

3.2 – Causal Inference and DAGs

ECON 480 • Econometrics • Fall 2021

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 [ryansafner/metricsF21](https://github.com/ryansafner/metricsF21)

 metricsF21.classes.ryansafner.com



Outline

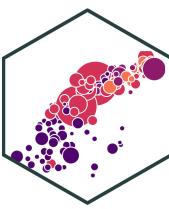


Correlation vs. Causation

Causal Diagrams

DAG Rules

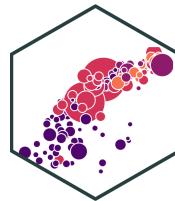
You Don't Need an RCT to Talk About Causality



- Statistics profession is obstinant that we cannot say anything about causality
- But you have to! It's how the human brain works!
- We can't conceive of (spurious) correlation without some causation



The Causal Revolution



Laura Hatfield
@laura_tastic

Wow: this comment from fresh page proofs.

Guess all of us researching causal inference in observational data need to find new jobs?

3

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Commented [DT1]: Causal language (including use of terms such as effect, efficacy, and predictor) should be used only for randomized clinical trials. For all other study designs, methods and results should be described in terms of association or, if appropriate tests were used, correlation, and should avoid cause-and-effect wording. We have eliminated causal language from the manuscript.



Seva
@SevaUT

normal person: this rain is making us wet

me, RCT genius: whoa there! First, take twenty walks and randomly apply the rain treatment



Laura Hatfield @laura_tastic · Jan 16

Wow: this comment from fresh page proofs.

Guess all of us researching causal inference in observational data need to find new jobs?

3

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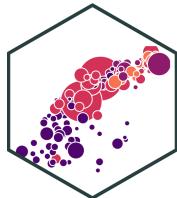
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RCTs and Evidence-Based Policy



- Should we *ONLY* base policies on the evidence from Randomized Controlled Trials?

 Dr Ellie Murray, ScD 
@EpiEllie 

IF U DONT SMOKE,
U ALREADY
BELIEVE IN
CAUSAL INFERENCE
WITHOUT
RANDOMIZED TRIALS

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#HistorianSignBunny #Epidemiology

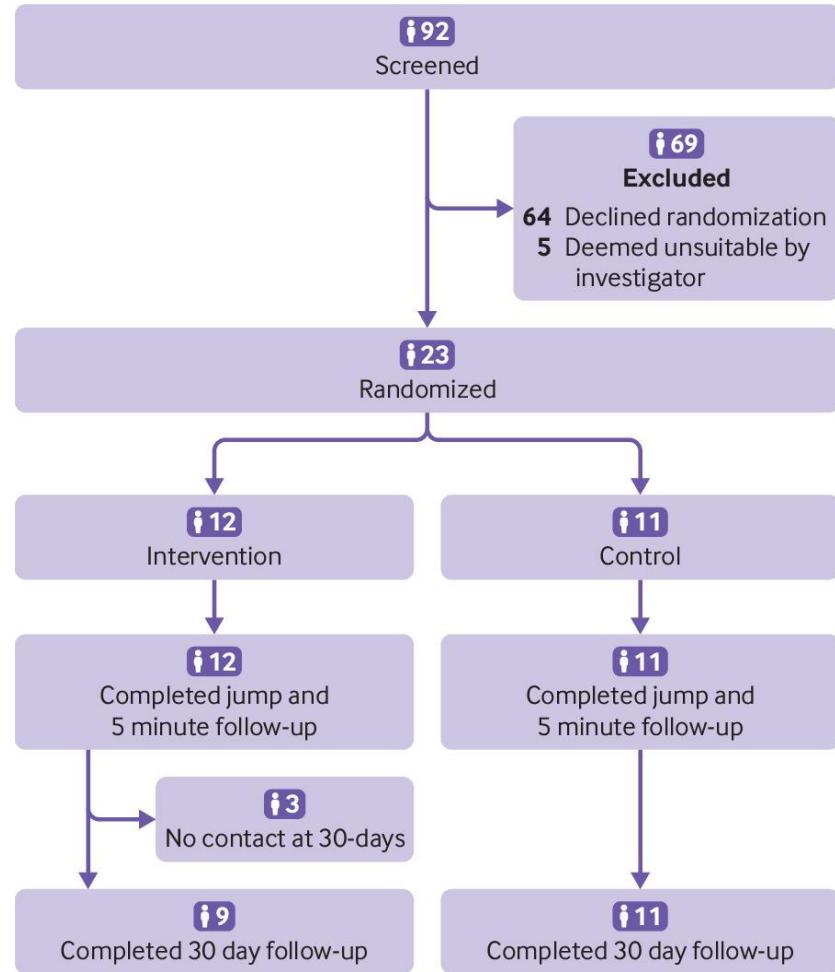
12:13 AM · Jul 13, 2018 

 940  33  Copy link to Tweet

 Tweet your reply

Source: [British Medical Journal](#)

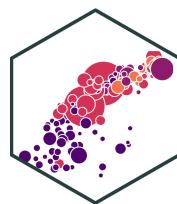
RCTs and Evidence-Based Policy III





Correlation vs. Causation

Correlation and Causation I



David Robinson @drob · Jun 22, 2017

Correlation implies causation, don't @ me



David Robinson
@drob

"Correlation implies causation," the dean whispered as he handed me my PhD.

"But then why-

"Because if they knew, they wouldn't need us."

3:46 PM · Jun 22, 2017 from Manhattan, NY



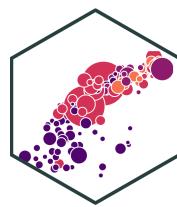
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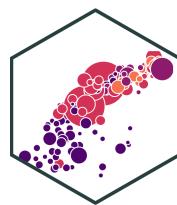
What Does Causation Mean?



- “Correlation does not imply causation”
 - this is exactly backwards!
 - this is just pointing out that **exogeneity is violated**



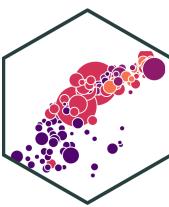
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- “Correlation does not imply causation”
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- “Correlation implies causation”
 - for an association, there must be *some* causal chain that relates X and Y
 - but not necessarily *merely* $X \rightarrow Y$



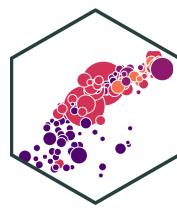
What Does Causation Mean?



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- “Correlation implies causation”
 - for an association, there must be *some* causal chain that relates X and Y
 - but not necessarily *merely* $X \rightarrow Y$
- “Correlation plus exogeneity is causation.”



Correlation and Causation



- **Correlation:**

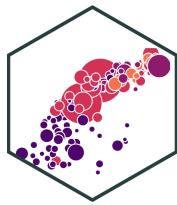
- Math & Statistics
- Computers, AI, Machine learning can figure this out (better than humans)

- **Causation:**

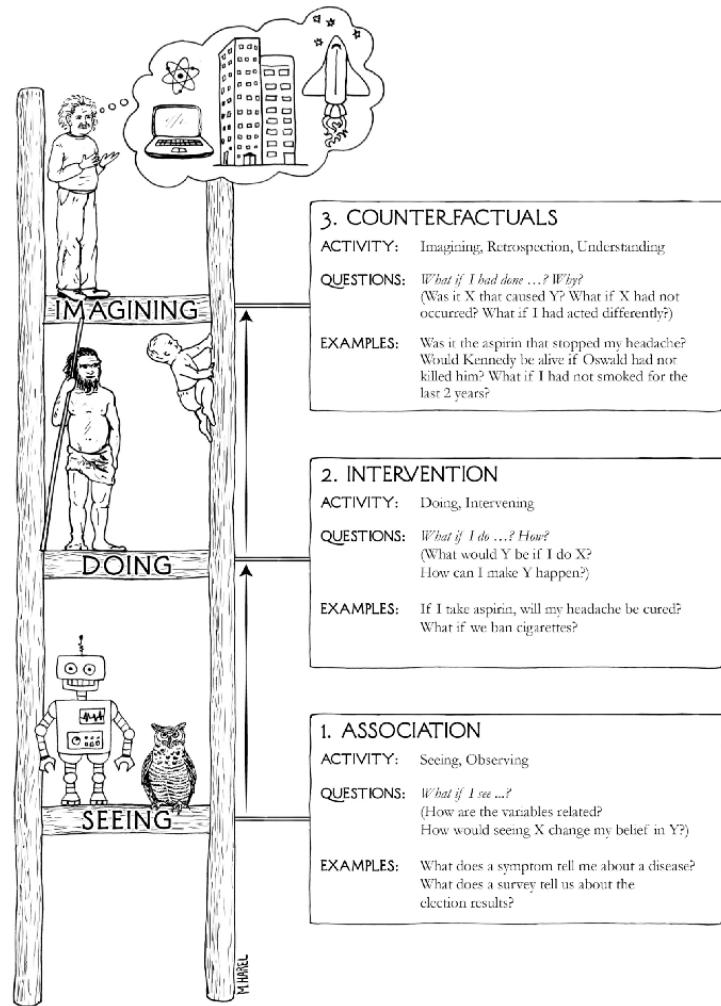
- Philosophy, Intuition, Theory
- **Counterfactual thinking**, unique to humans (vs. animals or computers)
- Computers cannot (yet) figure this out



The Causal Revolution



Causation Requires Counterfactual Thinking



JUDEA PEARL
WINNER OF THE TURING AWARD
AND DANA MACKENZIE

THE BOOK OF WHY



THE NEW SCIENCE
OF CAUSE AND EFFECT



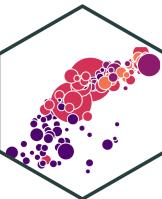
Causal Inference



- We will seek to understand what causality *is* and how we can approach finding it
- We will also explore the different common **research designs** meant to **identify** causal relationships
- **These skills**, more than supply & demand, constrained optimization models, ISLM, etc, **are the tools and comparative advantage of a modern research economist**



“The Credibility Revolution”



BREAKING NEWS:

