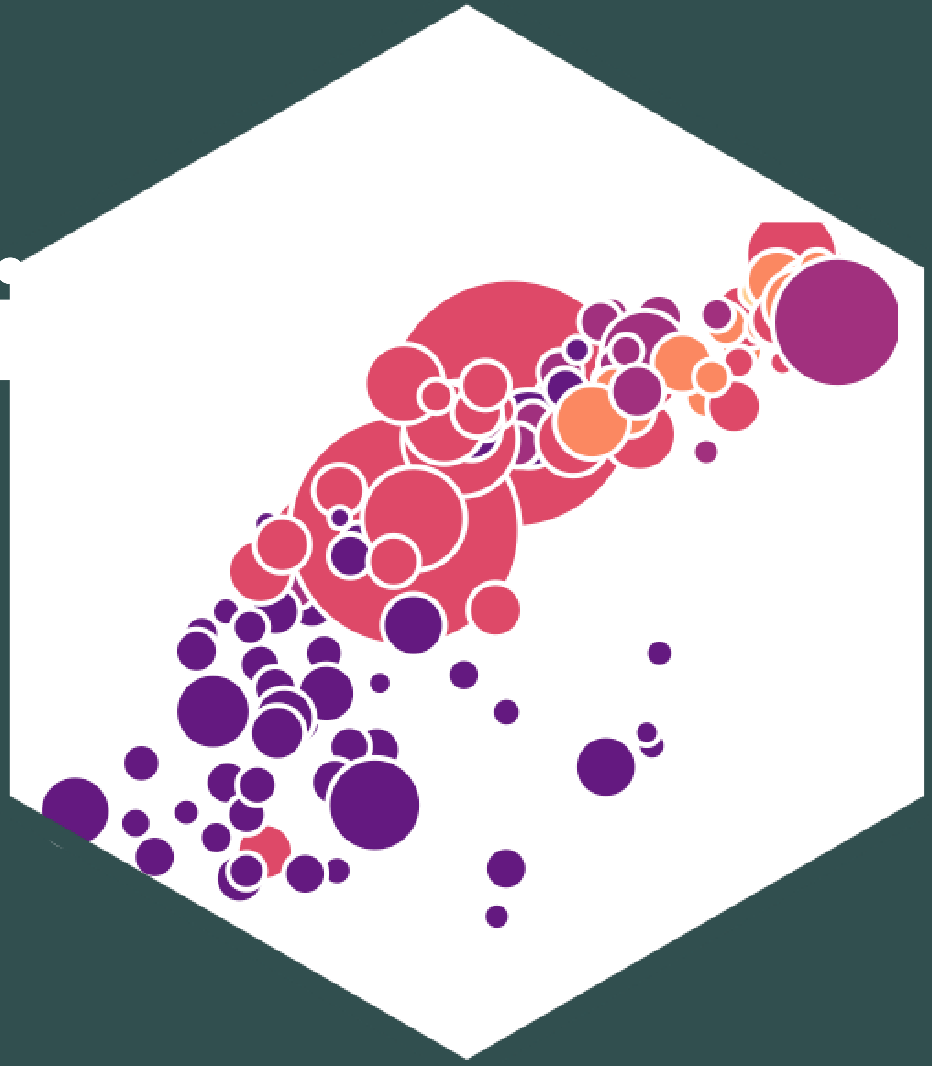
2.1 — Data 101 & Descriptive Statisti



ECON 480 • Econometrics • Fall 2022

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Contents

The Two Big Problems with Data

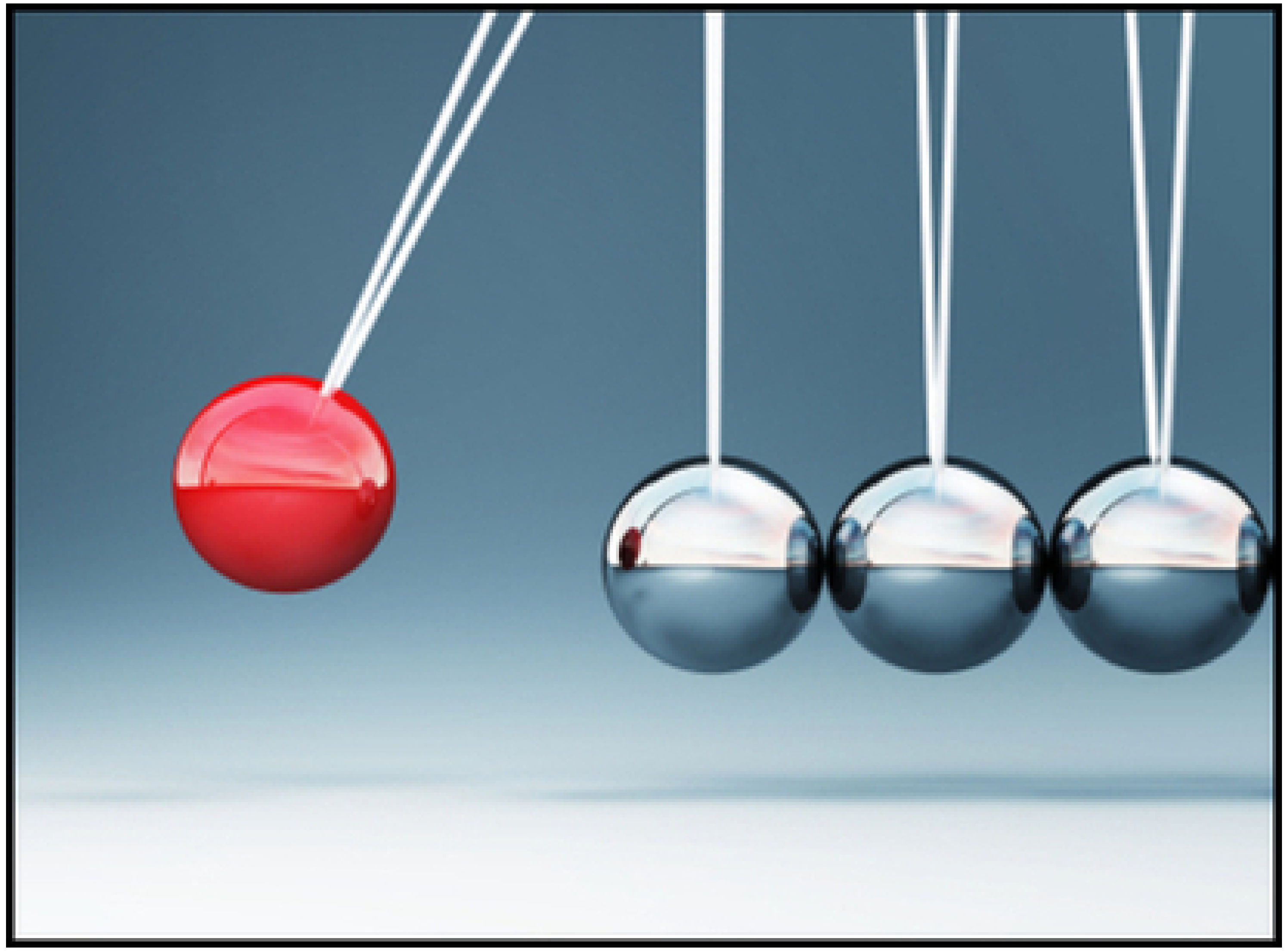
Two Big Problems with Data

- We want to use econometrics to **identify** causal relationships and make **inferences** about them

1. Problem for **identification**: **endogeneity**
2. Problem for **inference**: **randomness**

Identification Problem: Endogeneity

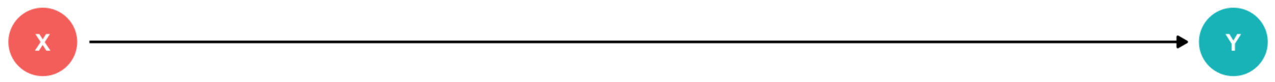
- An independent variable (X) is **exogenous** if its variation is **unrelated** to other factors that affect the dependent variable (Y)
- An independent variable (X) is **endogenous** if its variation is **related** to other factors that affect the dependent variable (Y)
- Note: unfortunately this is different from how economists talk about "endogenous" vs. "exogenous" variables in theoretical models...



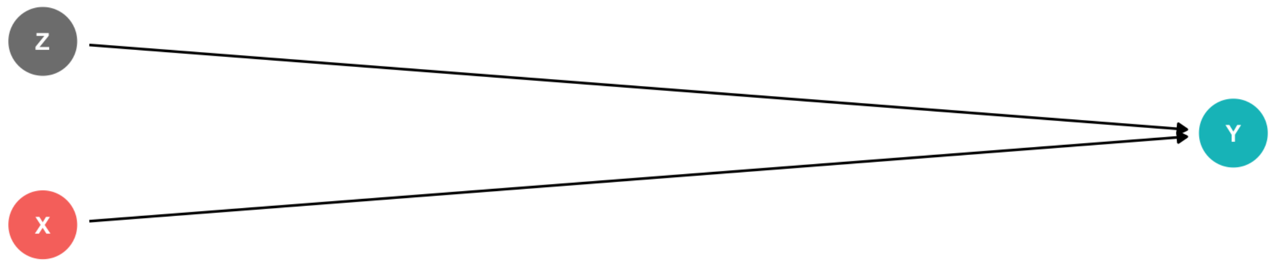
Identification Problem: Endogeneity

- An independent variable (X) is **exogenous** if its variation is **unrelated** to other factors that affect the dependent variable (Y)

X causes Y

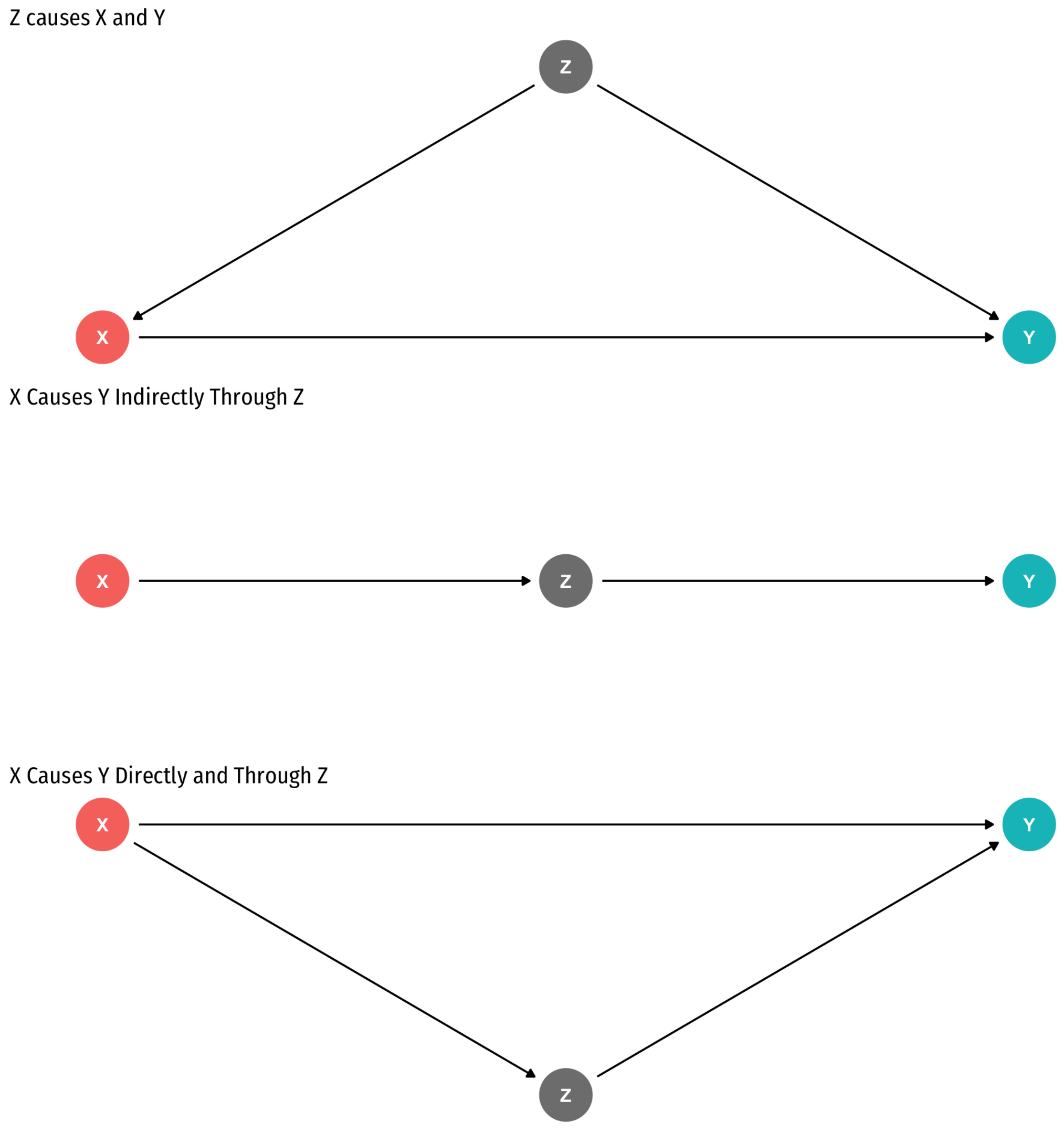


X and Z (independently) cause Y



Identification Problem: Endogeneity

- An independent variable (X) is **endogenous** if its variation is **related** to other factors that affect the dependent variable (Y), e.g. Z



