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Interfaces with Other Disciplines

Portfolio rebalancing model using multiple criteria

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ABSTRACT

In order to achieve greater flexibility in portfolio selection, transaction cost, short selling and higher moments should be considered, and actual transactions should be reflected. In this paper, five portfolio rebalancing models, with consideration of transaction cost and consisting of some or all criteria, including risk, return, short selling, skewness, and kurtosis, are compared to determine the important design criteria for a portfolio model. Two examples are used to perform simulated transactions, and the results indicate that the investment strategy of 'buy and hold' does not produce better returns for all the portfolios in the first example, and the models with higher moments or adopting short selling strategy perform better while rebalancing in the second example.

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1. Introduction

The mean–variance model of the portfolio selection, originally developed by Markowitz (1952), is one of the most important portfolio selection models in contemporary financing. Subsequently, numerous refinements have been proposed to improve the performances of investment portfolios, resulting in a large number of research focused on risk versus return, and diversification in investment strategies (Yusen et al., 2000). Markowitz's mean–variance model involves only two criteria; however, other criteria are taken into account by most investors, each with their own portfolio considerations.

Multiple criteria decision aids (MCDA) have been more frequently applied in daily life (Brans, 2004), as decisions formed upon thoughts of multiple criteria can satisfy a variety of issues and reach compromises that lead to greater harmony. In practical application, diverse criteria and preferences are usually resulted from different decision making strategies, as investors are generally concerned with more than two criteria. An investment strategy depends on the investors' preferences (Steuer and Na, 2003), and the selection of a proper portfolio is critical in achieving the investors' various objectives. Hallerbach and Spronk (1997) pointed out that most models fail to incorporate the multidimensional nature of portfolio selection issues, and only outlined a framework view focused on portfolio management. Spronk et al. (2005) proposed seven criteria for portfolio considerations; however, the discussion remains in the conceptual stage and has not yet been implemented.

As suggested, portfolio selection must consist of more criteria than only risk and return in order to provide investors with additional choices. For example, maximizing the criteria of skewness in a portfolio would result in better return (Konno and Suzuki, 1995; Yoshimoto, 1996). Chunnachinda et al. (1997) indicated that skewness values of efficient mean–variance-skewness portfolios are higher than those of efficient mean–variance portfolios. In the probability theory and statistics, skewness is a measure of asymmetry within the probability distribution of random variables. In finance, the criterion of skewness in a portfolio must be considered.

Empirical findings suggest that the incorporation of skewness into an investor's portfolio may result in an improved optimal portfolio (Prakash et al., 2003; Joro and Na, 2006; Yu et al., 2008; Li et al., 2010). Excluding skewness may lead to an inefficient portfolio (Leung et al., 2001). Sun and Yan (2003) pointed out that positively skewed portfolios of individual stocks would exhibit a greater extent of persistence. From practical evidence, most distributions of stock market returns are not normal distributions, and are characterized by significant skewness and kurtosis (Tang and Shum, 2003; Gondzio and Grothey, 2007). Most researches address the importance of considering skewness and kurtosis when evaluating the performance of a portfolio (Zakamouline and Koekebakker, 2009; Díaz et al., 2009) or risk management (Hong et al., 2009). However, short selling is rarely included in the above research when considering higher moments.

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White (1990) pointed out that the rationale of short selling is highly speculative, and thus, is more appropriate for use with low investment risks. From a normative viewpoint, Kwan (1997) argued that short selling has good potential to improve a portfolio's risk-return trade-off. Although short selling has inherent high risks, it is adopted by most investors to obtain interest arbitrage (Angel et al., 2003). However, in order to avoid high risks, the proportion of short selling is expected to be minimized. Jacobs et al. (2005) presented a portfolio optimization scheme that includes short selling, with an algorithm that shows how an investor can sell short and buy long. However, they only focused on algorithms for short selling, without mentioning other criteria.

Arnott and Wanger (1990) suggested that ignoring transaction costs would lead to an inefficient portfolio; whereas, adding transaction costs would assist decision makers to better understand the behavior of an efficient frontier (Sadjadi et al., 2004). Fang et al. (2006) proposed a portfolio rebalancing model with transaction costs, but did not consider short selling. Transaction costs play a crucial role in transactions (Zhang and Zhang, 2009; Chen and Wang, 2008). Moreover, constructing more realistic models, which incorporate market frictions, are also important (Choi et al., 2007; Kozhan and Schmid, 2009).

As decision makers must consider many factors in order to meet the requirements of real transactions, developing a comprehensive portfolio selection model that includes additional factors and the preferences of the investors, is a very important issue (Branke et al., 2009). In addition, multi-period portfolio selection models, which can periodically generate rebalancing if the situation are subject to future changes, is essential (Fang et al., 2006; Bertsimas and Pachamanovab, 2008; Yu et al., 2010; Çanakoglu and Özekici, 2010). To our knowledge, researches that consider transaction cost, short selling, and higher moment are few. Therefore, this paper aims to propose five portfolio rebalancing models that consider the transaction cost and consist of some or all criteria, including risk, return, short selling, skewness, and kurtosis, in order to determine important design criteria for a portfolio model that can be presented to investors.

The remainder of this paper is organized as follows. First, a mean–variance model of a portfolio selection is briefly reviewed in Section 2. In Section 3, five models are developed for portfolio selection to assist investors to solve problems. Fuzzy multi-objective programming is employed to address the issues of the five proposed portfolio models. Section 4 illustrates the proposed methods, and demonstrates two numerical examples. Finally, the conclusions and future research directions are presented in Section 5.