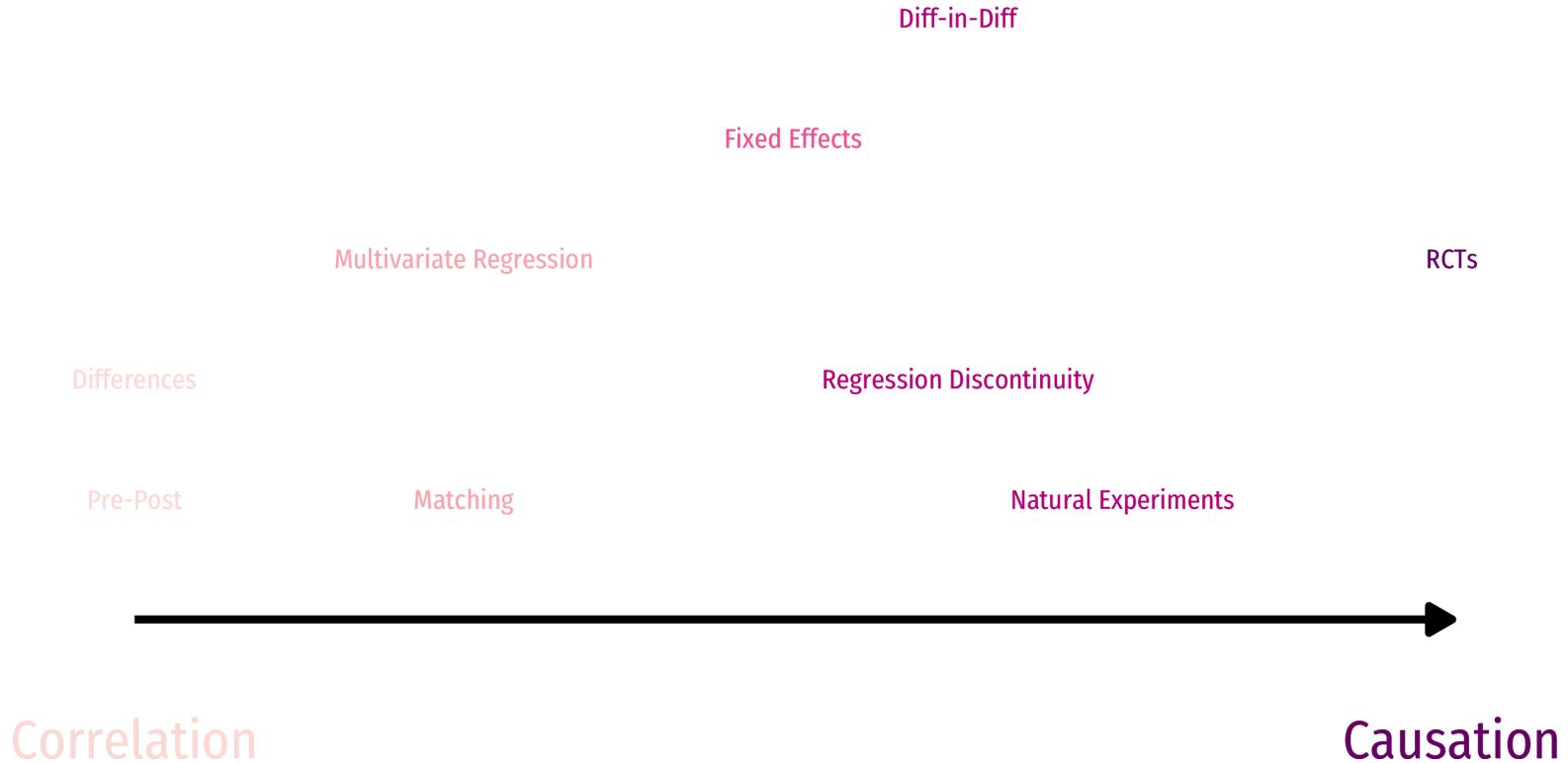
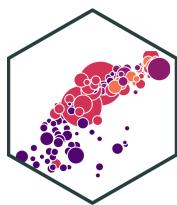
The 2021 Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel has been awarded with one half to David Card and the other half jointly to Joshua D. Angrist and Guido W. Imbens.

#NobelPrize

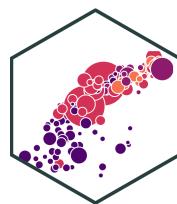


- Simultaneous “credibility revolution” in econometrics (c.1990s–2000s)
- Use clever research designs to approximate natural experiments
- Note: major disagreements between Pearl & Angrist/Imbens, etc.!

Clever Research Designs Identify Causality



Correlation and Causation



John B. Holbein @JohnHolbein1 · Apr 7, 2018



Causality isn't binary; it's a continuum.



John B. Holbein
@JohnHolbein1

Causality isn't achieved; it's approached.

11:05 AM · Apr 7, 2018



7



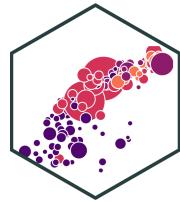
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Copy link to Tweet

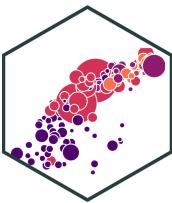
[Tweet your reply](#)

What Then IS Causation?



- X causes Y if we can intervene and change X without changing anything else, and Y changes
- Y “listens to” X
 - X may not be the only thing that causes Y !

What Then IS Causation?



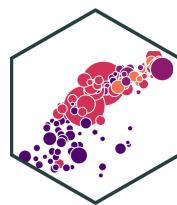
- X causes Y if we can intervene and change X without changing anything else, and Y changes
- Y “listens to” X
 - X may not be the only thing that causes Y !

Example

If X is a light switch, and Y is a light:

- Flipping the switch (X) causes the light to go on (Y)
- But NOT if the light is burnt out (No Y despite X)

Non-Causal Claims



- All of the following have non-zero correlations. Are they *causal*?

Example

- Greater ice cream sales → more violent crime
- Rooster crows → the sun rises in the morning
- Taking Vitamin C → colds go away a few days later
- Political party X in power → economy performs better/worse

Counterfactuals



- The *sine qua non* of causal claims are **counterfactuals**: what would Y have been if X had been different?
- It is **impossible** to make a counterfactual claim from data alone!
- Need a (theoretical) causal model of the data-generating process!



Counterfactuals and RCTs



- Again, RCTs are invoked as the gold standard for their ability to make counterfactual claims:
- Treatment/intervention (X) is **randomly assigned** to individuals

If person i who received treatment *had not received* the treatment, we can predict what his outcome *would have been*

If person j who did not receive treatment *had received treatment*, we can predict what her outcome *would have been*



- We can say this because, on average, treatment and control groups are *the same before treatment*

From RCTs to Causal Models



- RCTs are but the best-known method of a large, growing science of **causal inference**
- We need a **causal model** to describe the **data-generating process (DGP)**
- Requires us to make some **assumptions**



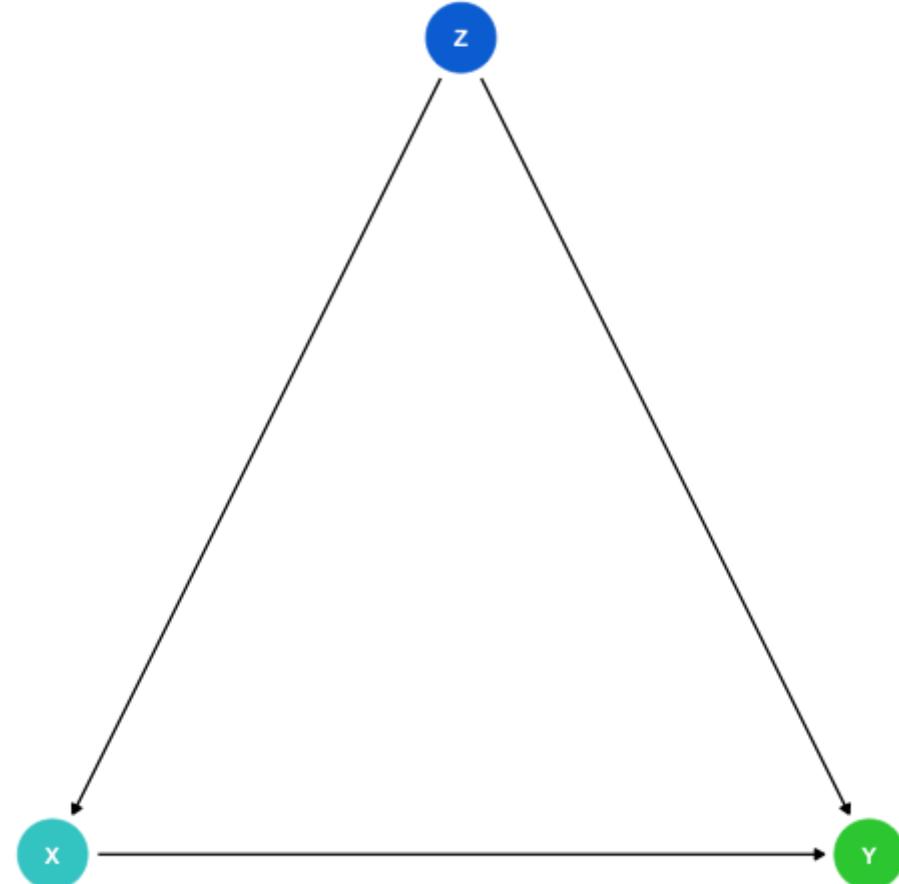


Causal Diagrams

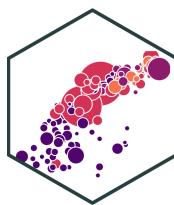
Causal Diagrams/DAGs



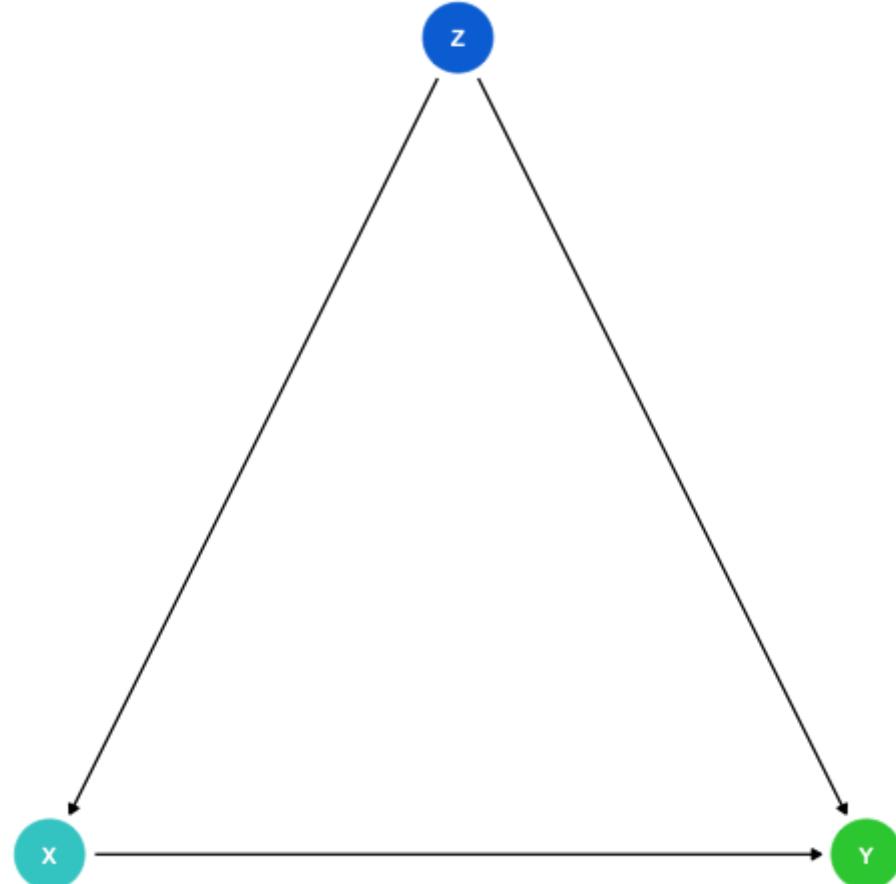
- A surprisingly simple, yet rigorous and powerful method of modeling is using a **causal diagram** or **DAG**:
 - **Directed**: Each node has arrows that points only one direction
 - **Acyclic**: Arrows only have one direction, and cannot loop back
 - **Graph**



