# 2.5 — Precision and Diagnostics

## ECON 480 • Econometrics • Fall 2022


Dr. Ryan Safner

Associate Professor of Economics

safner@hood.edu

ryansafner/metricsF22

metricsF22.classes.ryansafner.com

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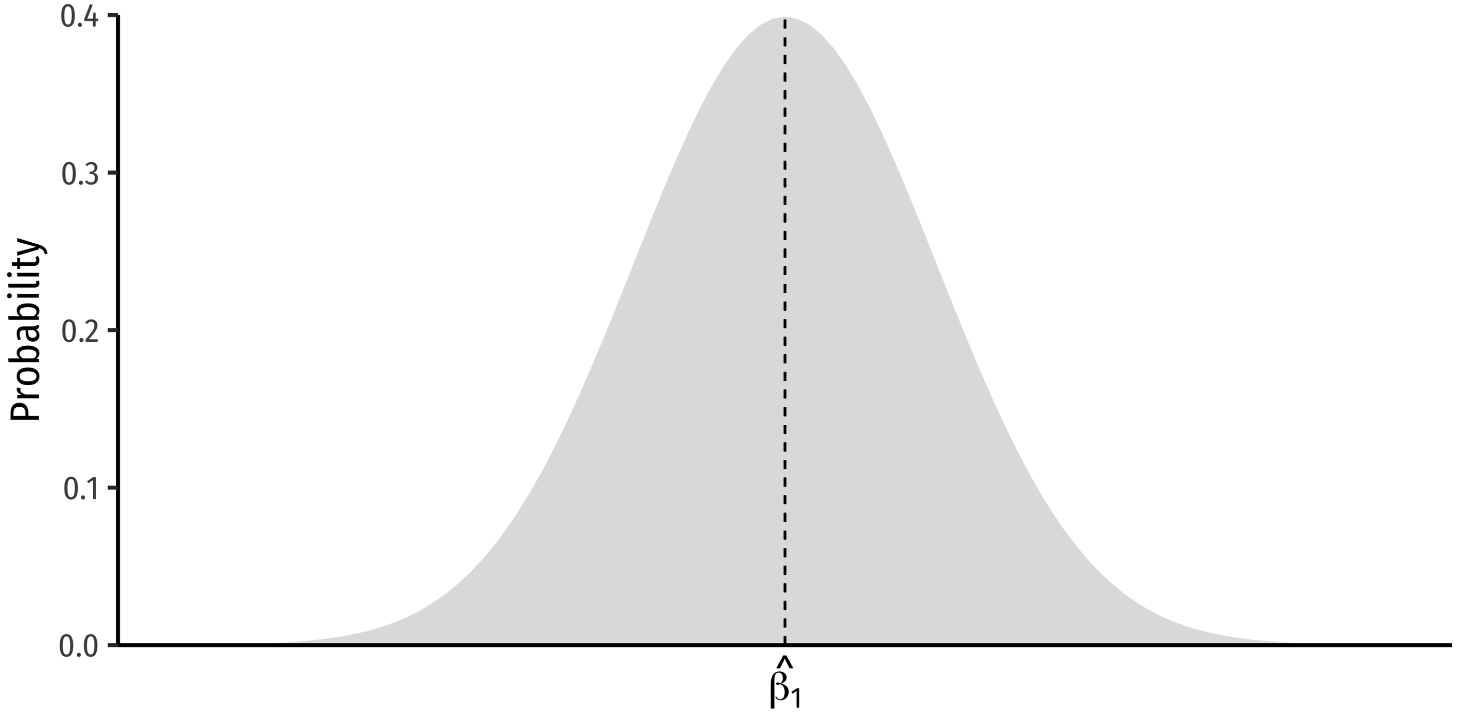
# The Sampling Distribution of $\hat{\beta}_1$

$$\hat{\beta}_1 \sim N(\mathbb{E}[\hat{\beta}_1], \sigma_{\hat{\beta}_1})$$



# The Sampling Distribution of $\hat{\beta}_1$

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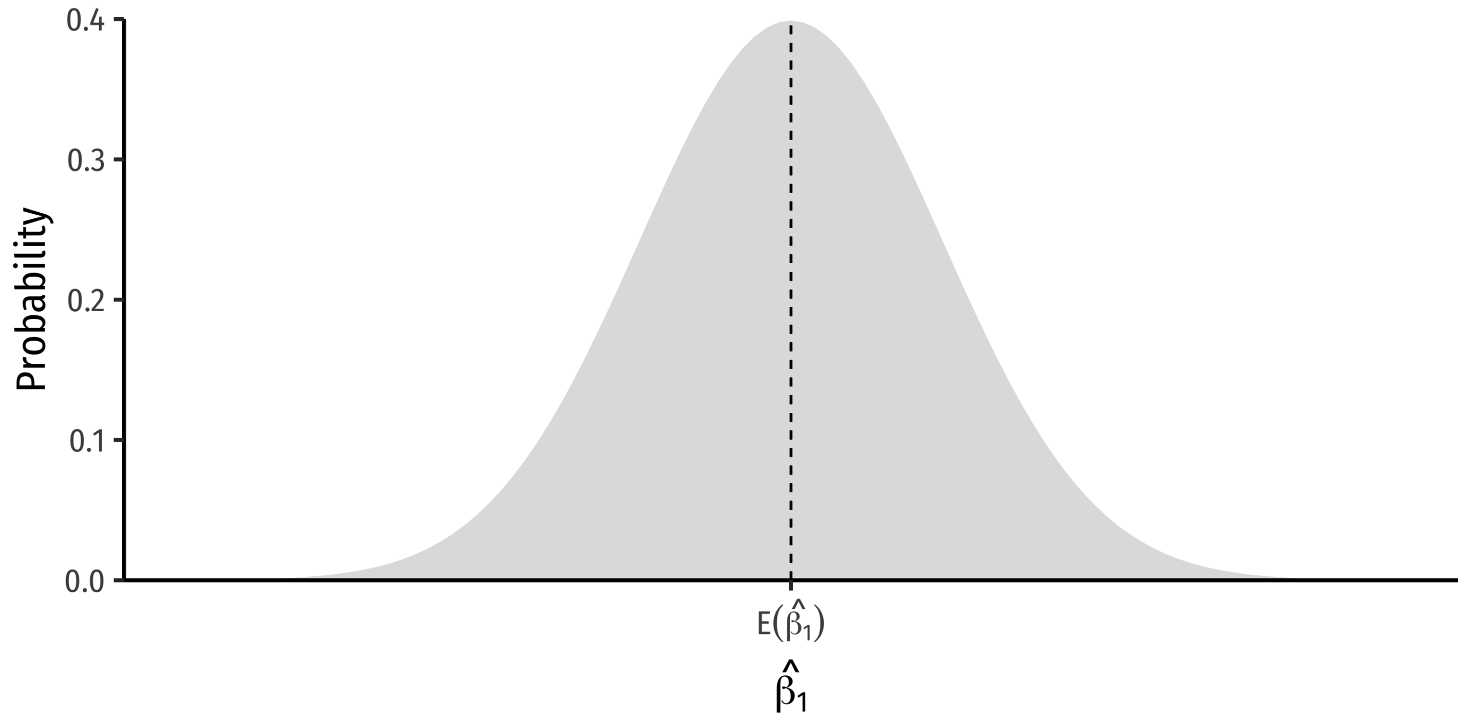




1. Under the 4 assumptions about $u$, particularly, $cor(X, u) = 0$

# The Sampling Distribution of $\hat{\beta}_1$

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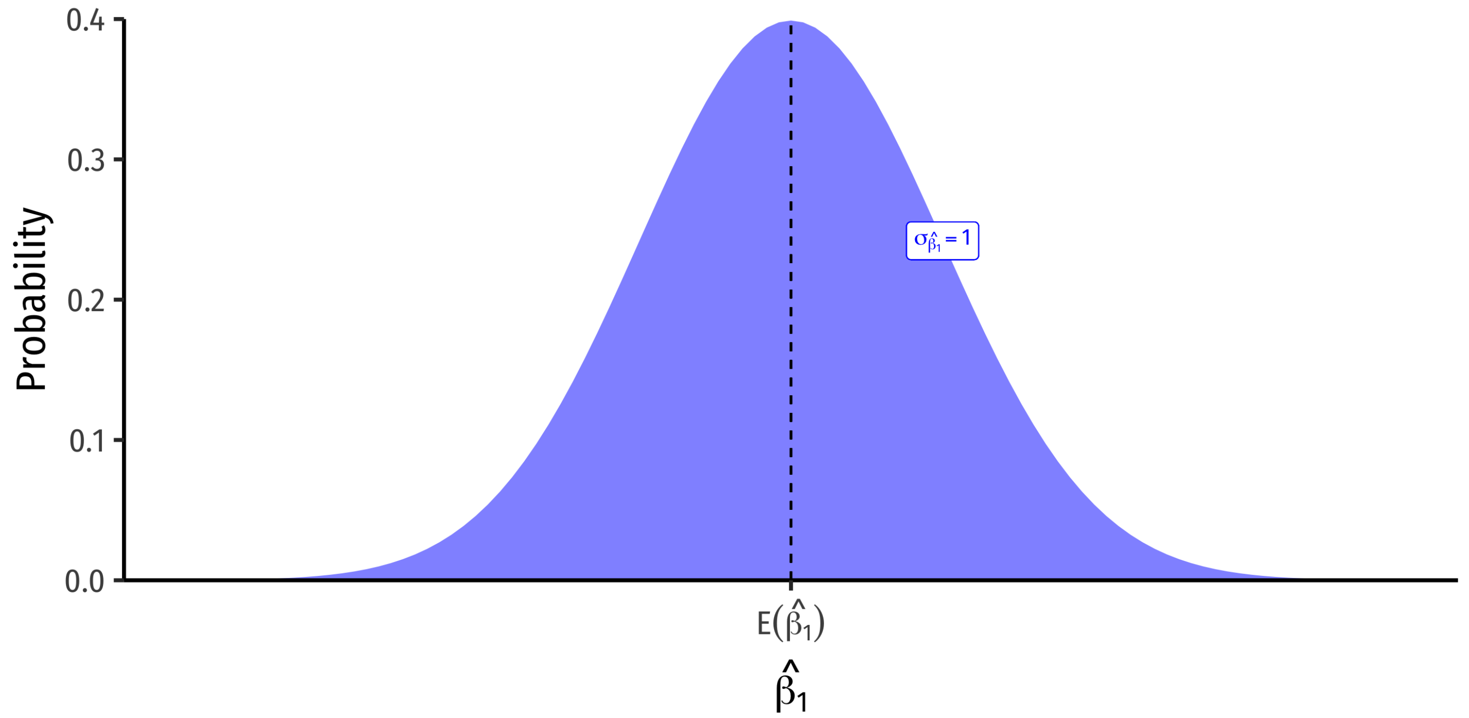
   - **Variance** $\sigma^2_{\hat{\beta}_1}$
   - **Standard error**[2] $\sigma_{\hat{\beta}_1} = \sqrt{var(\hat{\beta}_1)}$




1. Under the 4 assumptions about $u$, particularly, $cor(X, u) = 0$
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# The Sampling Distribution of $\hat{\beta}_1$