2. The mean–variance model

The mean–variance model (Markowitz, 1952) addresses portfolio selection problems and determines the composition for a portfolio of n securities, which minimizes risks while achieving a given level of expected returns, as follows:

$$\text{Min } \sigma_p = \sum_{i=1}^n w_i^2 \sigma_i^2 + \sum_{i=1}^n \sum_{j=1(j \neq i)}^n \sigma_{ij} w_i w_j, \quad (1)$$

$$\text{s.t. } \sum_{i=1}^n r_i w_i \geq \mu, \quad (2)$$

$$\sum_{i=1}^n w_i = 1, \quad (3)$$

$$w_i \geq 0, \quad i = 1, 2, \dots, n,$$

where n is the number of available securities; w_i is the investment portion in i securities for $i = 1, \dots, n$; r_i is the return on securities i ; μ is the expected portfolio return; σ_i^2 is the variance of the return on securities i ; and σ_{ij} is the covariance between the returns of securities i and j . The first constraint expresses the requirements of a portfolio return, while the second is the budget constraint. From $w_i \geq 0$, we can assume that short selling is not allowed. On the contrary, short selling is taken into consideration in the proposed model for multi-periods. Therefore, in the proposed models, w_i becomes an unrestricted sign regarding short selling.

3. The proposed models

Five multi-objective models, which consist of criteria including return, risk, proportion of short selling, skewness, and kurtosis, are evaluated. The four rebalancing multi-objective models with short selling are the MVS (mean, variance, and short selling), the MVS_S (mean, variance, short selling, and skewness), the MVS_K (mean, variance, short selling, and kurtosis), the MVS_SK (mean, variance, short selling, skewness, and kurtosis), while the model MV (mean and variance) does not involve short selling. The five models are presented and compared in order to determine which models perform the best.

- (1) The model MVS consists of four objectives, as shown in Eqs. (4)–(7), namely, the maximization of portfolio return, the minimization of portfolio risk as measured by the portfolio variance, the minimization of the short selling proportion of the portfolio, and the minimization of transaction cost. In order to consider short selling, w_i is an unrestricted sign, which is decomposed into $w_i^+ - w_i^-$. The details of the model MVS are as follows:

Model MVS:

$$\text{Max } \sum_{i=1}^n r_i (w_i^+ - w_i^-), \quad (4)$$

$$\text{Min } \sum_{i=1}^n (w_i^+ - w_i^-)^2 \sigma_i^2 + \sum_{i=1}^n \sum_{j=1(j \neq i)}^n \sigma_{ij} (w_i^+ - w_i^-) (w_j^+ - w_j^-), \quad (5)$$

$$\text{Min } \sum_{i=1}^n w_i^-, \quad (6)$$

$$\text{Min } \sum_{i=1}^n (p_1 l_i^+ + p_2 l_i^- + p_3 s_i^+ + p_4 s_i^-), \quad (7)$$

$$\text{s.t. } \sum_{i=1}^n (w_i^+ + k w_i^- + p_1 l_i^+ + p_2 l_i^- + p_3 s_i^+ + p_4 s_i^-) = 1, \quad (8)$$

$$w_i^+ = w_{i,0}^+ + l_i^+ - l_i^-, \quad (9)$$

$$w_i^- = w_{i,0}^- + s_i^+ - s_i^-, \quad (10)$$

$$0.05 u_i \leq w_i^+ \leq 0.2 u_i, \quad (11)$$

$$0.05 v_i \leq w_i^- \leq 0.2 v_i, \quad (12)$$

$$u_i + v_i = y_i, \quad (13)$$

$$\text{for } i = 1, \dots, n,$$

where $w_{i,0}^+$ is the proportion of i securities bought by investors prior to portfolio rebalancing; $w_{i,0}^-$ is the proportion of i securities sold short by investors prior to portfolio rebalancing; w_i^+ is the total proportion of i securities desired by the investors upon portfolio rebalancing; and w_i^- is the total proportion of i securities sold short by investors upon portfolio rebalancing. With each rebalancing, l_i^+ is the proportion of i securities bought by investors; l_i^- is the proportion of i securities sold by investors; s_i^+ is the proportion of i securities sold short by investors, and s_i^- is the proportion of i securities repurchased by investors, respectively; u_i is the binary variable that indicates whether the i securities are selected for buying; v_i is the binary variable that indicates whether the i securities are selected for sell short; and k is the initial margin requirement for short selling; p_j , for $j = 1, 2, 3$, and 4 indicates the transaction costs of buying, selling, selling short, and repurchasing, respectively; k and p_j are given constants.

Constraint (8) indicates the budget allocated to buying, short selling, and transaction cost. Constraint (9) shows the current long position after rebalancing. Constraint (10) represents the current short selling position after rebalancing. Constraints (11) and (12) state the upper and lower bounds of the total positions of each security in buying and short selling, respectively. Eq. (13) aims to control the number of invested securities for both buying and short selling, as shown in Table 1. Therefore, the proposed model is able to indicate whether the chosen securities should be held long or sold short.

- (2) The model MVS_S includes all the criteria of the model MVS as well as skewness. In addition to criteria of risk, return, short selling, and transaction cost, this study adds an objective to maximize the skewness of the portfolio (Eq. (14)) through the model MVS_S, as follows.