Inference Problem: Randomness

- Data is **random** due to **natural sampling variation**
  - Taking one sample of a population will yield slightly different information than another sample of the same population
- Common in statistics, *easy to fix*
- **Inferential Statistics**: making claims about a wider population using sample data
  - We use common tools and techniques to deal with randomness

The Two Problems: Where We're Heading...Ultimately

$$\textbf{Sample} \xrightarrow{\text{statistical inference}} \textbf{Population} \xrightarrow{\text{causal indentification}} \textbf{Unobserved Parameters}$$

- We want to **identify** causal relationships between **population** variables
  - Logically first thing to consider
  - **Endogeneity problem**
- We'll use **sample** *statistics* to **infer** something about population *parameters*
  - In practice, we'll only ever have a finite *sample distribution* of data
  - We *don't* know the *population distribution* of data
  - **Randomness problem**

Data 101

Data 101

- **Data** are information with context
- **Individuals** are the entities described by a set of data
  - e.g. persons, households, firms, countries

Data 101

- **Variables** are particular characteristics about an individual
  - e.g. age, income, profits, population, GDP, marital status, type of legal institutions
- **Observations** or **cases** are the separate individuals described by a collection of variables
  - e.g. for one individual, we have their age, sex, income, education, etc.
- individuals and observations are *not necessarily* the same:
  - e.g. we can have multiple observations on the same individual over time

Categorical Variables

- **Categorical variables** place an individual into one of several possible *categories*
  - e.g. sex, season, political party
  - may be responses to survey questions
  - can be quantitative (e.g. age, zip code)
- In R: `character` or `factor` type data
  - `factor` $\implies$ specific possible categories

Question	Categories or Responses
Do you invest in the stock market?	___ Yes ___ No
What kind of advertising do you use?	___ Newspapers ___ Internet ___ Direct mailings
What is your class at school?	___ Freshman ___ Sophomore ___ Junior ___ Senior
I would recommend this course to another student.	___ Strongly Disagree ___ Slightly Disagree ___ Slightly Agree ___ Strongly Agree
How satisfied are you with this product?	___ Very Unsatisfied ___ Unsatisfied ___ Satisfied ___ Very Satisfied

Categorical Variables: Visualizing I

```
diamonds %>%
  count(cut) %>%
  mutate(frequency = n / sum(n),
         percent = round(frequency * 100, 2))
```

Summary of diamonds by cut

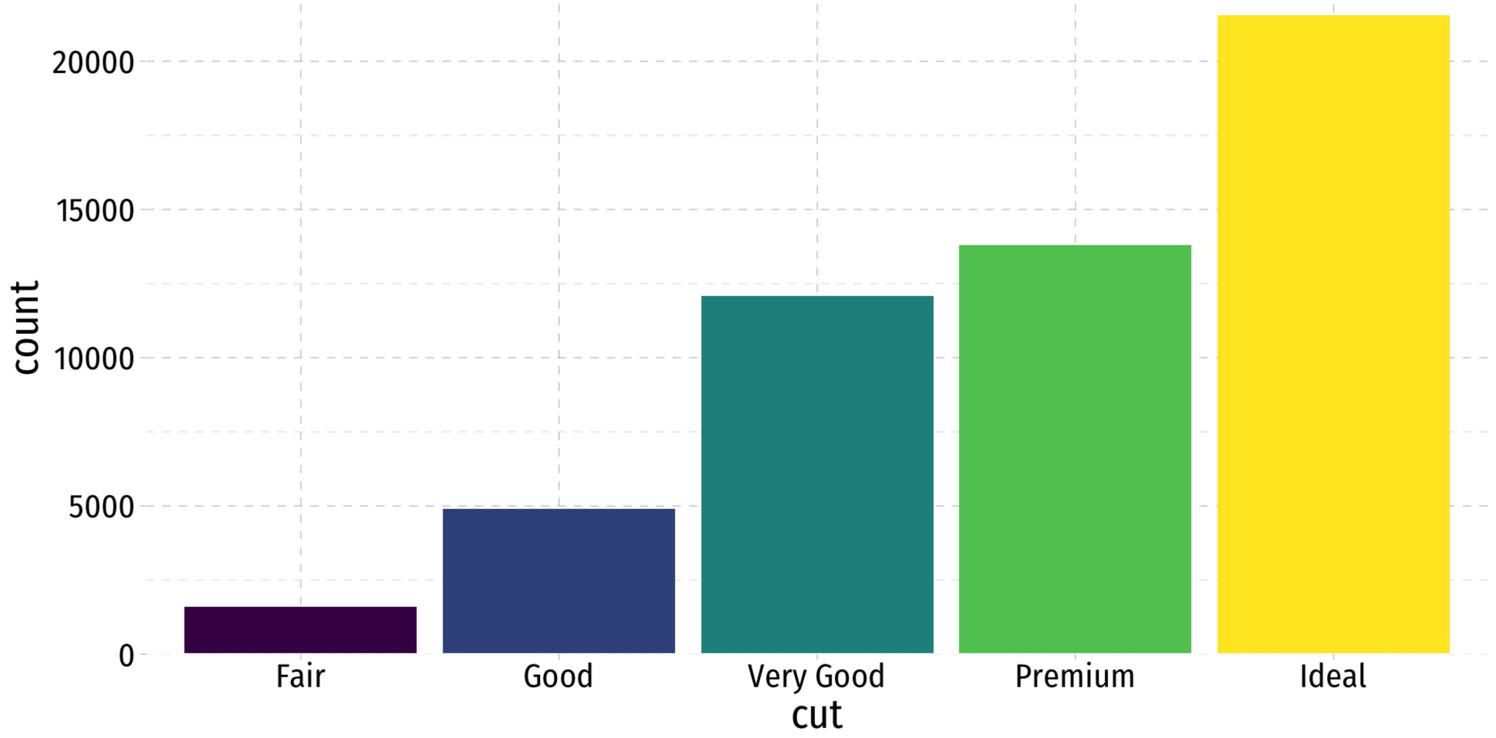
cut	n	frequency	percent
Fair	1610	0.0298480	2.98
Good	4906	0.0909529	9.10
Very Good	12082	0.2239896	22.40
Premium	13791	0.2556730	25.57
Ideal	21551	0.3995365	39.95

- Good way to represent categorical data is with a **frequency table**
- **Count (n)**: total number of individuals in a category
- **Frequency**: **proportion** of a category's occurrence relative to all data
  - Multiply proportions by 100% to get **percentages**

Categorical Variables: Visualizing II

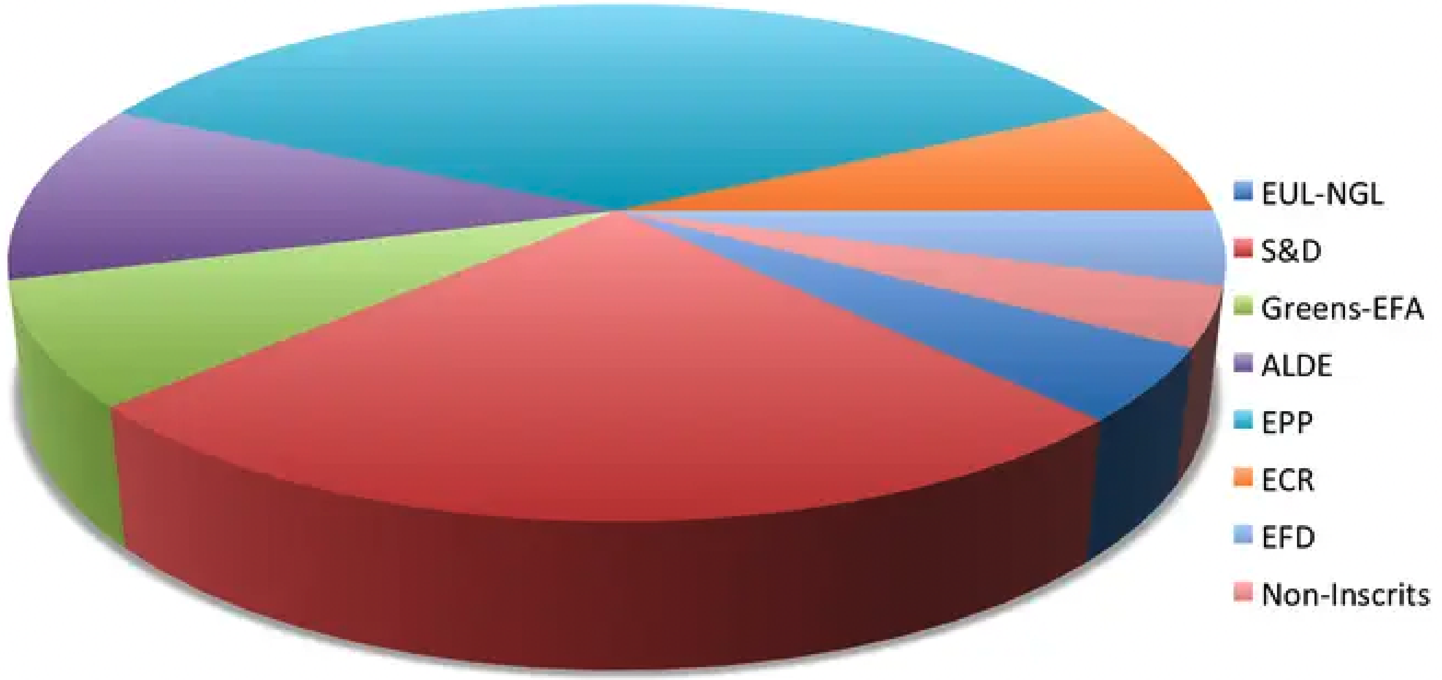
- **Charts and graphs are *always* better ways to visualize data**
- A **bar graph** represents categories as bars, with lengths proportional to the count or relative frequency of each category

```r
ggplot(diamonds, aes(x=cut,
                     fill=cut))+
  geom_bar()+
  guides(fill=F)+
  theme_pander(base_family = "Fira Sans Condensed"
           base_size=20)
```



Categorical Data: Pie Charts

- Avoid pie charts!
- People are *not* good at judging 2-d differences (angles, area)
- People *are* good at judging 1-d differences (length)



