Experiment 5: Lead-Lag Control of a DC Servo Motor

1 OBJECTIVES

1. To design lead-lag compensator for a DC servo motor system
2. To analyze the transient response of the system with or without lead-lag compensators.

2 BASIC KNOWLEDGE

In this experiment, we introduce mathematical models for a DC servomechanism and lead-lag compensator design using the root-locus method

2.1 Background information on DC servo motor system

A block diagram for a typical servomechanism is shown in Fig. 1. The action of the servomechanism is to track a desired position (or speed) despite the presence of disturbance inputs to the process and despite errors in the sensor data.

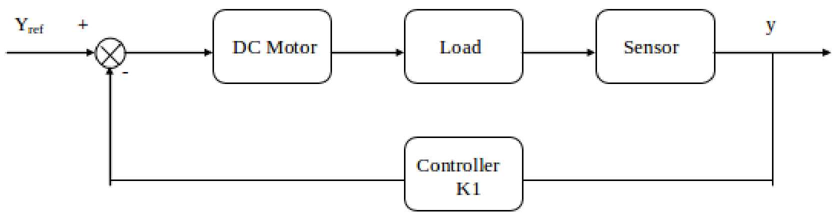

Figure 1: Servomechanism block diagram

2.1.1 Model Development

We begin by developing a simplified linear model of an armature controlled DC servo motor and load. Fig. 2 shows the schematic diagram of the motor and load.

Neglecting the inductance of the armature circuit, the armature voltage V_a produces a current I_a as given by:

$$I_a = \frac{V_a - V_b}{R_a} \tag{1}$$

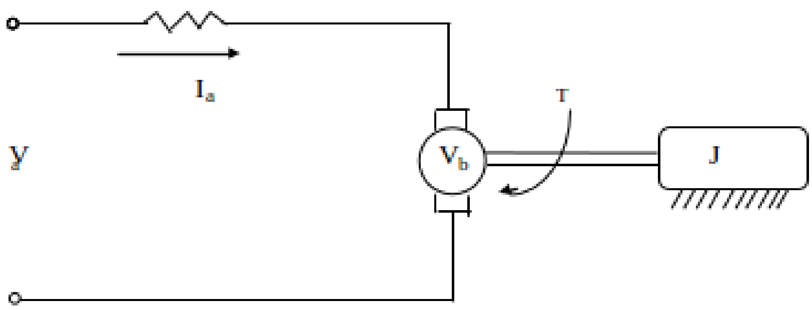

Figure 2: schematic diagram of the DC servo motor

Here V_b denotes the back emf of the motor and R_a is the armature resistance.

The motor torque T is proportional to I_a:

$$T = K_T I_a \tag{2}$$

But from Newton's law, assuming an inertia load:

$$T = J\ddot{\theta} \tag{3}$$

combining equations 1, 2, and 3 we obtain:

$$K_T \left[\frac{V_a - V_b}{R_a} \right] = J\ddot{\theta} \tag{4}$$

Using the relationship $V_b = K_b\dot{\theta}$ and, letting $V_a = U$ (input signal), we have:

$$\frac{K_T}{R_a} \left[U - K_b\dot{\theta} \right] = J\ddot{\theta} \tag{5}$$

$$\frac{JR_a}{K_T}\ddot{\theta} + K_b\dot{\theta} = U \tag{6}$$

$$\ddot{\theta} + \frac{K_b K_T}{JR_a}\dot{\theta} = \frac{K_T}{JR_a}U \tag{7}$$

or letting $\omega = \dot{\theta}$, and

$$\dot{\omega} + \frac{K_b K_T}{JR_a}\omega = \frac{K_T}{JR_a}U \tag{8}$$

Equations 7 and 8 are the differential equations for the DC motor.