Causal Diagrams/DAGs



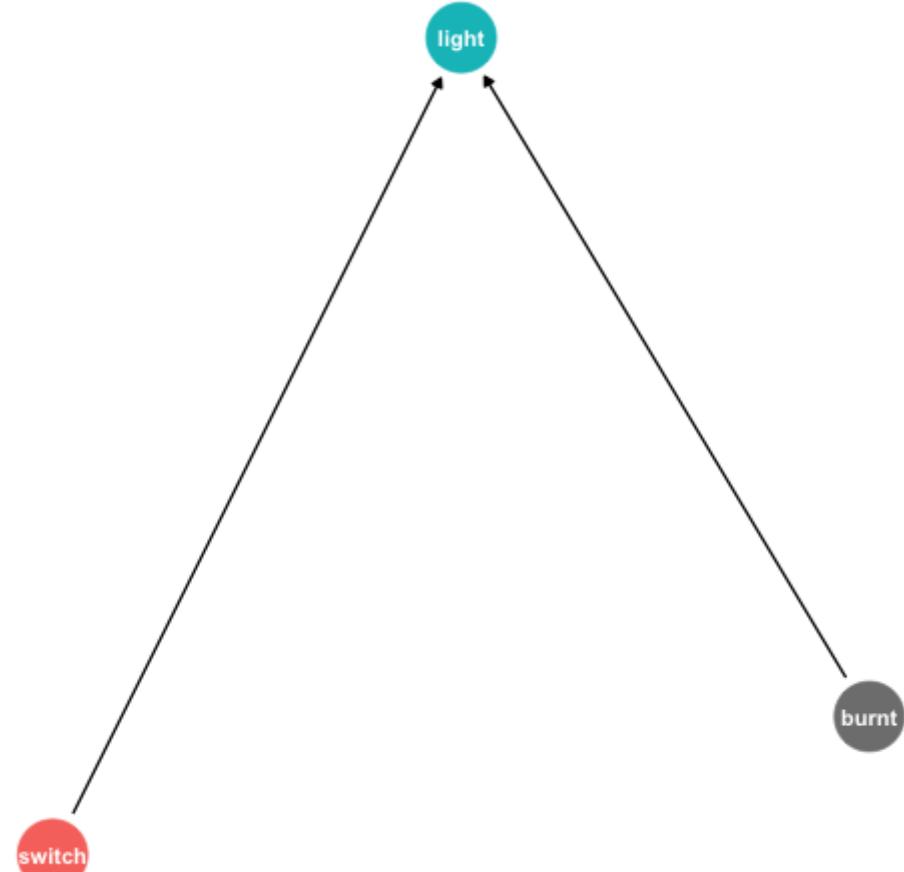
- A visual model of the data-generating process, encodes our understanding of the causal relationships
- Requires some common sense/economic intuition
- Remember, all models are wrong, we just need them to be *useful!*



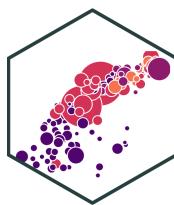
Causal Diagrams/DAGs



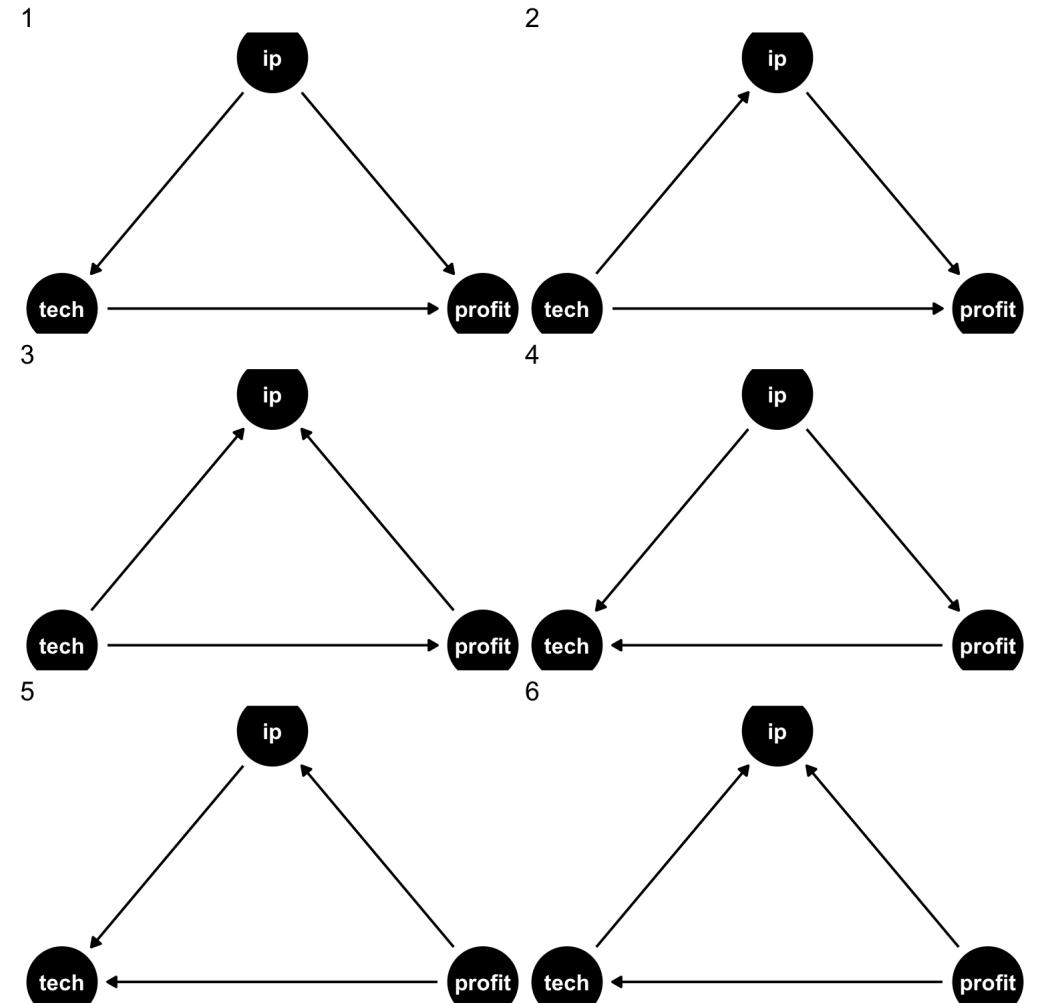
- Our light switch example of causality



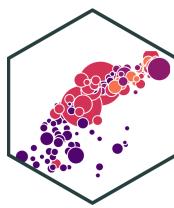
Drawing a DAG: Example



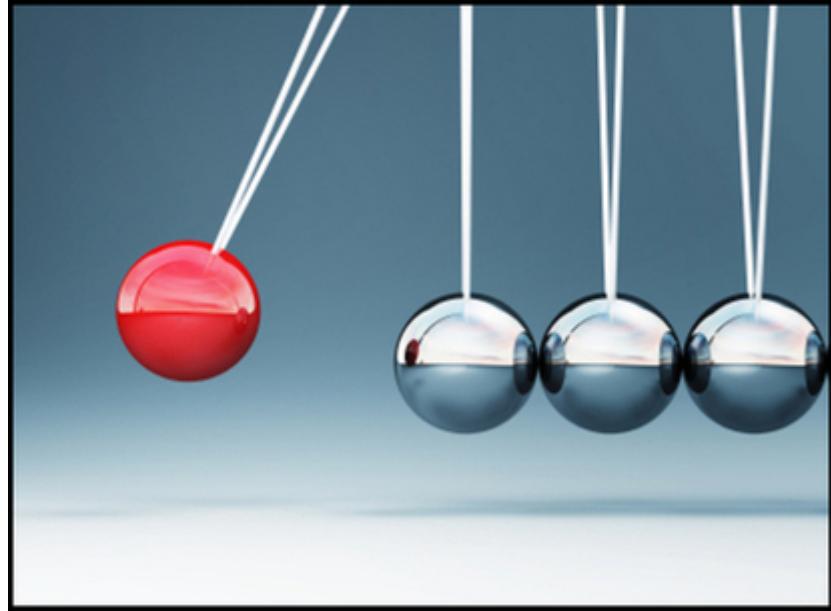
- Suppose we have data on three variables
 - IP : how much a firm spends on IP lawsuits
 - tech : whether a firm is in tech industry
 - profit : firm profits
- They are all correlated with each other, but what's are the causal relationships?
- We need our own **causal model** (from theory, intuition, etc) to sort
 - Data alone will not tell us!



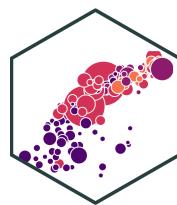
Drawing a DAG:



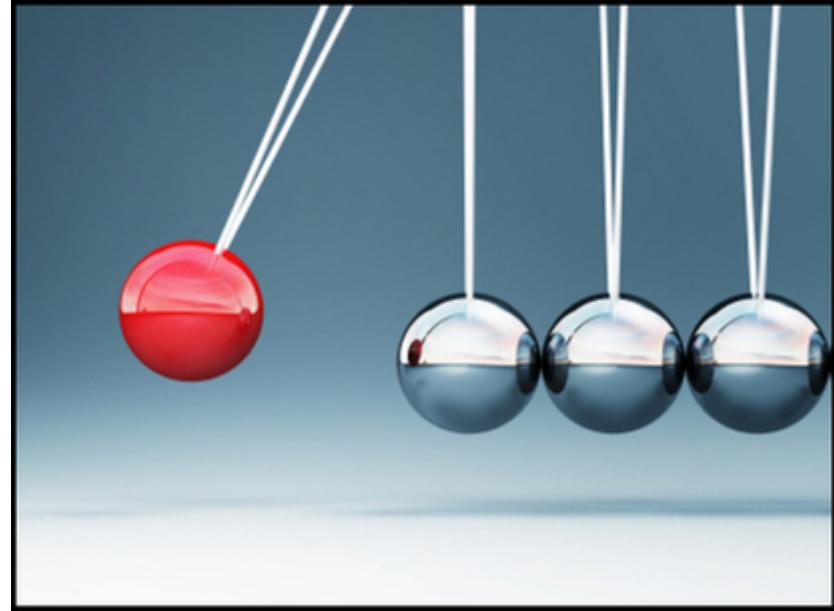
1. Consider all the variables likely to be important to the data-generating process (including variables we can't observe!)
2. For simplicity, combine some similar ones together or prune those that aren't very important
3. Consider which variables are likely to affect others, and draw arrows connecting them
4. Test some testable implications of the model (to see if we have a correct one!)



Side Notes



- Drawing an arrow requires a direction - making a statement about causality!
- *Omitting* an arrow makes an equally important statement too!
 - In fact, we will *need* omitted arrows to show causality!
- If two variables are correlated, but neither causes the other, likely they are both caused by another (perhaps **unobserved**) variable - add it!
- There should be no *cycles* or *loops* (if so, there's probably another missing variable, such as time)



DAG Example I

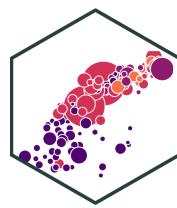


Example: what is the effect of education on wages?