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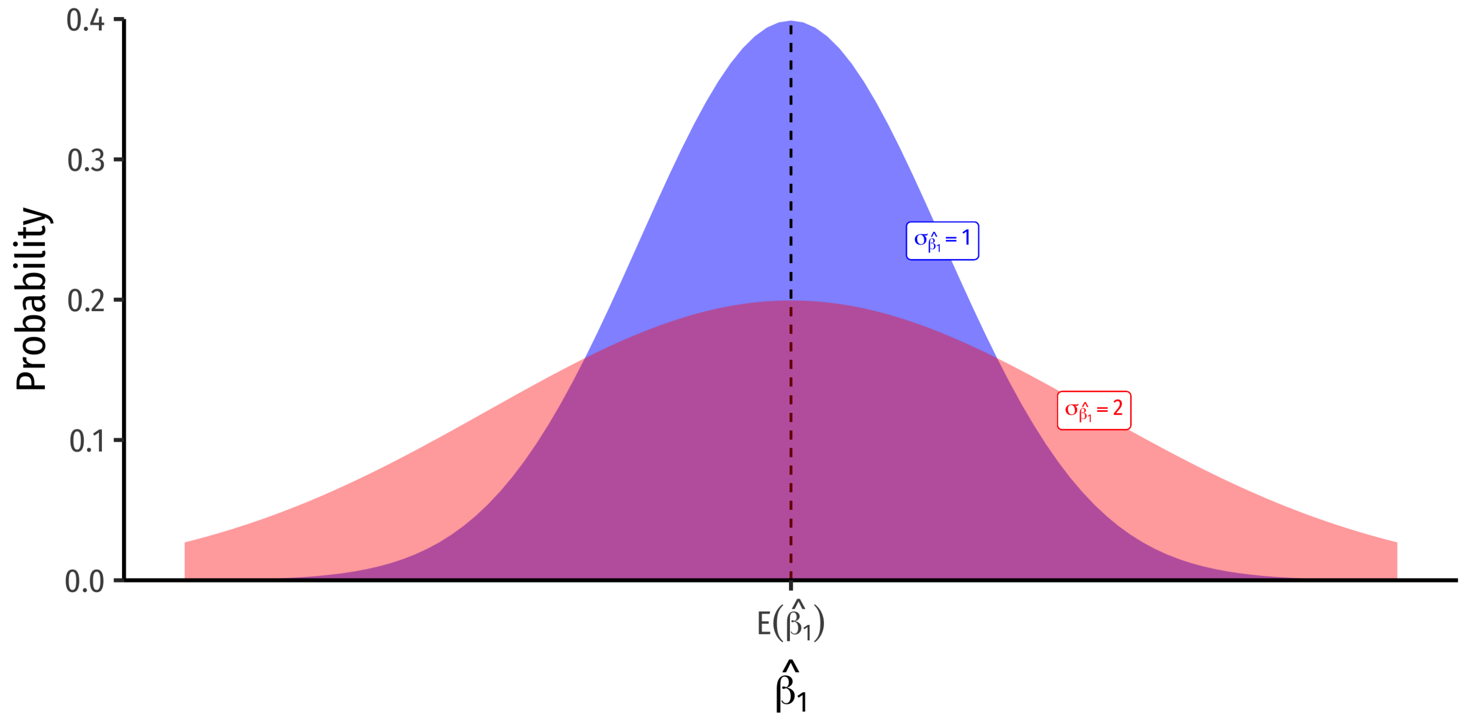



1. Under the 4 assumptions about $u$, particularly, $cor(X, u) = 0$
2. Standard **"error"** is the analog of standard *deviation* when talking about the *sampling distribution* of a sample statistic (such as $\bar{X}$ or $\hat{\beta}_1$).

# Variation in $\hat{\beta}_1$

# What Affects Variation in $\hat{\beta}_1$

$$var(\hat{\beta}_1) = \frac{(SER)^2}{n \times var(X)}$$

$$se(\hat{\beta}_1) = \sqrt{var(\hat{\beta}_1)} = \frac{SER}{\sqrt{n} \times sd(X)}$$

- Variation in $\hat{\beta}_1$ is affected by 3 things:

1. **Goodness of fit of the model (SER)**[1] ]

- Larger $SER \rightarrow$ larger $var(\hat{\beta}_1)$

2. **Sample size, n** • Larger $n \rightarrow$ smaller $var(\hat{\beta}_1)$

3. **Variance of X**

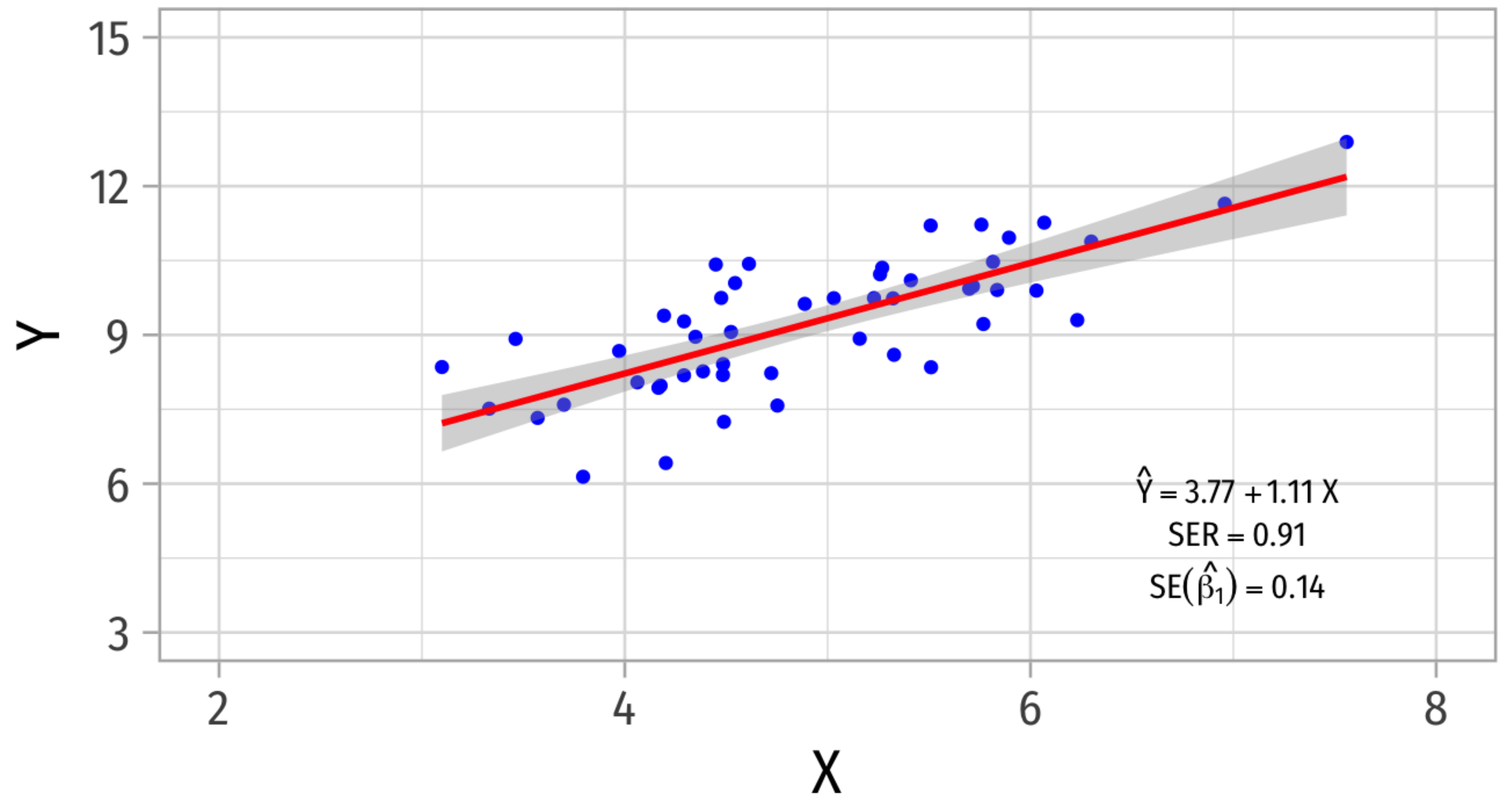
- Larger $var(X) \rightarrow$ smaller $var(\hat{\beta}_1)$

1. .magenta[†] Recall from last class, the **S**tandard **E**rror of the **R**egression $\hat{\sigma}_u = \sqrt{\frac{\sum \hat{u}_i^2}{n-2}}$