Model MVS_S:

$$\text{Max } \sum_{i=1}^n r_i (w_i^+ - w_i^-), \quad (4)$$

$$\text{Min } \sum_{i=1}^n (w_i^+ - w_i^-)^2 \sigma_i^2 + \sum_{i=1}^n \sum_{j=1(j \neq i)}^n \sigma_{ij} (w_i^+ - w_i^-) (w_j^+ - w_j^-), \quad (5)$$

$$\text{Min } \sum_{i=1}^n w_i^-, \quad (6)$$

$$\text{Min } \sum_{i=1}^n (p_1 l_i^+ + p_2 l_i^- + p_3 s_i^+ + p_4 s_i^-), \quad (7)$$

$$\text{Max } E[w^T (r - \bar{r})]^3, \quad (14)$$

$$\text{s.t. Constraints (8) } \sim (13),$$

where E is the expectation operator; T (superscript) is the transposing operator; $w = (w_1, \dots, w_n)^T$ is the vector of the portfolio weights; $r = (r_1, \dots, r_n)^T$ is the vector of the returns and $(\bar{r} = \bar{r}_1, \dots, \bar{r}_n)^T$ is the vector of the expected returns.

- (3) The model MVS_K includes all criteria from the model MVS and kurtosis. In addition to criteria of risk, return, short selling, and transaction cost, the objective of minimizing kurtosis is added Eq. (15) in the model MVS_K.

Model MVS_K:

$$\text{Max } \sum_{i=1}^n r_i (w_i^+ - w_i^-), \quad (4)$$

$$\text{Min } \sum_{i=1}^n (w_i^+ - w_i^-)^2 \sigma_i^2 + \sum_{i=1}^n \sum_{j=1(j \neq i)}^n \sigma_{ij} (w_i^+ - w_i^-) (w_j^+ - w_j^-), \quad (5)$$

$$\text{Min } \sum_{i=1}^n w_i^-, \quad (6)$$

$$\text{Min } \sum_{i=1}^n (p_1 l_i^+ + p_2 l_i^- + p_3 s_i^+ + p_4 s_i^-), \quad (7)$$

Table 1
The total position control mechanism of security i .

Action	Binary variable
Long	$u_i = 1, v_i = 0$ then $y_i = 1$
Short selling	$u_i = 0, v_i = 1$ then $y_i = 1$
No investing	$u_i = 0, v_i = 0$ then $y_i = 0$

$$\begin{aligned} & \text{Min } E[w^T(r - \bar{r})]^4, \\ & \text{s.t. Constraints(8)} \sim (13). \end{aligned} \quad (15)$$

- (4) The model MVS_KS includes all criteria of the model MVS, skewness, and kurtosis. In addition to criteria of risk, return, short selling, and transaction cost, the objectives of maximizing skewness while minimizing kurtosis Eqs. (14) and (15) are added into the model MVS_SK.

Model MVS_SK:

$$\text{Max } \sum_{i=1}^n r_i(w_i^+ - w_i^-), \quad (4)$$

$$\text{Min } \sum_{i=1}^n (w_i^+ - w_i^-)^2 \sigma_i^2 + \sum_{i=1}^n \sum_{j=1(j \neq i)}^n \sigma_{ij}(w_i^+ - w_i^-)(w_j^+ - w_j^-), \quad (5)$$

$$\text{Min } \sum_{i=1}^n w_i^-, \quad (6)$$

$$\text{Min } \sum_{i=1}^n (p_1 l_i^+ + p_2 l_i^- + p_3 s_i^+ + p_4 s_i^-), \quad (7)$$

$$\text{Max } E[w^T(r - \bar{r})]^3, \quad (14)$$

$$\text{Min } E[w^T(r - \bar{r})]^4, \quad (15)$$

s.t. Constraints (8) ~ (13).

- (5) The model MV is a mean–variance model that considers rebalancing, transaction cost, and multi-criteria decision making. It is represented as a triple objective decision making problem.

Model MV:

$$\text{Max } \sum_{i=1}^n r_i w_i, \quad (16)$$

$$\text{Min } \sum_{i=1}^n w_i^2 \sigma_i^2 + \sum_{i=1}^n \sum_{j=1(j \neq i)}^n w_i w_j \sigma_{ij}, \quad (1)$$

$$\text{Min } \sum_{i=1}^n (p_1 l_i^+ + p_2 l_i^-), \quad (17)$$

$$\text{s.t. } \sum_{i=1}^n (w_i + p_1 l_i^+ + p_2 l_i^-) = 1, \quad (18)$$

$$w_i = w_{i,0} + l_i^+ - l_i^-, \quad (19)$$

$$0.05u_i \leq w_i \leq 0.2u_i, \quad (20)$$

$$\text{all } w_i \geq 0 \text{ for } i = 1, \dots, n.$$

The above multiple criteria models are solved by fuzzy multi-objective programming (Zimmermann, 1978; Lee and Li, 1993) in order to transfer the multi-objective model into a single-objective model. Fuzzy multi-objective programming is based on the concept of fuzzy set, which uses a min operator to calculate the membership function value of the aspiration level, λ . The program forces each goal to achieve its aspiration level, and then provides a trade-off concern among the conflicting objectives or criteria. The ideal and anti-ideal solutions must be obtained in advance. By employing fuzzy multi-objective programming, the model MVS is used as an example for reformulation as a single objective model, as follows.

