Lab 2: Model Identification and Transient Response of a DC Servo Motor System

1 OBJECTIVES:

- To model and identify some of the parameters of a DC servo motor system.
- To illustrate the basic idea of closed-loop feedback control.
- To illustrate the transient response of a second order system to a step input and find the relationship between the parameters of the transient response and the parameters of the closed-loop DC motor system.

2 BASIC KNOWLEDGE

In this lab we introduce mathematical models for a DC servomechanism and basic identification methods.

A block diagram for a typical servomechanism is shown in Fig. 1. The action of the servomechanism is to track a desired position (or speed) despite the presence of disturbance inputs to the system and errors in the sensor data.

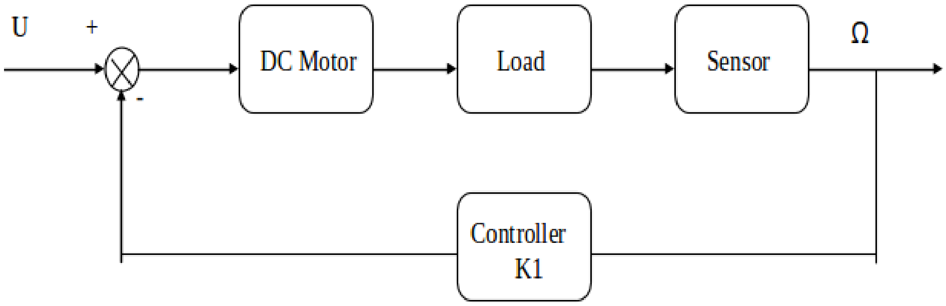

Figure 1: Servomechanism block diagram

2.1 Model Development

We begin by developing a simplified linear model of an armature controlled DC servo motor and load. Fig. 2 shows the schematic diagram of the motor and load.

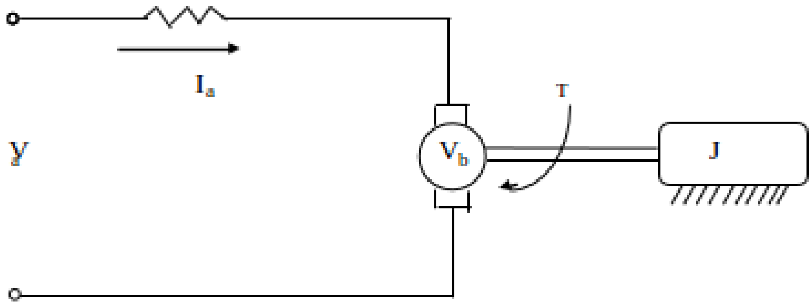

Figure 2: schematic diagram of the DC servo motor

Neglecting the inductance of the armature circuit, the armature voltage V_a produces a current I_a as given by:

$$I_a(t) = \frac{V_a(t) - V_b(t)}{R_a} \tag{1}$$

Here $V_b(t)$ denotes the back emf of the motor and R_a is the armature resistance.

The motor torque $T(t)$ is proportional to $I_a(t)$:

$$T(t) = K_T I_a(t) \tag{2}$$

From Newton's law, assuming an inertia load:

$$T(t) = J\ddot{\theta}(t) \tag{3}$$

combining equations 1, 2, and 3 we obtain:

$$K_T \left[\frac{V_a(t) - V_b(t)}{R_a} \right] = J\ddot{\theta}(t) \tag{4}$$

Using the relationship $V_b = K_b\dot{\theta}$ and, letting $V_a(t) = u(t)$ (the input signal), we have:

$$\frac{K_T}{R_a} \left[u(t) - K_b\dot{\theta}(t) \right] = J\ddot{\theta}(t) \tag{5}$$

$$\frac{JR_a}{K_T}\ddot{\theta}(t) + K_b\dot{\theta}(t) = u(t) \tag{6}$$

$$\ddot{\theta} + \frac{K_b K_T}{JR_a}\dot{\theta} = \frac{K_T}{JR_a}u(t) \tag{7}$$

or letting $\omega(\mathrm{t}) = \dot{\theta}(t)$, and

$$\dot{\omega}(t) + \frac{K_b K_T}{J R_a}\omega(t) = \frac{K_T}{J R_a}U(t) \tag{8}$$

Equations 7 and 8 are the differential equations for the DC motor.