- Education (X , “treatment” or “exposure”)
- Wages (Y , “outcome” or “response”)



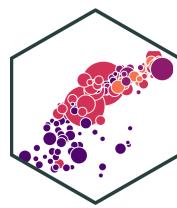
DAG Example I



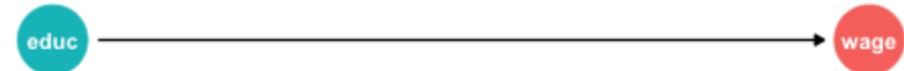
- What other variables are important?
 - Ability
 - Socioeconomic status
 - Demographics
 - Phys. Ed. requirements
 - Year of birth
 - Location
 - Schooling laws
 - Job connections



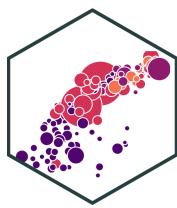
DAG Example I



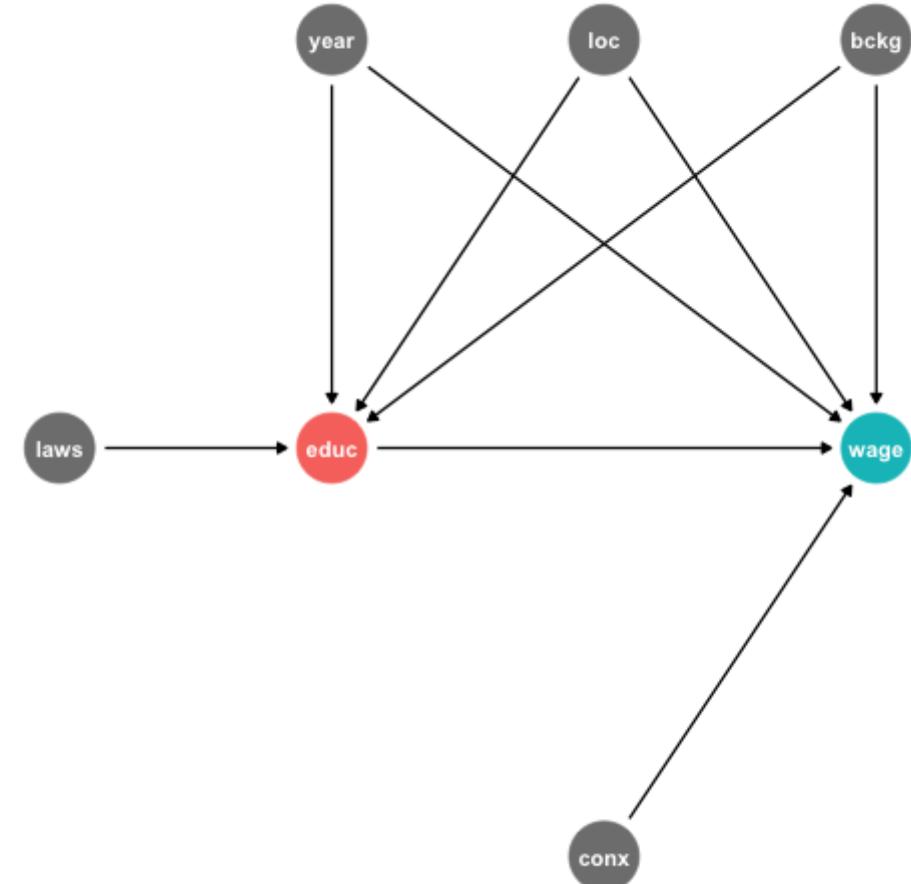
- In social science and complex systems, 1000s of variables could plausibly be in DAG!
- So simplify:
 - Ignore trivial things (Phys. Ed. requirement)
 - Combine similar variables (Socioeconomic status, Demographics, Location) → Background



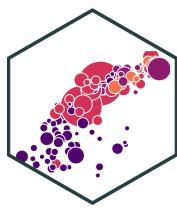
DAG Example II



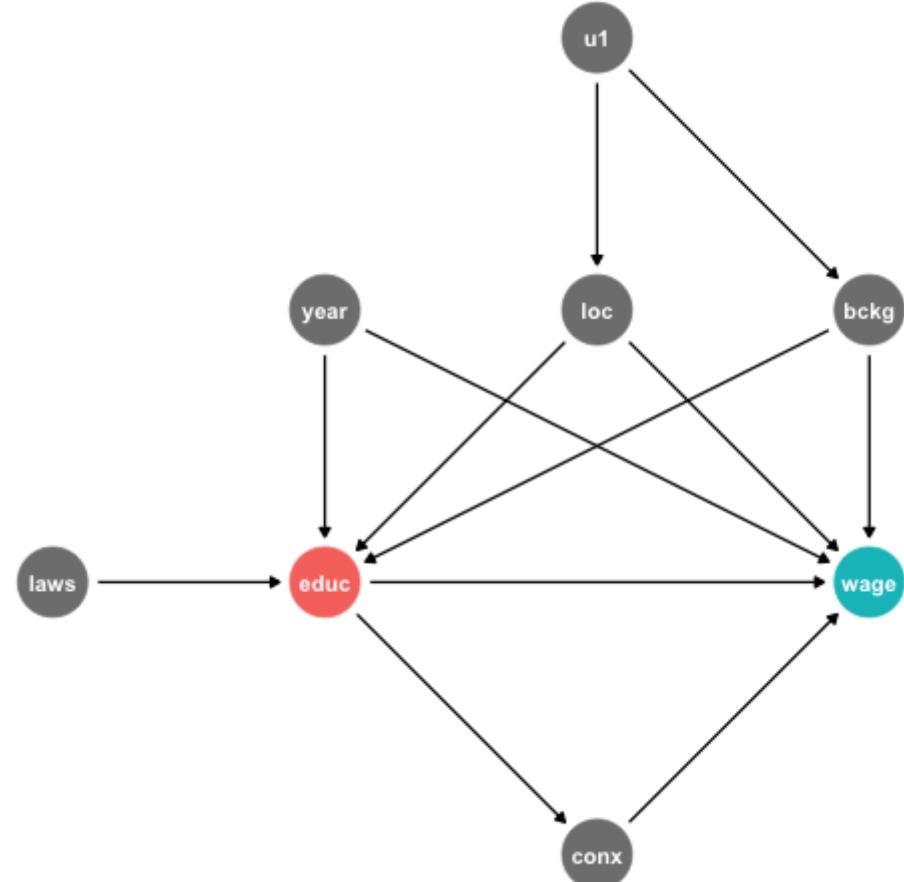
- Background, Year of birth, Location, Compulsory schooling, all cause education
- Background, year of birth, location, job connections probably cause wages



DAG Example III



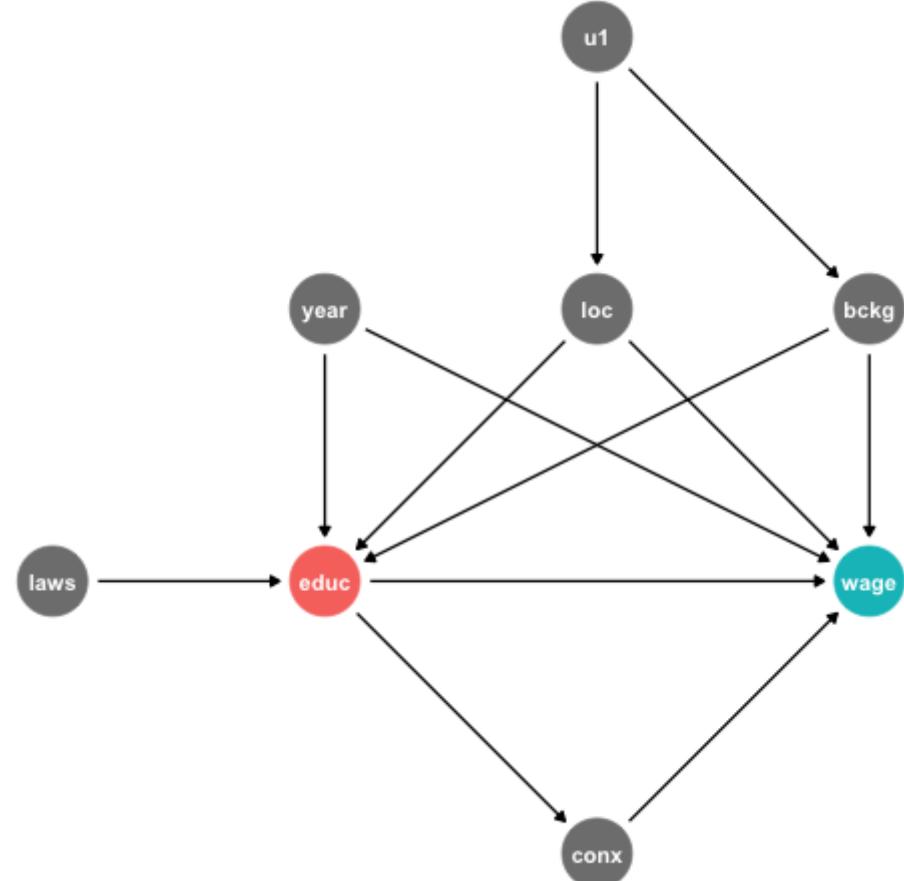
- Background, Year of birth, Location, Compulsory schooling, all cause education
- Background, year of birth, location, job connections probably cause wages
- Job connections in fact is probably caused by education!
- Location and background probably both caused by unobserved factor (`u1`)



DAG Example IV



- This is messy, but we have a causal model!
- Makes our assumptions **explicit**, and many of them are **testable**
- DAG suggests certain relationships that will *not* exist:
 - all relationships between `laws` and `conx` go through `educ`
 - so if we controlled for `educ`, then `cor(laws, conx)` should be zero!



Let the Computer Do It: Dagitty.net I



The screenshot shows the DAGitty.net homepage. At the top, there's a navigation bar with icons for back, forward, search, and other browser controls. The main title is "Not Secure — dagitty.net". Below the title, there are three main buttons: "Launch" (with a small icon of a graph), "Download" (with a download arrow icon), and "Learn" (with a question mark icon). To the right of these is a sidebar titled "Versions" which lists various software releases. Below the sidebar, there's a section titled "What is this?" with a detailed explanation of DAGitty as a causal modeling tool. At the bottom, there's information about the developer, Johannes Textor, and a note about the algorithms implemented.

Welcome to DAGitty!

Not Secure — dagitty.net

Launch
Launch DAGitty online in your browser

Download
Download DAGitty's source for offline use

Learn
Learn more about DAGs and DAGitty

Code
The R package "dagitty" is available on CRAN or github

Versions

The following versions of DAGitty are available:

- [Development version](#)
Recent development snapshot. May contain new features, but could also contain new bugs.
- [Experimental version](#)
Most recent development snapshot. May not even work.
- [2.3: Released 2015-08-19](#)
- [2.2: Released 2014-10-30](#)
- [2.1: Released 2014-02-06](#)
- [2.0: Released 2013-02-12](#)
- [1.1: Released 2011-11-29](#)
- [1.0: Released 2011-03-24](#)
- [0.9b: Released 2010-11-24](#)
- [0.9a: Released 2010-09-01](#)

News on Twitter
[#dagitty Tweets](#)
[Changelog](#)

2018-04-04
Updated the development version and preparing for a long overdue release!

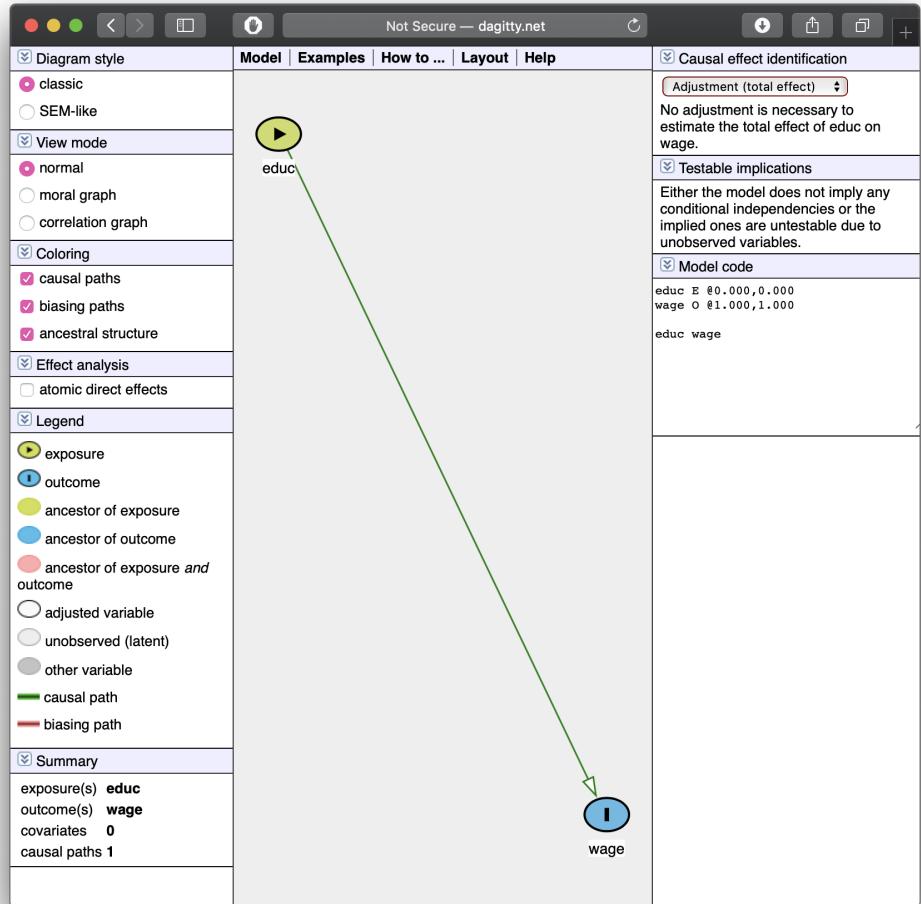
2015-08-19

DAGitty is a browser-based environment for creating, editing, and analyzing causal models (also known as directed acyclic graphs or causal Bayesian networks). The focus is on the use of causal diagrams for minimizing bias in empirical studies in epidemiology and other disciplines. For background information, see the "[learn](#)" page.

