Lab 4: Lead-Lag Control of a DC Servo Motor

1 OBJECTIVES

1. To understand the concept of system type and its implication on tracking ability.
2. To design a lead-lag compensator for a DC servo motor system.
3. To analyze the transient response of the system with or without lead-lag compensators.

2 BASIC KNOWLEDGE

In this experiment, we introduce mathematical models for a DC servomechanism and lead-lag compensator design using the root-locus method

A block diagram for a typical servomechanism is shown in Fig. 1. The action of the servomechanism is to track a desired position (or speed) despite the presence of disturbance inputs to the system and errors in the sensor data.

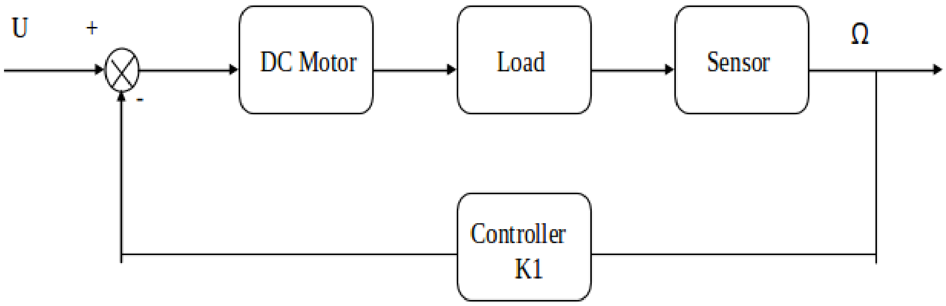

Figure 1: Servomechanism block diagram

2.1 Model Development

We begin by developing a simplified linear model of an armature controlled DC servo motor and load. Fig. 2 shows the schematic diagram of the motor and load.

Neglecting the inductance of the armature circuit, the armature voltage V_a produces a current I_a as given by:

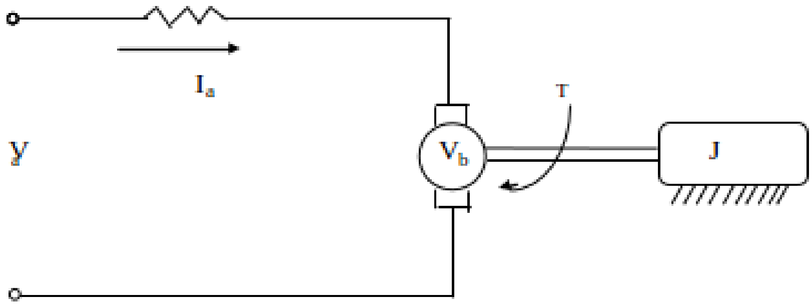

Figure 2: schematic diagram of the DC servo motor

$$I_a(t) = \frac{V_a(t) - V_b(t)}{R_a} \tag{1}$$

Here $V_b(t)$ denotes the back emf of the motor and R_a is the armature resistance.

The motor torque $T(t)$ is proportional to $I_a(t)$:

$$T(t) = K_T I_a(t) \tag{2}$$

From Newton's law, assuming an inertia load:

$$T(t) = J\ddot{\theta}(t) \tag{3}$$

combining equations 1, 2, and 3 we obtain:

$$K_T \left[\frac{V_a(t) - V_b(t)}{R_a} \right] = J\ddot{\theta}(t) \tag{4}$$

Using the relationship $V_b = K_b\dot{\theta}$ and, letting $V_a(t) = u(t)$ (the input signal), we have:

$$\frac{K_T}{R_a} \left[u(t) - K_b\dot{\theta}(t) \right] = J\ddot{\theta}(t) \tag{5}$$

$$\frac{JR_a}{K_T}\ddot{\theta}(t) + K_b\dot{\theta}(t) = u(t) \tag{6}$$

$$\ddot{\theta} + \frac{K_b K_T}{JR_a}\dot{\theta} = \frac{K_T}{JR_a}u(t) \tag{7}$$

or letting $\omega(t) = \dot{\theta}(t)$, and

$$\dot{\omega}(t) + \frac{K_b K_T}{J R_a}\omega(t) = \frac{K_T}{J R_a}U(t) \tag{8}$$

Equations 7 and 8 are the differential equations for the DC motor.