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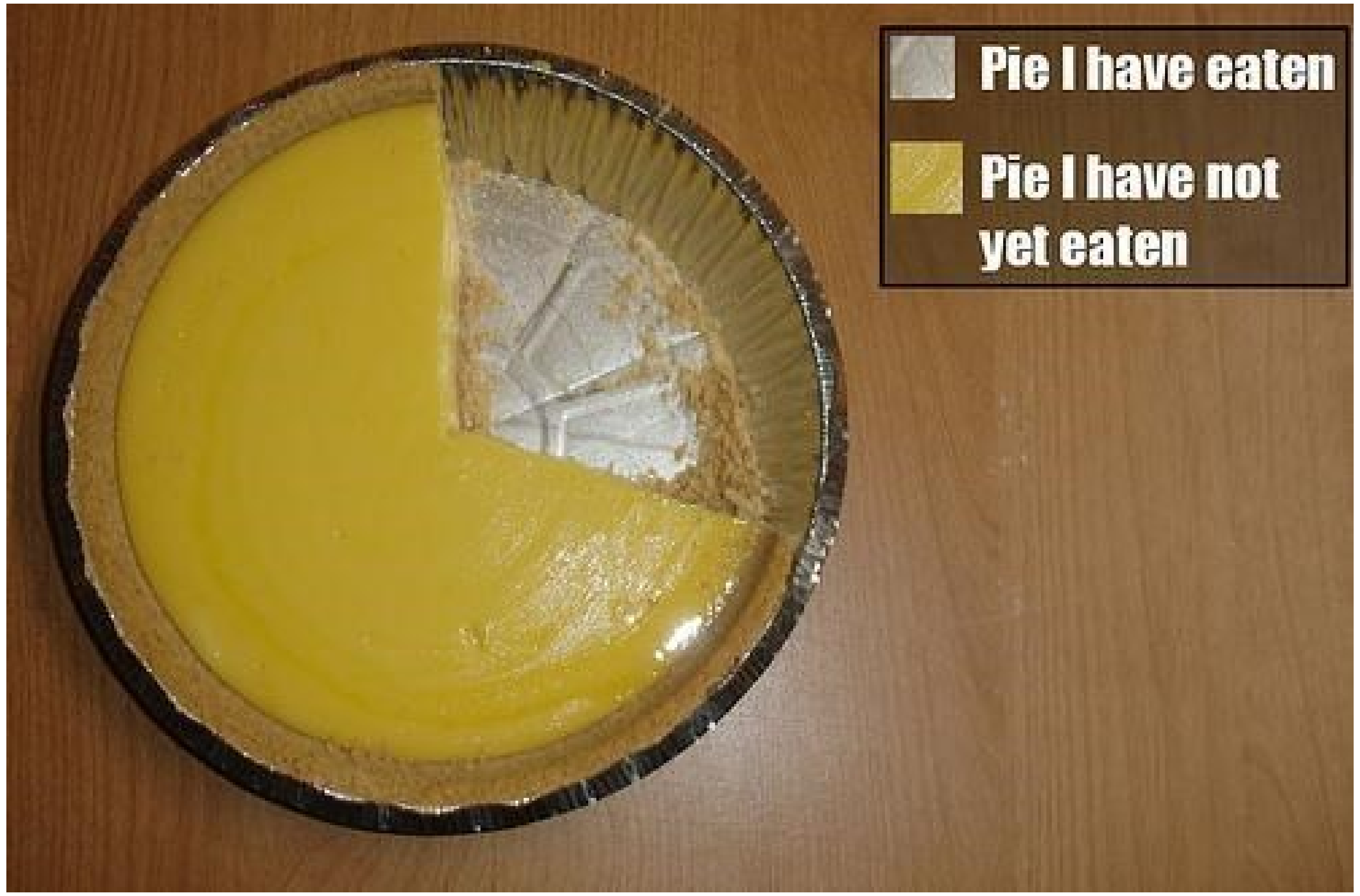
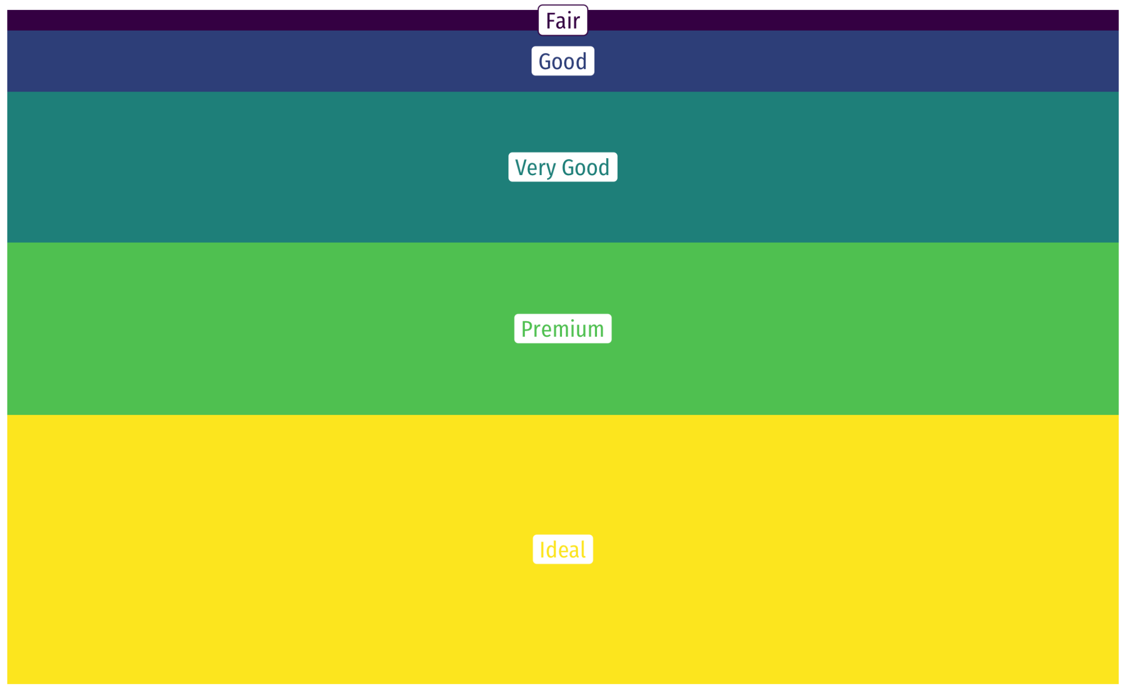


Categorical Data: Alternatives to Pie Charts I

- Try something else: a *stacked bar chart*

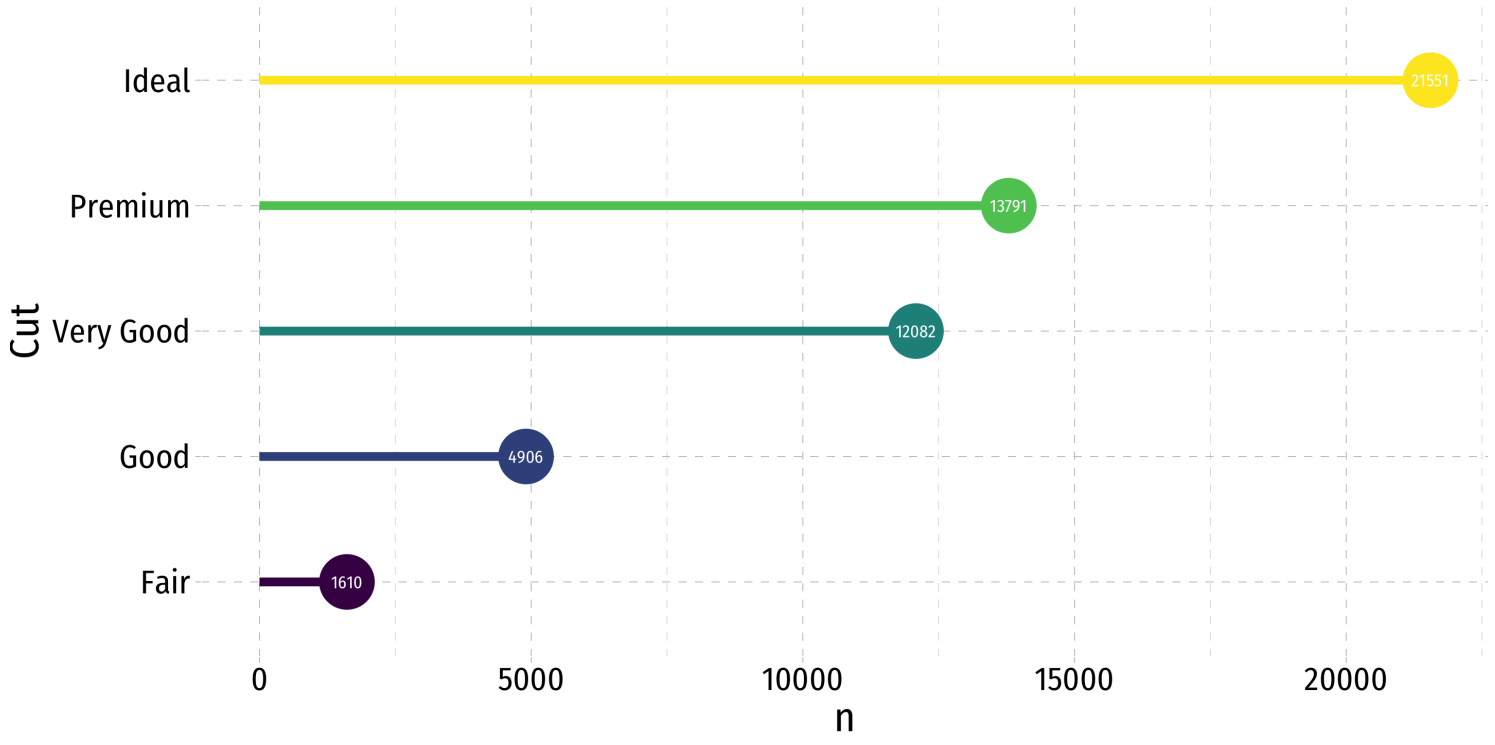
```
diamonds %>%
  count(cut) %>%
ggplot(data = .)+
  aes(x = "",
      y = n)+
  geom_col(aes(fill = cut))+
  geom_label(aes(label = cut,
                 color = cut),
             position = position_stack(vjust = 0.5
             )+
  guides(color = F,
         fill = F)+
  theme_void()
```



Categorical Data: Alternatives to Pie Charts II

- Try something else: a *lollipop chart*

```r
diamonds %>%
  count(cut) %>%
  mutate(cut_name = as.factor(cut)) %>%
ggplot(., aes(x = cut_name, y = n, color = cut))+
 geom_point(stat="identity",
            fill="black",
            size=12)  +
  geom_segment(aes(x = cut_name, y = 0,
                   xend = cut_name,
                   yend = n), size = 2)+
  geom_text(aes(label = n),color="white", size=3)
  coord_flip()+
  labs(x = "Cut")+
  theme_pander(base_family = "Fira Sans Condensed"
               base_size=20)+
  guides(color = F)
```