Let $\frac{K_b K_T}{J R_a} = \frac{1}{\tau}$, $\frac{K_T}{J R_a} = \frac{K_{0s}}{\tau}$. Here, K_{0s} is the static gain and τ is the time constant of the system from input voltage to output speed. From D.E. 7 we can have the following transfer function:

$$\frac{\theta(s)}{U(s)} = \frac{K_{0s}}{s(\tau s+1)} \tag{9}$$

And from D.E. 8 we obtain the transfer function of the DC motor from U to $\dot{\theta} = \omega$:

$$\frac{\omega(s)}{U(s)} = \frac{K_{0s}}{(\tau s+1)} \tag{10}$$

Considering the measurements of speed and position, the system can be depicted as in Fig. 3

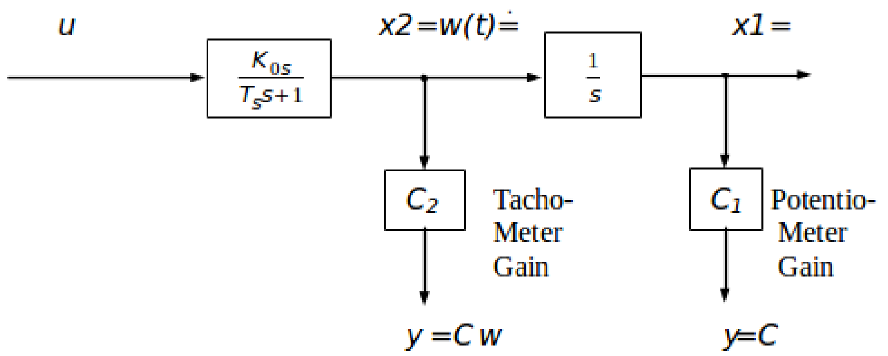

Figure 3: schematic of the DC motor system with measurements

Then, the second order D.E. of the DC motor can be rewritten in the state-space format:

$x = A\dot{x} + Bu$ $y = Cx$

where, $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, $u = u$, $y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$, and $A = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{1}{\tau} \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ \frac{K_s}{\tau} \end{bmatrix}$, $C = \begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix}$

Further, the input/output transfer function of the DC motor is:

$$G(s) = \frac{Y(s)}{U(s)} = \begin{bmatrix} G_1(s) \\ G_2(s) \end{bmatrix} = \begin{bmatrix} \frac{C_1 K_{0x}}{s(\tau s+1)} \\ \frac{C_2 K_{0s}}{\tau s+1} \end{bmatrix} = \begin{bmatrix} \frac{\left(\frac{C_1}{C_2}\right) * C_2 K_{0s}}{s(\tau s+1)} \\ \frac{C_2 K_{0s}}{\tau s+1} \end{bmatrix} \tag{11}$$

Define: $C_s = C_1/C_2 and K_s = C_2 K_{0s}$, then

$$G(s) = \frac{Y(s)}{U(s)} = \begin{bmatrix} G_1(s) \\ G_2(s) \end{bmatrix} = \begin{bmatrix} \frac{C_s K_s}{s(\tau s+1)} \\ \frac{K_s}{\tau s+1} \end{bmatrix} \tag{12}$$

The model has one pole at the origin and one pole on the negative real axis. The problem is to identify parametesr K_s, τ and C_s. From 12, the transfer function from input u to output y_2 is:

$$\frac{Y_2(s)}{U(s)} = \frac{K_s}{\tau s + 1} \tag{13}$$

The step response of the first order system is shown in Fig. 4:

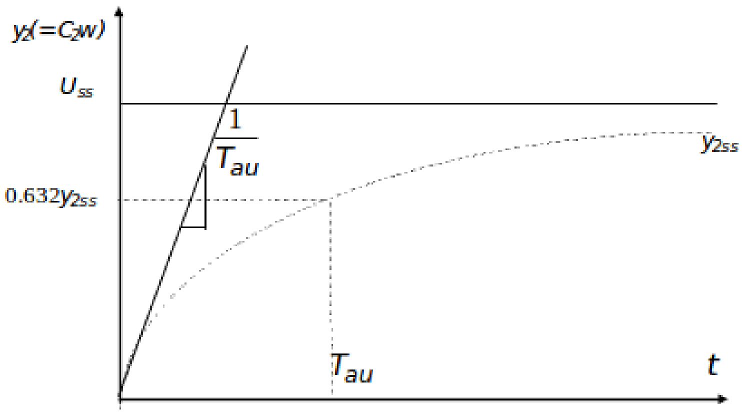

Figure 4: step response of the DC servo motor

From this diagram, we can determine that the time constant of the motor, τ, is the time it takes y_2 to reach 63.2% of its steady state value y_{2ss}. We can also obtain the steady-state gain $K_s = \frac{y_{2ss}}{U_{ss}}$.