# Variation in $\hat{\beta}_1$: Goodness of Fit

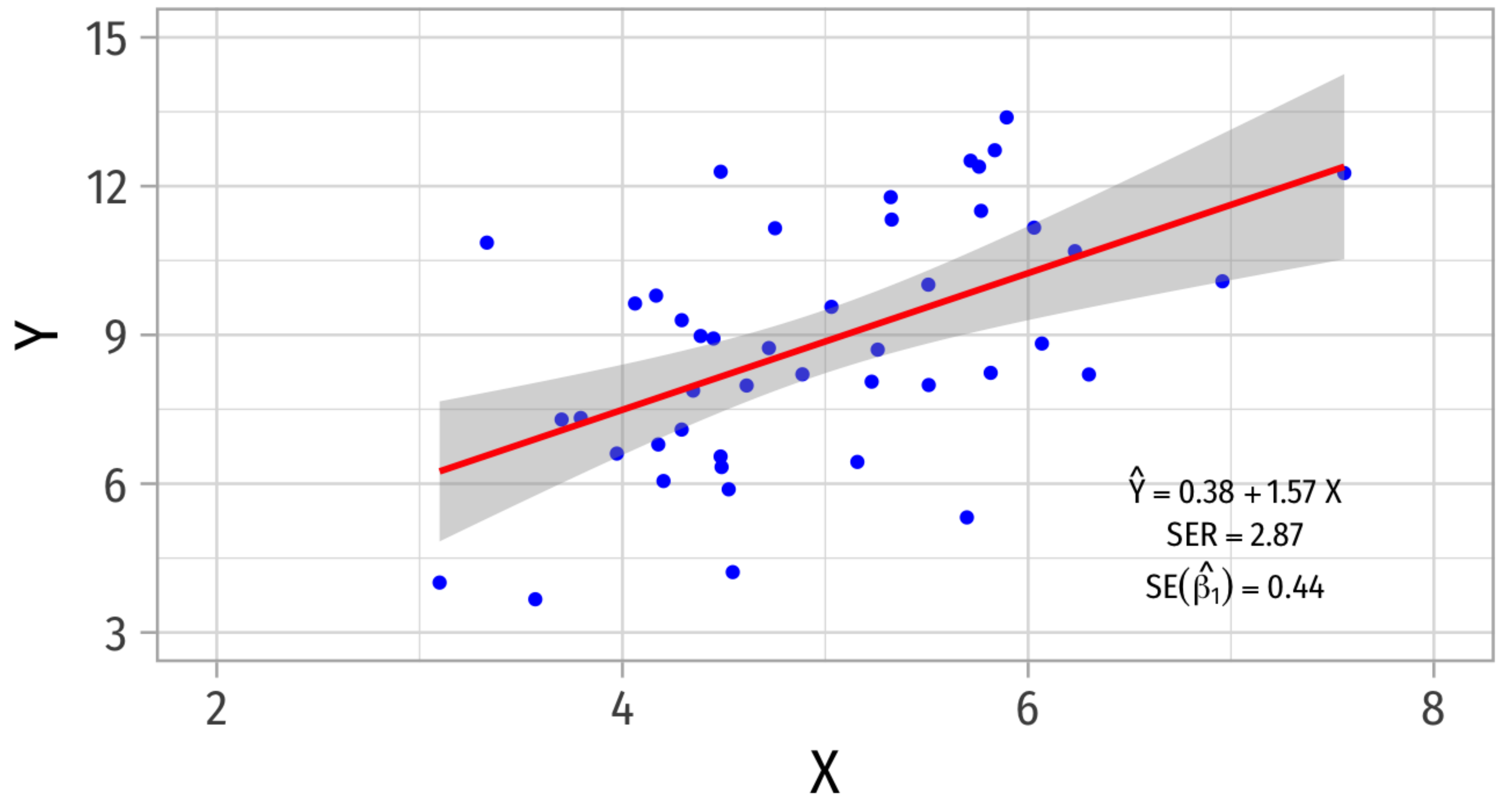
## Model With Better Fit

Lower SER lowers variation in $\hat{\beta}_1$

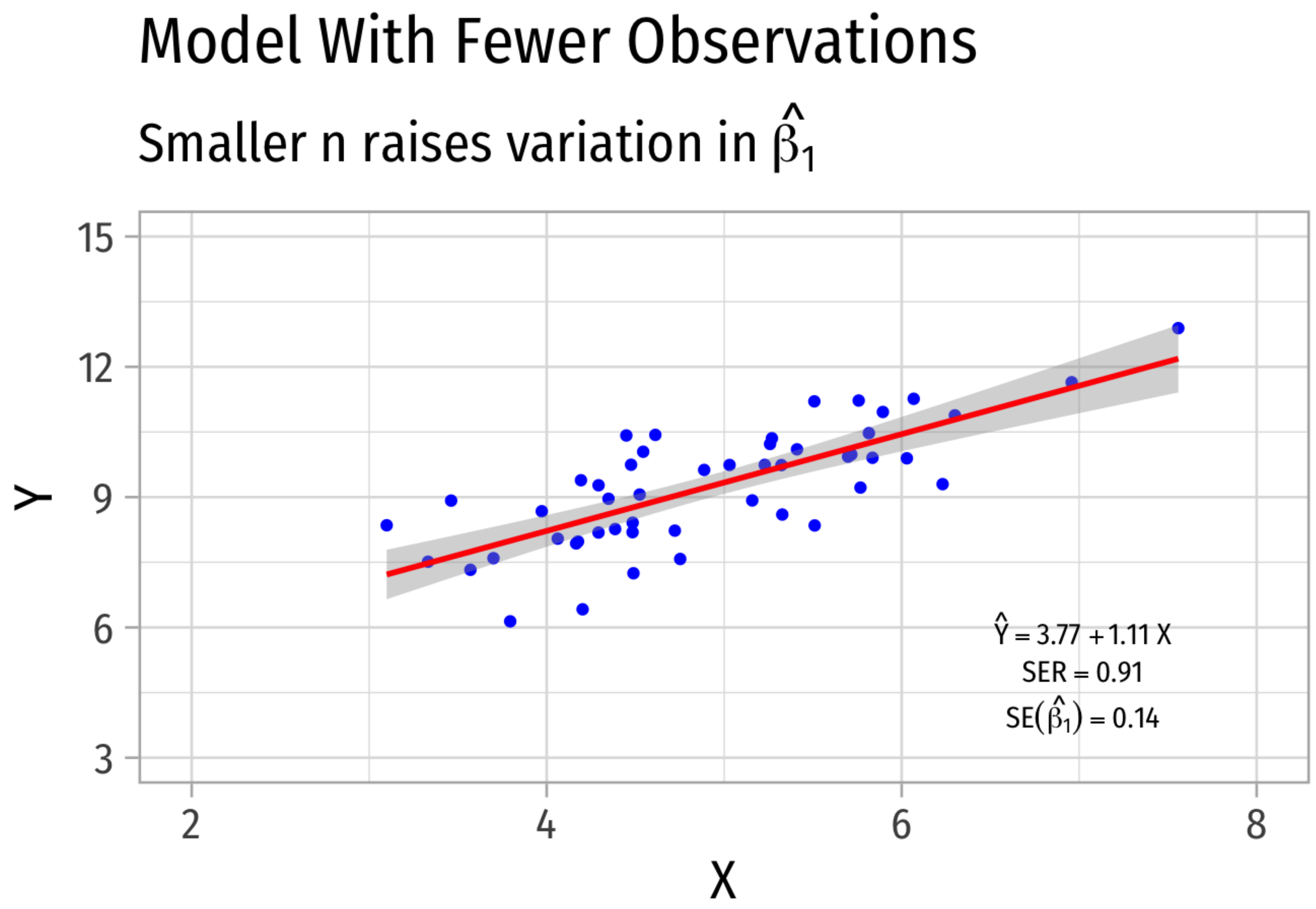


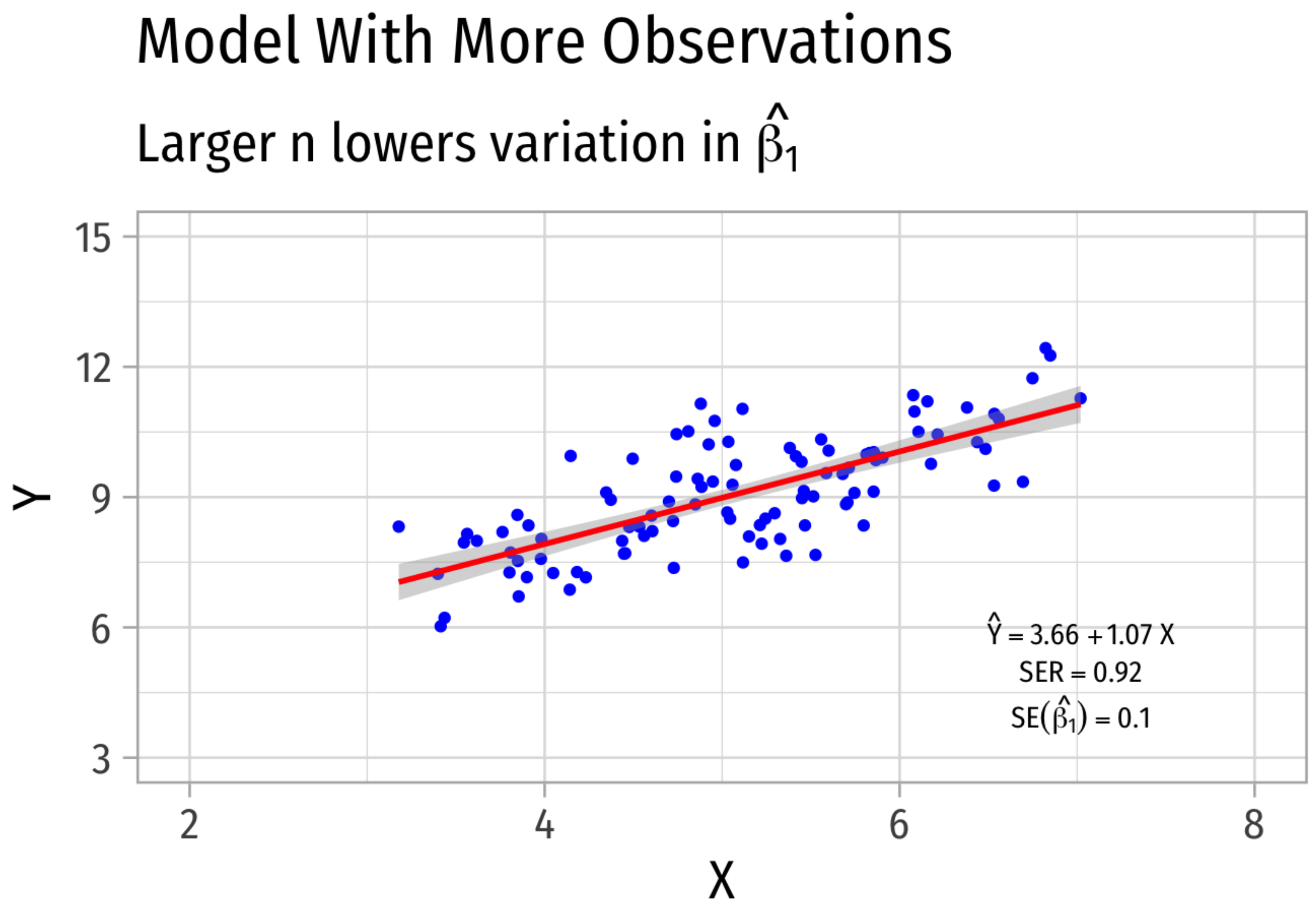

## Model With Worse Fit

Higher SER raises variation in $\hat{\beta}_1$

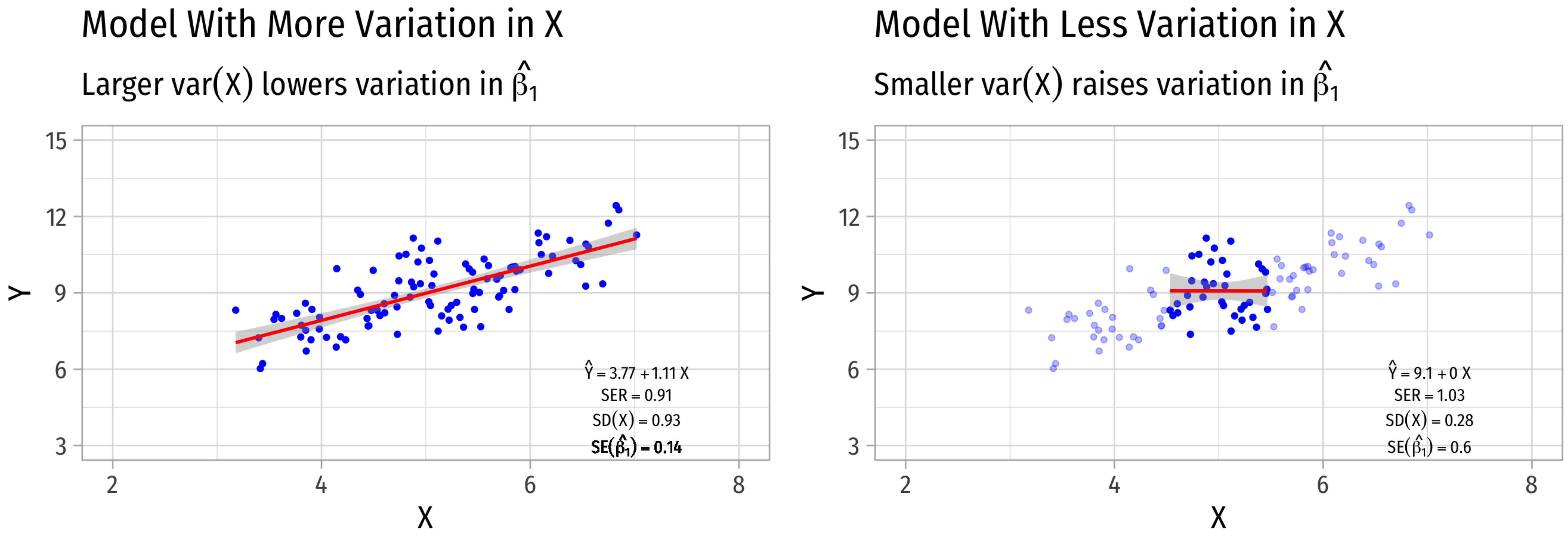


# Variation in $\hat{\beta}_1$: Sample Size



# Variation in $\hat{\beta}_1$: Variation in $X$



# Presenting Regression Results

# Our Class Size Regression

```r
school_reg %>% summary()
```

```

Call:
lm(formula = testscr ~ str, data = ca_school)

Residuals:
    Min      1Q  Median      3Q     Max 
-47.727 -14.251   0.483  12.822  48.540 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 698.9330     9.4675  73.825  < 2e-16 ***
str          -2.2798     0.4798  -4.751 2.78e-06 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

- How can we present all of this information in a tidy way?

# Our Class Size Regression

```r
library(broom)
school_reg %>% tidy()
```

| term | estimate | std.error | statistic | p.value |
|---|---|---|---|---|
| (Intercept) | 698.932952 | 9.4674914 | 73.824514 | 0.0e+00 |
| str | -2.279808 | 0.4798256 | -4.751327 | 2.8e-06 |

```r
school_reg %>% glance()
```

| r.squared | adj.r.squared | sigma | statistic | p.value | df | logLik | AIC | BIC | d |
|---|---|---|---|---|---|---|---|---|---|
| 0.0512401 | 0.0489703 | 18.58097 | 22.57511 | 2.8e-06 | 1 | -1822.25 | 3650.499 | 3662.62 | 1 |

- Better (?), but still not how you see regressions reported in reports...especially when you have many regression models!

# Regression Tables

- Professional journals and papers often have a **regression table**, including:
  - Estimates of $\hat{\beta}_0$ and $\hat{\beta}_1$
  - Standard errors of $\hat{\beta}_0$ and $\hat{\beta}_1$ (often below, in parentheses)
  - Indications of statistical significance (often with asterisks)
  - Measures of regression fit: $R^2$, $SER$, etc
- Later: multiple rows & columns for multiple variables & models

| | **Test Score** |
|---|---|
| Constant | 698.93*** |
| | (9.47) |
| STR | -2.28*** |
| | (0.48) |
| n | 420 |
| $R^2$ | 0.05 |
| SER | 18.54 |

* p < 0.1, ** p < 0.05, *** p < 0.01

# Regression Output Tables

- A number of packages (and documentation/guides) that will make nice regression output tables for you:
  - modelsummary
  - stargazer (and a good cheat sheet)
  - huxtable

| | Test Score |
|---|---|
| Constant | 698.93*** |
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| * p < 0.1, ** p < 0.05, *** p < 0.01 | |

# Using `modelsummary` I

- You will need to first `install.packages("modelsummary")`
- Load with `library(modelsummary)`
- Command: `modelsummary()`
- Main argument is the name of your `lm` regression object
- Default output is *fine*, but often we want to customize a bit!

```r
# install.packages("modelsummary") # install first
# load package
library(modelsummary)

modelsummary(school_reg) # our regression
```

| | **Model 1** |
|---|---|
| (Intercept) | 698.933 |
| | (9.467) |
| str | −2.280 |
| | (0.480) |
| Num.Obs. | 420 |
| R2 | 0.051 |
| R2 Adj. | 0.049 |
| AIC | 3650.5 |
| BIC | 3662.6 |
| F | 22.575 |
| RMSE | 18.54 |

# Using `modelsummary` II

- Whole command is `modelsummary()`, everything will go in `()`

1. `models`, a `list()` of models to use, can give a name to each model, will show up as column title in table

```r
models = list("Test Score" = school_reg) # set name to "Test Score"
```

2. `coef_rename` if you want to rename any independent variables as something nicer than their names in the dataset
   - `"old name" = "new name"` (yes annoying!)

