Proposition 0.1. *(Opt-out buyer strategy) Define, for any allocation a,*

$$e_i^j(a) \triangleq \frac{a_i^j}{\varsigma_i^j}, \tag{1}$$

and define,

$$\ell_i = \underset{\mathcal{I}' \subset \mathcal{I}, |\mathcal{I}'|=n}{\arg\max} \sum_{j \in \mathcal{I}'} d^j.$$

Buyer i chooses its seller pool by determining n, where

$$n = \arg\min_{\ell_i}(j \in \ell_i : \sum_n \frac{d^n}{d_i} = d_i\}, \tag{2}$$

which produces the minimal subset

$$\mathcal{I}_i = \{j \in \mathcal{I} : j < n\} \subset \mathcal{I}. \tag{3}$$

Now let $j^* = n < I$, and define,

$$e_i(a) \triangleq e_i^{j^*}(a). \tag{4}$$

Then, we have that e_i is an optimal feasible strategy for buyer i.

Proof: In the case that there exists a seller who can completely satisfy a buyers' demand, $j^* = 1$, $|\mathcal{I}_i| = 1$ and (6) holds. If such a buyer does not exist, as the set ℓ_i is an ordered set, i may discover j^* by computing ℓ_i. In the case that $d_i > \sum_{j \in \mathcal{I}} d^j / d_i$, then $j^* > I$ and we consider the buyers' demand infeasible (CAN DO BETTER!). We also note that $\mathcal{I}_i$ is not the only possible minimum subset $\in \mathcal{I}$ able to satisfy i's demand, it is the minimal subset where a coordinated bid is possible. We will show optimality through the analysis in Section 5.

Lemma 0.1. *(Opt-out buyer coordination) Let $i \in \mathcal{I}$ be a opt-out buyer. For any profile $s_i = (d_i, p_i)$, let $a_i \equiv \sum_j a_i^j(s)$ be the resulting data tranfer. For a fixed s_{-i}, a better reply for i is $x_i = (z_i, p_i)$, where $\mathcal{I}_i$ is computed as in the buyer strategy,*

$$z_i^j = e_i(a)$$

and

$$a_i^j(z_i, p_i) = z_i^{j^*}, \tag{5}$$

Proof: As s_{-i} is fixed, we omit it, in addition, we will use $u \equiv u_i \equiv u_i(s_i) \equiv u_i(s_i; s_{-i})$. We have, by (5) and (27), $\forall\ j \in \mathcal{I}_i$,

$$\begin{aligned} z_i^{j^*} = z_i^j = e_i(a) &= e_i^{j^*}(a) \\ &\leq \left[d^{j^*} - \sum_{p_k^j > y} d_k^{j^*} \right]^+ / \varsigma_i^{j^*}, \end{aligned}$$

and so

$$a_i^j(z_i, p_i) = a_i^{j^*}(z_i, p_i) = z_i^{j^*} = e_i(a).$$

In order to determine that i has no loss of utility, we will show that

$$u_i(d_i, p_i) \leq u_i(z_i, p_i).$$

We address two cases:
1. *There exists a seller who can fully satisfy i's demand.*
In this case, $|\mathcal{I}_i| = 1$, and the case is trivial as no coordination is necessary for a single bid.
2. *Buyer i's demand can only be satisfied by a minimal subset of sellers.*
Buyer i maintains ordered set ℓ_i where the sellers with the largest bid are considered first, the seller j^* defines the minimal subset $\mathcal{I}_i$ where a coordinated bid is possible. From (29) and (31), we have that, $\forall\ j \in \mathcal{I}_i$,

$$e_i^j(z_i, p_i) = \left[d^j - \sum_{p_k^j > y} d_k^j \right]^+ / \varsigma_i^j$$

We have $e_i^j(a(z_i, p_i)) = a_i^{j^*}(z_i, p_i)/\varsigma_i^{j^*} = e_i(a)$, which implies that

$$\theta_i \circ e_i(a(z_i, p_i)) = \theta_i \circ e_i(a).$$

Therefore, by the definition of utility (**??**),

$$\begin{aligned}\theta_i \circ e_i(a(z_i, p_i)) - \theta_i(a))\\ = c_i^j(s) - c_i^{j^*}(z_i, p_i).\end{aligned}$$

Now using the definition of the buyers' valuation (3), we have $\forall\ i \in \mathcal{I}$,

$$\begin{aligned}u_i(z_i, p_i) - u_i\\ = \sum_{\mathcal{I}_j} \left(\theta_i(a_i^j) - \theta_i(a_i^{j^*}(z_i, p_i)) \right)\end{aligned}$$

Now, from (29), $\forall\ j \in \mathcal{I}_i$, $a_i(z_i, p_i) \leq z_i^j \leq a_i^j$ and $\theta_i \geq 0 \Rightarrow u_i(z_i, p_i) - u_i \geq 0$, $\forall\ i \in \mathcal{I}$, as is shown by the definition of the buyers' utility, (CAN USE THIS! VERY STRONG)

$$\sum_{\mathcal{I}_j} \left(\frac{\sigma(a_i^j)^{1-\alpha}}{1-\alpha} - \frac{\sigma(a_i^j(z_i, p_i))^{1-\alpha}}{1-\alpha} \right) \geq 0.$$

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