Max λ

$$\text{s.t. } \lambda \leq \frac{(r^* - r_l)}{r_g - r_l}, \quad (21)$$

$$\lambda \leq \frac{(\sigma^* - \sigma_l)}{(\sigma_g - \sigma_l)}, \quad (22)$$

$$\lambda \leq \frac{(w^* - w_l^-)}{(w_g^- - w_l^-)}, \quad (23)$$

$$\lambda \leq \frac{(c^* - c_l)}{(c_g - c_l)}, \quad (24)$$

$$r^* = \sum_{i=1}^n r_i(w_i^+ - w_i^-), \quad (25)$$

$$\sigma^* = \sum_{i=1}^n (w_i^+ - w_i^-)^2 \sigma_i^2 + \sum_{i=1}^n \sum_{j=1(j \neq i)}^n \sigma_{ij}(w_i^+ - w_i^-)(w_j^+ - w_j^-), \quad (26)$$

$$w^{-*} = \sum_{i=1}^n w_i^{-}, \quad (27)$$

$$c^* = \sum_{i=1}^n (p_1 l_i^+ + p_2 l_i^- + p_3 s_i^+ + p_4 s_i^-), \quad (28)$$

s.t. constraints(8) ~ (13),

where r^* is the return of the portfolio; r_l is the anti-ideal return of the portfolio; r_g is the ideal return of the portfolio that maximizes the objective; σ^* is the inherent risk of the portfolio; σ_l is the anti-ideal risk of the portfolio; σ_g is the ideal risk of the portfolio; and w^{-*} is the short selling proportion of the portfolio; w_l^{-} is the anti-ideal short selling proportion of the portfolio; w_g^{-} is the ideal short selling proportion of the portfolio; and c^* is the transaction cost of the portfolio; c_l is the anti-ideal transaction cost of the portfolio; while c_g is the ideal transaction cost of the portfolio.

Constraints (21)–(24) respectively achieve the goals of maximizing the portfolio return, while maintaining the minimization of the portfolio risk, the short selling proportion, and the transaction cost. Both ideal and anti-ideal values of the criteria are required from investors or history. In our cases, the ideal and anti-ideal solutions for return, risk, skewness, and kurtosis of the portfolio are pre-specified by the best and worst historical data, taken from the n securities at each rebalancing. The ideal and anti-ideal proportions of short selling are 0 and 1, respectively; while 0 and 0.01 are the ideal and anti-ideal values for the transaction cost of the proposed portfolio. The rate of initial margin requirement for short selling k is 0.9; and p_1 , p_2 , p_3 , and p_4 are 0.001425, 0.004425, 0.005225, and 0.001425, respectively, in the Taiwan Stock Market. The other four models are transformed in the same manner, through a fuzzy multi-objective programming into single objective.

4. Numerical examples

This paper presents two examples for simulated transaction. The first example demonstrates a buy and hold strategy, while the second one provides monthly stock weight updates for the investors. Finally, the results of both examples are compared with the Taiwan 50 Index (TSE50), which is the benchmark for the two examples. The historical data are downloaded from the website (<http://veri.library.ncnu.edu.tw/tejcount>).

This study first considers 45 different stocks investments listed on the Taiwan Stock Exchange. The exchange codes of the 45 stocks are listed in Table 2. The analyzed data of the 45 stocks are sourced from November 1, 2006 to October 22, 2009. The historical data of the first 60 transaction days are used to build the initial models. For the monthly updates of the second example, 20 transaction days are set as a sliding window. Therefore, rebalancing takes place on 34 occasions before October 22, 2009. This study assumes an investment of \$1 million NTD in the Taiwan Stock Market. For investments based on the weights generated by the initial models, the first business day is January 25, 2007. The models are executed on a Genuine Intel CPU T2400 1.83 GHz and a 1 GB RAM notebook computer, with Lingo 11.0 software (Schrage, 2002).

Example 1. “Buy and hold” is an investment strategy where securities are purchased and held for long periods. This example uses buy and hold to validate the five proposed models. Investments are made on January 25, 2007; however, the investment weight of each stock remains constant until October 22, 2009. The average return, risk, skewness, and kurtosis between November 1, 2006 and January 24, 2007 are listed in Table 3. The five proposed portfolio weights for each stock are compared in Table 4.

As shown in Table 4, there are 10, 8, 7, 6, and 12 stocks selected for investment by the models MV, MVS, MVS_S, MVS_K, and MVS_SK, respectively. The stocks for short selling are 1 (No. 2498), and 4 (No. 1722, 2353, 2354, and 2498), as selected by the models MVS_S and MVS_SK, respectively. As seen in Table 3, the short selling of stock is reasonable, as the average return of stock 2498 is negative, and with a high risk during that period. Although the returns of stocks No. 1722, 2353, 2354 are positive, their returns are smaller. The investment of \$1 million NTD on the TSE50 is completed as a benchmark. Finally, this study balances the five portfolios for comparison with the TSE50 on July 25, 2007 (short term), December 17 (mid term), and October 22, 2009 (long term), respectively. The results are listed in Table 5.

The market value of portfolio MVS_S is the best among the five proposed portfolios from October 22, 2009, as the model criteria with skewness generates a higher return than the others. On October 22, 2009, the resulting order of balances of the models is: MVS_S > MVS_SK > MV > TSE50 > MVS_K > MVS. It is therefore assumed that if the portfolio weights do not change over time, they will not obtain good results, as they must adapt to an ever changing rebalancing environment.

Example 2. In this example, the original investment strategy is changed to a sliding window, in which the weights of the portfolios are changed by rebalancing for every 20 transaction days, as shown in Fig. 1. The first duration of historical data is from November 1, 2006 to January 24, 2007, and the second duration is from November 29, 2006 to March 3, 2007. The updated stock weights for every 20 transaction days depend on the 60 historical data updates.

