and since the subsets are disjoint, additivity holds. Notice the expression also applies to disjoint sets ■

13.7.3. Multiplication Rule

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Notice that:

$$\begin{aligned} P(A \cap B) &= P(B)P(A|B) \\ &= P(A)P(B|A) \end{aligned}$$

And for 3 events we have:

$$\begin{aligned} P[(A \cap B) \cap C] &= P(A \cap B)P(C|A \cap B) \\ &= P(A)P(B|A)P(C|A \cap B) \end{aligned}$$

More generally:

$$\boxed{P\left(\bigcap_{i=1}^{n} A_i\right) = P(A_1) \prod_{i=2}^{n} P\left(A_i \middle| \bigcap_{j=1}^{i-1} A_j\right)}$$

A particular intersection of events would be represented as a full path in a probability tree.

Companies, Investors, and Flow of Capital
The Balance Sheet: A Statement of Financial Position
The Income Statement
Statement of Cash Flows
Assets
Securities
Shareholder's Equity
WACC

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