```r
coef_rename = list("(Intercept)" = "Constant",
                   "str" = "Student Teacher Ratio")
```

# Using `modelsummary` III

- Whole command is `modelsummary()`, everything will go in `()`

3. `gof_map`: a `list()` of goodness of fit statistics, can customize what you want to include/exclude, what you want to label them in the table...a bit advanced, here's what I like:

```
gof_map = list(
  list("raw" = "nobs", "clean" = "n", "fmt" = 0),
  list("raw" = "r.squared", "clean" = "R<sup>2</sup>", "fmt" = 2),
  #list("raw" = "adj.r.squared", "clean" = "Adj. R<sup>2</sup>", "fmt" = 2), # we'll want this later!
  list("raw" = "rmse", "clean" = "SER", "fmt" = 2)
  )
```

4. Other minor options (combine with commas):

```
fmt = 2, # round to 2 decimals
output = "html" # depending on type of document creating; pdf would be "latex"
escape = FALSE # allows formatting of things like <sup>2</sup>
stars = c('*' = .1, '**' = .05, '***' = 0.01) # show significance levels if set to true, I don't like the c
```

# Using `modelsummary` IV

```r
modelsummary(models = list("Test Score" = school_reg),
             fmt = 2, # round to 2 decimals
             output = "html",
             coef_rename = c("(Intercept)" = "Constant",
                             "str" = "STR"),
             gof_map = list(
               list("raw" = "nobs", "clean" = "n", "fmt" = 0)
               list("raw" = "r.squared", "clean" = "R<sup>2</
               #list("raw" = "adj.r.squared", "clean" = "Adj.
               list("raw" = "rmse", "clean" = "SER", "fmt" =
             ),
             escape = FALSE,
             stars = c('*' = .1, '**' = .05, '***' = 0.01)
)
```

| | Test Score |
|---|---|
| Constant | 698.93*** |
| | (9.47) |
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| n | 420 |
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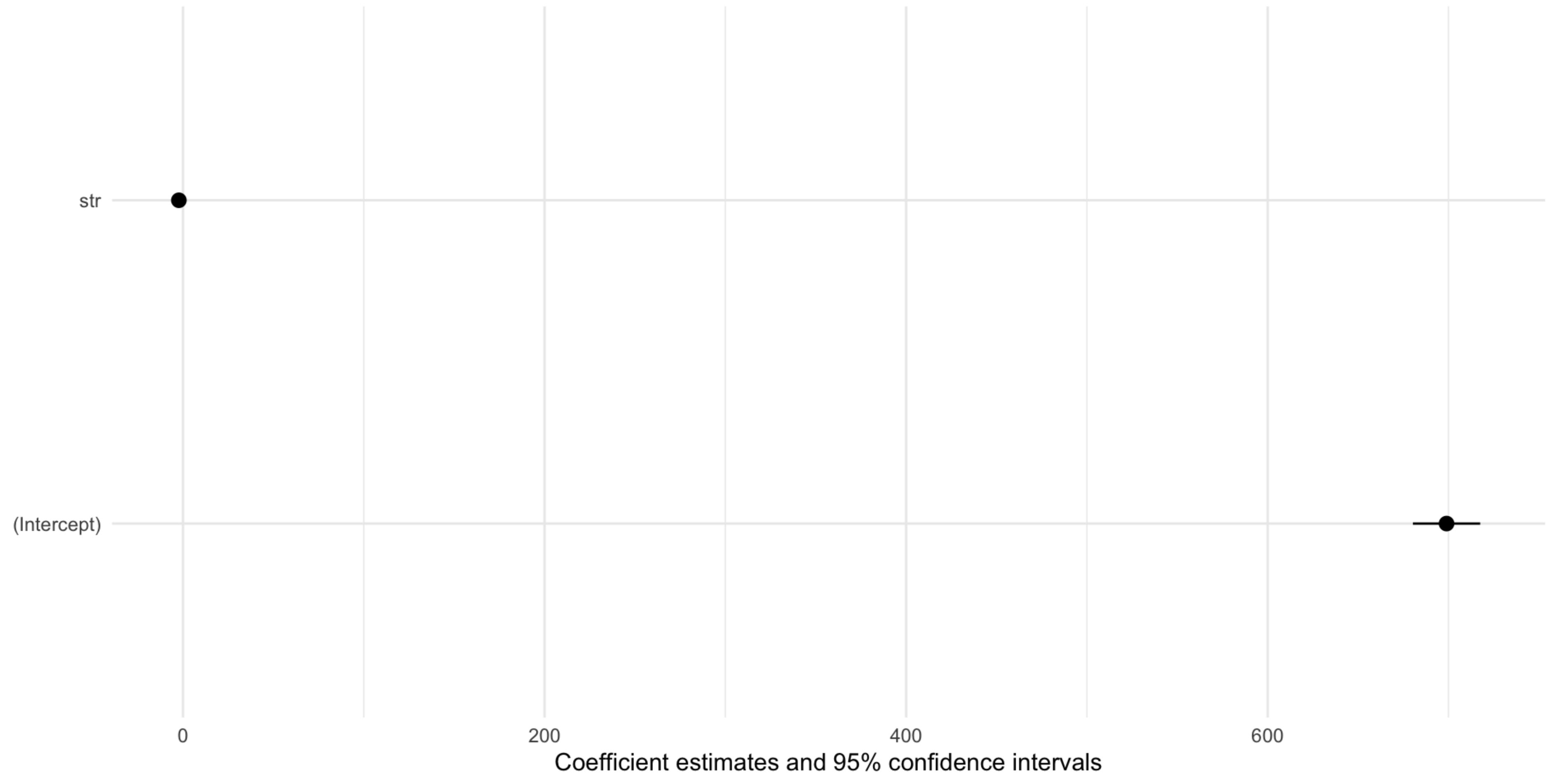
* p < 0.1, ** p < 0.05, *** p < 0.01

# `modelplot()` in `modelsummary`

Also nice about the `modelsummary` package is the command `modelplot()`

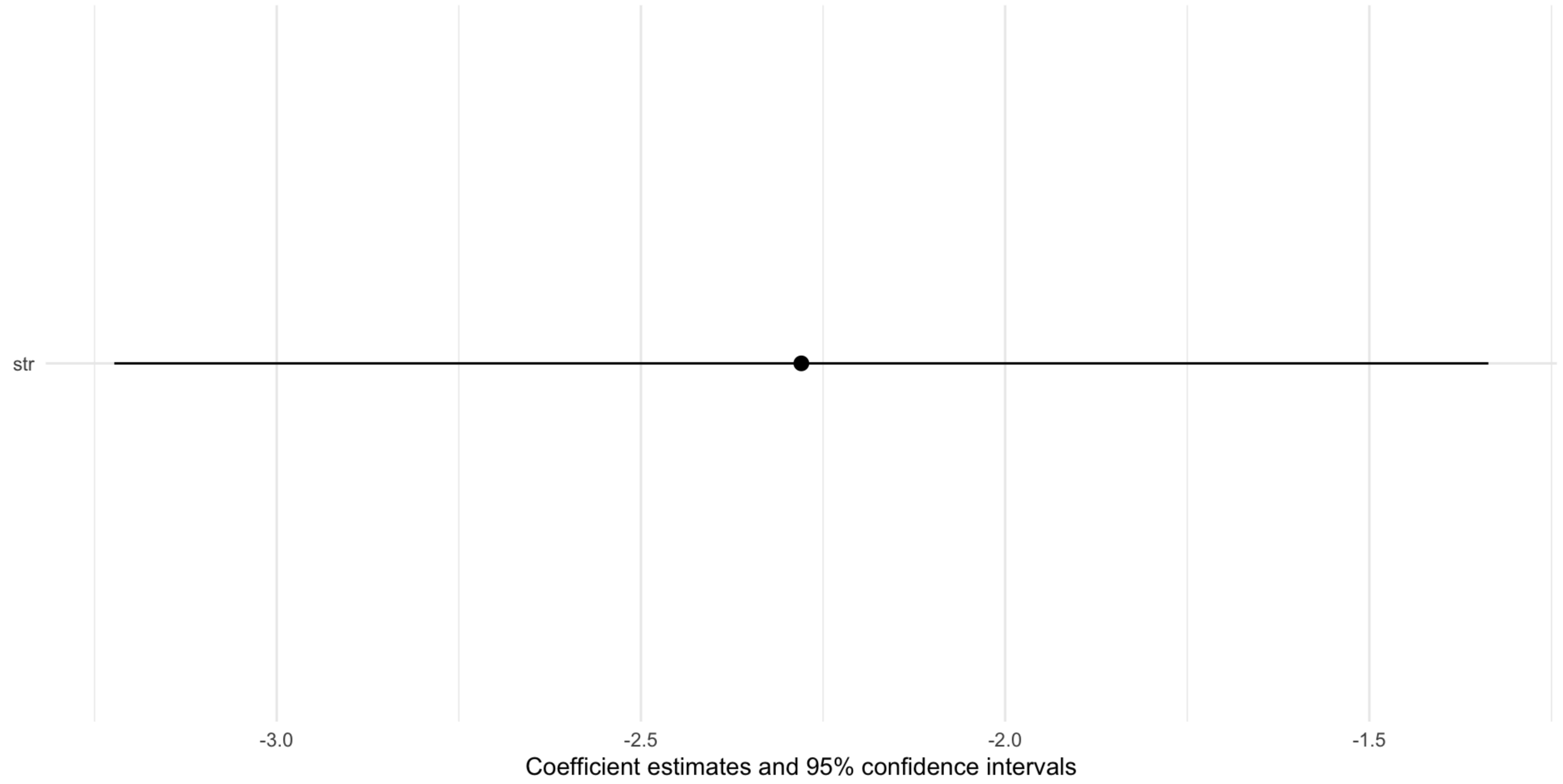
```
modelplot(school_reg)
```



# modelplot() in modelsummary

Also nice about the modelsummary package is the command modelplot()

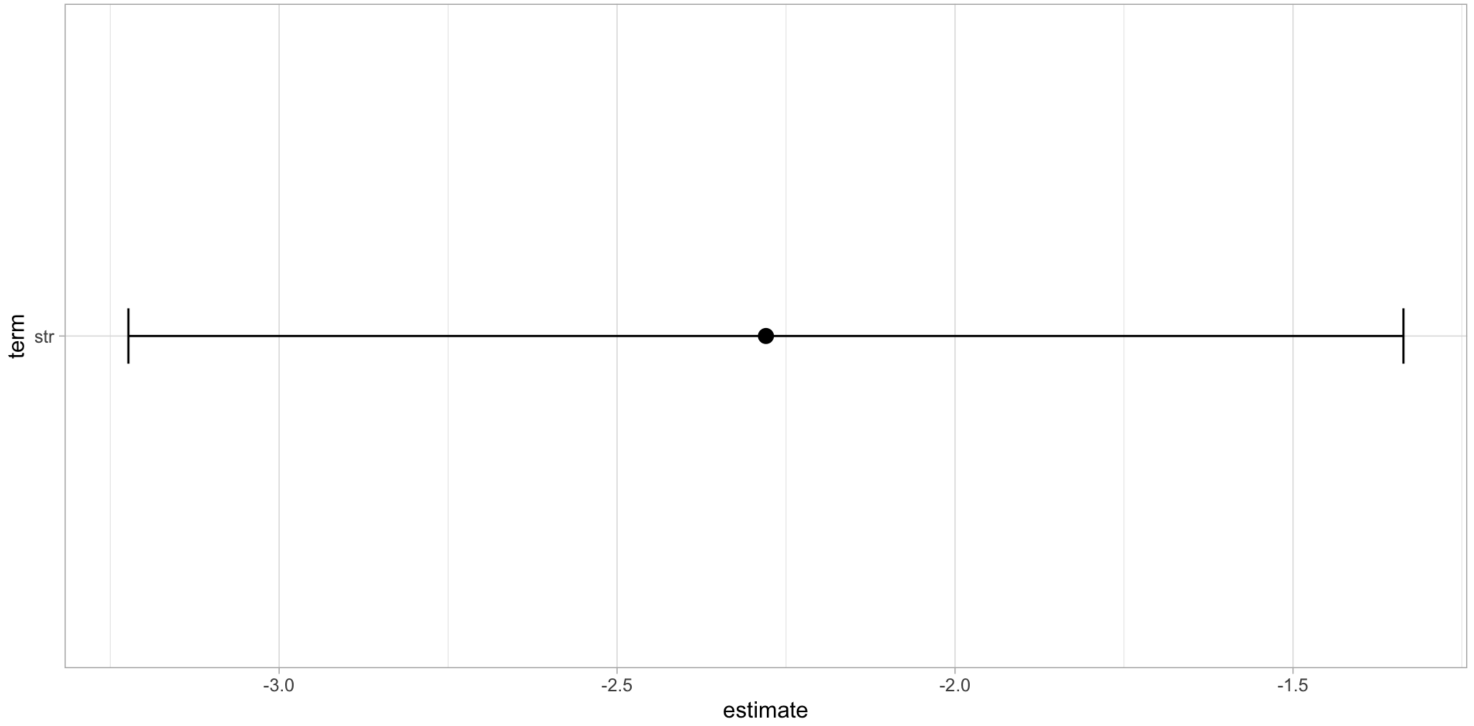
```
modelplot(school_reg,
          coef_omit = 'Intercept') # don't show intercept
```



# Though You Could Make It Yourself in `ggplot`

- Use the `conf.low` and `conf.high` (from a `tidy` regression) as `xmin` and `xmax` `aes`thetics inside `geom_errorbarh()`.

