Experiment 6: State Feedback Control of a DC Servo Motor System

1 OBJECTIVES

- To demonstrate how properties of a closed-loop system are influenced by the closed-loop roots.
- To introduce pole placement by state feedback design for a continuous system.
- To obtain the state space model for DC servo system.

2 BASIC KNOWLEDGE

In this section we introduce mathematical models in state space for a DC servomechanism.

A block diagram for a typical servomechanism is shown in Fig. 1. The action of the servomechanism is to track a desired position (speed) despite the presence of disturbance inputs to the process.

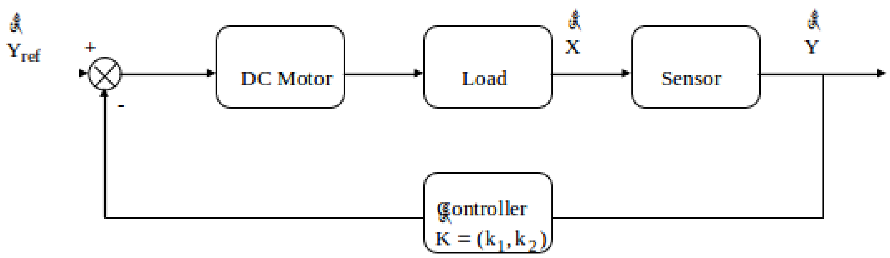

Figure 1: Servomechanism block diagram

2.1 Model Development

We begin by developing a simplified linear model of an armature controlled DC-Servo motor and load. Fig. 2 shows the schematic diagram of the motor and load.

Neglecting the inductance of the armature circuit, the armature voltage V_a produces a current I_a as given by:

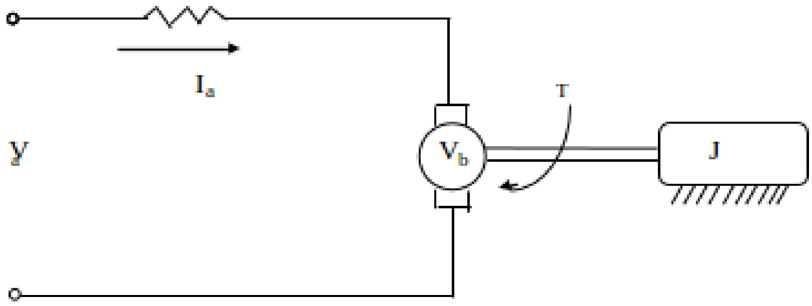

Figure 2: schematic diagram of the DC servo motor

$$I_a = \frac{V_a - V_b}{R_a} \tag{1}$$

Here V_b denotes the back emf of the motor and R_a is the armature resistance.

$$T = K_T I_a \tag{2}$$

But from Newton's law, assuming an inertia load and neglecting load damping:

$$T = J\ddot{\theta} \tag{3}$$

combining equations 1, 2, and 3 we obtain:

$$K_T \left[\frac{V_a - V_b}{R_a}\right] = J\ddot{\theta} \tag{4}$$

Using the relationship $V_b = K_b\dot{\theta}$ and, letting $V_a = U$ (input signal), we have:

$$\frac{K_T}{R_a}\left[U - K_b\dot{\theta}\right] = J\ddot{\theta} \tag{5}$$

$$\frac{JR_a}{K_T}\ddot{\theta} + K_b\dot{\theta} = U \tag{6}$$

$$\ddot{\theta} + \frac{K_b K_T}{JR_a}\dot{\theta} = \frac{K_T}{JR_a}U \tag{7}$$

Equation 7 is the differential equation for the DC motor.

Let $\frac{K_b K_T}{J R_a} = \frac{1}{T_s}$, $\frac{K_T}{J R_a} = \frac{K_{0s}}{s(T_s s+1)}$. Here K_{0s} is the static gain and T_s is the time constant of the system from input voltage to output speed. From D.E. 7 we can have the following transfer function:

$$\frac{\theta(s)}{U(s)} = \frac{K_{0s}}{s(T_s s+1)} \tag{8}$$

Considering the measurements of speed and position, the system can be depicted as in Fig. 3:

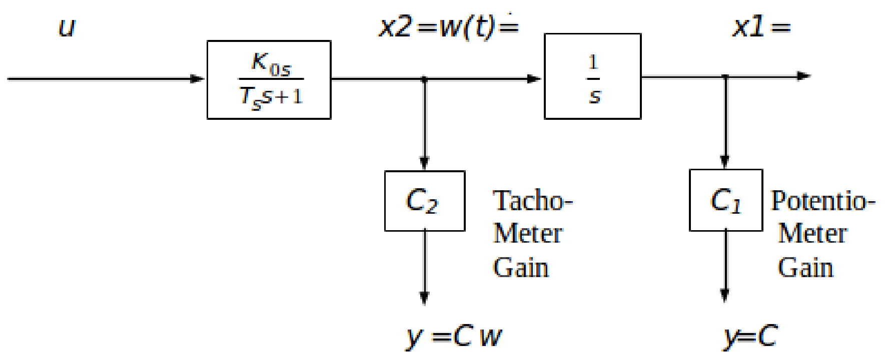

Figure 3: schematic of the DC motor system with measurements

Then, the second order D.E. of the DC-motor can be rewritten in state-space format:

$$\dot{x} = Ax + Buy = Cx \tag{9}$$

where, $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, $u = u$, $y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$, and $A = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{1}{\tau} \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ \frac{K_s}{\tau} \end{bmatrix}$, $C = \begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix}$

2.2 State space design method

Using a state feedback controller, we want to to position the closed-loop poles of of the system. There are different ways of achieving this. One of the design methods is described below.

The continuous-time system is represented by the state equation:

$\dot{x} = Ax + Bu$ $y = Cx$

A,B,C is a 2 by 2 matrix defined as above.

The state controller realizes a linear feedback control law of the form:

$$u = -K(y_d - y) = -KC(x_d - x) \tag{10}$$

Where K = $[k_1 k_2]$ is a feedback gain vector.

We require that the roots of the closed-loop system be equal to λ_1, λ_2. The design method consists of finding a K such that the roots of the closed-loop system are in the desired locations. It can be shown that there exists a linear feedback law that gives a closed-loop system with roots specified only if the pair (A,B) is controllable.

Define $C_s = C_1/C_2$ and $K_s = C_2 K_{0s}$, then the state matrix of the closed-loop system is: $A_c = A - BKC = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{1}{T_s} \end{bmatrix} - \begin{bmatrix} 0 \\ -\frac{K_{0s}}{T_s} \end{bmatrix} \begin{bmatrix} k_1 & k_2 \end{bmatrix} \begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix} =$
$\begin{bmatrix} 0 & 1 \\ -\frac{k_1 C_s K_{0s}}{T_s} & -\frac{1+k_2 C_2 K_{0s}}{T_s} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k_1 \frac{C_1}{C_2} C_2 K_{0s}}{T_s} & -\frac{1+k_2 C_2 K_{0s}}{T_s} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k_1 C_s K_s}{T_s} & -\frac{1+k_2 K_s}{T_s} \end{bmatrix}$
and the characteristic equation is:

$$C(s) = \det(\lambda I - A_c) = \lambda^2 + \lambda \frac{1 + K_s k_2}{T_s} + \frac{K_s k_1 C_s}{T_s} = 0 \qquad (11)$$

By means of the feedback gains, the roots of the characteristic equation may be changed. From Vieta's formula we obtain:

$$\lambda_1 \cdot \lambda_2 = \frac{K_s C_s k_1}{T_s} \qquad \lambda_1 + \lambda_2 = -\frac{1 + K_s k_2}{T_s} \qquad (12)$$

and we can calculate k_1 and k_2 from:

$$k_1 = \frac{\lambda_1 \lambda_2 T_s}{K_s C_s} \qquad k_2 = -\frac{(\lambda + \lambda_2) T_s + 1}{K_s} \qquad (13)$$

Then you can use the parameters (K_s,T_s,C_s) that you determined for the servo motor in Lab 5 to design the feedback gains.

3 EXPERIMENTAL PROCEDURES

We will be designing state controllers to position the closed-loop poles of the DC servo system in several different places.

1. Open Matlab and open the Simulink model **Lab6_StateFB.slx**.
2. Design a controller according to the procedure of section 2.2 to position the roots as follows: $\lambda_1 = -2, \lambda_2 = -4$. Enter the values for k_1 and k_2 in the gain blocks.
3. Run the simulation
4. Find the scope data in the workspace and save it to a .mat file using the save command.
5. Repeat the above for the following pairs of root locations:

(a) $\lambda_1 = -4, \lambda_2 = -5$

(b) $\lambda_{1,2} = -2 \pm 2j$

(c) $\lambda_{1,2} = -2 \pm 4\sqrt{6}j$

4 EXERCISES

1. Fill in the following table:

Poles	k_1	k_2	ζ	ω_n
			N/A	N/A
			N/A	N/A

2. Explain the results.

3. Compute the rank of [B AB].