Table 2
The exchange codes of the 45 stocks.

No	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Code	1101	1102	1216	1301	1303	1326	1402	1722	2002	2105	2308	2311	2317	2324	2325
No	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Code	2330	2347	2353	2354	2357	2382	2409	2454	2498	2603	2801	2880	2881	2882	2883
No	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
Code	2885	2886	2888	2890	2891	2892	2912	3009	3231	3474	3481	5854	6505	8046	9904

Table 3

The average return, risk, skewness, and kurtosis of the 45 stocks.

No	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Code	1101	1102	1216	1301	1303	1326	1402	1722	2002	2105	2308	2311	2317	2324	2325
Return	0.002	0.003	0.002	0.002	0.002	0.003	0.003	0.002	0.002	0.001	0.004	0.004	0.002	0.001	0.004
Risk	0.017	0.018	0.02	0.008	0.008	0.01	0.02	0.022	0.011	0.021	0.019	0.014	0.016	0.012	0.022
Skewness	0.846	0.839	0.307	0.037	0.048	0.413	0.366	0.486	0.572	0.486	0.169	1.86	1.303	−0.24	0.775
Kurtosis	4.68	5.921	3.494	3.269	3.145	3.347	2.973	3.811	4.747	4.097	5.306	10.278	7.221	3.493	4.061
No	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Code	2330	2347	2353	2354	2357	2382	2409	2454	2498	2603	2801	2880	2881	2882	2883
Return	0.0025	0.0048	0.0011	0.0049	0.0021	0.0025	−0.0002	0.0027	−0.0082	−0.0001	0.0012	0.0025	0.002	0.0029	0.0033
Risk	0.0144	0.0292	0.0222	0.027	0.0139	0.0176	0.012	0.0179	0.0327	0.0127	0.0149	0.0103	0.0181	0.0169	0.015
Skewness	0.184	0.6863	0.1933	0.5657	0.4443	−0.1047	0.3357	0.6659	−0.1908	0.8169	−0.009	0.1723	0.2644	0.0421	1.3006
Kurtosis	4.346	3.136	5.579	4.118	3.227	2.108	2.7	5.357	2.75	5.683	4.033	4.32	4.935	4.677	8.248
No	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
Code	2885	2886	2888	2890	2891	2892	2912	3009	3231	3474	3481	5854	6505	8046	9904
Return	0.002	0.0002	0.0027	0.0003	0.0028	0.0012	0.0009	−0.0011	0.0043	0.0022	0.0065	0.0016	0.0006	0.0007	0.0047
Risk	0.0155	0.0143	0.0195	0.0092	0.0135	0.0091	0.0125	0.0139	0.0209	0.0174	0.0263	0.0126	0.0112	0.0177	0.0227
Skewness	0.6722	−0.3651	0.6019	0.2012	0.2552	−0.1093	0.4134	−0.629	0.7424	0.8214	0.6308	0.4125	−0.042	0.5702	0.812
Kurtosis	6.901	2.95	3.317	2.272	3.325	3.474	4.735	7.089	3.818	6.327	3.171	3.679	3.762	4.41	4.132

Table 4

The proportion of each stock in the five proposed portfolios.

Index	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Code	1101	1102	1216	1301	1303	1326	1402	1722	2002	2105	2308	2311	2317	2324	2325
MV	0	0	0.05	0.069	0	0	0	0	0.052	0	0.05	0.05	0	0	0
MVS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.186
MVS_S	0	0	0	0	0	0	0	0	0	0	0.2	0.2	0	0	0.2
MVS_K	0	0	0	0	0	0	0.2	0	0.075	0	0	0	0	0	0
MVS_SK	0	0	0	0	0	0	0	−0.0702	0	0.0634	0	0.0912	0	0	0
No	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Code	2330	2347	2353	2354	2357	2382	2409	2454	2498	2603	2801	2880	2881	2882	2883
MV	0	0.2	0	0.078	0	0	0	0.05	0	0	0	0	0	0	0
MVS	0	0.134	0	0.129	0	0	0	0	0	0	0	0	0	0	0
MVS_S	0.0698	0.1377	0	0	0	0	0	0	−0.05	0	0	0	0	0	0
MVS_K	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.157
MVS_SK	0	0	−0.0648	−0.0523	0	0	0	0	−0.0681	0	0	0	0	0	0.1567
No	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
Code	2885	2886	2888	2890	2891	2892	2912	3009	3231	3474	3481	5854	6505	8046	9904
MV	0	0	0	0	0	0	0	0	0	0	0.2	0	0	0	0.2
MVS	0	0	0	0	0	0	0.05	0	0	0	0.2	0	0.05	0.05	0.2
MVS_S	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.1458
MVS_K	0.0877	0	0	0	0	0	0	0	0	0.2	0	0.167	0	0	0
MVS_SK	0.0877	0	0	0	0	0	0.0942	0	0	0	0	0	0	0	0.0952

The five portfolios are balanced according to the TSE50 every 20 transaction days, and the balance of the first month is placed into the next month for accumulation, so forth. The details of the first four rebalanced weights of the five portfolios are shown in the Appendix. Table 6 is the comparison of the balance of the five models with the TSE50. At the 34th rebalancing, the balance of the MVS_S (NT 1,697,489) portfolio is higher than the other 4 portfolios.

Fig. 1 shows the fluctuating trends of all models over time; however, as all market value increase each month, it becomes higher than the monthly balance of the TSE50 steadily. The model MVS_S, with the criteria of maximizing skewness and return, while minimizing risk and the short selling proportion, performs much better than the other portfolio models, even in the bear market during 2008–2009, as shown in Fig. 2, as compared to the Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX).