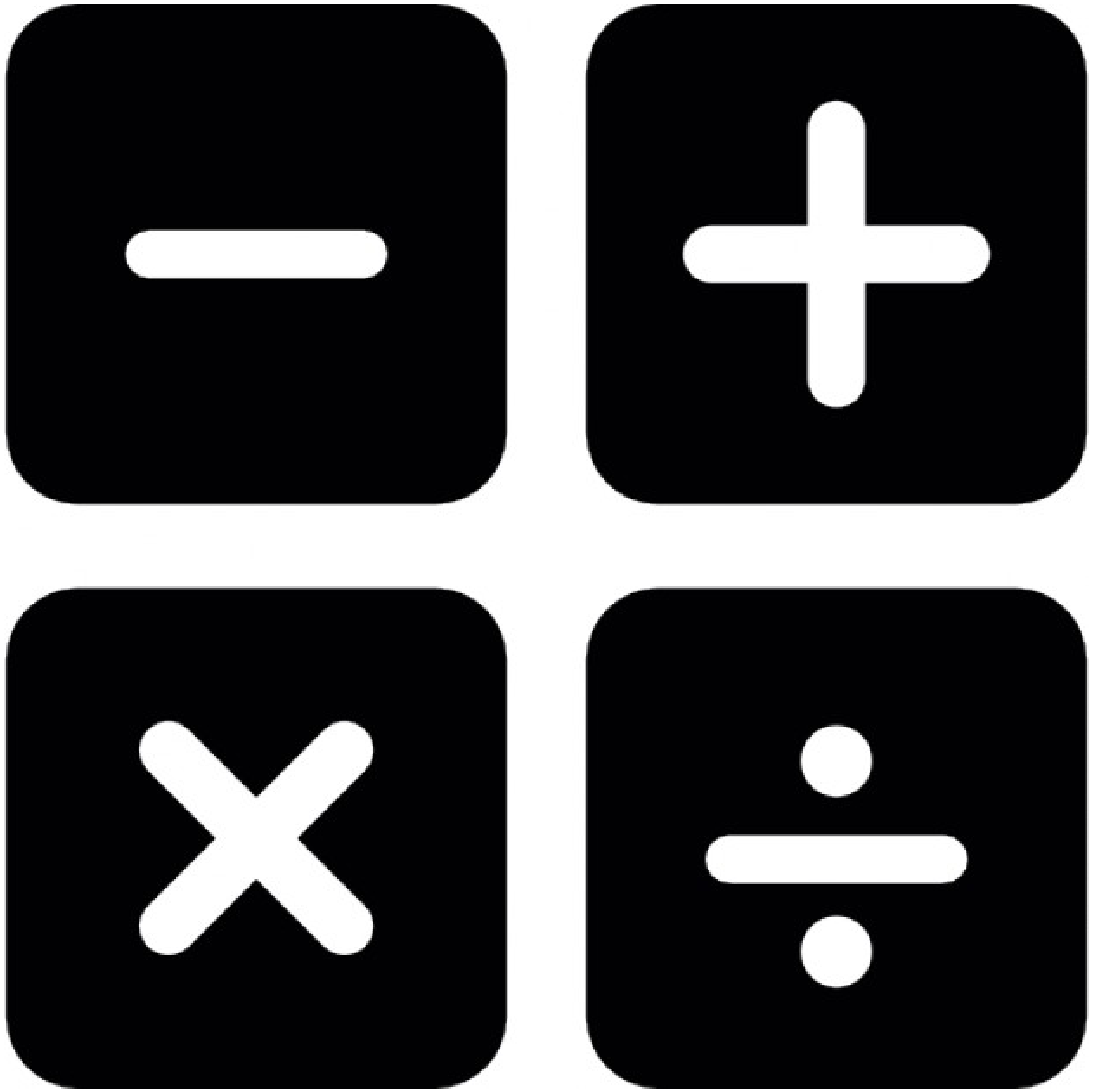
Categorical Data: Alternatives to Pie Charts III

- Try something else: a *treemap*

```
library(treemapify)
diamonds %>%
  count(cut) %>%
ggplot(., aes(area = n, fill = cut)) +
  geom_treemap() +
  guides(fill = FALSE) +
  geom_treemap_text(aes(label = cut),
                    colour = "white",
                    place = "topleft",
                    grow = TRUE)
```

Quantitative Data I

- **Quantitative variables** take on numerical values of equal units that describe an individual
  - Units: points, dollars, inches
  - Context: GPA, prices, height
- We can mathematically manipulate *only* quantitative data
  - e.g. sum, average, standard deviation
- In R: `numeric` type data
  - `integer` if whole number
  - `double` if has decimals



Discrete Data

- **Discrete data** are finite, with a countable number of alternatives
- **Categorical**: place data into categories
  - e.g. letter grades: A, B, C, D, F
  - e.g. class level: freshman, sophomore, junior, senior
- **Quantitative**: integers
  - e.g. SAT Score, number of children, age (years)

Continuous Data

- **Continuous data** are infinitely divisible, with an uncountable number of alternatives
  - e.g. weight, length, temperature, GPA
- Many discrete variables may be treated as if they are continuous
  - e.g. SAT scores (whole points), wages (dollars and cents)

Spreadsheets

id	name	age	sex	income
1	John	23	Male	41000
2	Emile	18	Male	52600
3	Natalya	28	Female	48000
4	Lakisha	31	Female	60200
5	Cheng	36	Male	81900

- The most common data structure we use is a **spreadsheet**
  - In *R*: a `data.frame` or `tibble`
- A **row** contains data about all variables for a single **individual**
- A **column** contains data about a single **variable** across all individuals

Spreadsheets: Indexing

id	name	age	sex	income
1	John	23	Male	41000
2	Emile	18	Male	52600
3	Natalya	28	Female	48000
4	Lakisha	31	Female	60200
5	Cheng	36	Male	81900

- Each .hi-purple[cell] can be referenced by its row and column (in that order!), `df[row,column]`

```
example[3,2] # value in row 3, column 2
```

```
# A tibble: 1 × 1
  name
  <chr>
1 Natalya
```

- Recall with `tidyverse` you can do this with `select()` and `filter()` or `slice()`

Spreadsheets: Notation

- It is common to use some notation like the following:
- Let $\{x_1, x_2, \cdots, x_n\}$ be a simple data series on variable X
  - n individual observations
  - x_i is the value of the i^{th} observation for $i = 1, 2, \cdots, n$

Quick Check

Let x represent the score on a homework assignment:

$$75, 100, 92, 87, 79, 0, 95$$

1. What is n?
2. What is x_1?
3. What is x_6?

Datasets: Cross-Sectional

id	name	age	sex	income
1	John	23	Male	41000
2	Emile	18	Male	52600
3	Natalya	28	Female	48000
4	Lakisha	31	Female	60200
5	Cheng	36	Male	81900

- **Cross-sectional data**: observations of individuals at a given point in time
- Each observation is a unique individual

$$x_i$$

- Simplest and most common data
- A **"snapshot"** to compare differences across individuals

Datasets: Time-Series

Year	GDP	Unemployment	CPI
1950	8.2	0.06	100
1960	9.9	0.04	118
1970	10.2	0.08	130
1980	12.4	0.08	190
1985	13.6	0.06	196

- **Time-series data**: observations of the *same* individual(s) over time
- Each observation is a time period

$$x_t$$

- Often used for macroeconomics, finance, and forecasting
- Unique challenges for time series
- A **"moving picture"** to see how individuals change over time

Datasets: Panel

City	Year	Murders	Population
Philadelphia	1986	5	3.700
Philadelphia	1990	8	4.200
D.C.	1986	2	0.250
D.C.	1990	10	0.275
New York	1986	3	6.400

- **Panel**, or **longitudinal** dataset: a time-series for *each* cross-sectional entity
  - Must be *same* individuals over time
- Each obs. is an individual in a time period

$$x_{it}$$

- More common today for serious researchers; unique challenges and benefits
- A **combination** of "snapshot" comparisons over time

Descriptive Statistics

Variables and Distributions

- Variables take on different values, we can describe a variable’s .hi[distribution] (of these values)
- We want to *visualize* and *analyze* distributions to search for meaningful patterns using **statistics**

Two Branches of Statistics

- Two main branches of statistics:

1. **Descriptive Statistics:** describes or summarizes the properties of a sample
2. **Inferential Statistics:** infers properties about a larger population from the properties of a sample[1]

1. We'll encounter inferential statistics mainly in the context of regression later

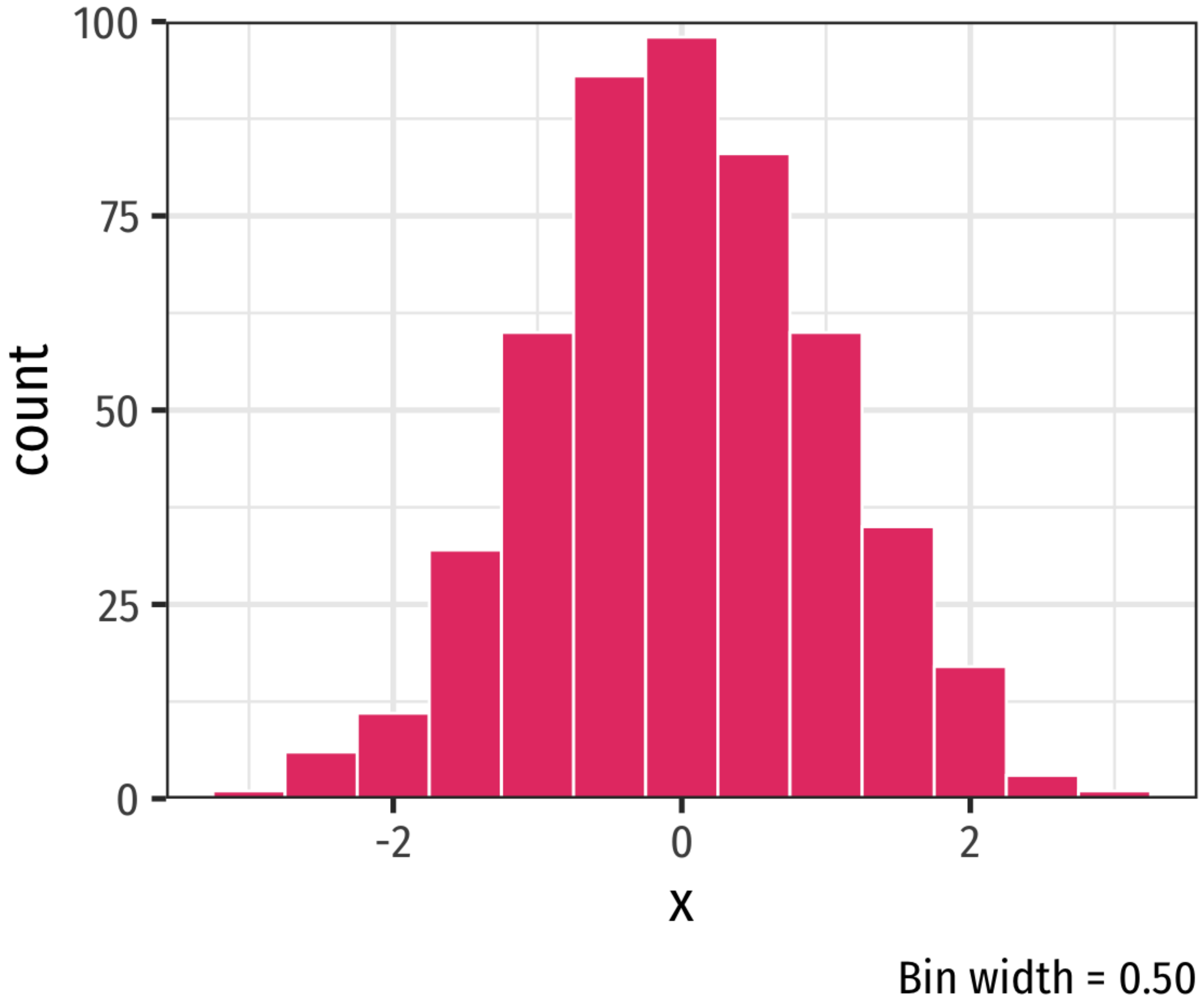
Histogram

- A common way to present a *quantitative* variable's distribution is a **histogram**
  - The quantitative analog to the bar graph for a categorical variable
- Divide up values into **bins** of a certain size, and count the number of values falling within each bin, representing them visually as bars

Bin width = 0.20

Histogram: Bin Size

- A common way to present a *quantitative* variable's distribution is a **histogram**
  - The quantitative analog to the bar graph for a categorical variable
- Divide up values into **bins** of a certain size, and count the number of values falling within each bin, representing them visually as bars
  - Changing the **bin-width** will affect the bars