DAGitty is developed and maintained by [Johannes Textor](#) ([Tumor Immunology Lab](#) and [Institute for Computing and Information Sciences, Radboud University Nijmegen](#)). The algorithms implemented in DAGitty were developed in close

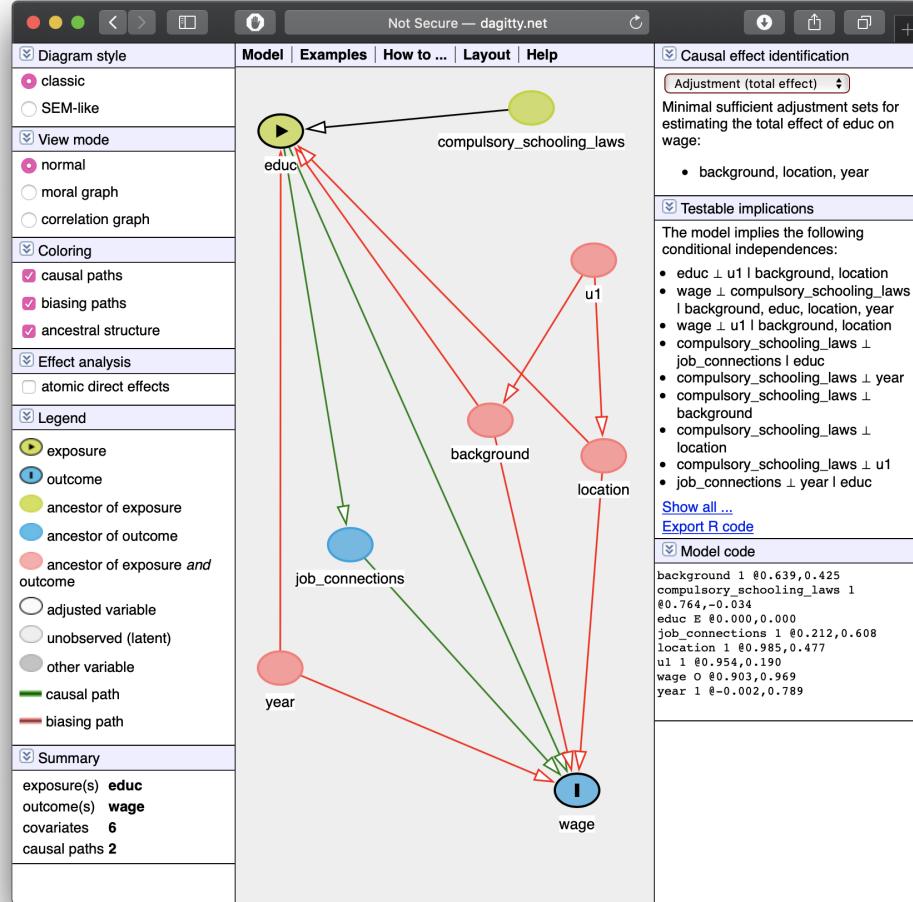
- [Dagitty.net](#) is a great tool to make these and give you testable implications
- Click Model -> New Model
- Name your "exposure" variable (X of interest) and "outcome" variable (Y)

Let the Computer Do It: Dagitty.net II



- Click and drag to move nodes around
- Add a new variable by double-clicking
- Add an arrow by double-clicking one variable and then double-clicking on the target (do again to remove arrow)

Let the Computer Do It: Dagitty.net III



Causal effect identification

Adjustment (total effect)

Minimal sufficient adjustment sets for estimating the total effect of educ on wage:

- background, location, year

Testable implications

The model implies the following conditional independences:

- educ $\perp\!\!\!\perp$ u1 | background, location
- wage $\perp\!\!\!\perp$ compulsory_schooling_laws | background, educ, location, year
- wage $\perp\!\!\!\perp$ u1 | background, location
- compulsory_schooling_laws $\perp\!\!\!\perp$ job_connections | educ
- compulsory_schooling_laws $\perp\!\!\!\perp$ background
- compulsory_schooling_laws $\perp\!\!\!\perp$ location
- compulsory_schooling_laws $\perp\!\!\!\perp$ u1
- job_connections $\perp\!\!\!\perp$ year | educ

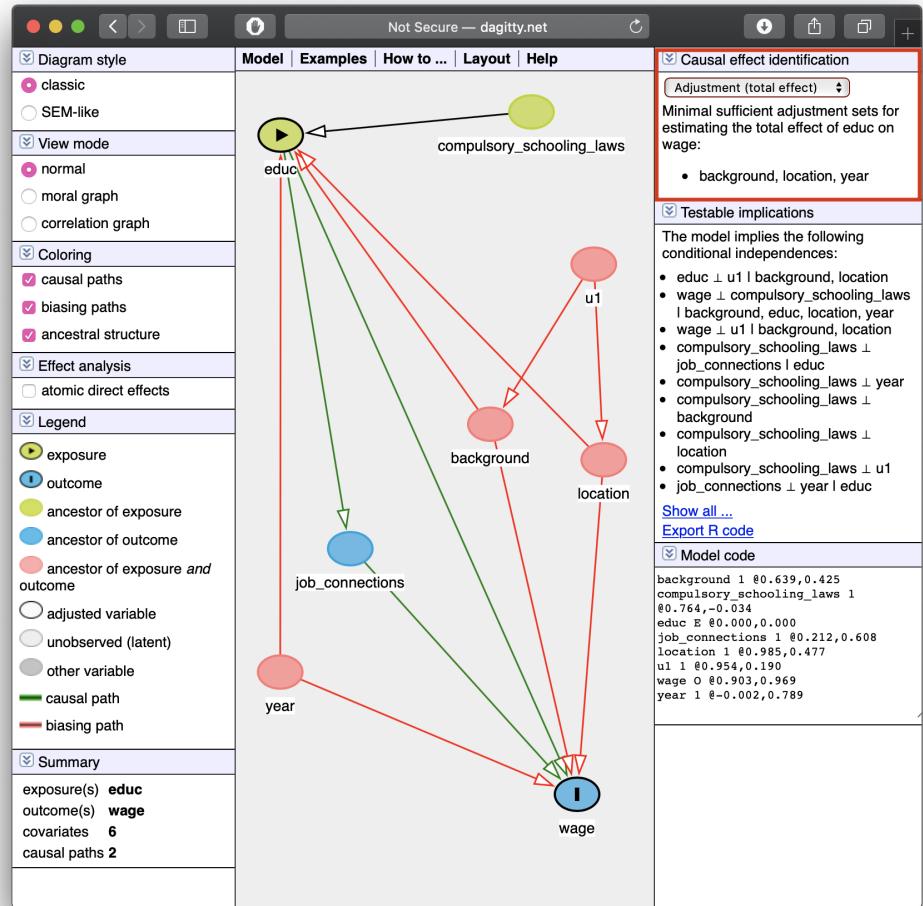
Show all ...

Export R code

Model code

```
background 1 @0.639, 0.425
compulsory_schooling_laws 1
@0.764, -0.034
educ E @0.000, 0.000
job_connections 1 @0.212, 0.608
location 1 @0.985, 0.477
u1 1 @0.954, 0.190
wage O @0.903, 0.969
year 1 @-0.002, 0.789
```

Let the Computer Do It: Dagitty.net III

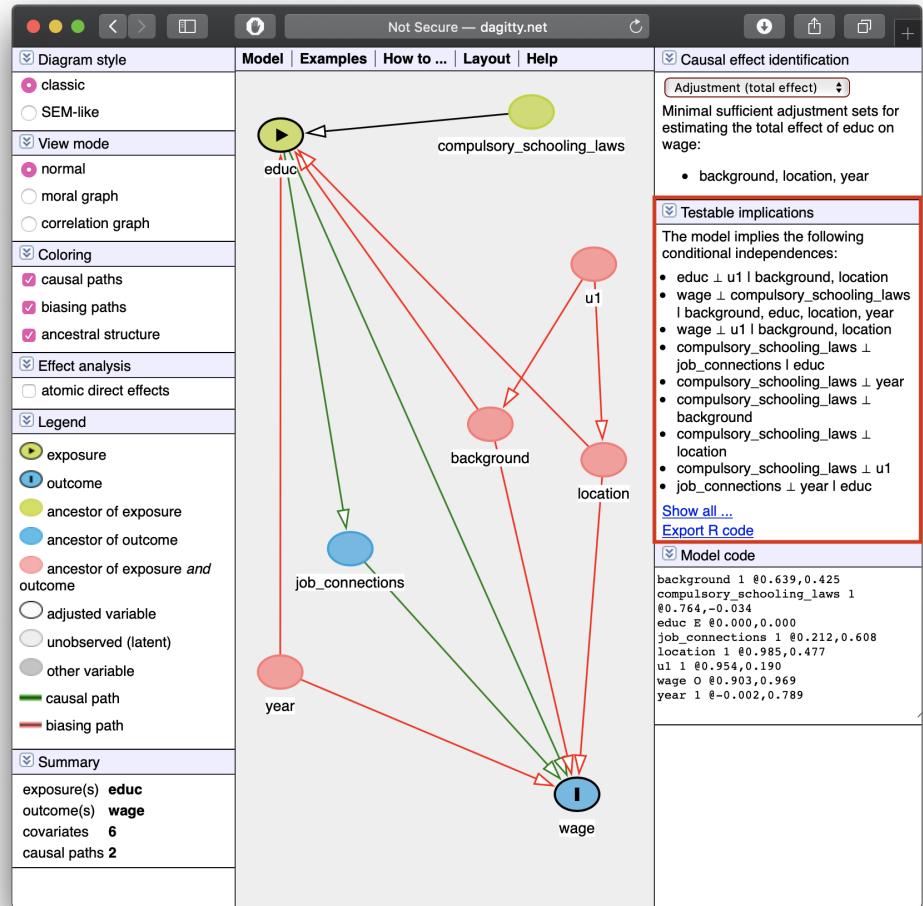
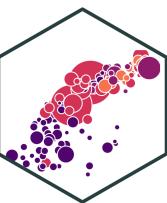


- Tells you **how to identify your effect!** (upper right)

Minimal sufficient adjustment sets

containing background, location, year for estimating the total effect of `educ` on `wage`: background, location, year