On October 22, 2009, the performance of the five rebalanced portfolios and TSE50 are ranked as $MVS_S > MVS_SK > MVS > MVS_K > TSE50 > MV$ accordingly, as shown in Fig. 1 and Table 6. The performance of the TSE50 has the worst performance, though no transaction costs occurred from buying and holding between January 25, 2007 and October 22, 2009. All models adopting short selling strategy are superior to both the benchmark TSE50 and model MV. Apparently, MVS_S and MVS_SK, the models with higher moments, perform better than either the model MVS or MV, which have no higher moments. In addition, the criterion of skewness is more important than kurtosis, and the models adopting short selling strategy perform better than other models, especially in the bear market.

Table 5

The market values of the five portfolios and the TSE50.

Portfolio	MV	MVS	MVS_S	MVS_K	MVS_SK	TSE 50
7/25/2007	1,521,379	1,439,877	1,355,191	1,388,977	1,343,448	1,186,224
12/17/2007	1,183,812	1,118,437	1,145,686	1,092,697	1,075,303	964,286
10/22/2009	1,015,085	907,223	1,188,454	909,478	1,017,326	913,265

(Unit: NTD).

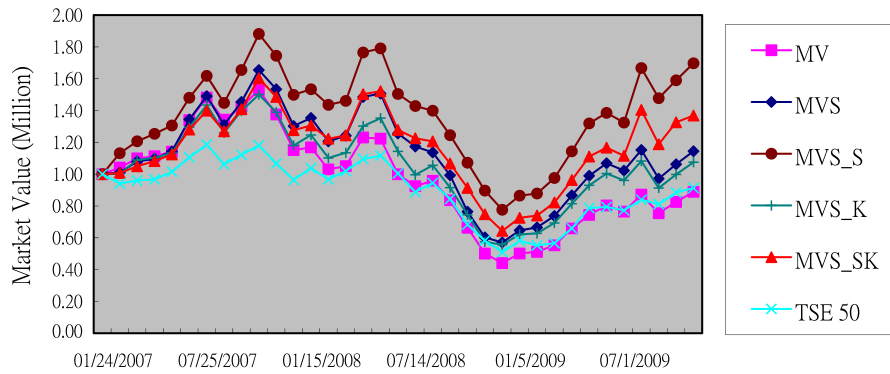


Fig. 1. The market values of the five portfolios and TSE50 for each rebalancing.

Table 6

The monthly market values of the five models of the TSE50.

No	Date	MV	MVS	MVS_S	MVS_K	MVS_SK	TSE50
0	1/24/2007	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000
1	3/3/2007	1,041,604	1,010,209	1,132,067	1,002,769	1,007,508	941,327
2	3/30/2007	1,099,438	1,079,949	1,206,947	1,089,494	1,049,786	960,034
3	4/30/2007	1,113,025	1,096,458	1,253,920	1,103,611	1,082,289	967,687
4	5/29/2007	1,142,191	1,142,500	1,306,455	1,132,976	1,124,132	1,015,306
5	6/27/2007	1,339,721	1,344,047	1,480,581	1,295,015	1,280,581	1,105,442
6	7/25/2007	1,482,093	1,490,505	1,618,209	1,434,874	1,399,479	1,186,224
7	8/22/2007	1,342,399	1,309,823	1,448,241	1,260,934	1,269,617	1,062,075
8	9/20/2007	1,423,883	1,454,421	1,655,159	1,398,826	1,408,395	1,122,449
9	10/22/2007	1,528,825	1,654,915	1,881,944	1,501,044	1,602,538	1,181,122
10	11/19/2007	1,375,334	1,534,110	1,744,564	1,389,703	1,485,556	1,066,327
11	12/17/2007	1,151,474	1,302,881	1,499,006	1,180,249	1,276,455	964,286
12	1/15/2008	1,168,729	1,353,699	1,534,212	1,246,486	1,306,434	1,038,265
13	2/20/2008	1,030,458	1,206,437	1,436,153	1,102,008	1,222,933	967,687
14	3/20/2008	1,050,911	1,242,173	1,460,975	1,134,851	1,244,070	1,019,558
15	4/18/2008	1,230,878	1,485,527	1,765,297	1,302,203	1,503,211	1,095,238
16	5/19/2008	1,223,522	1,506,801	1,791,009	1,352,234	1,521,851	1,116,497
17	6/16/2008	999,924	1,256,141	1,504,412	1,143,262	1,278,353	1,005,102
18	7/14/2008	924,709	1,173,748	1,428,639	996,418	1,225,828	887,755
19	8/12/2008	959,022	1,136,647	1,399,354	1,055,800	1,206,384	944,728
20	9/9/2008	836,524	992,964	1,245,217	915,727	1,066,956	844,388
21	10/8/2008	663,695	764,560	1,072,262	744,870	913,570	683,503
22	11/6/2008	500,881	602,744	897,114	574,108	747,963	582,143
23	12/4/2008	441,387	570,682	776,356	556,605	643,406	510,374
24	1/5/2009	501,528	646,870	865,087	621,043	725,663	581,633
25	2/9/2009	511,776	665,263	878,632	627,788	739,991	552,721
26	3/9/2009	552,921	737,264	976,132	693,298	822,098	566,667
27	4/6/2009	659,195	866,937	1,143,763	814,555	963,276	659,694
28	5/5/2009	743,836	990,021	1,319,274	930,206	1,111,091	787,415
29	6/4/2009	803,556	1,068,835	1,385,636	1,004,258	1,166,981	794,218
30	7/1/2009	764,776	1,022,709	1,324,542	960,920	1,115,528	772,959
31	7/29/2009	871,666	1,152,163	1,667,609	1,082,552	1,404,458	839,286
32	8/27/2009	754,801	971,846	1,477,764	913,129	1,188,277	812,075
33	9/24/2009	825,527	1,062,593	1,590,405	998,393	1,326,254	884,354
34	10/22/2009	889,105	1,144,426	1,697,489	1,075,282	1,368,630	913,265