In addition, notice that regarding C_s there is a conformity that has to be satisfied:

$$C_s = \frac{\frac{dy_1}{dt}}{y_2} \tag{14}$$

This gives the way to identify C_s.

2.1.2 Closed Loop Control for a DC servomechanism

The block diagram of Fig. 1 for the servomechanism can be simplified into the block diagram given in Fig. 5.

This configuration is referred to as a negative feedback closed loop configuration where G(s) is the forward loop transfer function and H(s) is the feedback loop transfer function. The equivalent transfer function between

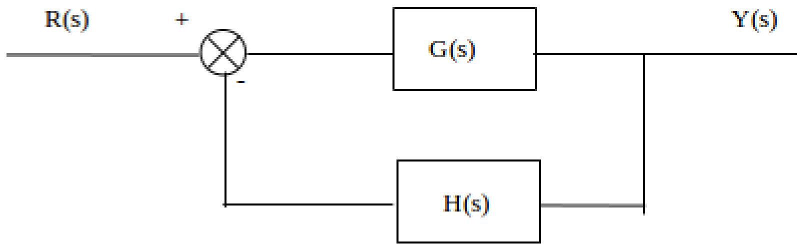

Figure 5: simplified block diagram

the input r(t) and the output y(t) as shown in Fig. 4 can be represented as the transfer function G'(s) shown in Fig. 6.

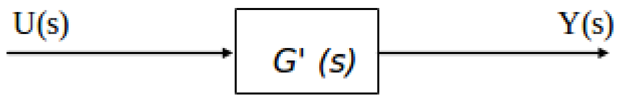

Figure 6: eqivalent closed-loop transfer function

$$G'(s) = \frac{Y(s)}{U(s)} = \frac{G(s)}{1 + G(s)H(s)} \tag{15}$$

Considering the servomechanism transfer function:
From Eq. 9: $G(s) = \frac{C_s K_s}{s(\tau s+1)}$,
H(s) = K_1 (Output feedback controller gain),

$$G'(s) = \frac{\frac{C_s K_s}{\tau}}{s^2 + \frac{s}{\tau} + \frac{K_1 K_s C_s}{\tau}} \tag{16}$$

We observe that the transfer function of the closed-loop system, G'(s) is a second order system with one free controller parameter K_1. Changing the gain K_1 can be used to alter the transient dynamics of the second order system.

2.2 Background information on lead-lag compensator design

Lead and lag compensators are used quite extensively in control. A lead compensator can increase the stability or speed of response of a system; a lag compensator can reduce (but not eliminate) the steady state error. If improvements in both transient response and steady-state response are desired, then both a lead compensator and a lag compensator may be used simultaneously. Depending on the effect desired, one or more lead and lag compensators may be used in various combinations.

3 EXPERIMENTAL PROCEDURES

3.1 Identify K_s, τ, C_s:

1. Run **MATLAB** and open the model **Lab5_identification.slx**. The *DC servo* block contains a masked model of a DC servo motor. Run the model. A unit step input is applied to the motor winding at t = 1s. Open the scope and look at the angular velocity trace. Determine the steady-state speed of the motor. This is the value of K_s, the steady-state gain. Determine the time at which the speed reaches 63.2% of its final value. Find the elapsed time since the step function was applied. This value is the time constant τ.

2. Again using the scope, determine the slope of the asymptote of the angle trace. This is another way of measuring the angular velocity which allows us to determine C_s (see Eq. 14).

3.2 Lead Compensator Design

1. From **3.1** we know system parameters K_s, τ, C_s, with which we can estimate the transfer function of the motor and design a controller accordingly.