Let the Computer Do It: Dagitty.net III



- Tells you some **testable implications** of your model

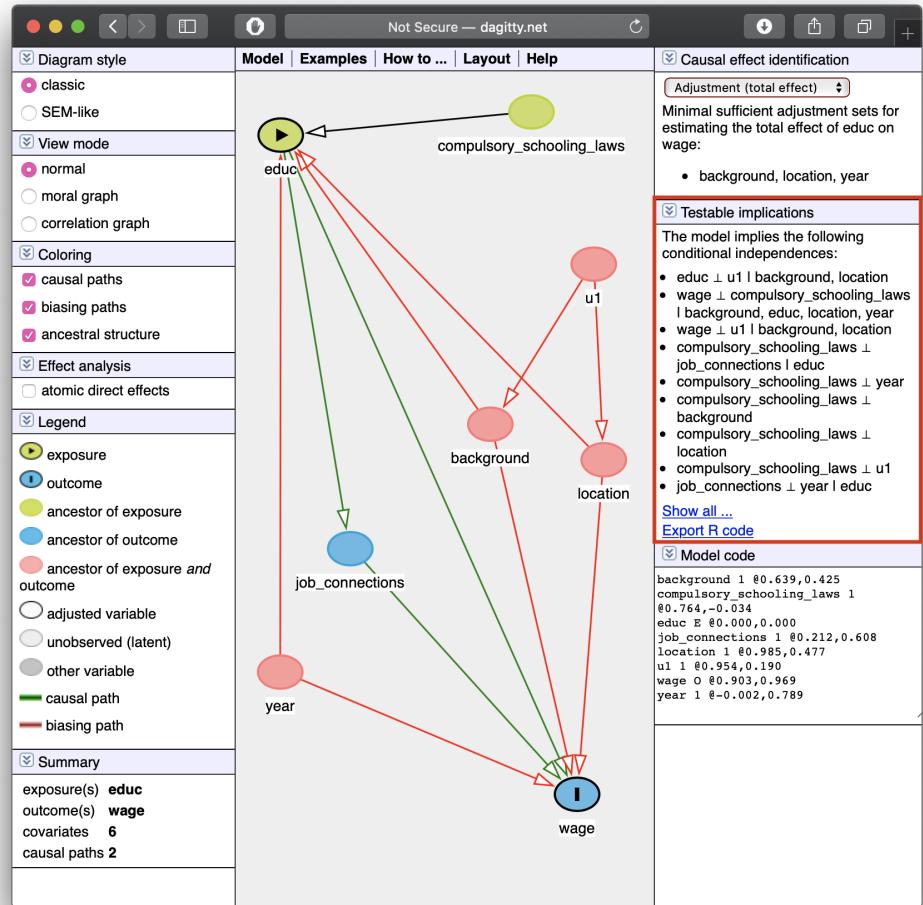
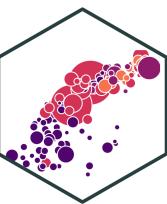
- These are independencies or **conditional independencies**:

$$X \perp\!\!\!\perp Y | Z$$

"X is independent of Y, given Z"

- Implies that by controlling for Z, X and Y should have no correlation

Let the Computer Do It: Dagitty.net III



- Tells you some **testable implications** of your model

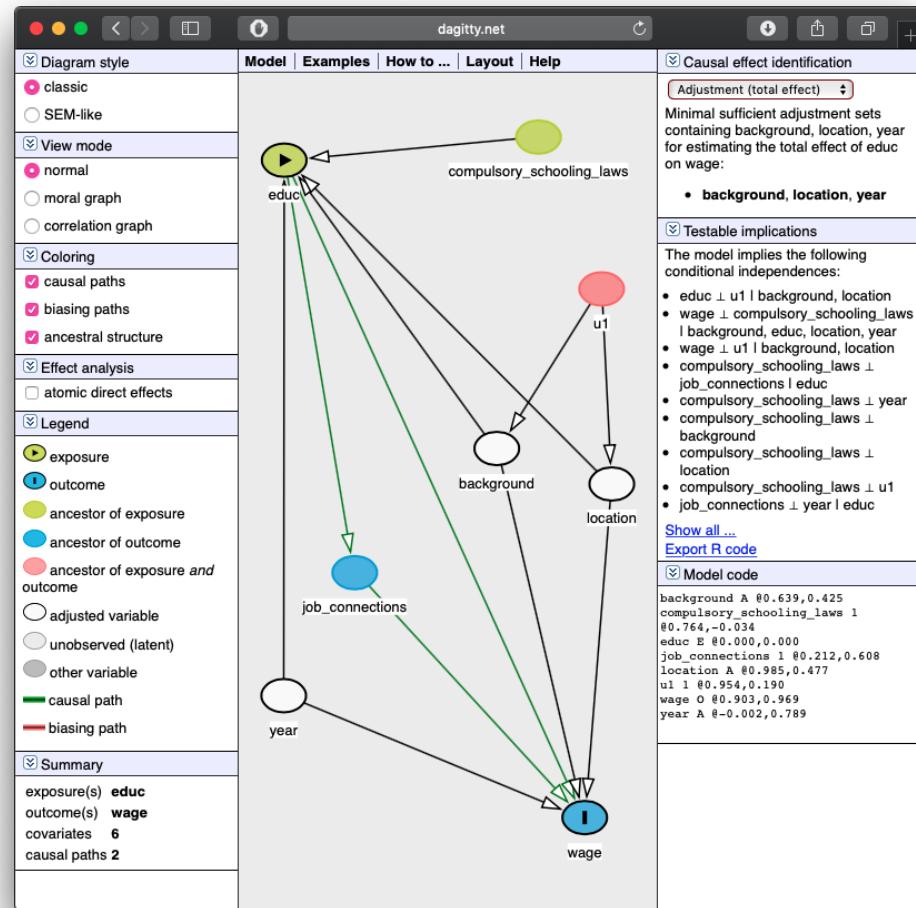
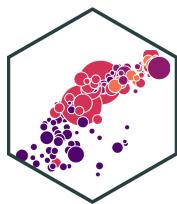
- Example:** look at the last one listed:

`job_connections` $\perp\!\!\!\perp \text{year} \mid \text{educ}$

“Job connections are independent of year, controlling for education”

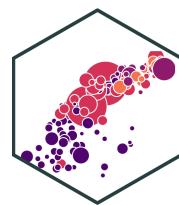
- Implies that by controlling for `educ`, there should be no correlation between `job_connections` and `year` – can test this with data!

Causal Effect



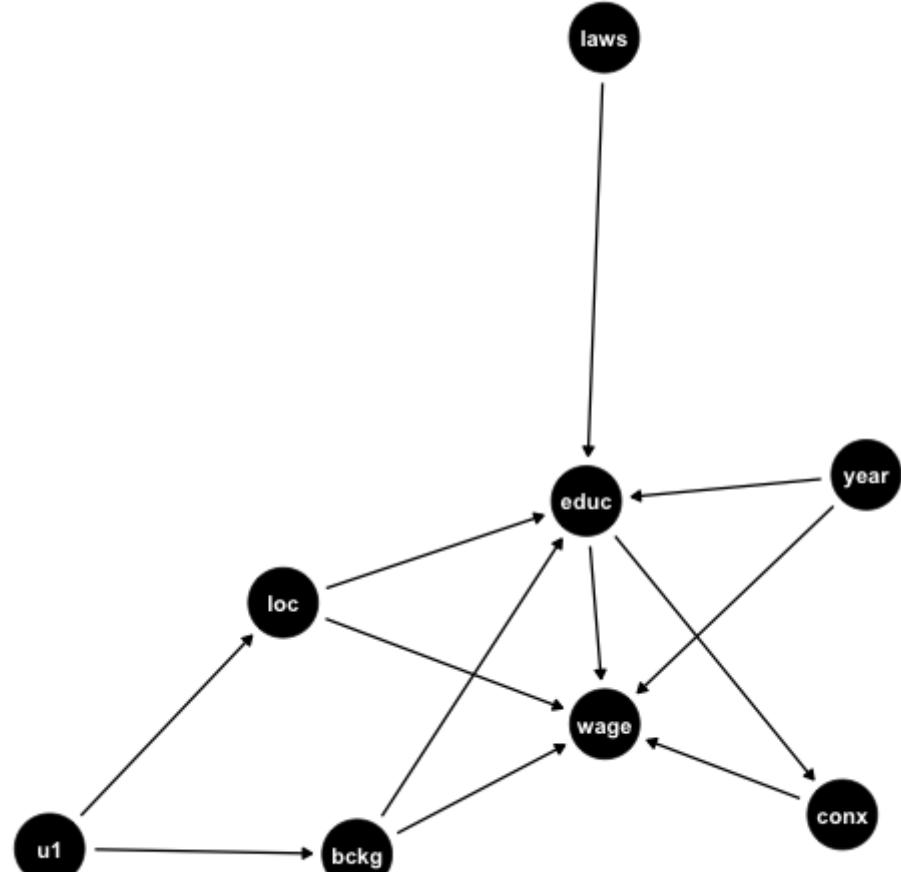
- If we control for background, location, and year, we can identify the causal effect of educ \rightarrow wage.

You Can Draw DAGs In R

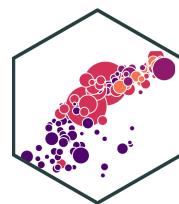


- New package: `ggdag`
- Arrows are made with formula notation:
`Y~X+Z` means "Y is caused by X and Z"

```
# install.packages("ggdag")
library(ggdag)
dagify(wage~educ+conx+year+bckg+loc,
       educ~bckg+year+loc+laws,
       conx~educ,
       bckg~u1,
       loc~u1,
       exposure = "educ", # optional: define X
       outcome = "wage" # optional: define Y
) %>%
ggdag()+
theme_dag()
```

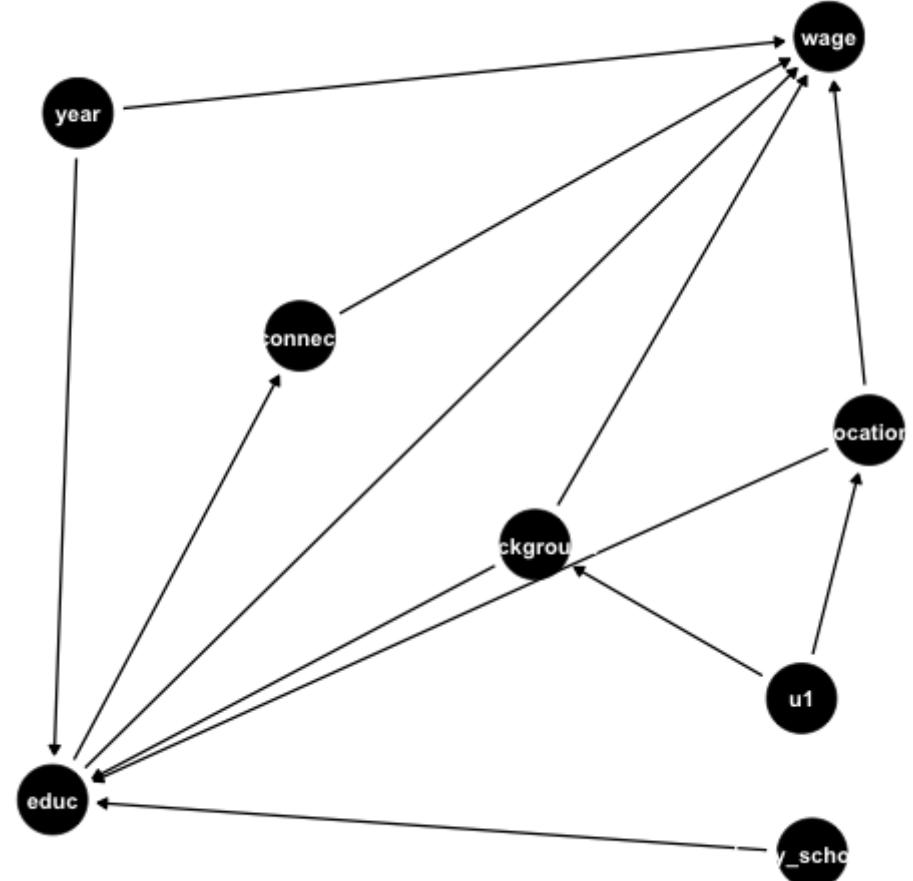


You Can Draw DAGs In R



- Or you can just copy the code from [dagitty.net!](#)
- Use `dagitty()` from the [dagitty](#) package, and paste the code in quotes

```
library(dagitty)
dagitty('dag {
bb="0,0,1,1"
background [pos="0.413,0.335"]
compulsory_schooling_laws [pos="0.544,0.076"]
educ [exposure,pos="0.185,0.121"]
job_connections [pos="0.302,0.510"]
location [pos="0.571,0.431"]
u1 [pos="0.539,0.206"]
wage [outcome,pos="0.552,0.761"]
year [pos="0.197,0.697"]
background -> educ
background -> wage
compulsory_schooling_laws -> educ
educ -> job_connections
educ -> wage
job_connections -> wage
location -> educ
location -> wage
u1 -> background
u1 -> location
year -> educ
year -> wage
}') %>%
  ggdag()+
  theme_dag()
```