```
school_reg %>%
  tidy(conf.int = TRUE) %>%
  filter(term == "str") %>%
  ggplot()+
  aes(x = estimate,
      y = term)+
  geom_point(size = 3)+
  geom_errorbarh(aes(xmin = conf.low, xmax = conf.
                 height = 0.1)+ # height of whiske
  theme_light()
```



# Diagnostics About Regression

# Diagnostics: Residuals I

- We often look at the residuals of a regression to get more insight about its **goodness of fit** and its **bias**
- Recall `broom`'s `augment` creates some useful new variables
  - `.fitted` are fitted (predicted) values from model, i.e. $\hat{Y}_i$
  - `.resid` are residuals (errors) from model, i.e. $\hat{u}_i$

# Diagnostics: Residuals II

- Often a good idea to store in a new object (so we can make some plots)

```r
aug_reg <- augment(school_reg)

aug_reg %>% head()
```

| testscr | str | .fitted | .resid | .hat | .sigma | .cooksd | .std.resid |
|---:|---:|---:|---:|---:|---:|---:|---:|
| 690.80 | 17.88991 | 658.1474 | 32.65260 | 0.0044244 | 18.53408 | 0.0068925 | 1.7612148 |
| 661.20 | 21.52466 | 649.8608 | 11.33917 | 0.0047485 | 18.59490 | 0.0008927 | 0.6117112 |
| 643.60 | 18.69723 | 656.3069 | -12.70689 | 0.0029742 | 18.59279 | 0.0006996 | -0.6848850 |
| 647.70 | 17.35714 | 659.3620 | -11.66198 | 0.0058575 | 18.59441 | 0.0011673 | -0.6294767 |
| 640.85 | 18.67133 | 656.3659 | -15.51592 | 0.0030072 | 18.58766 | 0.0010548 | -0.8363024 |
| 605.55 | 21.40625 | 650.1308 | -44.58076 | 0.0044603 | 18.47411 | 0.0129531 | -2.4046387 |

# Recall: Assumptions about Errors

- We make **4 critical assumptions about $u$**:

1. The expected value of the errors is 0

$$\mathbb{E}[u] = 0$$

2. The variance of the errors over $X$ is constant:

$$var(u|X) = \sigma_u^2$$

3. Errors are not correlated across observations:

$$cor(u_i, u_j) = 0 \quad \forall i \neq j$$

4. There is no correlation between $X$ and the error term:

$$cor(X, u) = 0 \text{ or } E[u|X] = 0$$

# Assumptions 1 and 2: Errors are i.i.d.

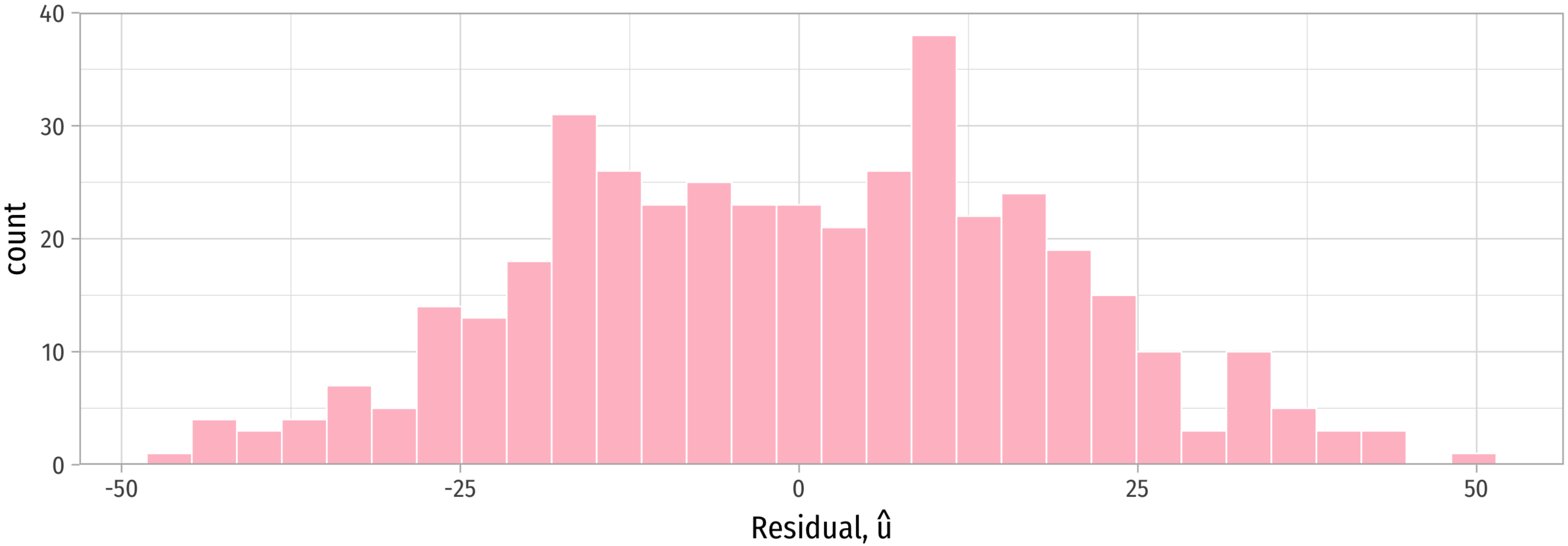
- Assumptions 1 and 2 assume that errors are coming from the same (*normal*) distribution

$$u \sim N(0, \sigma_u)$$

- Assumption 1: $E[u] = 0$
- Assumption 2: $sd(u|X) = \sigma_u$
  - virtually always unknown...
- We often can visually check by plotting a **histogram** of $u$

# Plotting a Histogram of Residuals

Plot | Code



# Checking the Distribution of Residuals

```r
school_reg %>% summary()
```

```

Call:
lm(formula = testscr ~ str, data = ca_school)