Let $\frac{K_b K_T}{J R_a} = \frac{1}{\tau}$, $\frac{K_T}{J R_a} = \frac{K_{0s}}{\tau}$. Here, K_s is the static gain and τ is the time constant of the system. The D.E. 7 is a model of the system from the input voltage to the output speed. Using the Laplace transform we can convert to the following transfer function from input $(u(t))$ to output position $(\theta(t)$:

$$\frac{\Theta(s)}{U(s)} = \frac{K_s}{s(\tau s + 1)} \tag{9}$$

From the D.E. 8 the transfer function of the DC motor from input $u(t)$ to output speed $\dot{\theta}(t) = \omega(t)$ is:

$$\frac{\Omega(s)}{U(s)} = \frac{K_s}{(\tau s + 1)} \tag{10}$$

The system can be depicted as in Fig. 3

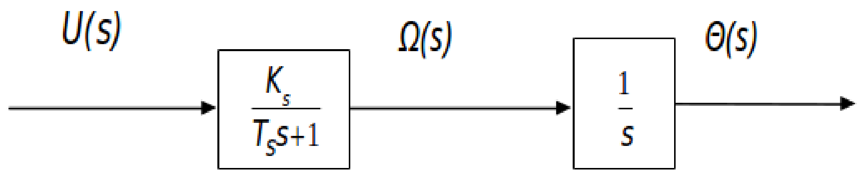

Figure 3: Schematic of the DC servo system

The model from input armature voltage to output position has one pole at the origin and one pole on the negative real axis. The problem is to identify the parameters $K_s, and\tau$. The transfer function from input $u(t)$ to output $\theta(t)$ is:

$$\frac{\Omega(s)}{U(s)} = \frac{K_s}{\tau s + 1} \tag{11}$$

The unit step response of the first order system is shown in Fig. 4:

From this response, we can determine that the time constant of the motor, τ, is the time it takes $\omega(t)$ to reach 63.2% of its steady-state value ω_{ss}. We can also obtain the steady-state gain $K_s = \frac{\omega_{ss}}{u_{ss}}$.

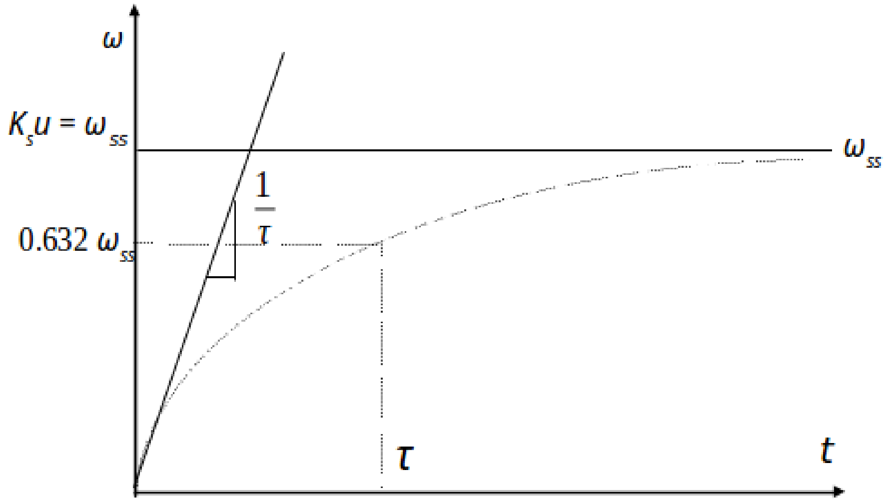

Figure 4: step response of the DC servo motor

2.2 Closed-Loop Control for a DC servomechanism

The block diagram of Fig. 1 for the servomechanism can be simplified into the block diagram given in Fig. 5.