Histogram: Example

Example

A class of 13 students takes a quiz (out of 100 points) with the following results:

$$\{0, 62, 66, 71, 71, 74, 76, 79, 83, 86, 88, 93, 95\}$$

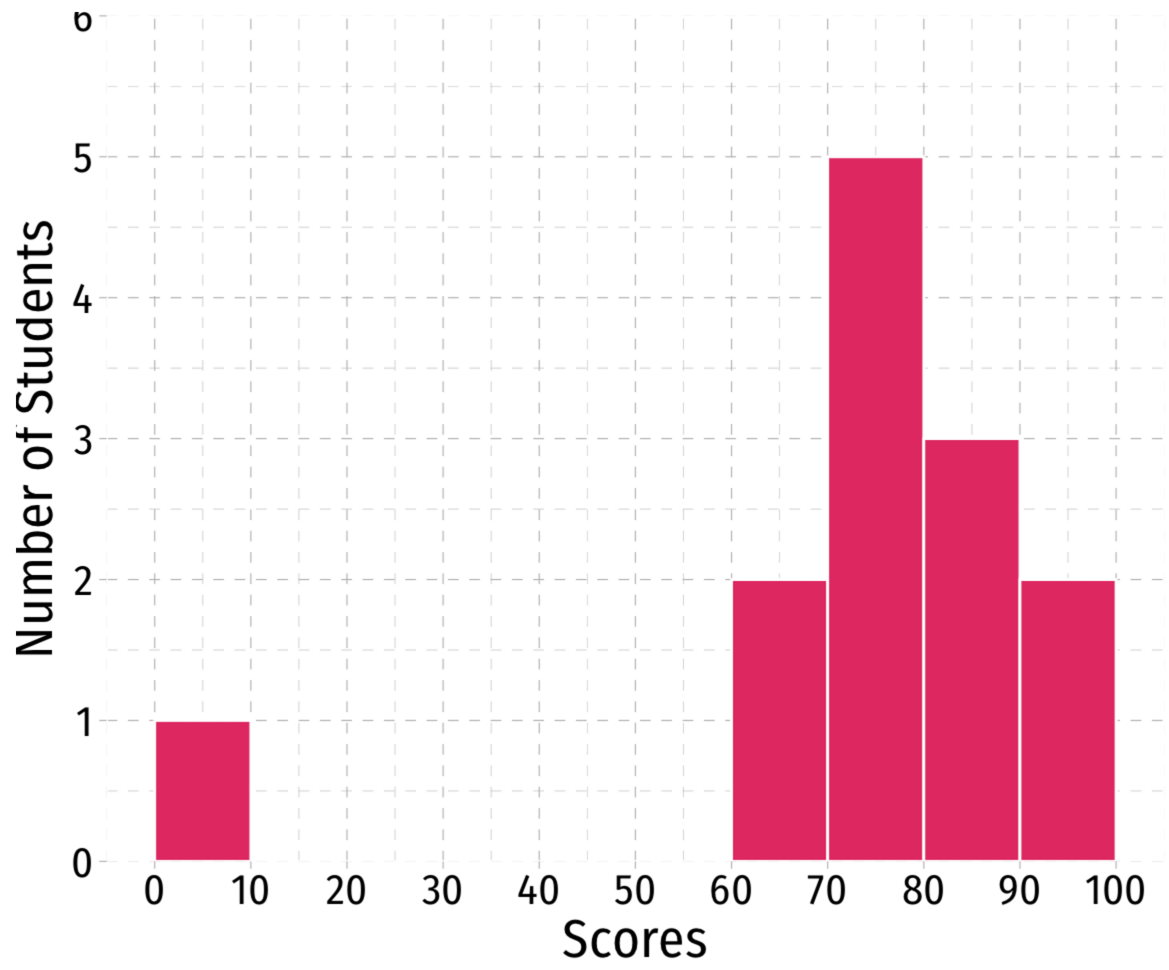
Histogram: Example

Example

A class of 13 students takes a quiz (out of 100 points) with the following results:

$$\{0, 62, 66, 71, 71, 74, 76, 79, 83, 86, 88, 93, 95\}$$

```
ggplot(quizzes,aes(x=scores))+
  geom_histogram(breaks = seq(0,100,10),
                 color = "white",
                 fill = "#e64173")+
  scale_x_continuous(breaks = seq(0,100,10))+
  scale_y_continuous(limits = c(0,6), expand = c(0
  labs(x = "Scores",
       y = "Number of Students")+
  theme_bw(base_family = "Fira Sans Condensed",
           base_size=20)
```



Descriptive Statistics

- We are often interested in the *shape* or *pattern* of a distribution, particularly:
  - Measures of **center**
  - Measures of **dispersion**
  - **Shape** of distribution

Measures of Center

Mode

- The .himode of a variable is simply its most frequent value
- A variable can have multiple modes

Example

A class of 13 students takes a quiz (out of 100 points) with the following results:

$$\{0, 62, 66, \mathbf{71}, \mathbf{71}, 74, 76, 79, 83, 86, 88, 93, 95\}$$

Mode

- There is no dedicated `mode()` function in R, surprisingly
- A workaround in `dplyr`:

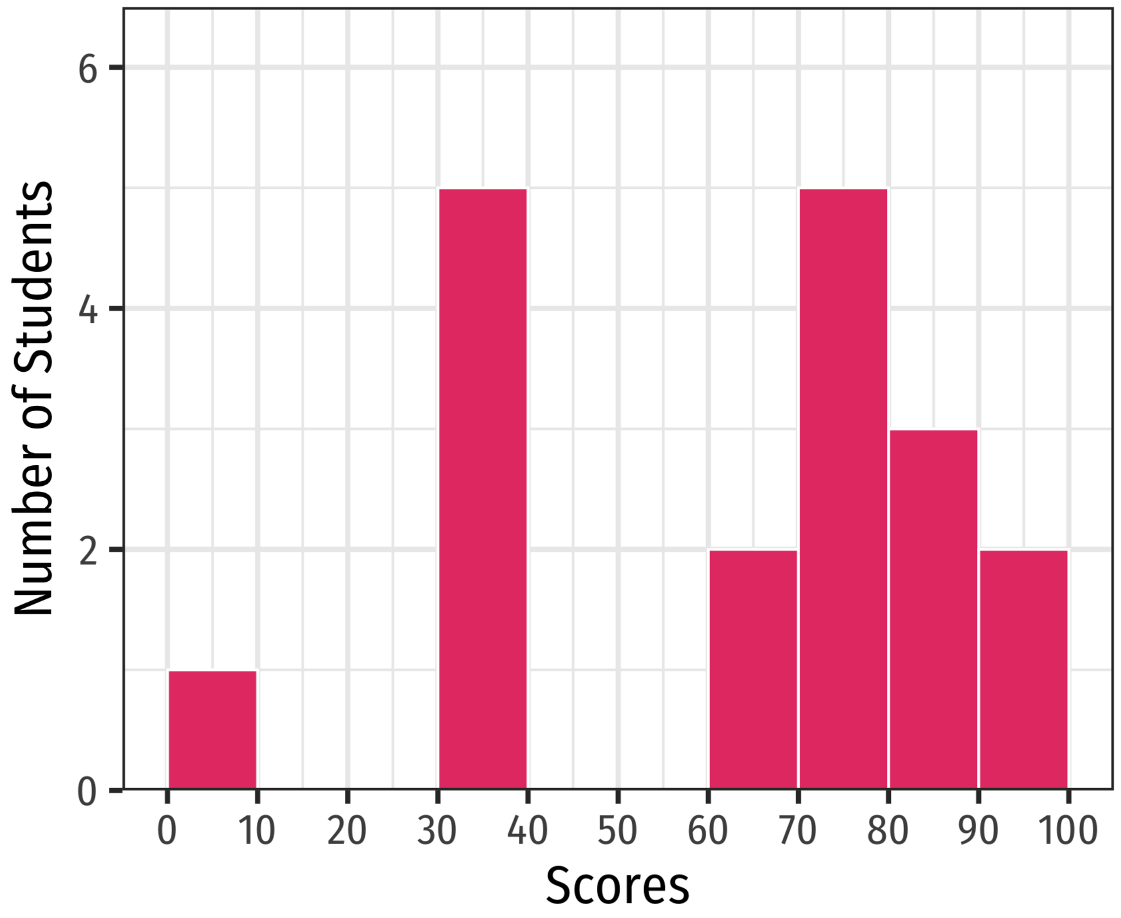
```
quizzes %>%
  count(scores) %>%
  arrange(desc(n))
```

scores <dbl>	n <int>
71	2
0	1
62	1
66	1
74	1

1-5 of 12 rows Previous **1** 2 3 Next

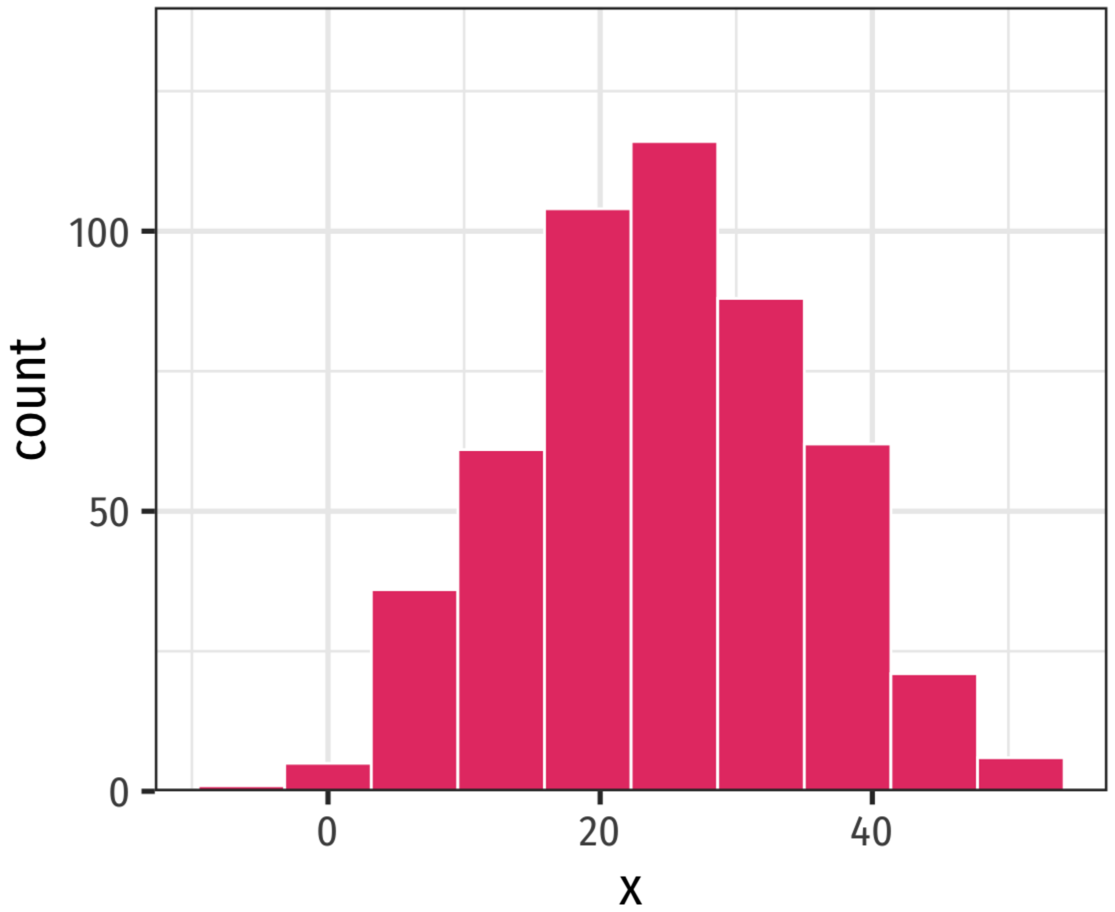
Multi-Modal Distributions

- Looking at a histogram, the modes are the "peaks" of the distribution
  - Note: depends on how wide you make the bins!
- May be unimodal, bimodal, trimodal, etc