Residuals:
    Min      1Q  Median      3Q     Max 
-47.727 -14.251   0.483  12.822  48.540 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 698.9330     9.4675  73.825  < 2e-16 ***
str          -2.2798     0.4798  -4.751 2.78e-06 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
aug_reg %>%
  summarize(E_u = mean(.resid),
            sd_u = sd(.resid))
```

| E_u | sd_u |
|---:|---:|
| 0 | 18.55878 |

# Residual Plot

- We often plot a **residual plot** to see any odd patterns about residuals
  - $x$-axis are $\hat{Y}_i$ values (`.fitted`)
  - $y$-axis are $u_i$ values (`.resid`)

Plot | Code

# Heteroskedasticity

# Homoskedasticity

- "**Homoskedasticity**:" variance of the residuals over $X$ is constant, written:

$$var(u|X) = \sigma_u^2$$

- Knowing the value of $X$ does not affect the variance (spread) of the errors

# Heteroskedasticity I

- "**Heteroskedasticity**:" variance of the residuals over $X$ is **NOT** constant:

$$var(u|X) \neq \sigma_u^2$$

- **This does not cause $\hat{\beta_1}$ to be biased**, but it does cause the standard error of $\hat{\beta_1}$ to be incorrect
- This **does** cause a problem for **inference**!
  - Specifically, it will make $se(\hat{\beta_1})$ wrong (often too small)[1]



1. Later, when we learn about hypothesis testing, having standard errors incorrectly too small will *overstate* the significance of the finding

# Heteroskedasticity II

- Recall the formula for the standard error of $\hat{\beta}_1$:

$$se(\hat{\beta}_1) = \sqrt{var(\hat{\beta}_1)} = \frac{SER}{\sqrt{n} \times sd(X)}$$

- This *assumes* homoskedasticity (Assumption 2)

# Heteroskedasticity III

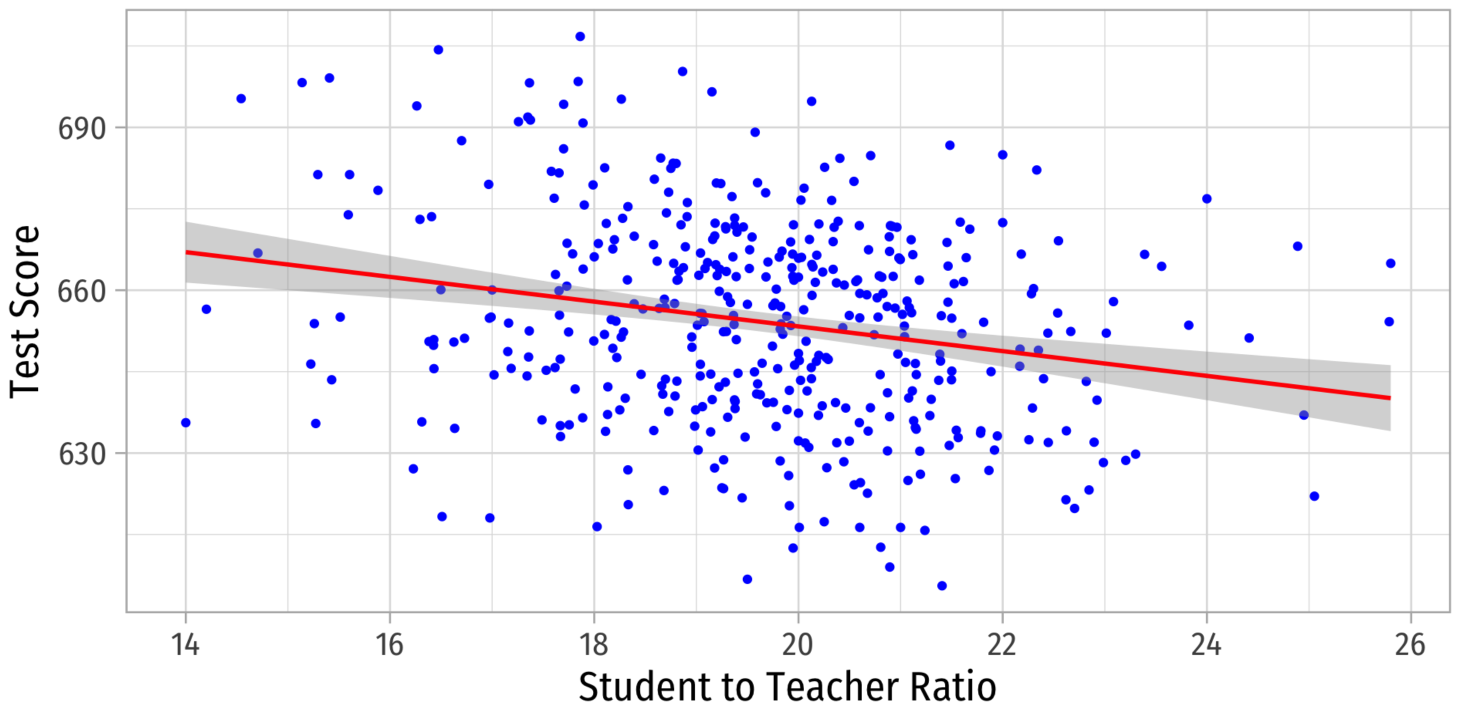
- A better formula for estimating standard errors that are **robust** to heteroskedasticity (called .hi["robust standard errors"]):

$$se(\hat{\beta}_1) = \sqrt{\frac{\displaystyle\sum_{i=1}^{n}(X_i - \bar{X})^2\hat{u}^2}{\left[\displaystyle\sum_{i=1}^{n}(X_i - \bar{X})^2\right]^2}}$$

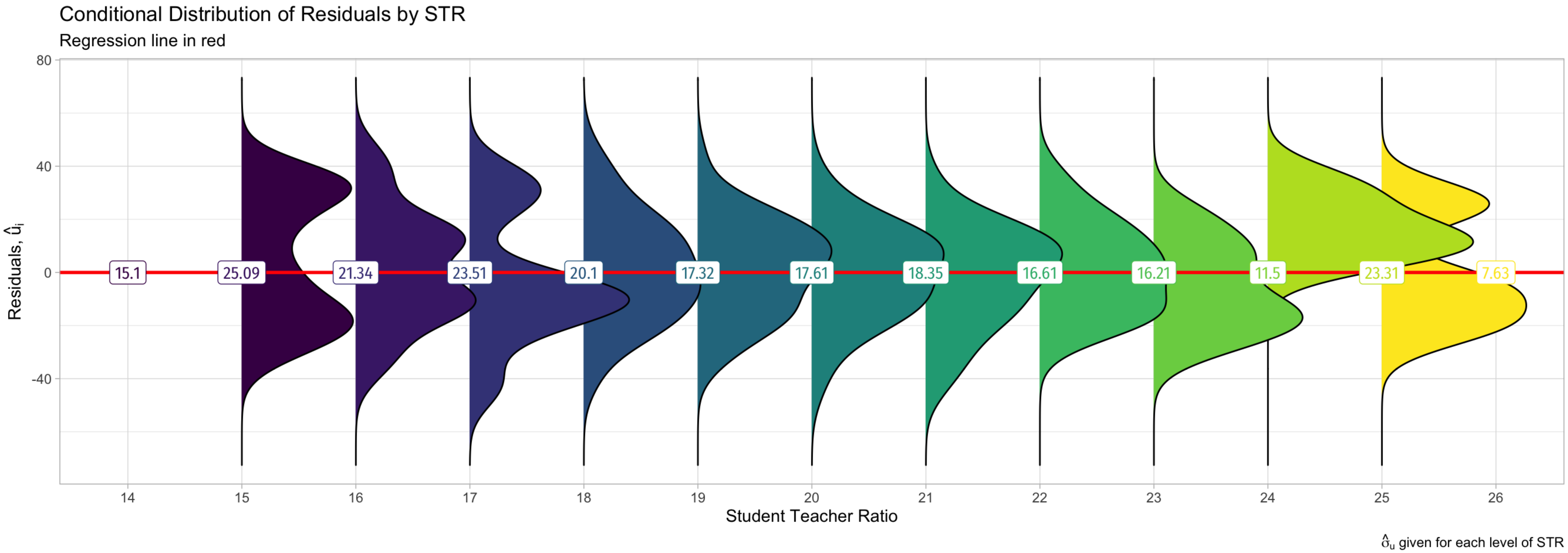
- Don't learn formula, **do learn what heteroskedasticity is and how it affects our model!**

# Visualizing Heteroskedasticity I

- Our original scatterplot with regression line
- Does the spread of the errors change over different values of $str$?
  - No: homoskedastic
  - Yes: heteroskedastic



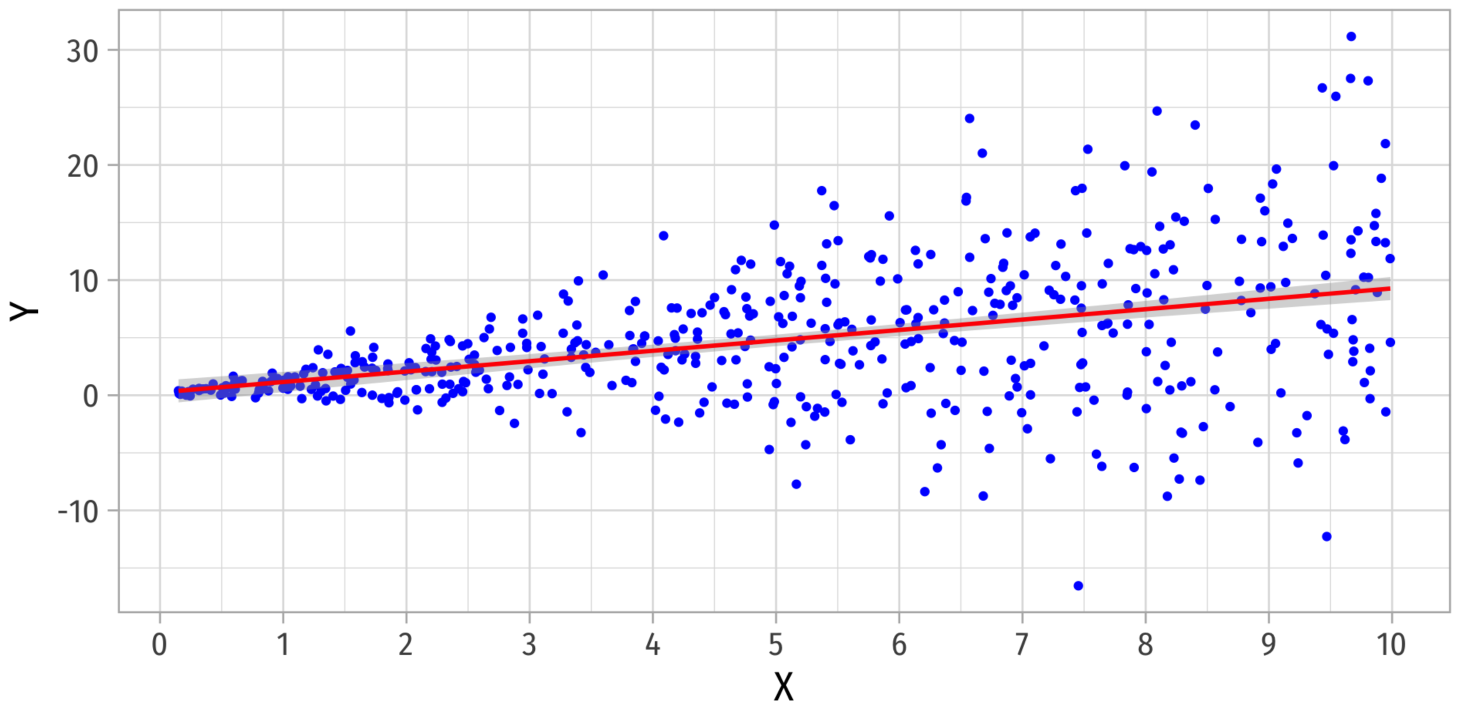
# Visualizing Heteroskedasticity



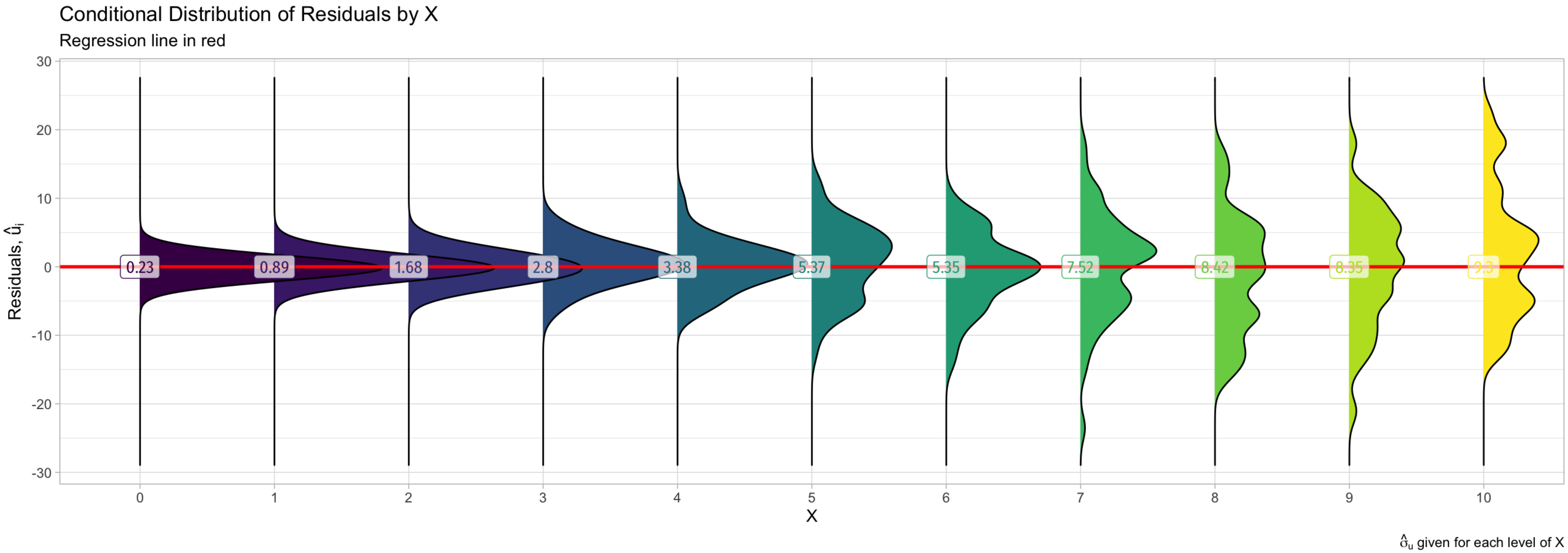

- Notice the distribution of $\hat{u}$, changes for different values of STR, and $\sigma_{\hat{u}}$ is not constant

# More Obvious Heteroskedasticity

- Visual cue: data is “fan-shaped”
  - Data points are closer to line in some areas
  - Data points are more spread from line in other areas

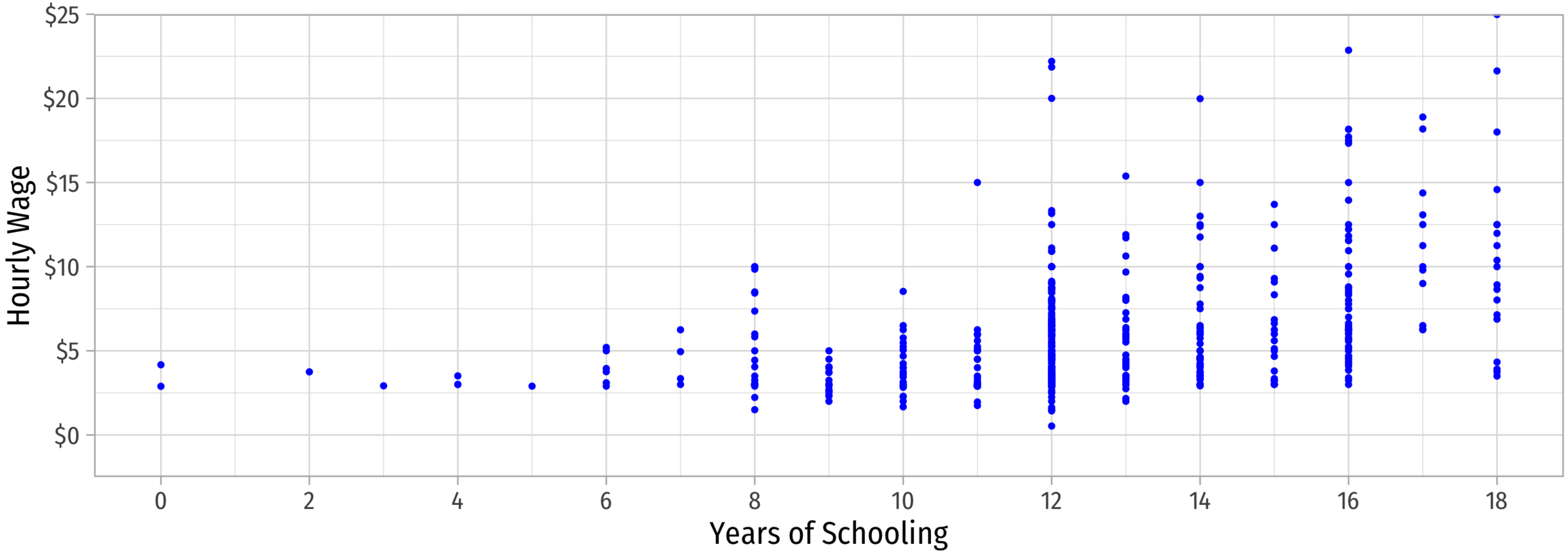


# More Obvious Heteroskedasticity



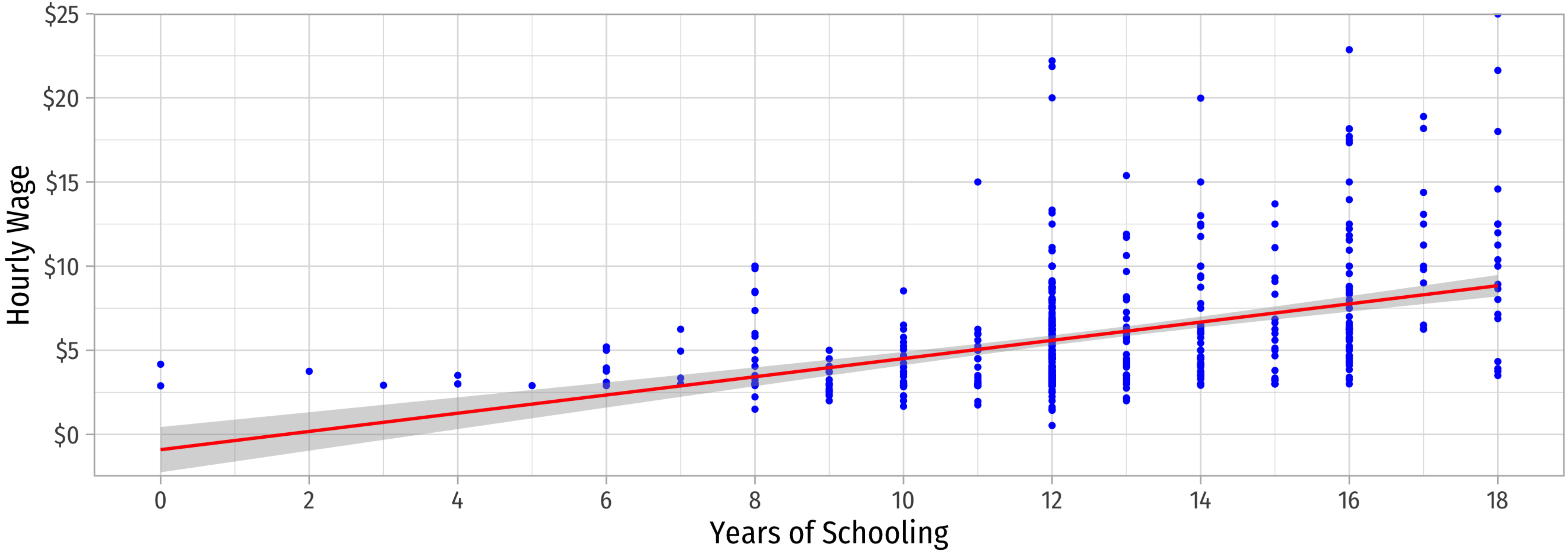
# What Might Cause Heteroskedastic Errors?

$$wage_i = \beta_0 + \beta_1 educ_i + u_i$$



# What Might Cause Heteroskedastic Errors?

$$wage_i = \beta_0 + \beta_1 educ_i + u_i$$

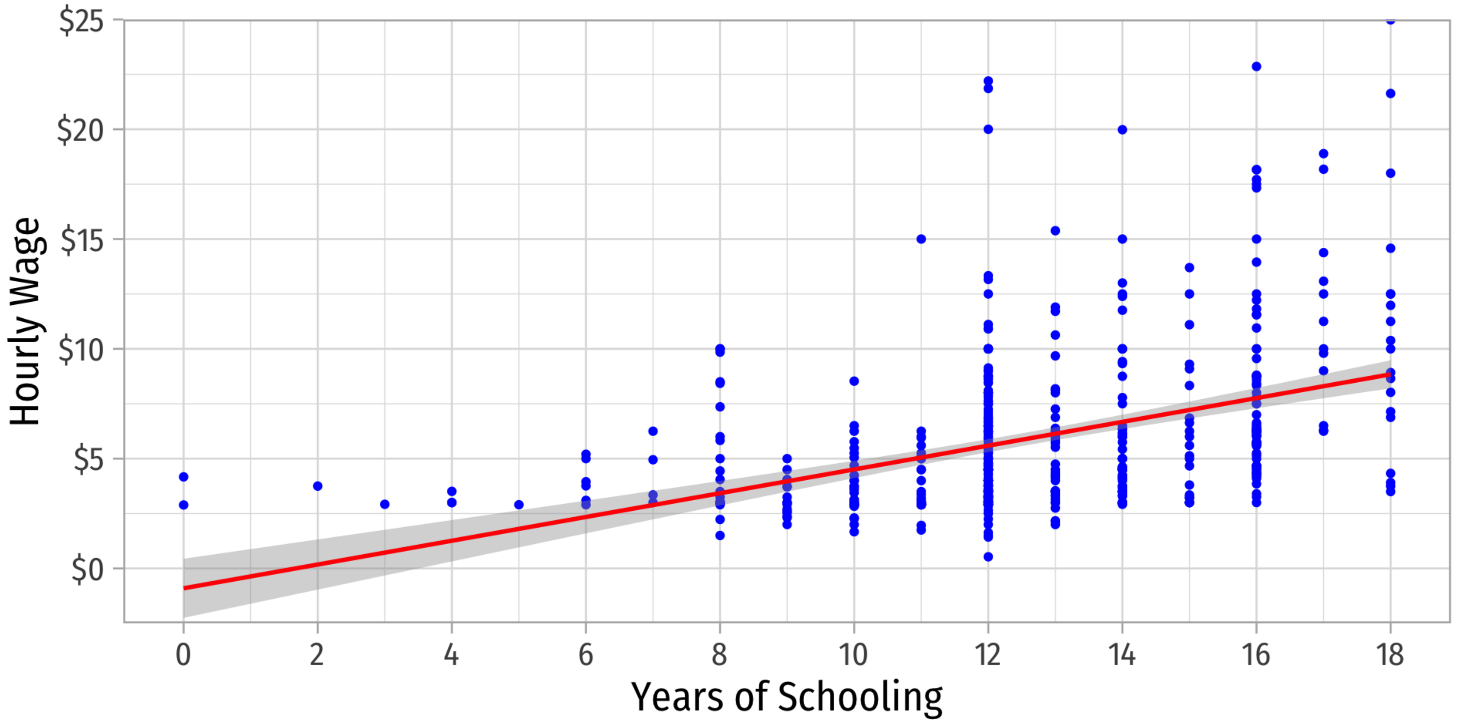


# What Might Cause Heteroskedastic Errors?

$$wage_i = \beta_0 + \beta_1 educ_i + u_i$$

| | Wage |
|---|---|
| Intercept | -0.90 |
| | (0.68) |
| Years of Schooling | 0.54*** |
| | (0.05) |
| n | 526 |
| $R^2$ | 0.16 |
| SER | 3.37 |

* p < 0.1, ** p < 0.05, *** p < 0.01



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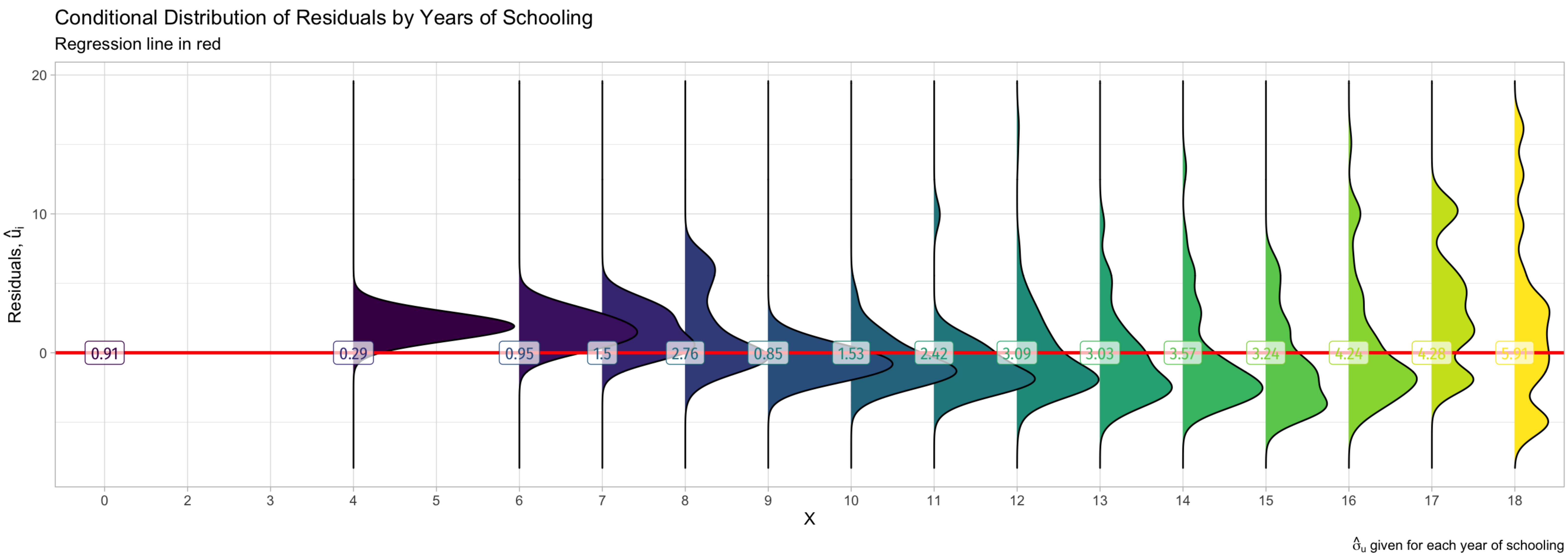


# Detecting Heteroskedasticity I

- Several tests to check if data is heteroskedastic
- One common test is **Breusch-Pagan test**
- Can use the `lmtest` package's function `bptest()`
  - $H_0$: homoskedastic[1]
  - If $p$-value < 0.05, reject $H_0 \implies$ heteroskedastic

