

x_5	x_4	x_3	x_2	x_1		1	1	-	1	8
Z	E	R	O	E	S	-	9	8	-	9
+		O	N	E	S	+				9
BINARY						-	0	-	-	2

Var($Z, E, R, O, S, N, B, I, A, Y$) Extn (x_1, x_2, x_3, x_4, x_5) Dom($\emptyset, 1$)
 Dom($\emptyset, 1, 2, 3, 4, 5, 6, 7, 8, 9$) (as variables are checked, the Dom will shrink)

Con(All d.s.s ($Z, E, R, O, S, N, B, I, A, Y$)) ($S+S=Y+(x_1 \cdot 10)$),
 $(E+E+x_1=R+(x_2 \cdot 10))$, $(O+N+x_2=A+(x_3 \cdot 10))$,
 $(R+O+x_3=N+(x_4 \cdot 10))$, $(E+x_4=I+(x_5 \cdot 10))$, $(Z+x_5=B)$,
 $(Z, O, B \neq \emptyset)$

$(Y(S+S=Y+(x_1 \cdot 10)))$ (integer plus itself is always even)

Dom($Y: \emptyset, 2, 4, 6, 8$)

$(x_5(Z+x_5=B, \text{All d.s.s}))$ (Z can't be equal to 0, therefore 1 is added)

$x_5=1$

$(x_4(E+x_4=I+(1 \cdot 10)))$ (If $I+(x_5 \cdot 10) \geq 10$, something is needed to get a higher number than available)

$(E(E+1=I+(1 \cdot 10)))$ (The only number in Dom that when added to 1 is greater than or equal to 10 is 9)

$E=9$

$(x_1(E+E+x_1=R+(x_2 \cdot 10), \text{All d.s.s}))$ ($9+9=18$ and $9+9+1=19$, but 19 would cause $R=9$ which breaks All d.s.s)

$x_1=0$

$(R(9+9+0=R+(x_2 \cdot 10)))$

$R=8$

$(x_2(9+9+0=8+(x_2 \cdot 10)))$

$x_2=1$

$(I(9+1=I+(1 \cdot 10)))$

$I=0$

$(Z(Z+1=B, \text{All d.s.s}))$

Dom($Z: 1, 2, 3, 4, 5, 6$)

$(S(S+S=Y+0, \text{All d.s.s}))$ ($S+S$ must be less than 10 since $x_1=0$, $Y \neq 8$, so $S+S$ can't sum to that)

Dom($S: 1, 2, 3$)

$(Y(8+S=Y+0))$

Dom($Y: 2, 4, 6$)

$(O(8+0+(0, 1)=N+(1 \cdot 10), \text{All d.s.s}))$ ($8+0+x_3$ must be greater than 10, as x_4 is 1 & $N \neq 0$, but less than 18)

Dom($O: 2, 3, 4, 5, 6, 7$)

$(N(8+0+(0, 1) \leq N+(1 \cdot 10), \text{All d.s.s}))$

Dom($N: 1, 2, 3, 4, 5, 6$)

$$0 = 2$$

$$+ N = \begin{pmatrix} 1 \\ 3 \\ 9 \end{pmatrix}, \begin{pmatrix} 3 \\ 6 \\ 7 \end{pmatrix}, \begin{pmatrix} 9 \\ 7 \\ 0 \end{pmatrix} \quad (+1)$$

$$0 = 3$$

$N = \{1, 2, 3, 4, 5, 6\}$
 $A = \{5, 6, 0, 0, 0, 1\}$

$$0 = y$$

$N = (1, 2, 3, 4, 5, 6)$
 $A = (6, 7, 8, 9, 10, 11)$

$$0 = 5$$

$$N = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 7 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}$$

$$0 = 0$$

$N = \cancel{1}, \cancel{2}, \cancel{3}, \boxed{4}, \boxed{5}, \cancel{6}$
 $H = \cancel{8}, \cancel{9}, \cancel{0}, \boxed{1}, \boxed{2}, \cancel{3}$
 x_2

$$0 = 7$$

$N = \{ \cancel{1}, \cancel{2}, 3, 4, 5, 6 \}$
 $A = \{ \cancel{1}, \cancel{2}, 1, 2, 3, 4 \}$

Checking answers of $O+N=A$
against $S+O=N$ based on X_2 value of A
 $8+2+0=10$
No valid assignment

$$8 + 3 + 0 = 11$$

$O, N, A = 3, 1, 5$ valid

$$8+4+0=12 \quad / \quad 8+4+1=13$$

$$8+4+0=12 \quad / \quad 8+4+1=13$$

0, 11, 14 = 4, 2, 2 valid

$$8+5+0=13 / 8+5+1=14$$

No valid assignment

$$8 + 6 + 1 = 15$$

$$8 + 6 + 1 = 15$$

$$8 + 7 + 1 = 16$$

$8+7+1=16$
 $0, N, A=7, 6, 9$ valid

$$\text{Dom}(O:3,4,6,7 \mid N:1,2,5,6 \mid A:2,4,5,7)$$
$$\text{Dom}(Z:1,2,3,4,5,6 \mid B:2,3,4,5,6,7)$$
$$\text{Dom}(S:1,2,3 \mid Y:2,4,6)$$

$$\begin{array}{r}
 1 \quad 1 \quad 0 \quad 1 \quad 0 \\
 6 \quad 9 \quad 8 \quad 3 \quad 9 \quad 2 \\
 + \quad \quad 3 \quad 1 \quad 9 \quad 2 \\
 \hline
 7 \quad 0 \quad 1 \quad 5 \quad 8 \quad 4
 \end{array}$$

Starting with 0 and 'checking Forward Checking'

$$0 = 3 \quad 0 + N = A \rightarrow \text{Dom}(N: 1 \mid A: 5)$$

$$\text{All d.s.s} \rightarrow \text{Dom}(S: 1, 2 \mid Y: 2, 4, 6 \mid Z: 1, 2, 4, 5, 6 \mid B: 2, 4, 5, 6, 7)$$

$$N = 1 \quad 0 + N = A \rightarrow \text{Dom}(A: 5)$$

$$\text{All d.s.s} \rightarrow \text{Dom}(S: 2 \mid Y: 2, 4, 6 \mid Z: 2, 4, 5, 6 \mid B: 2, 4, 5, 6, 7)$$

$$A = 5 \quad \text{All d.s.s} \rightarrow \text{Dom}(S: 2 \mid Y: 2, 4, 6 \mid Z: 2, 4, 6 \mid B: 2, 4, 6, 7)$$

$$S = 2 \quad S + S = Y \rightarrow \text{Dom}(Y: 4)$$

$$\text{All d.s.s} \rightarrow \text{Dom}(Z: 4, 6 \mid B: 4, 6, 7)$$

$$Y = 4 \quad \text{All d.s.s} \rightarrow \text{Dom}(Z: 6 \mid B: 6, 7)$$

$$Z = 6 \quad Z + 1 = B \rightarrow \text{Dom}(B: 7)$$

$$B = 7$$

$$Z = 6, E = 9, R = 8, O = 3, S = 2, N = 1, B = 7, I = 0, A = 5, Y = 4$$

$$698392$$

$$+ \quad 3192$$

$$\hline 701584$$