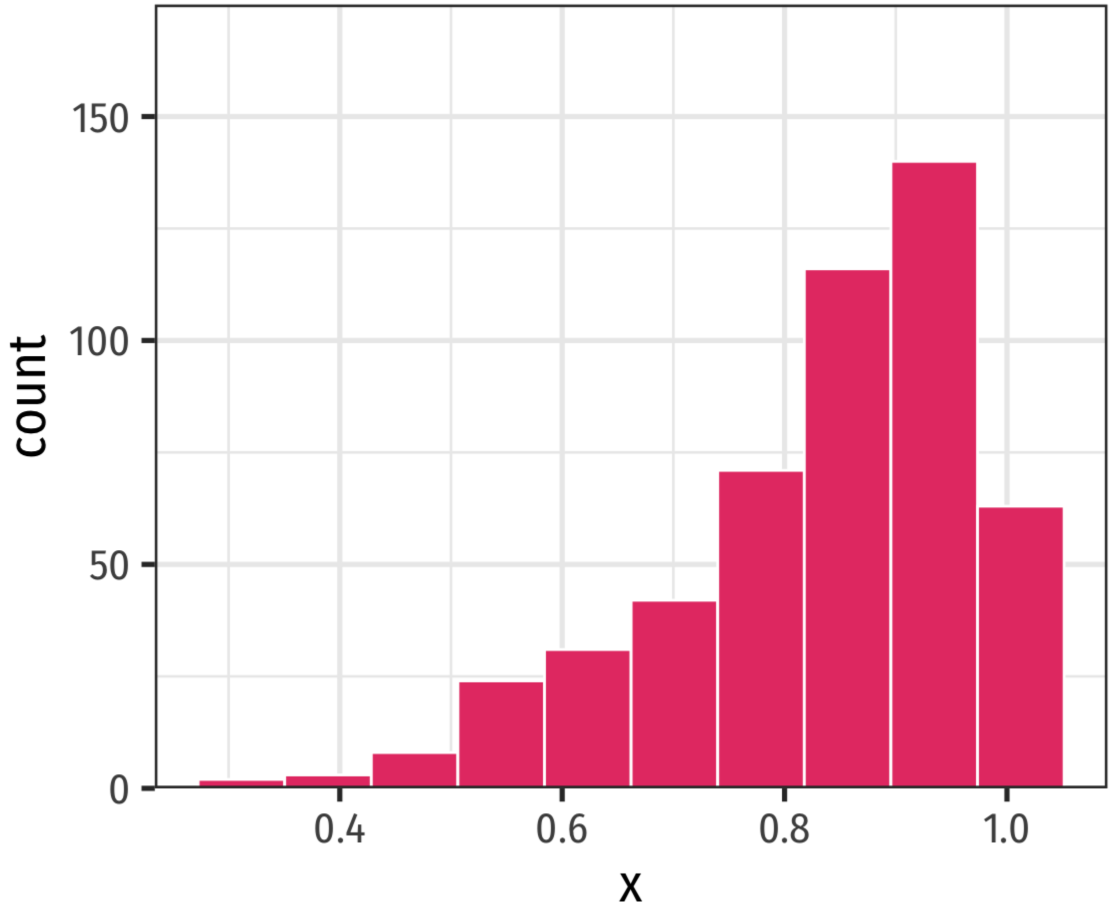
Symmetry and Skew I

- A distribution is **symmetric** if it looks roughly the same on either side of the “center”
- The thinner ends (far left and far right) are called the **tails** of a distribution



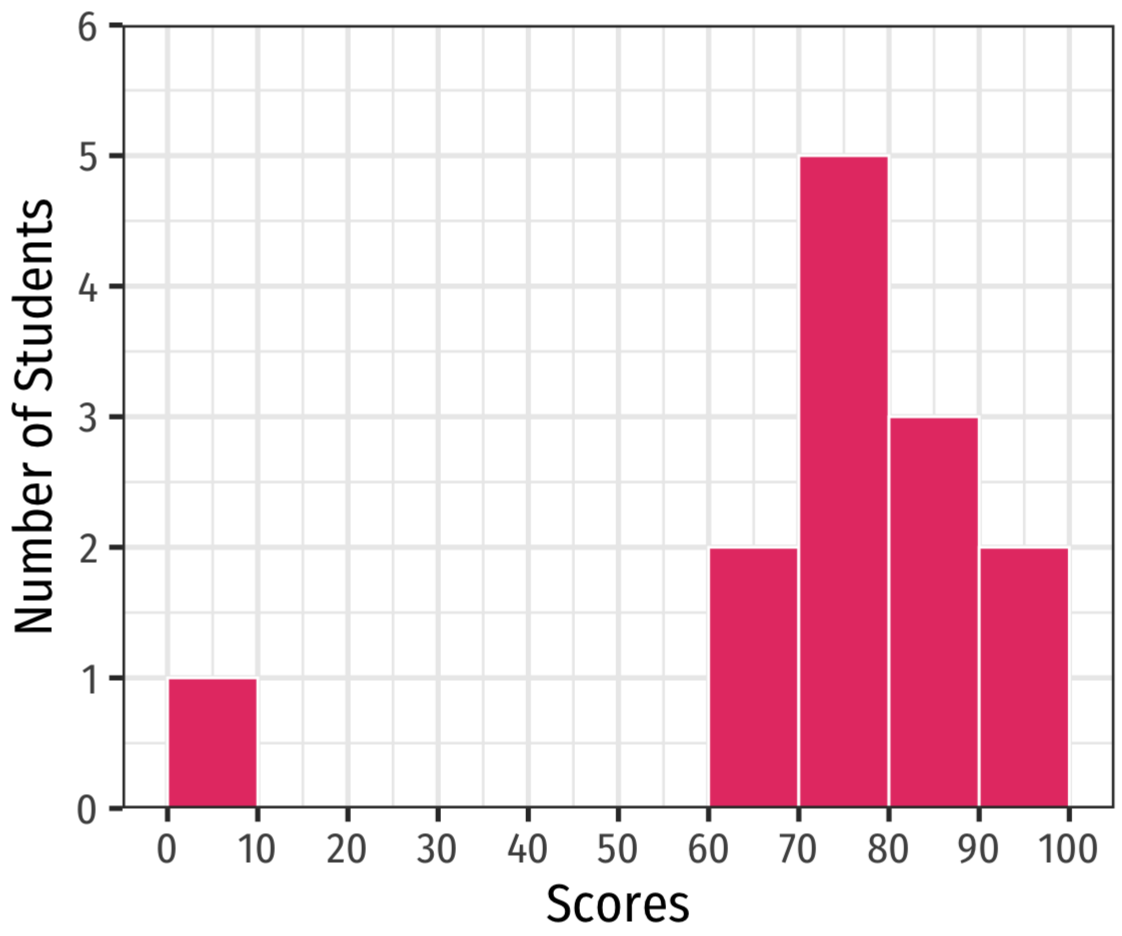
Symmetry and Skew I

- If one tail stretches farther than the other, distribution is **skewed** in the direction of the longer tail
  - In this example, skewed to the **left**



Outliers

- **Outlier**: "extreme" value that does not appear part of the general pattern of a distribution
- Can strongly affect descriptive statistics
- Might be the most informative part of the data
- Could be the result of errors
- Should always be explored and discussed!



Arithmetic Mean (Population)

- The natural measure of the center of a *population*'s distribution is its **"average"** or **arithmetic mean μ**

$$\mu = \frac{x_1 + x_2 + \ldots + x_N}{N} = \frac{1}{N}\sum_{i=1}^{N} x_i$$

- For N values of variable x, "mu" is the sum of all individual x values (x_i) from 1 to N, divided by the N number of values[1]
- See today's appendix for more about the **summation operator, $\sum$**, it'll come up again!

1. Note the mean need not be an actual value of the data!

Arithmetic Mean (Sample)

- When we have a *sample*, we compute the **sample mean $\bar{x}$**

$$\bar{x} = \frac{x_1 + x_2 + \ldots + x_n}{n} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

- For n values of variable x, "x-bar" is the sum of all individual x values (x_i) divided by the n number of values

Arithmetic Mean (Sample)

Example

$$\{0, 62, 66, 71, 71, 74, 76, 79, 83, 86, 88, 93, 95\}$$

$$\bar{x} = \frac{1}{13}(0 + 62 + 66 + 71 + 71 + 74 + 76 + 79 + 83 + 86 + 88 + 93 + 95)$$

$$\bar{x} = \frac{944}{13}$$

$$\bar{x} = 72.62$$

```r
quizzes %>%
  summarize(mean = mean(scores))
```

```
# A tibble: 1 × 1
   mean
  <dbl>
1  72.6
```

Arithmetic Mean: Affected by Outliers

Example: If we drop the outlier (0)

$$\{62, 66, 71, 71, 74, 76, 79, 83, 86, 88, 93, 95\}$$

$$\begin{aligned}\bar{x} &= \frac{1}{12}(62 + 66 + 71 + 71 + 74 + 76 + 79 + 83 + 86 + 88 + 93 + 95)\\ &= \frac{944}{12}\\ &= 78.67\end{aligned}$$

```r
quizzes %>%
  filter(scores > 0) %>%
  summarize(mean = mean(scores))
```

```
# A tibble: 1 × 1
   mean
  <dbl>
1  78.7
```

Median

$$\{0, 62, 66, 71, 71, 74, \mathbf{76}, 79, 83, 86, 88, 93, 95\}$$

- The **median** is the midpoint of the distribution
  - 50% to the left of the median, 50% to the right of the median
- Arrange values in numerical order
  - For odd n: median is middle observation
  - For even n: median is average of two middle observations

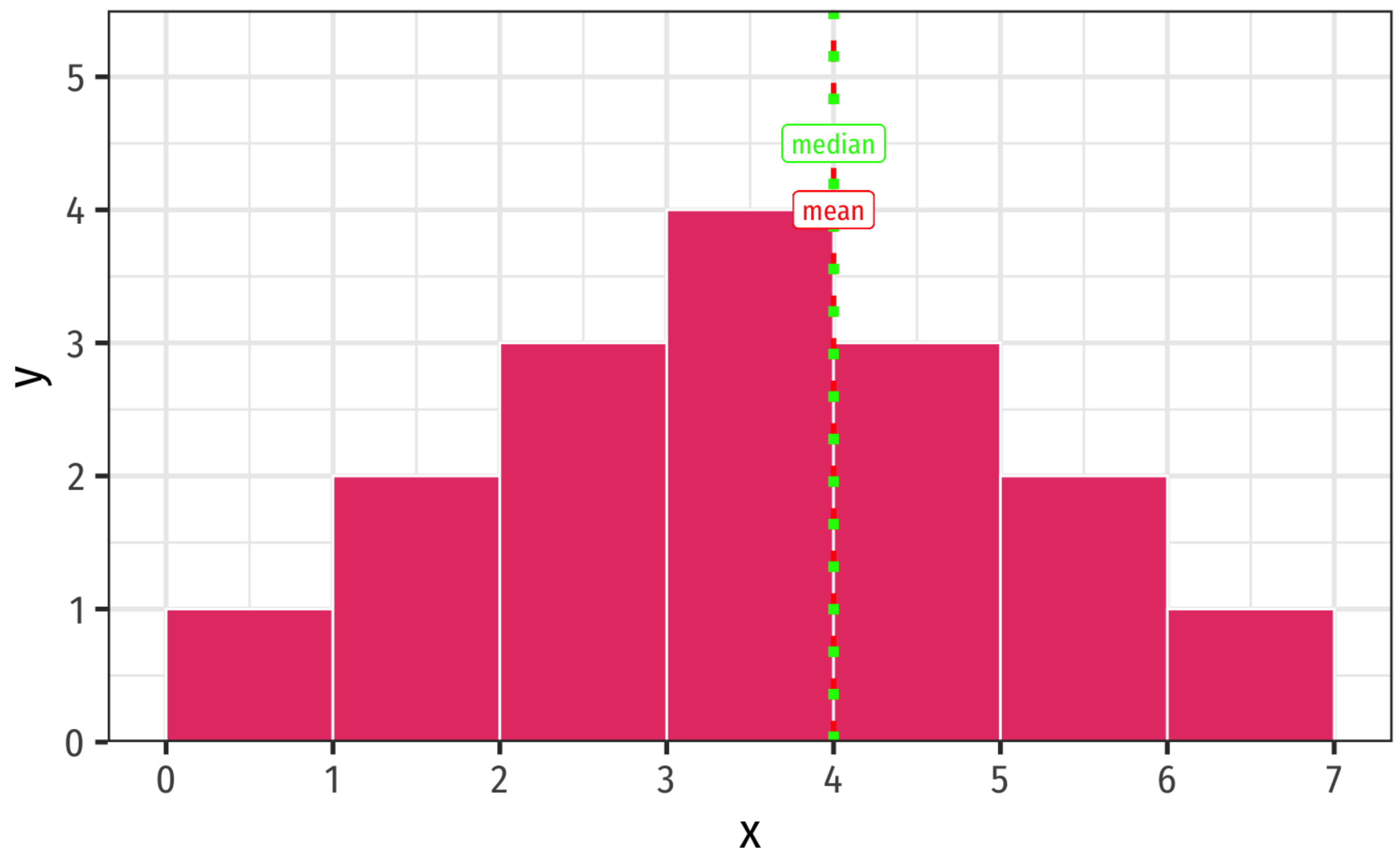
Mean, Median, and Outliers

Mean, Median, Symmetry, & Skew I

- Symmetric distribution: mean $\approx$ median

```
symmetric %>%
  summarize(mean = mean(x),
            median = median(x))
```

```
# A tibble: 1 × 2
   mean median
  <dbl>  <dbl>
1     4      4
```