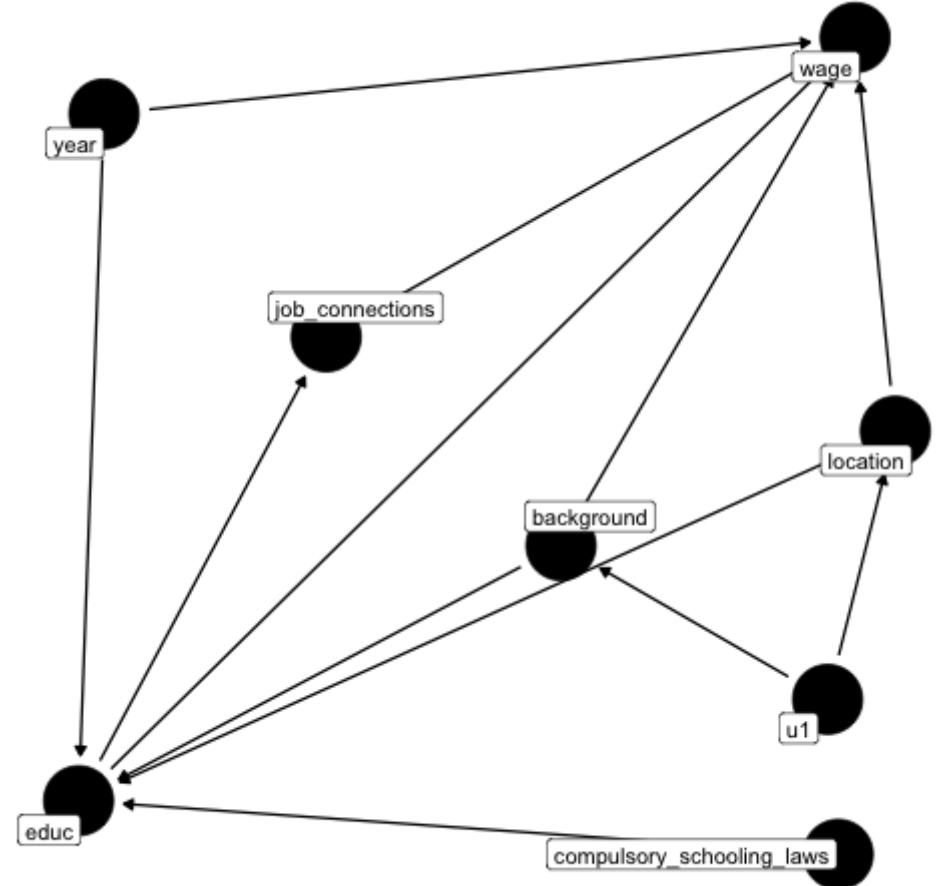


You Can Draw DAGs In R

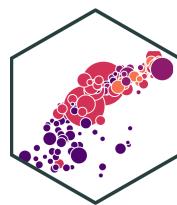


- It's not very pretty, but if you set `text = FALSE, use_labels = "name"` inside `ggdag()`, makes it easier to read

```
dagitty('dag {
bb="0,0,1,1"
background [pos="0.413,0.335"]
compulsory_schooling_laws [pos="0.544,0.076"]
educ [exposure,pos="0.185,0.121"]
job_connections [pos="0.302,0.510"]
location [pos="0.571,0.431"]
u1 [pos="0.539,0.206"]
wage [outcome,pos="0.552,0.761"]
year [pos="0.197,0.697"]
background -> educ
background -> wage
compulsory_schooling_laws -> educ
educ -> job_connections
educ -> wage
job_connections -> wage
location -> wage
u1 -> background
u1 -> location
year -> educ
year -> wage
}') %>%
  ggdag(., text = FALSE, use_labels = "name")+
  theme_dag()
```

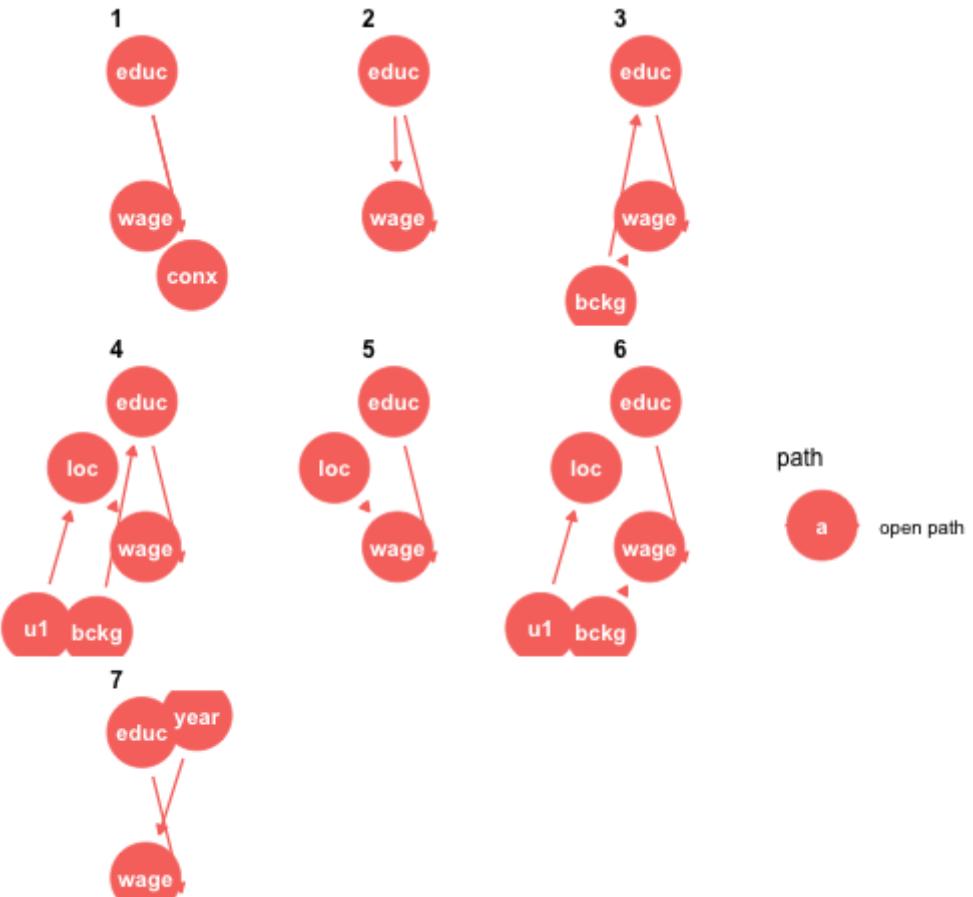


ggdag: Additional Tools

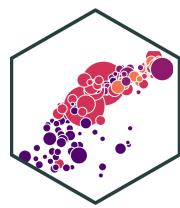


- If you have defined `X` (exposure) and `Y` (outcome), you can use `ggdag_paths()` to have it show all possible paths between X and Y !

```
dagify(wage~educ+conx+year+bckg+loc,  
       educ~bckg+year+loc+laws,  
       conx~educ,  
       bckg~u1,  
       loc~u1,  
       exposure = "educ",  
       outcome = "wage"  
     ) %>%  
tidy_dagitty(seed = 2) %>%  
  ggdag_paths() +  
  theme_dag()
```

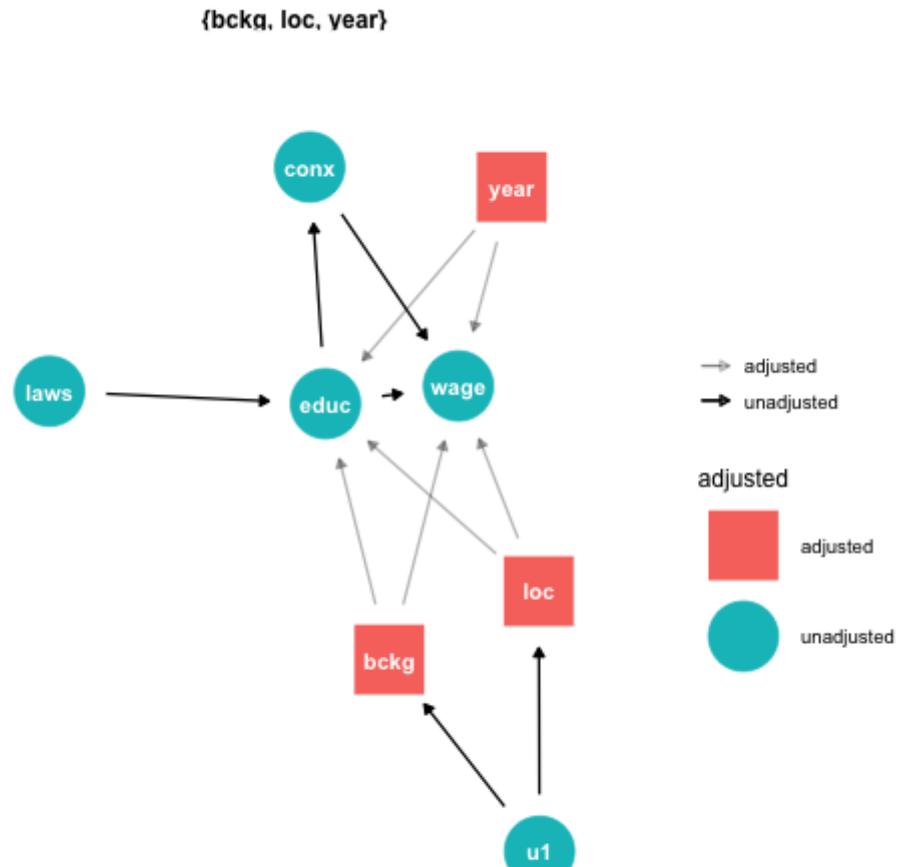


You Can Draw DAGs In R

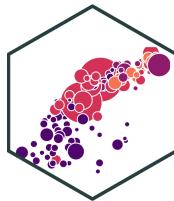


- If you have defined `X` (exposure) and `Y` (outcome), you can use `ggdag_adjustment_set()` to have it show you what you need to control for in order to identify $X \rightarrow Y$!

```
dagify(wage~educ+conx+year+bckg+loc,  
      educ~bckg+year+loc+laws,  
      conx~educ,  
      bckg~u1,  
      loc~u1,  
      exposure = "educ",  
      outcome = "wage"  
    ) %>%  
  
  ggdag_adjustment_set(shadow = T)+  
  theme_dag()
```



You Can Draw DAGs In R



- You can also use

```
impliedConditionalIndependencies()
```

from the `dagitty` package to have it show the testable implications from `dagitty.net`