```r
library("lmtest")
school_reg %>% bptest()
```

```
	studentized Breusch-Pagan test

data:  .
BP = 5.7936, df = 1, p-value = 0.01608
```

- Since $p < 0.05$, can reject $H_0$ that errors are homoskedastic and conclude they are *heteroskedastic*



1. More on hypothesis testing next week!

# How About the Wages Regression?

```r
wage_reg %>% bptest()
```

```
	studentized Breusch-Pagan test

data:  .
BP = 15.306, df = 1, p-value = 9.144e-05
```

# Fixing Heteroskedasticity I

- Heteroskedasticity is easy to fix with software that can calculate **robust standard errors** (using the more complicated formula above)

- Easiest method is to use `estimatr` package
  - `lm_robust()` command (*instead* of `lm`) to run regression
  - set `se_type = "stata"` to calculate robust SEs using the formula above[1]

```
#install.packages("estimatr")
library(estimatr)
```



1. This uses the method that Stata uses with the `robust` command, one advantage Stata has over R

# Fixing Heteroskedasticity II

```r
school_reg_robust <- lm_robust(testscr ~ str, data = ca_school,
                               se_type = "stata")

school_reg_robust
```

```
              Estimate Std. Error   t value      Pr(>|t|)   CI Lower   CI Upper
(Intercept) 698.932952 10.3643599 67.436191 9.486678e-227 678.560192 719.305713
str          -2.279808  0.5194892 -4.388557  1.446737e-05  -3.300945  -1.258671
             DF
(Intercept) 418
str         418
```

```r
school_reg_robust %>% summary()
```

```
Call:
lm_robust(formula = testscr ~ str, data = ca_school, se_type = "stata")

Standard error type:  HC1

Coefficients:
            Estimate Std. Error t value   Pr(>|t|) CI Lower CI Upper  DF
(Intercept)   698.93    10.3644  67.436 9.487e-227  678.560  719.306 418
str            -2.28     0.5195  -4.389  1.447e-05   -3.301   -1.259 418

Multiple R-squared:  0.05124 ,  Adjusted R-squared:  0.04897
F-statistic: 19.26 on 1 and 418 DF,  p-value: 1.447e-05
```

# Fixing Heteroskedasticity III

```
# can tidy, glance, augment, etc
school_reg_robust %>% tidy()
```

| term | estimate | std.error | statistic | p.value | conf.low | conf.high | df | ou |
|---|---|---|---|---|---|---|---|---|
| (Intercept) | 698.932952 | 10.3643599 | 67.436191 | 0.00e+00 | 678.560192 | 719.305713 | 418 | tes |
| str | -2.279808 | 0.5194892 | -4.388557 | 1.45e-05 | -3.300945 | -1.258671 | 418 | tes |

```
school_reg_robust %>% glance()
```

| r.squared | adj.r.squared | statistic | p.value | df.residual | nobs | se_type |
|---|---|---|---|---|---|---|
| 0.0512401 | 0.0489703 | 19.25943 | 1.45e-05 | 418 | 420 | HC1 |

# Showing The Effect of Heteroskedasticity (on $se(\hat{\beta}_1)$)

