Spin Evolution

Figure 1: At low temperatures we observed smaller groups of aligned spins. We concluded that the influence of the heat (i.e. $\tau\sigma$) on the free energy was low and therefore, that the energy U was minmised.

Given the partition function $Z = (2\cosh(\epsilon/\tau))^N$, we calculated the internal energy using,

$$U = \tau^2 \partial_\tau \ln(Z) \tag{1}$$
$$= \tau^2 \partial_\tau \ln\left(2\cosh\left(\frac{\epsilon}{\tau}\right)^N\right)$$
$$= N\tau^2 \partial_\tau \ln\left(2\cosh\left(\frac{\epsilon}{\tau}\right)\right)$$
$$= N\tau^2 \partial_\tau \left(2\cosh\left(\frac{\epsilon}{\tau}\right)\right) \frac{1}{2\cosh\left(\frac{\epsilon}{\tau}\right)}$$
$$= N\tau^2 \partial_\tau \left(\frac{\epsilon}{\tau}\right) \frac{\sinh\left(\frac{\epsilon}{\tau}\right)}{\cosh\left(\frac{\epsilon}{\tau}\right)}$$
$$= -\epsilon N \tanh\left(\frac{\epsilon}{\tau}\right). \tag{2}$$

We calculated the free energy of the system using,

$$F = -\tau \ln Z \tag{3}$$
$$= -\tau \ln\left(\left(2\cosh\left(\frac{\epsilon}{\tau}\right)\right)^N\right)$$
$$= -N\tau \ln\left(2\cosh\left(\frac{\epsilon}{\tau}\right)\right)$$
$$= -N\tau \ln\left(\exp\left(\frac{\epsilon}{\tau}\right) + \exp\left(-\frac{\epsilon}{\tau}\right)\right)$$
$$= -N\tau \ln\left(\exp\left(\frac{\epsilon}{\tau}\right)\left(1 + \exp\left(-2\frac{\epsilon}{\tau}\right)\right)\right)$$
$$= -N\tau \ln\left(\exp\left(\frac{\epsilon}{\tau}\right)\right) - N\tau \ln\left(1 + \exp\left(-2\frac{\epsilon}{\tau}\right)\right)$$
$$= -N\epsilon - N\tau \ln\left(1 + \exp\left(-2\frac{\epsilon}{\tau}\right)\right). \tag{4}$$

The entropy followed from the combination of Equation 4 and Equation 2 using Equation 5,

$$\tau\sigma = F - U \tag{5}$$
$$= -N\epsilon \tanh\left(\frac{\epsilon}{\tau}\right) + N\epsilon + N\tau \ln\left(1 + \exp\left(-2\frac{\epsilon}{\tau}\right)\right)$$
$$\sigma = \frac{\epsilon}{\tau}\left(1 - \tanh\left(\frac{\epsilon}{\tau}\right)\right) + \ln\left(1 + \exp\left(-2\frac{\epsilon}{\tau}\right)\right). \tag{6}$$

Finally, we determined the specific heat using Equation ??,

1