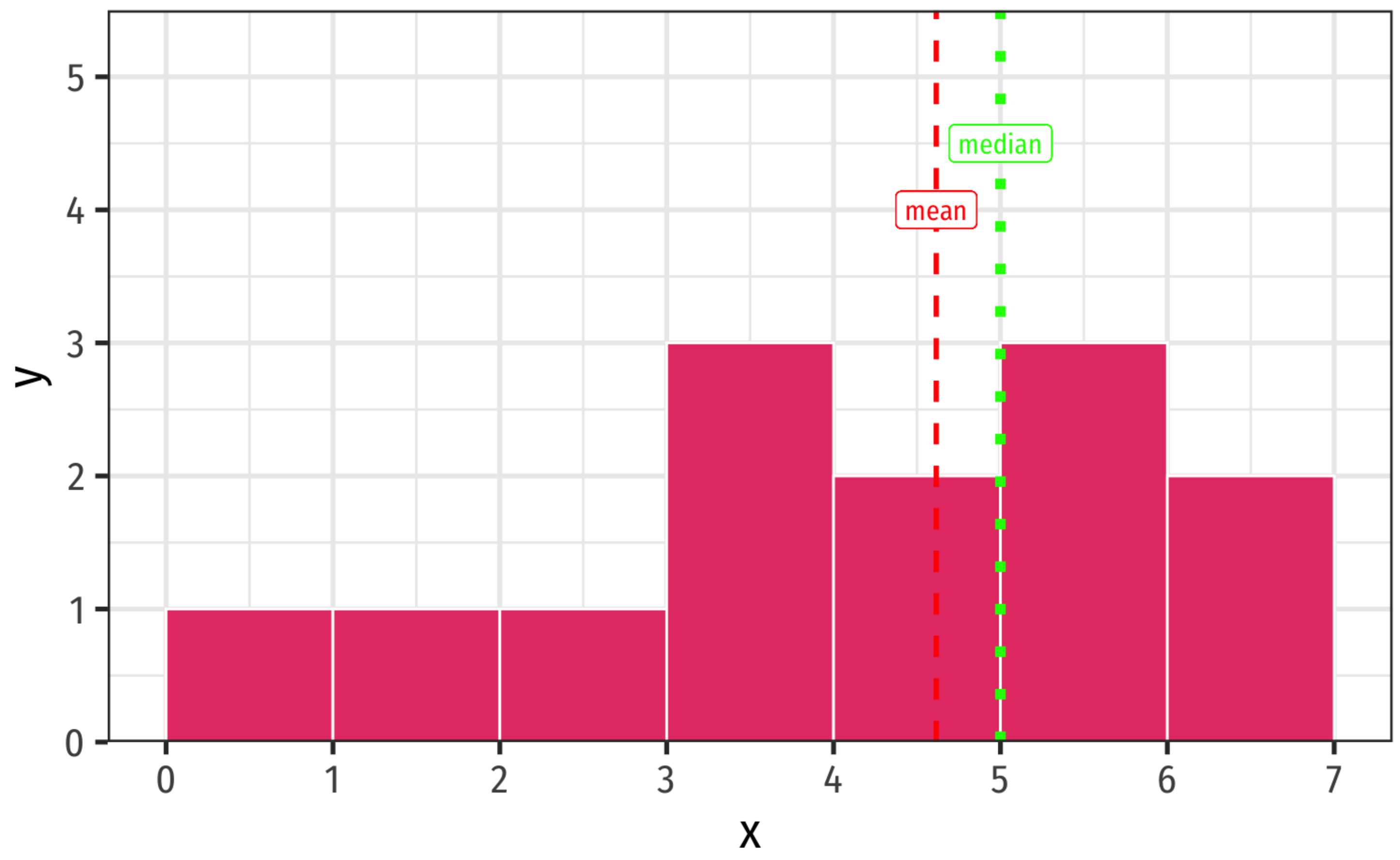


Mean, Median, Symmetry, & Skew II

- Left-skewed: mean < median

```r
leftskew %>%
  summarize(mean = mean(x),
            median = median(x))
```

```
# A tibble: 1 × 2
   mean median
  <dbl>  <dbl>
1  4.62      5
```

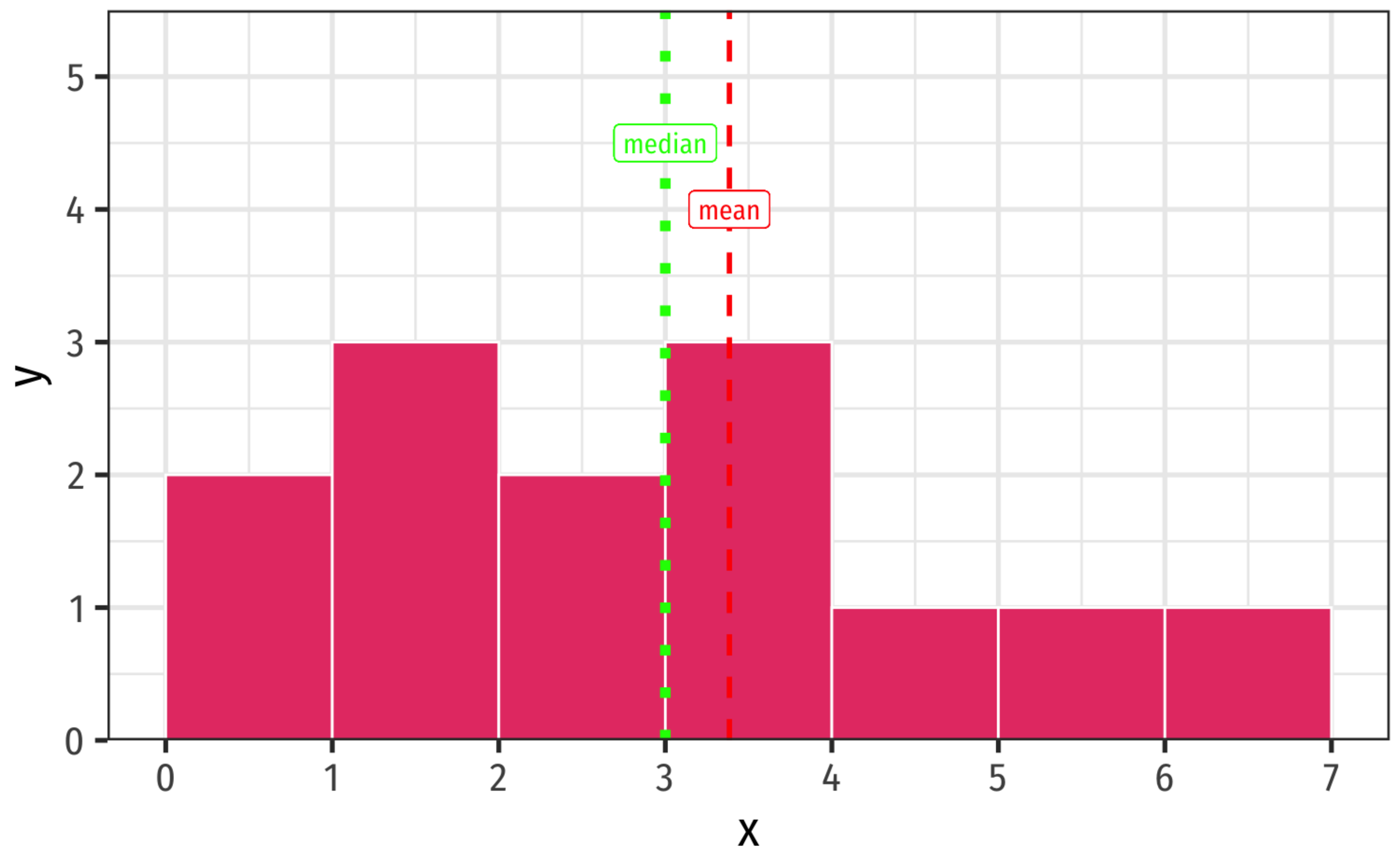


Mean, Median, Symmetry, & Skew III

- Right-skewed: mean > median

```
rightskew %>%
  summarize(mean = mean(x),
            median = median(x))
```

```
# A tibble: 1 × 2
   mean median
  <dbl>  <dbl>
1  3.38      3
```



Measures of Dispersion

Range

- The more *variation* in the data, the less helpful a measure of central tendency will tell us
- Beyond just the center, we also want to measure the spread
- Simplest metric is **range** $= max - min$

Five Number Summary I

- Common set of summary statistics of a distribution: **"five number summary"**:

1. Minimum value
2. 25th percentile (Q_1, median of first 50% of data)
3. 50th percentile (median, Q_2)
4. 25th percentile (Q_3, median of last 50% of data)
5. Maximum value

```r
# Base R summary command
summary(quizzes$scores)
```

```
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max. 
   0.00   71.00   76.00   72.62   86.00   95.00 
```

```r
quizzes %>% # dplyr
  summarize(Min = min(scores),
            Q1 = quantile(scores, 0.25),
            Median = median(scores),
            Q3 = quantile(scores, 0.75),
            Max = max(scores))
```

```
# A tibble: 1 × 5
    Min    Q1 Median    Q3   Max
  <dbl> <dbl>  <dbl> <dbl> <dbl>
1     0    71     76    86    95
```