```r
modelsummary(models = list("Normal SE" = school_re
                           "Robust SE" = school_re
             fmt = 2, # round to 2 decimals
             output = "html",
             coef_rename = c("(Intercept)" = "Cons
                             "str" = "STR"),
             gof_map = list(
               list("raw" = "nobs", "clean" = "n",
               list("raw" = "r.squared", "clean" =
               #list("raw" = "adj.r.squared", "cle
               list("raw" = "rmse", "clean" = "SER
             ),
             escape = FALSE,
             stars = c('*' = .1, '**' = .05, '***'
)
```

| | Normal SE | Robust SE |
|---|---|---|
| Constant | 698.93*** | 698.93*** |
| | (9.47) | (10.36) |
| STR | −2.28*** | −2.28*** |
| | (0.48) | (0.52) |
| n | 420 | 420 |
| $R^2$ | 0.05 | 0.05 |
| SER | 18.54 | 18.54 |

* p < 0.1, ** p < 0.05, *** p < 0.01

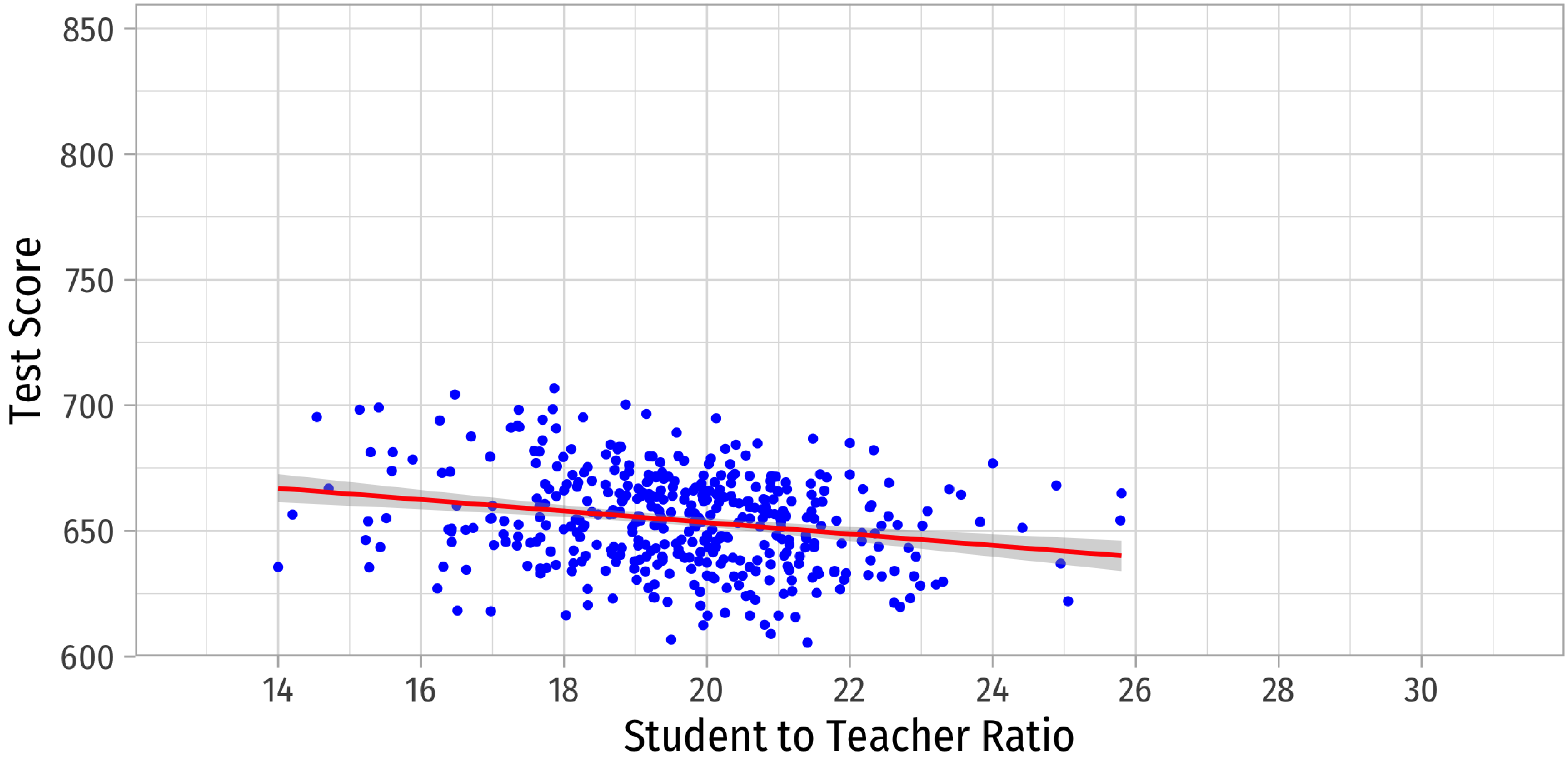
- What changed?

# Outliers

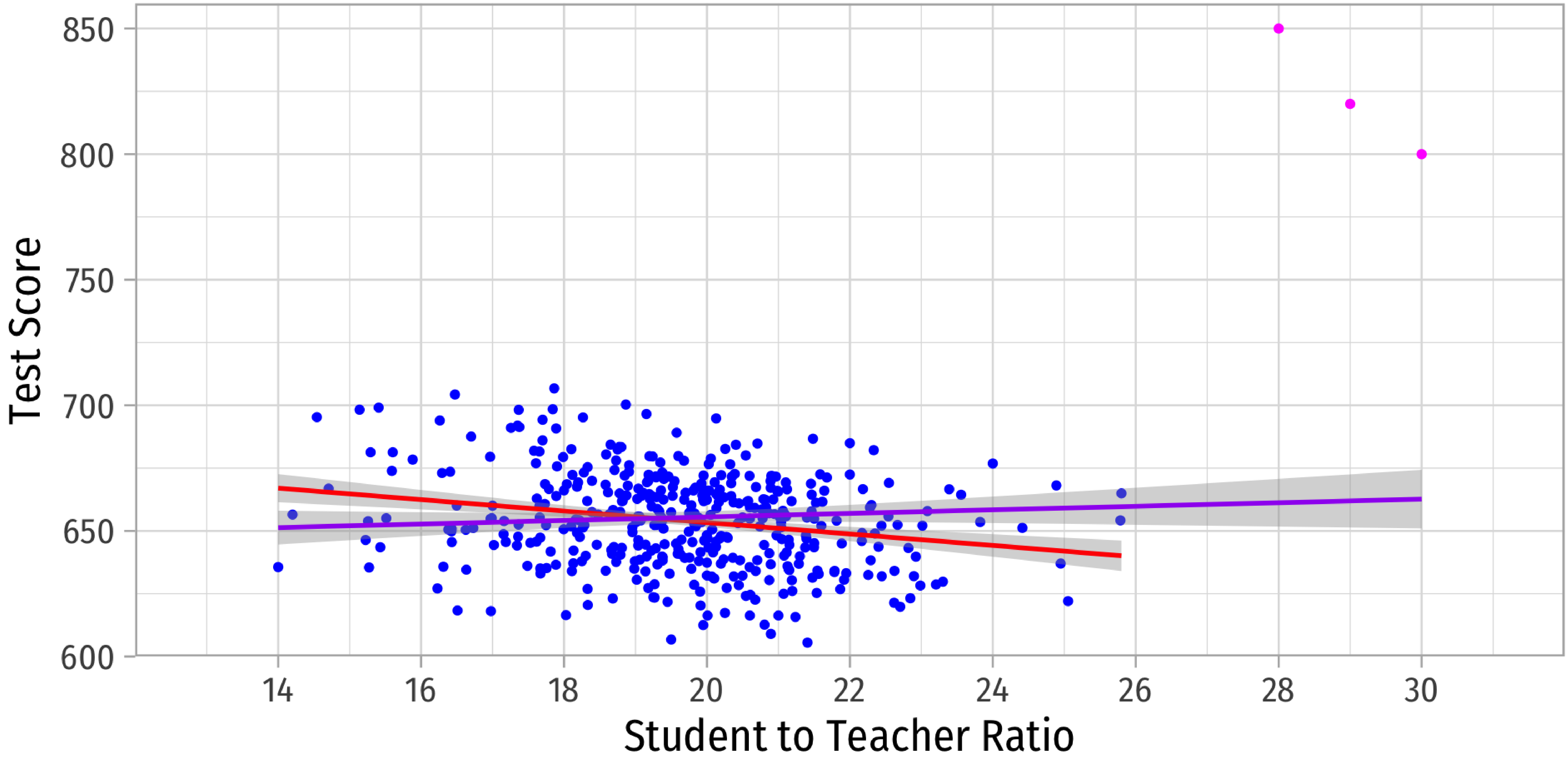
# Outliers Can Bias OLS! I

- **Outliers** can affect the slope (and intercept) of the line and add **bias**
  - May be result of human error (measurement, transcribing, etc)
  - May be meaningful and accurate
- In any case, compare how including/dropping outliers affects regression and always discuss outliers!

# Outliers Can Bias OLS! II



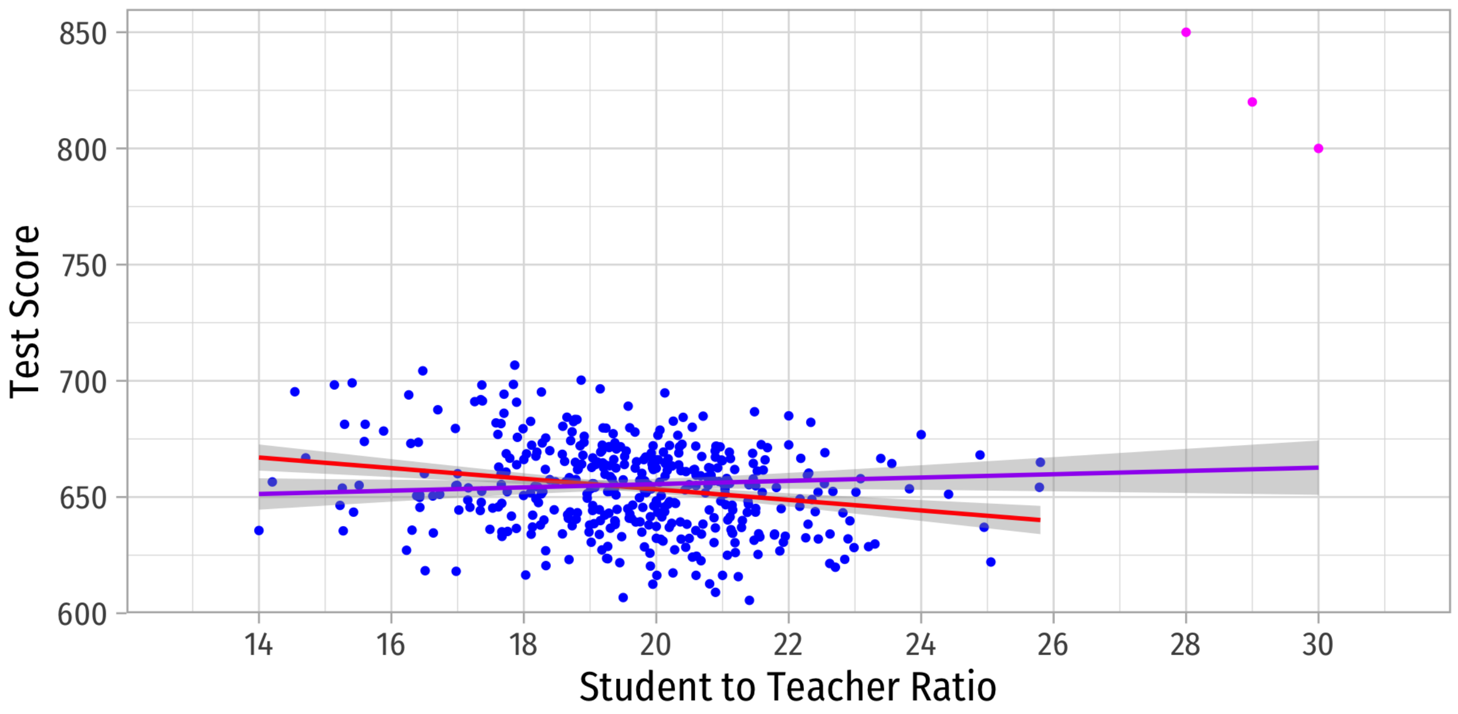
# Outliers Can Bias OLS! II



# Outliers Can Bias OLS! III

| | Original | With Outliers |
|---|---|---|
| Constant | 698.93*** | 641.40*** |
| | (9.47) | (11.21) |
| STR | -2.28*** | 0.71 |
| | (0.48) | (0.57) |
| n | 420 | 423 |
| $R^2$ | 0.05 | 0.00 |
| SER | 18.54 | 23.71 |

* p < 0.1, ** p < 0.05, *** p < 0.01



# Detecting Outliers

- The `car` package has an `outlierTest` command to run on the regression

```r
# install.packages("car")
library("car")

# Use Bonferonni test
outlierTest(school_outlier_reg) # will point out which obs #s seem outliers
```

```
    rstudent unadjusted p-value Bonferroni p
422 8.822768         3.0261e-17   1.2800e-14
423 7.233470         2.2493e-12   9.5147e-10
421 6.232045         1.1209e-09   4.7414e-07
```

```r
# find these observations
ca_school_outliers %>%
  slice(c(422,423,421)) # find observations 422, 423, 421
```

| observat | district | testscr | str |
|---:|---|---:|---:|
| 422 | Crazy District 2 | 850 | 28 |
| 423 | Crazy District 3 | 820 | 29 |
| 421 | Crazy District 1 | 800 | 30 |