Let $\frac{K_b K_T}{J R_a} = \frac{1}{\tau}$, $\frac{K_T}{J R_a} = \frac{K_{0s}}{\tau}$. Here, K_s is the static gain and τ is the time constant of the system. The D.E. 7 is a model of the system from the input voltage to the output speed. Using the Laplace transform we can convert to the following transfer function from input $(u(t))$ to output position $(\theta(t)$:

$$\frac{\Theta(s)}{U(s)} = \frac{K_s}{s(\tau s + 1)} \tag{9}$$

From the D.E. 8 the transfer function of the DC motor from input $u(t)$ to output speed $\dot{\theta}(t) = \omega(t)$ is:

$$\frac{\Omega(s)}{U(s)} = \frac{K_s}{(\tau s + 1)} \tag{10}$$

The system can be depicted as in Fig. 3

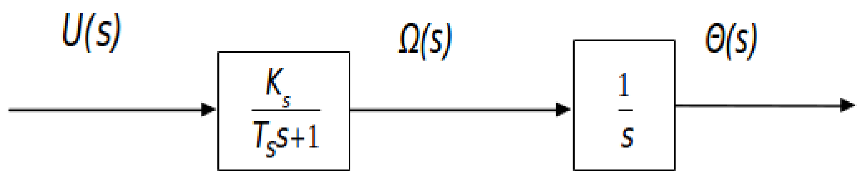

Figure 3: Schematic of the DC servo system

The model from input armature voltage to output position has one pole at the origin and one pole on the negative real axis. The problem is to identify the parameters $K_s, and\tau$. The transfer function from input $u(t)$ to output $\theta(t)$ is:

$$\frac{\Omega(s)}{U(s)} = \frac{K_s}{\tau s + 1} \tag{11}$$

The unit step response of the first order system is shown in Fig. 4:

From this response, we can determine that the time constant of the motor, τ, is the time it takes $\omega(t)$ to reach 63.2% of its steady-state value ω_{ss}. We can also obtain the steady-state gain $K_s = \frac{\omega_{ss}}{u_{ss}}$.

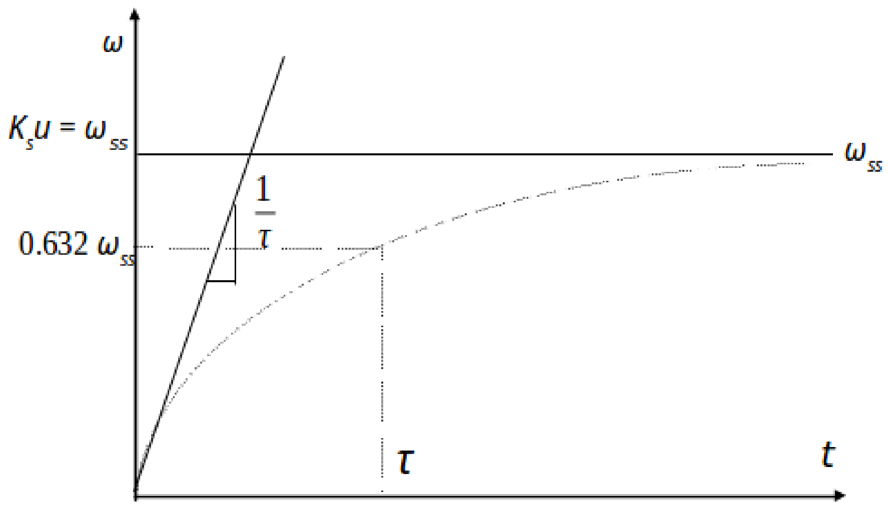

Figure 4: step response of the DC servo motor

2.2 Closed-Loop Control for a DC servomechanism

The block diagram of Fig. 1 for the servomechanism can be simplified into the block diagram given in Fig. 5.