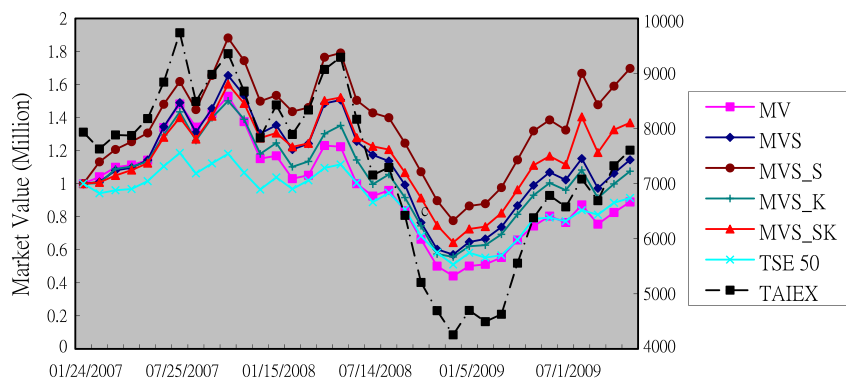


Fig. 2. The trends of the five portfolios, TSE50, and TAIEX.

5. Conclusions

The rebalancing models that consider transaction cost, including short selling cost, are more flexible and their results can reflect real transactions. This study uses a fuzzy multi-objective programming to construct multiple criteria portfolio rebalancing models with consideration on criteria of return, risk, short selling proportion, skewness, and kurtosis. The results of the first example indicate that, in comparison with the TSE50, the investment strategy of 'buy and hold' does not produce better returns in all portfolios, as the weight of each stock remains the same over time. However, in the second example, the models with higher moments or adopting short selling strategy perform better than the TSE50.

Some important issues remain for future research; for example, more criteria could be taken into consideration regarding financing. According to [Spronk et al. \(2005\)](#), there are seven criteria for financing, and the issue of how to quantify all qualitative considerations into criteria is a key problem. In addition future studies can further develop dynamic rebalancing intervals ([Yu et al., 2010](#); [Çanakoğlu and Özekici, 2010](#)). Moreover, rather than a portfolio selection based on historical return, a portfolio selection that is able to predict future return can be developed in order to meet this fast-changing environment.

Acknowledgements

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Appendix A. The weights of the five portfolios at the first four rebalances

Model MV															
Index	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Code	1101	1102	1216	1301	1303	1326	1402	1722	2002	2105	2308	2311	2317	2324	2325
1/24/2007	0	0	0.05	0.069	0	0	0	0	0.052	0	0.05	0.05	0	0	0
3/3/2007	0	0	0	0	0	0	0	0	0	0	0.0556	0	0	0	0
3/30/2007	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4/30/2007	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Index	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Code	2330	2347	2353	2354	2357	2382	2409	2454	2498	2603	2801	2880	2881	2882	2883
1/24/2007	0	0.2	0	0.078	0	0	0	0.05	0	0	0	0	0	0	0
3/3/2007	0	0.2	0	0.2	0	0	0	0	0	0	0	0	0	0	0
3/30/2007	0	0.2	0	0.2	0	0	0	0	0	0	0	0	0	0	0
4/30/2007	0	0.1562	0	0.1511	0	0	0	0	0	0	0	0	0	0	0
Index	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
Code	2885	2886	2888	2890	2891	2892	2912	3009	3231	3474	3481	5854	6505	8046	9904
1/24/2007	0	0	0	0	0	0	0	0	0	0	0.2	0	0	0	0.2
3/3/2007	0	0	0	0	0	0	0	0	0.1424	0	0.2	0	0	0	0.2
3/30/2007	0	0	0	0	0	0	0	0	0.1995	0	0.2	0	0	0	0.2
4/30/2007	0	0	0	0	0.098	0	0	0	0.1982	0	0.2	0	0	0	0.1959
Model MVS															
Index	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Code	1101	1102	1216	1301	1303	1326	1402	1722	2002	2105	2308	2311	2317	2324	2325
1/24/2007	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.186
3/3/2007	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.199
3/30/2007	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.158
4/30/2007	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.155
Index	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Code	2330	2347	2353	2354	2357	2382	2409	2454	2498	2603	2801	2880	2881	2882	2883
1/24/2007	0	0.134	0	0.129	0	0	0	0	0	0	0	0	0	0	0
3/3/2007	0	0.125	0	0.187	0	0	0	0	−0.116	0	0	0	0	0	0
3/30/2007	0	0.19	0	0.2	0	0	0	0	−0.058	0	0	0	0	0	0
4/30/2007	0	0.149	0	0.163	0	0	0	0	−0.05	0	0	0	0	0	0
Index	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
Code	2885	2886	2888	2890	2891	2892	2912	3009	3231	3474	3481	5854	6505	8046	9904
1/24/2007	0	0	0	0	0	0	0.05	0	0	0	0.2	0	0.05	0.05	0.2
3/3/2007	0	0	0	0	0	0	0	0	0	0	0.2	0	0	0	0.182
3/30/2007	0	0	0	0	0	0	0	0	0	0	0.2	0	0	0	0.2
4/30/2007	0	0	0	0	0.092	0	0	0	0	0	0.2	0	0	0	0.195