Five Number Summary II

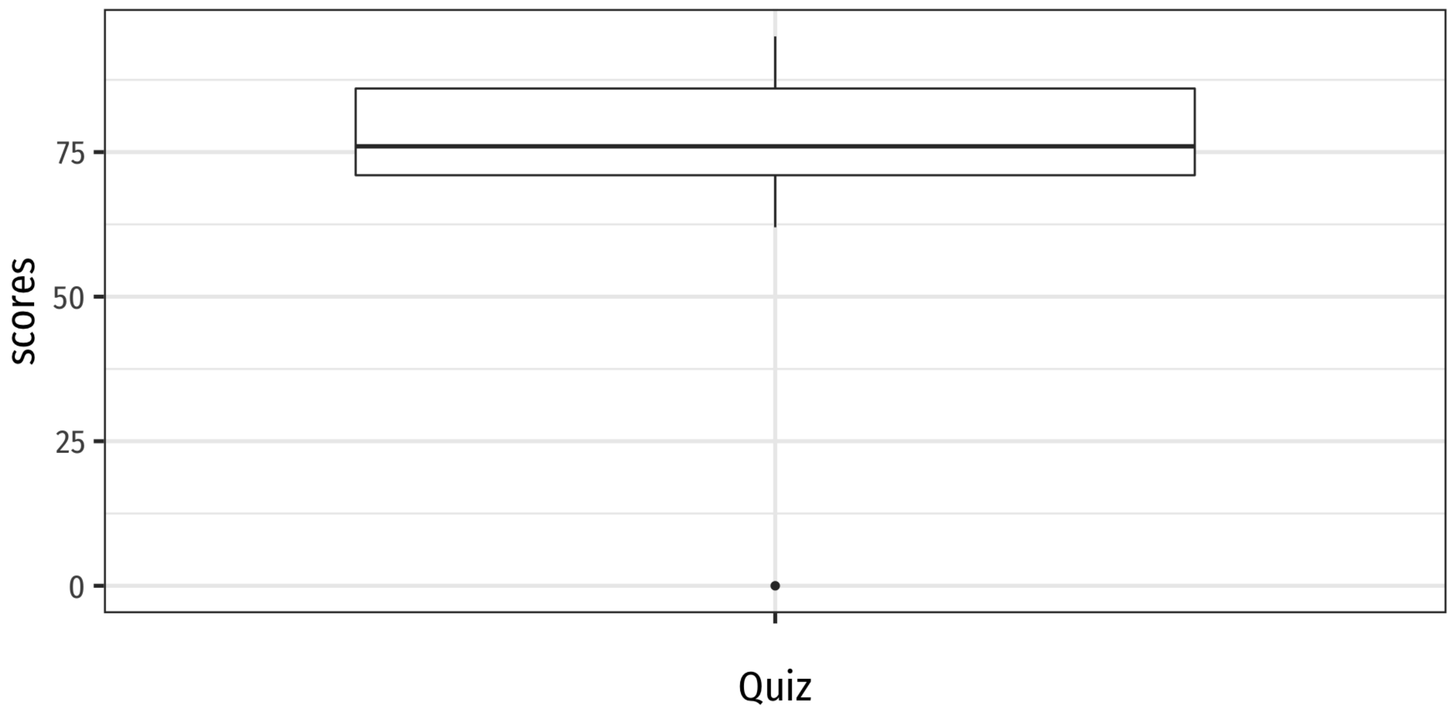
- The n^{th} **percentile** of a distribution is the value that places n percent of values beneath it

```
quizzes %>%
  summarize("37th percentile" = quantile(scores,0.37))
```

```
# A tibble: 1 × 1
  `37th percentile`
              <dbl>
1              72.3
```

Boxplot I

- **Boxplots** are a great way to visualize the 5 number summary
- **Height of box**: Q_1 to Q_3 (known as **interquartile range (IQR)**, middle 50% of data)
- **Line inside box**: median (50^{th} percentile)
- **"Whiskers"** identify data within $1.5 \times IQR$
- Points *beyond* whiskers are **outliers**
  - common definition: Outlier $> 1.5 \times IQR$



Boxplot Comparisons I

- Boxplots (and five number summaries) are great for comparing two distributions

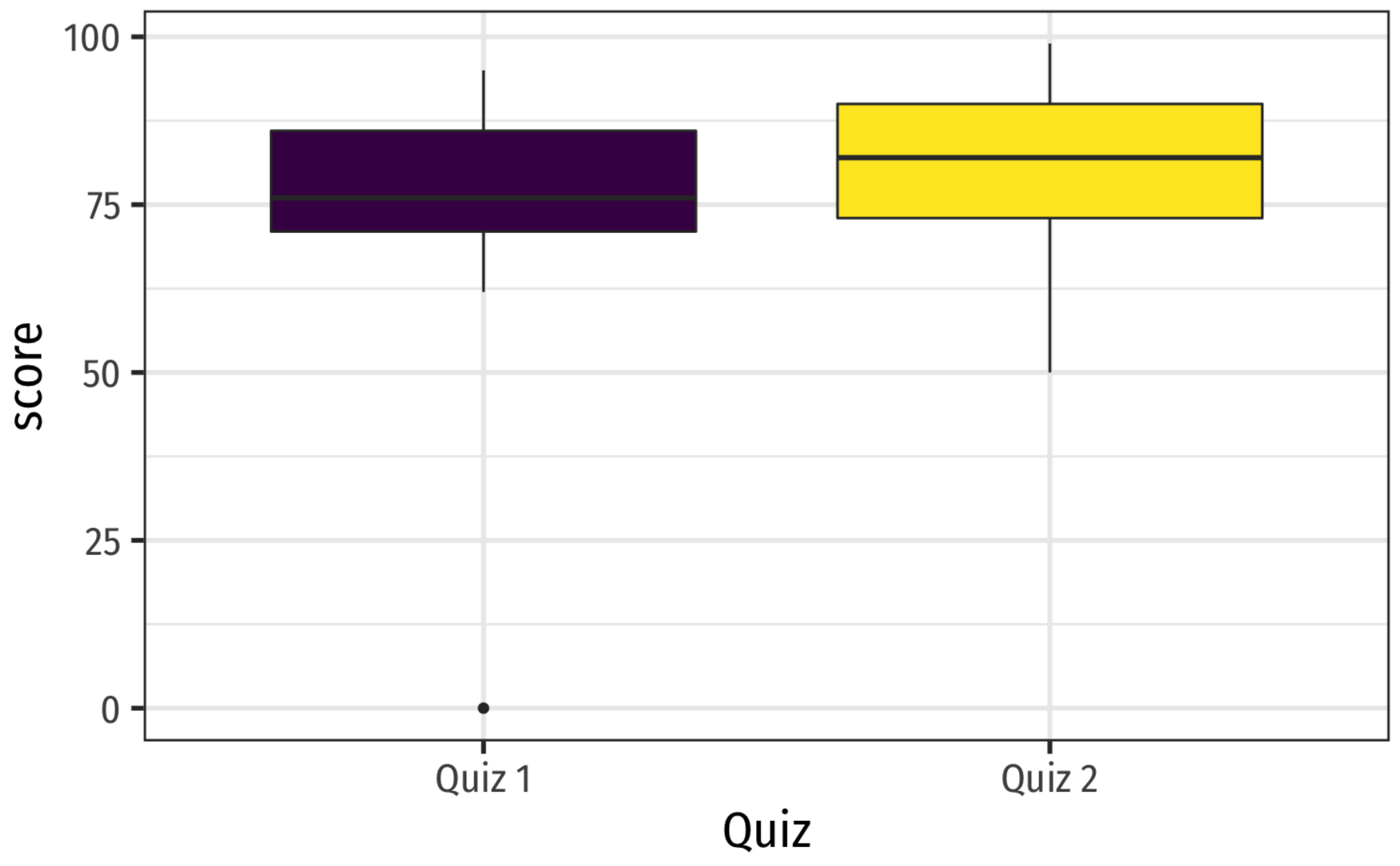
Example

$$\begin{aligned}\text{Quiz 1} &: \{0, 62, 66, 71, 71, 74, 76, 79, 83, 86, 88, 93, 95\}\\ \text{Quiz 2} &: \{50, 62, 72, 73, 79, 81, 82, 82, 86, 90, 94, 98, 99\}\end{aligned}$$

Boxplot Comparisons II

```
quizzes_new %>% summary()
```

```
    student          quiz_1          quiz_2     
 Min.   : 1   Min.   : 0.00   Min.   :50.00  
 1st Qu.: 4   1st Qu.:71.00   1st Qu.:73.00  
 Median : 7   Median :76.00   Median :82.00  
 Mean   : 7   Mean   :72.62   Mean   :80.62  
 3rd Qu.:10   3rd Qu.:86.00   3rd Qu.:90.00  
 Max.   :13   Max.   :95.00   Max.   :99.00  
```



Aside: Making Nice Summary Tables I

- I don't like the options available for printing out summary statistics
- So I wrote my own `R function` called `summary_table()` that makes nice summary tables (it uses `dplyr` and `tidyr`!). To use:

1. Download the `summaries.R` file from the website[1] and move it to your working directory/project folder
2. Load the function with the `source()` command:[2]

```
source("summaries.R")
```

1. One day I'll make this part of a package I'll write.
2. If it *was* a package, then you'd load with `library()`. But you can run a single .R script with `source()`

Aside: Making Nice Summary Tables II

3. The function has at least 2 arguments: the `data.frame` (automatically piped in if you use the pipe!) and then all variables you want to summarize, separated by commas[1]

```
mpg %>%
  summary_table(hwy, cty, cyl)
```

```
# A tibble: 3 × 9
  Variable   Obs   Min    Q1 Median    Q3   Max  Mean `Std. Dev.`
  <chr>    <dbl> <dbl> <dbl>  <dbl> <dbl> <dbl> <dbl>       <dbl>
1 cty        234     9    14     17    19    35 16.9         4.26
2 cyl        234     4     4      6     8     8  5.89        1.61
3 hwy        234    12    18     24    27    44 23.4         5.95
```

1. There is one restriction: No variable name can have an underscore (`_`) in it. You will have to rename them or else you will break the function!

Aside: Making Nice Summary Tables III

4. When `rendered` in `Quarto`, it looks nicer:

```
mpg %>%
  summary_table(hwy, cty, cyl) %>%
  knitr::kable(., format="html")
```

Variable	Obs	Min	Q1	Median	Q3	Max	Mean	Std. Dev.
cty	234	9	14	17	19	35	16.86	4.26
cyl	234	4	4	6	8	8	5.89	1.61
hwy	234	12	18	24	27	44	23.44	5.95

Measures of Dispersion: Deviations

- Every observation i **deviates** from the mean of the data:

$$deviation_i = x_i - \mu$$

- There are as many deviations as there are data points (n)
- We can measure the *average* or **standard deviation** of a variable from its mean
- Before we get there...

Variance (Population)

- The **population variance** σ^2 of a *population* distribution measures the average of the *squared* deviations from the *population* mean (μ)

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2$$

- Why do we square deviations?
- What are these units?

Standard Deviation (Population)

- Square root the variance to get the **population standard deviation σ**, the average deviation from the population mean (in same units as x)

$$\sigma = \sqrt{\sigma^2} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2}$$

Variance (Sample)

- The **sample variance** s^2 of a *sample* distribution measures the average of the *squared* deviations from the *sample* mean $(\bar{x})$

$$\sigma^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

- Why do we divide by $n-1$?

Standard Deviation (Sample)

- Square root the sample variance to get the **sample standard deviation** s, the average deviation from the *sample* mean (in same units as x)

$$s = \sqrt{s^2} = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2}$$

Sample Standard Deviation: Example

Example

Calculate the sample standard deviation for the following series:

$$\{2, 4, 6, 8, 10\}$$

```r
sd(c(2,4,6,8,10))
```

```
[1] 3.162278
```

The Steps to Calculate sd(), Coded I

```r
#  first let's save our data in a tibble
sd_example <- tibble(x = c(2,4,6,8,10))
```

```r
# first find the mean (just so we know)

sd_example %>%
  summarize(mean(x))
```

```
# A tibble: 1 × 1
  `mean(x)`
      <dbl>
1         6
```

```r
# now let's make some more columns:
sd_example <- sd_example %>%
  mutate(deviations = x - mean(x), # take deviations from mean
         deviations_sq = deviations^2) # square them
```

The Steps to Calculate `sd()`, Coded II

```
sd_example # see what we made
```

```
# A tibble: 5 × 3
      x deviations deviations_sq
  <dbl>      <dbl>         <dbl>
1     2         -4            16
2     4         -2             4
3     6          0             0
4     8          2             4
5    10          4            16
```

The Steps to Calculate `sd()`, Coded III

```r
sd_example %>%
  # sum the squared deviations
  summarize(sum_sq_devs = sum(deviations_sq),
            # divide by n-1 to get variance
            variance = sum_sq_devs/(n()-1),
            # square root to get sd
            std_dev = sqrt(variance))
```

```
# A tibble: 1 × 3
  sum_sq_devs variance std_dev
        <dbl>    <dbl>   <dbl>
1          40       10    3.16
```

Sample Standard Deviation: You Try

Example

Calculate the sample standard deviation for the following series:

$$\{1, 3, 5, 7\}$$

```r
sd(c(1,3,5,7))
```

```
[1] 2.581989
```

Descriptive Statistics: Populations vs. Samples

Population parameters

- **Population size**: N
- **Mean**: μ
- **Variance**: $\sigma^2 = \frac{1}{N} \sum_{i=1}^{N} (x_i - \mu)^2$
- **Standard deviation**: $\sigma = \sqrt{\sigma^2}$

Sample statistics

- **Population size**: n
- **Mean**: $\bar{x}$
- **Variance**: $s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2$
- **Standard deviation**: $s = \sqrt{s^2}$