This configuration is referred to as a negative feedback closed-loop control system where $G(s)$ is the forward loop transfer function and H(s) is the feedback loop transfer function. The equivalent transfer function between the input r(t) and the output c(t) as shown in Fig. 4 can be represented as the transfer function $G'(s)$ shown in Fig. 6.

$$G'(s) = \frac{Y(s)}{U(s)} = \frac{G(s)}{1 + G(s)H(s)} \tag{12}$$

Considering the servomechanism transfer function:
From Eq. 9: $G(s) = \frac{K_s}{s(\tau s+1)}$,
H(s) = K_1 (Output feedback controller gain),

$$G'(s) = \frac{\frac{K_s}{\tau}}{s^2 + \frac{s}{\tau} + \frac{K_1 K_s}{\tau}} \tag{13}$$

We observe that the transfer function of the closed-loop system, $G'(s)$, is a second order system with one free controller parameter K_1. Changing

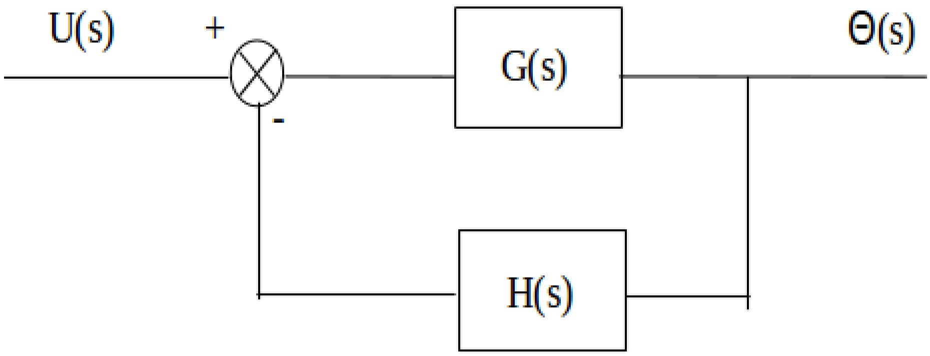

Figure 5: Simplified block diagram

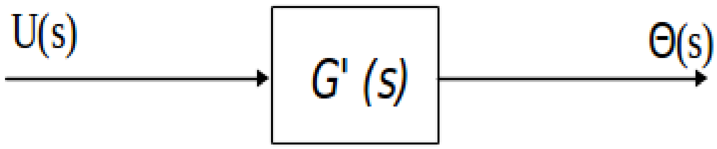

Figure 6: Equivalent closed-loop transfer function

the gain K_1 can be used to alter the transient response dynamics of the closed-loop second order system.

3 EXPERIMENTAL PROCEDURES:

3.1 Model Identification:

1. Run **MATLAB** and open the model **Lab2_Identification.slx**. The *DC servo* block contains a masked model of a DC servo motor. Run the model. A unit step input voltage is applied to the motor winding at t = 1s. Open the scope and look at the angular velocity trace. Determine the steady-state speed of the motor. This is can be used to determine the value of K_s, the steady-state gain. Determine the time at which the speed reaches 63.2% of its final value. Find the

elapsed time since the step function was applied. This value is the time constant τ.

2. Open the model **Lab2_Comparison.slx**. Update the servo transfer function block to match the parameters you determined above. Compare the behavior of your model transfer function to that of the DC Servo block. See if you can more closely match the DC Servo block by making small changes in your transfer function parameters.

3.2 Transient Response:

In this part the closed loop servomechanism system for position control is being implemented as explained in section 2.2. The main goal is to change the controller gain K_1, observe the characteristics of the transient response and relate these changes to the parameters of the closed-loop system and the parameters that define the transient response. The following objectives should be met:

1. Understanding the relationship between the system parameters: damping ratio (ζ) and natural frequency (ω_n), the transient response parameters: %overshoot (M_p), settling time ($T_{settling}$), peak time (T_p), rise time (T_r), and the closed-loop poles, with the gain K_1.

2. Understanding the qualitative effect of the controller gain K_1 on the transient response of the servomechanism.

3. Obtaining a basic understanding of closed-loop proportional control for the DC-servo system.

Procedure:

1. Open the model **Lab2_Transient.slx** and run it. Save the theta trajectories associated with each of the different values of K_1. You will need this data for an exercise.

4 EXERCISES:

1. Fill in the following table for each case.

Gain	Poles	%overshoot	ζ	ω_n	$T_{settling}$	T_{peak}	T_r

Note

2. Using the table from exercise 1, plot the following:

    (a) Poles in the s-plane for all values of the gain constant K_1.

    (b) The damping ratio vs. K_1.

    (c) % overshoot vs. ζ.

    (d) Settling time, $T_{settling}$ vs. ζ.

3. Using the graphs from exercise 2, explain (qualitatively) the effect of changing K_1 on:

    (a) Poles of the closed-loop system

    (b) Overshoot of the closed-loop system

    (c) Settling time of the closed-loop system