(continued on next page)

Appendix A (continued)

Model MVS_S															
Index	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Code	1101	1102	1216	1301	1303	1326	1402	1722	2002	2105	2308	2311	2317	2324	2325
1/24/2007	0	0	0	0	0	0	0	0	0	0	0.2	0.2	0	0	0.2
3/3/2007	0	0	0	0	0	0	0	0	0	0	0.2	0.116	0	0	0.192
3/30/2007	0	0	0	0	0	0	0	0	0	0	0.198	0	0	0	0.185
4/30/2007	0	0	0	0	0	0	0	0	0	0	0.2	0	0	0	0.177
Index	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Code	2330	2347	2353	2354	2357	2382	2409	2454	2498	2603	2801	2880	2881	2882	2883
1/24/2007	0.07	0.138	0	0	0	0	0	0	−0.05	0	0	0	0	0	0
3/3/2007	0	0.115	0	0	0	0	0	0	−0.183	0	0	0	0	0	0
3/30/2007	0.05	0.132	0	0	0	0	0	0	−0.095	0	0	0	0	0	0
4/30/2007	0.147	0.131	0	0	0	0	0	0	−0.05	0	0	0	0	0	0
Index	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
Code	2885	2886	2888	2890	2891	2892	2912	3009	3231	3474	3481	5854	6505	8046	9904
1/24/2007	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.146
3/3/2007	0	0	0	0	0	0	0	0	0	0	0.092	0	0	0	0.119
3/30/2007	0	0	0	0	0	0	0	0	0	0	0.149	0	0	0	0.2
4/30/2007	0	0	0	0	0	0	0	0	0	0	0.108	0	0	0	0.191
Model MVS_K															
Index	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Code	1101	1102	1216	1301	1303	1326	1402	1722	2002	2105	2308	2311	2317	2324	2325
1/24/2007	0	0	0	0	0	0	0	0	0	0	0.2	0.2	0	0	0.2
3/3/2007	0	0	0	0	0	0	0	0	0	0	0.1998	0.1161	0	0	0.1916
3/30/2007	0	0	0	0	0	0	0	0	0	0	0.1977	0	0	0	0.1851
4/30/2007	0	0	0	0	0	0	0	0	0	0	0.2	0	0	0	0.177
Index	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Code	2330	2347	2353	2354	2357	2382	2409	2454	2498	2603	2801	2880	2881	2882	2883
1/24/2007	0.0698	0.1377	0	0	0	0	0	0	−0.05	0	0	0	0	0	0
3/3/2007	0	0.1147	0	0	0	0	0	0	−0.1832	0	0	0	0	0	0
3/30/2007	0.05	0.1324	0	0	0	0	0	0	−0.0948	0	0	0	0	0	0
4/30/2007	0.1469	0.1314	0	0	0	0	0	0	−0.05	0	0	0	0	0	0
Index	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
Code	2885	2886	2888	2890	2891	2892	2912	3009	3231	3474	3481	5854	6505	8046	9904
1/24/2007	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.1458
3/3/2007	0	0	0	0	0	0	0	0	0	0	0.0924	0	0	0	0.1186
3/30/2007	0	0	0	0	0	0	0	0	0	0	0.1485	0	0	0	0.2
4/30/2007	0	0	0	0	0	0	0	0	0	0	0.1083	0	0	0	0.1909
Model MVS_SK															
Index	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Code	1101	1102	1216	1301	1303	1326	1402	1722	2002	2105	2308	2311	2317	2324	2325
1/24/2007	0	0	0	0	0	0	0	−0.0702	0	0.0634	0	0.0912	0	0	0
3/3/2007	0	0	0	0	0	0	0	0	0	0	0.2	0.123	0	0	0
3/30/2007	0	0	0	0	0	0	0	0	0	0	0.2	0.1145	0	0	0.05
4/30/2007	0	0	0	0	0	0	0	0	0	0	0.2	0.109	0	0	0.0514
Index	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Code	2330	2347	2353	2354	2357	2382	2409	2454	2498	2603	2801	2880	2881	2882	2883
1/24/2007	0	0	−0.0648	−0.0523	0	0	0	0	−0.0681	0	0	0	0	0	0.1567
3/3/2007	0	0	0	0	0	0	0	0	−0.163	0	0	0	0	0	0.153
3/30/2007	0	0.0843	0	0	0	0	0	0	−0.1026	0	0	0	0	0	0.0579
4/30/2007	0.1123	0.0843	0	0	0	0	0	0	−0.0574	0	0	0	0	0	0.054

Appendix A (continued)

Model MVS_SK															
Index	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
Code	2885	2886	2888	2890	2891	2892	2912	3009	3231	3474	3481	5854	6505	8046	9904
1/24/2007	0.0877	0	0	0	0	0	0.0942	0	0.0792	0	0.1001	0	0	0	0.0952
3/3/2007	0	0	0	0	0	0	0.05	0	0.0734	0	0.0945	0	0	0	0.1574
3/30/2007	0	0	0	0	0	0	0	0	0.0767	0	0.1314	0	0	0	0.1919
4/30/2007	0	0	0	0	0	0	0	0	0.0748	0	0.0774	0	0	0	0.1846

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