Improving some bounds and whatnot.

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Abstract

Part I

Introduction

1 Introduction

Let Γ be a finite group with a subset S. The *Cayley digraph*, denoted Cay(Γ,A), is a digraph with vertex set Γ, such that (x,y) is a directed edge if and only if $yx^{-1} \in$ A. In this paper we will be working with $\mathbb{Z}_m$ as our vertex set, and will denote these Cayley graphs as Cay(m,A).

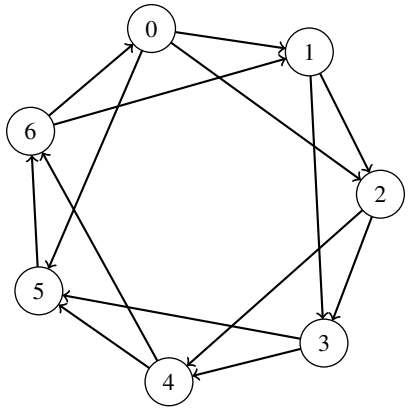

Fig. 1: Cay($\mathbb{Z}_7$, {1,2}).

For any positive integer d we define

$$\text{m(d,A)} = \max\{\text{m} \mid \text{d(m,A)} \leq \text{d}\},$$

the largest positive integer m such that the diameter, d(m,A), of the Cayley digraph Cay(m,A) is less than or equal to d. For positive integers d and k,

$$\text{m(d,k)} = \max\{\text{m(d,A)} \mid \text{there exists a set A with } |\text{A}| = \text{k }\},$$

the maximum modulus m such that there exists a generating set with cardinality equal to k and the diameter of the Cayley digraph is less than or equal to d.

Current known bounds include

$$\text{m(1,k)} = \text{k+1},$$

$$\text{m(d,1)} = \text{d+1, and}$$

$$\text{m(d,2)} = \lfloor \tfrac{d(d+4)}{3} \rfloor + 1 \text{ for d} \geq 2.$$

In this paper we will examine the case when k = 3. A current lower bound for this case is

$$\text{m(d,3)} \geq \tfrac{176}{2197} d^3 + \text{O}(d^3).$$

Part II

Results

2 What else?