This configuration is referred to as a negative feedback closed-loop control system where $G(s)$ is the forward loop transfer function and H(s) is the feedback loop transfer function. The equivalent transfer function between the input r(t) and the output c(t) as shown in Fig. 4 can be represented as the transfer function $G'(s)$ shown in Fig. 6.

$$G'(s) = \frac{Y(s)}{U(s)} = \frac{G(s)}{1 + G(s)H(s)} \tag{12}$$

Considering the servomechanism transfer function:
From Eq. 9: $G(s) = \frac{K_s}{s(\tau s+1)}$,
H(s) = K_1 (Output feedback controller gain),

$$G'(s) = \frac{\frac{K_s}{\tau}}{s^2 + \frac{s}{\tau} + \frac{K_1 K_s}{\tau}} \tag{13}$$

We observe that the transfer function of the closed-loop system, $G'(s)$, is a second order system with one free controller parameter K_1. Changing

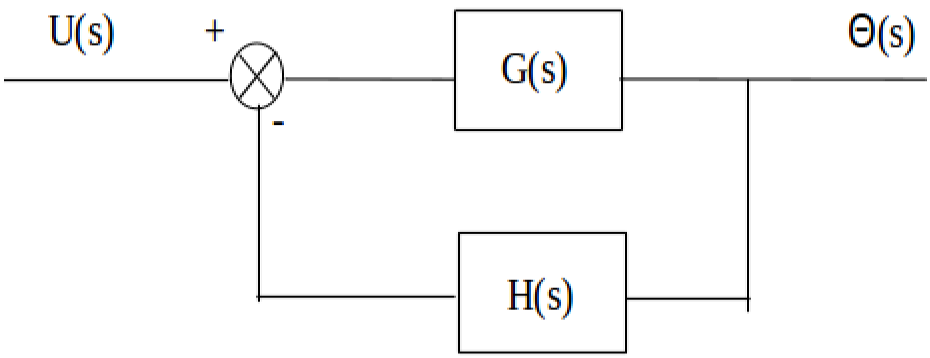

Figure 5: Simplified block diagram

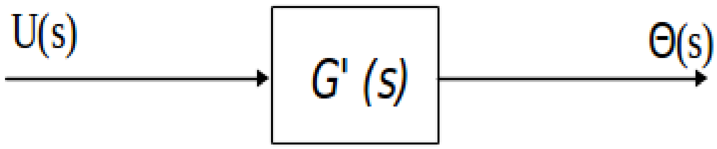

Figure 6: Equivalent closed-loop transfer function

the gain K_1 can be used to alter the transient response dynamics of the closed-loop second order system.

2.3 Background information on lead-lag compensator design

Lead and lag compensators are used quite extensively in control. A lead compensator can increase the stability or speed of response of a system; a lag compensator can reduce (but not eliminate) the steady state error. If improvements in both transient response and steady-state response are desired, then both a lead compensator and a lag compensator may be used simultaneously. Depending on the effect desired, one or more lead and lag compensators may be used in various combinations.

3 EXPERIMENTAL PROCEDURES

3.1 System Type and Tracking Error

In this portion of the procedure, we explore the effect of system type on our ability to track reference signals of various types. The Simulink model used in this portion includes five different cases. First, we have position feedback used to track a step, ramp, and parabolic reference. The remaining two cases use speed feedback with step and ramp references.

Procedure:

1. Open the model **Lab4_SystemType.slx** and run it. For each case, observe the following: Does the system state track the reference? If not, characterize the error (is the error constant, increasingly linearly, increasing quadratically, etc.). How does the system's ability to track the same signal differ depending on whether speed or position is being tracked?

2. If a reference signal is not being tracked, adjust the gain for that case. What happens to the error? Is it possible to eliminate the error?

3.2 Identify K_s and τ

1. Run MATLAB and open the model **Lab4_identification.slx**. The DC servo block contains a masked model of a DC servo motor. Run the model. A unit step input is applied to the motor winding at t = 1s. Open the scope and look at the angular velocity trace. Determine the steady-state speed of the motor. This is the value of K_s, the steady-state gain. Determine the time at which the speed reaches 63.2% of its final value. Find the elapsed time since the step function was applied. This value is the time constant τ.