2. The transient response specifications are as follows:

   (a) Maximum percentage overshoot less than 5

   (b) Settling time (2% band) less than 1 second

   Calculate the corresponding restrictions on the values of the damping ratio ζ and the natural frequency ω_n.

3. Determine our desired closed loop dominant pole locations from the ζ and ω_n calculated in the previous step. You might try selecting the points at the intersections of the constraining curves from the previous step.

4. The open loop system consisting of the controller and plant has 1 zero and 3 poles. The open loop plant poles are fixed at zero and $1/\tau$. The compensator pole and zero must be chosen so that the root locus intersects the desired pole location. The feedback gain associated with this point on the root locus must then be determined. *You will have to show this work in your report.*

   Suggested procedure:

   (a) Find the real axis projection of the desired pole location.
   (b) Place the controller zero at or to the left of this point on the real axis.
   (c) Determine where on the real axis the controller pole must be placed using the *angle criterion*. Remember that a lead compensator has its pole to the left of its zero.
   (d) If no valid angle can be found, go back to step 4b and choose a different zero location.
   (e) Find the feedback gain. You might use the *magnitude criterion* or the matlab command rlocfind(tf).

5. Record the zero, pole, and gain values.

3.3 Study the transient response of the servo motor system with or without lead-lag compensator

Part 1:

1. Open the Simulink model **Lab5_ConstantFeedback.slx**, which contains our servo model with fixed gain feedback. Set the gain to unity to start with.

2. Run the simulation. How long does it take for the angle of the motor to settle within about 2% of the reference?

3. Increase the gain and improve on the performance. What is the shortest settling time you can obtain with a constant feedback that meets our overshoot requirements? You might use MATLAB's rlocus() to plot the root locus, sgrid() to get the boundary associated with our zeta constraint, and rlocfind() to get the gain associated with your desired closed loop pole locations.

4. Find the scope data output in the Matlab workspace and save it as a .mat file using the save command.

Part 2:

1. Open the Simulink model **Lab5_Lead.slx**, which contains our servo model with a lead compensator transfer function in the feedback path.
2. Modify the lead compensator transfer function to match the zero, pole, and gain values previously determined.
3. Calculate the steady-state gain of the lead compensator and scale our reference signal accordingly. The reference we are providing in this configuration corresponds to the desired value of the compensator output, not the desired state measurement.
4. Run the simulation. Does the system trajectory meet the specifications? The servo model block includes a maximum control effort, which constrains the rise time. This could be an issue for some choices of pole location.
5. Find the scope data output in the Matlab workspace and save it as a .mat file using the save command.

Part 3:

1. Open the Simulink model **Lab5_Lag.slx**, which contains our servo model with a lag compensator transfer function in the forward path.
2. Note how long it takes to settle.
3. Find the scope data output in the Matlab workspace and save it as a .mat file.

3.4 Lead vs Lead-Lag compensation

1. Open the Simulink model **Lab5_LeadLag.slx**. This model is intended to demonstrate an application for a lag compensator. The steady state error in theta for the servo is zero for a step input since the theta-input transfer function is a type 1 system. However, the steady state error for a lead-compensated system is not zero for a ramp input. A lag compensator can reduce this error.
2. Modify the lead compensator transfer function blocks to match the zero, pole, and gain values previously determined.
3. Run the simulation.
4. Confirm that the transient response of the step-input system is almost the same as before the addition of the lag compensator.
5. Look at the ramp input systems. Compare the error of the lead compensated system to that of the lead-lag compensated system.

4 REPORT

1. Plot the transients responses from 3.3. Compare the results of the three situations. Comments are needed to describe the effects of the lead compensator and lag compensator and illustrate why they have those effects.

2. Show the detailed procedure of designing the lead compensator based on the root-locus approach. What parameters did you determine for the servo motor? What values did you calculate for ζ and ω_n? What about the zero and pole locations and the gain for the lead compensator? Were you able to meet the specifications laid out in 3.2?

3. Design the lead compensator in a more simple way. Design a lead-lag compensator and show how and why it becomes a lead or lead-lag compensator.

Question: We determined that the time constant of the motor, τ, is the time it takes for y_2 to reach 63.2% of its steady state value y_{2ss}, why?