3.3 Proportional Feedback

1. Open the Simulink model **Lab4_ConstantFeedback.slx**, which contains our servo model with fixed gain feedback.

2. What is the shortest settling time you can obtain with proportional feedback that results in a peak overshoot of less than 5%? You might use MATLAB's rlocus() to plot the root locus, sgrid() to get the boundary associated with our zeta constraint, and rlocfind() to get the gain associated with your desired closed loop pole locations.

3.4 Designing a Lead Compensator for Better Performance

1. From **3.1** we know system parameters K_s and τ, with which we can estimate the transfer function of the motor and design a controller

accordingly.

2. The transient response specifications are as follows:

   (a) Maximum percentage overshoot less than 5
   (b) Settling time (2% band) less than 1.2 seconds

   Calculate the corresponding restrictions on the values of the damping ratio ζ and the natural frequency ω_n.

3. Determine our desired closed loop dominant pole locations from the ζ and ω_n calculated in the previous step. You might try selecting the points at the intersections of the constraining curves from the previous step.

4. The open loop system consisting of the controller and plant has 1 zero and 3 poles. The open loop plant poles are fixed at zero and $1/\tau$. The compensator pole and zero must be chosen so that the root locus intersects the desired pole location. The feedback gain associated with this point on the root locus must then be determined. *You will have to show this work in your report.*

   Suggested procedure:

   (a) Find the real axis projection of the desired pole location.
   (b) Place the controller zero at or to the left of this point on the real axis, but not too far (consider the limit in which the lead pole goes to $-\infty$ and has an angle of 0).
   (c) Determine where on the real axis the controller pole must be placed using the *angle criterion*. Remember that a lead compensator has its pole to the left of its zero.
   (d) If no valid angle can be found, go back to step 4b and choose a different zero location.
   (e) Find the feedback gain. You might use the *magnitude criterion* and check your result with the matlab command rlocfind().

5. Record the zero, pole, and gain values.

6. Open the Simulink model **Lab4_Lead.slx**. This model contains the following five systems:

System	Input	Control
1	step	lead in feedback path
2	step	lead in forward path
3	ramp	lead in feedback path
4	ramp	lead in forward path
5	ramp	proportional

7. For systems 1-4, change the gains and lead compensator transfer function blocks to match the controller parameters just determined. For system 5, set the G4 block to a proportional gain constant determined in the previous part, or another value that meets the maximum overshoot constraint.

8. Run the simulation. Regarding the step-fed systems, does the system trajectory meet the specifications? How do the responses of the feedback path compensated system and the forward path compensated system differ? What is the impact on the closed loop transfer function of the compensator configuration and offer a mathematical explanation for the difference.

   Regarding the ramp-fed systems, how does the lead compensator affect the steady state error?

3.5 Lag Compensation

1. Open the Simulink model **Lab4_Lag.slx**, which contains our servo model with a lag compensator transfer function in the forward path.

2. Plot the root locus of the lag compensated system.

3. Try and find a gain that does not exceed our overshoot limit of 5%. What is the settling time? What is the effect of the lag compensator on the root locus and the resultant transient behavior.

3.6 Lead-Lag Compensation: Transient Performance with Steady-State Error Reduction

1. Open the Simulink model **Lab4_LeadLag.slx**. This model is intended to demonstrate an application for a lag compensator. The steady state error in theta for the servo is zero for a step input since the theta-input transfer function is a type 1 system. However, the steady state error for a lead-compensated system is not zero for a ramp input. A lag compensator can reduce this error.

2. Modify the lead and lag compensator transfer function blocks to match the zero, pole, and gain values previously determined.

3. Run the simulation.

4. Confirm that the transient response of the step-input system is almost the same as before the addition of the lag compensator.

5. Look at the ramp input systems. Compare the error of the lead compensated system to that of the lead-lag compensated system. Does

the lag compensator reduce the steady state error without detriment to transient performance?

6. Look at the step speed reference system at the bottom of the model diagram. How does the lead-lag compensated system performance compare to that of the lead compensated system?

4 REPORT

1. Explain the tracking errors observed in the System Type and Tracking Error in terms of the Final Value Theorem.