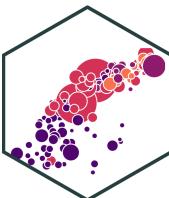
```
library(dagitty)
dagify(wage~educ+conx+year+bckg+loc,
      educ~bckg+year+loc+laws,
      conx~educ,
      bckg~u1,
      loc~u1,
      exposure = "educ",
      outcome = "wage"
    ) %>
impliedConditionalIndependencies()
```

```
## bckg _||_ conx | educ
## bckg _||_ laws
## bckg _||_ loc | u1
## bckg _||_ year
## conx _||_ laws | educ
## conx _||_ loc | educ
## conx _||_ u1 | bckg, loc
## conx _||_ u1 | educ
## conx _||_ year | educ
## educ _||_ u1 | bckg, loc
## laws _||_ loc
## laws _||_ u1
## laws _||_ wage | bckg, educ, loc, year
## laws _||_ year
## loc _||_ year
## u1 _||_ wage | bckg, loc
## u1 _||_ year
```



DAG Rules

DAG Rules



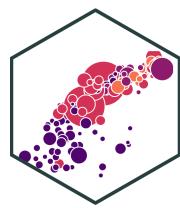
- How does dagitty.net and `ggdag` know how to identify effects, or what to control for, or what implications are testable?
- Comes from fancy math called “do-calculus”

JUDEA PEARL
WINNER OF THE TURING AWARD
AND DANA MACKENZIE

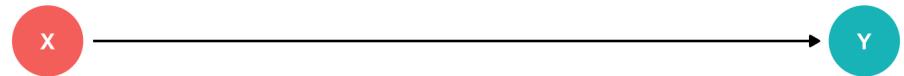
THE
BOOK OF
WHY



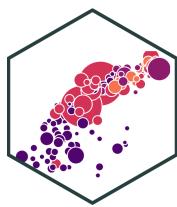
DAGs I



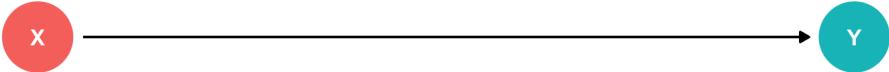
- Typical notation:
- X is independent variable of interest
 - Epidemiology: "intervention" or "exposure"
- Y is dependent or "response" variable
- Other variables use other letters
- You can of course use words instead of letters!



DAGs and Causal Effects



- Arrows indicate causal effect (& direction)
- Two types of causal effect:
 1. Direct effects: $X \rightarrow Y$



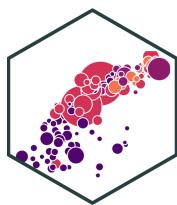
DAGs and Causal Effects



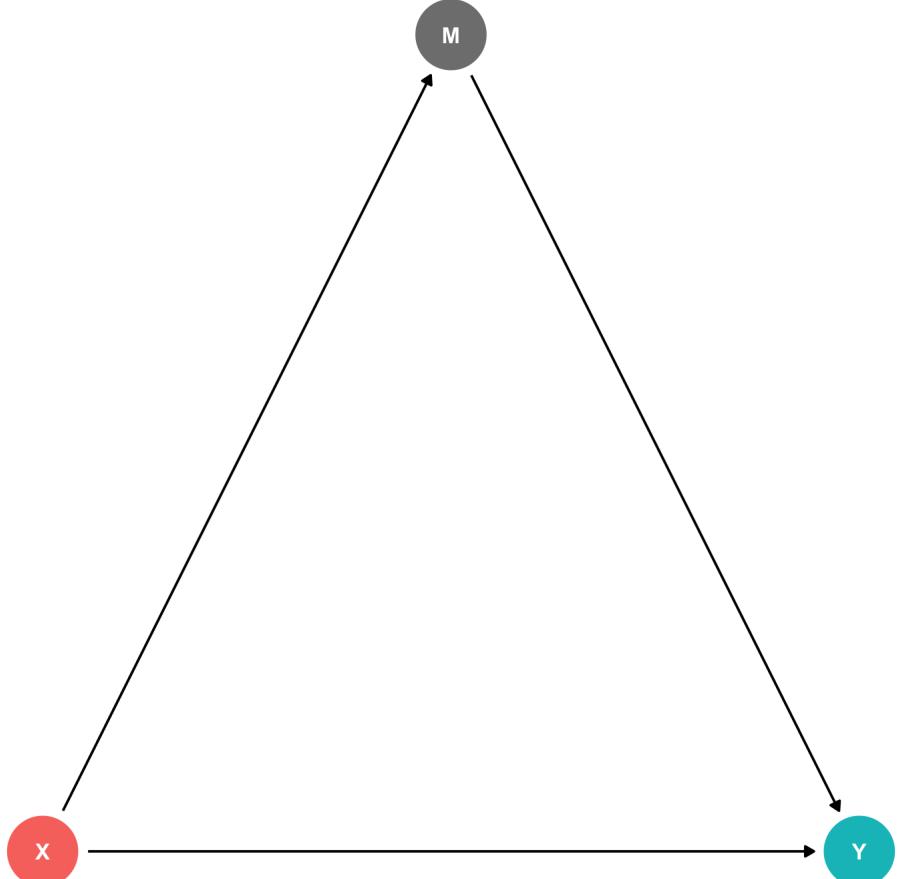
- Arrows indicate causal effect (& direction)
- Two types of causal effect:
 1. Direct effects: $X \rightarrow Y$
 2. Indirect effects: $X \rightarrow M \rightarrow Y$
 - M is a “**mediator**” variable, the **mechanism** by which X affects Y



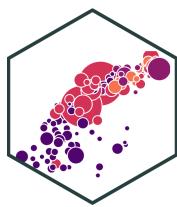
DAGs and Causal Effects



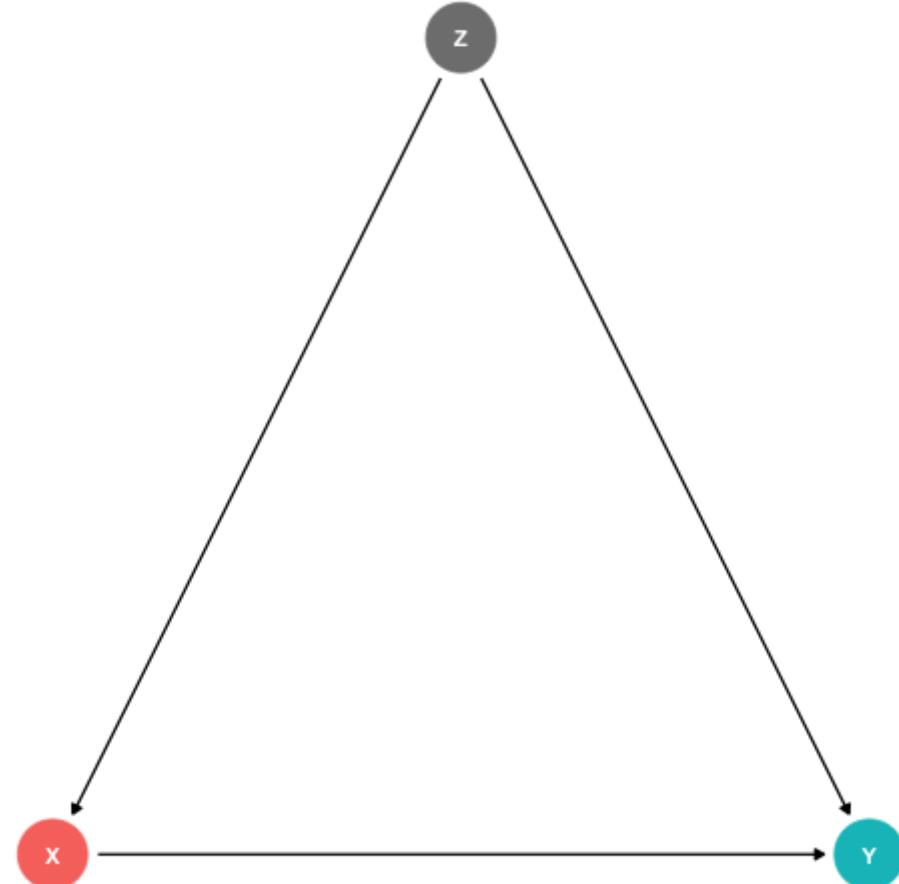
- Arrows indicate causal effect (& direction)
- Two types of causal effect:
 1. Direct effects: $X \rightarrow Y$
 2. Indirect effects: $X \rightarrow M \rightarrow Y$
 - M is a “**mediator**” variable, the **mechanism** by which X affects Y
- 3. You of course might have both!



Confounders



- Z is a “confounder” of $X \rightarrow Y$, it causes *both X and Y*
- $\text{cor}(X, Y)$ is made up of two parts:
 1. Causal effect of $(X \rightarrow Y)$
 2. Z causing both the values of X and Y
- Failing to control for Z will **bias** our estimate of the causal effect of $X \rightarrow Y$!



Confounders

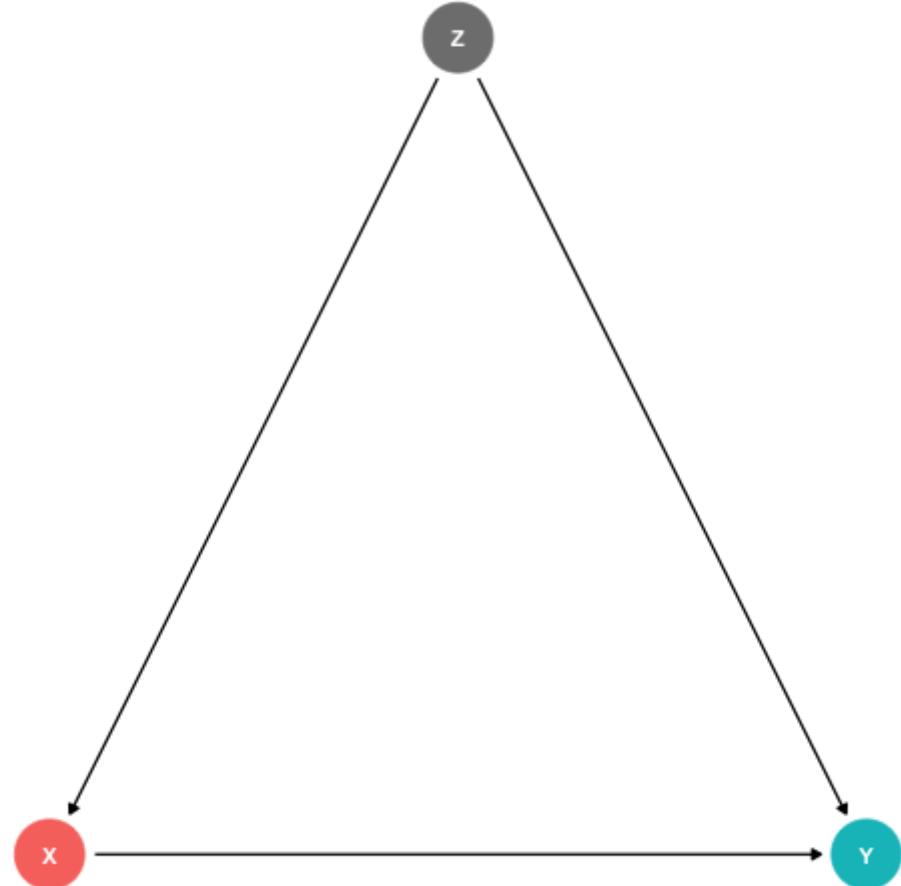


- Confounders are the DAG-equivalent of **omitted variable bias** (next class)

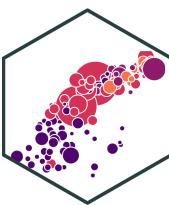
$$Y_i = \beta_0 + \beta_1 X_i$$

- By leaving out Z_i , this regression is **biased**
- $\hat{\beta}_1$ picks up *both*:

- $X \rightarrow Y$
- $X \leftarrow Z \rightarrow Y$



“Front Doors” and “Back Doors”



- With this DAG, there are 2 paths that connect X and Y^{\dagger} :

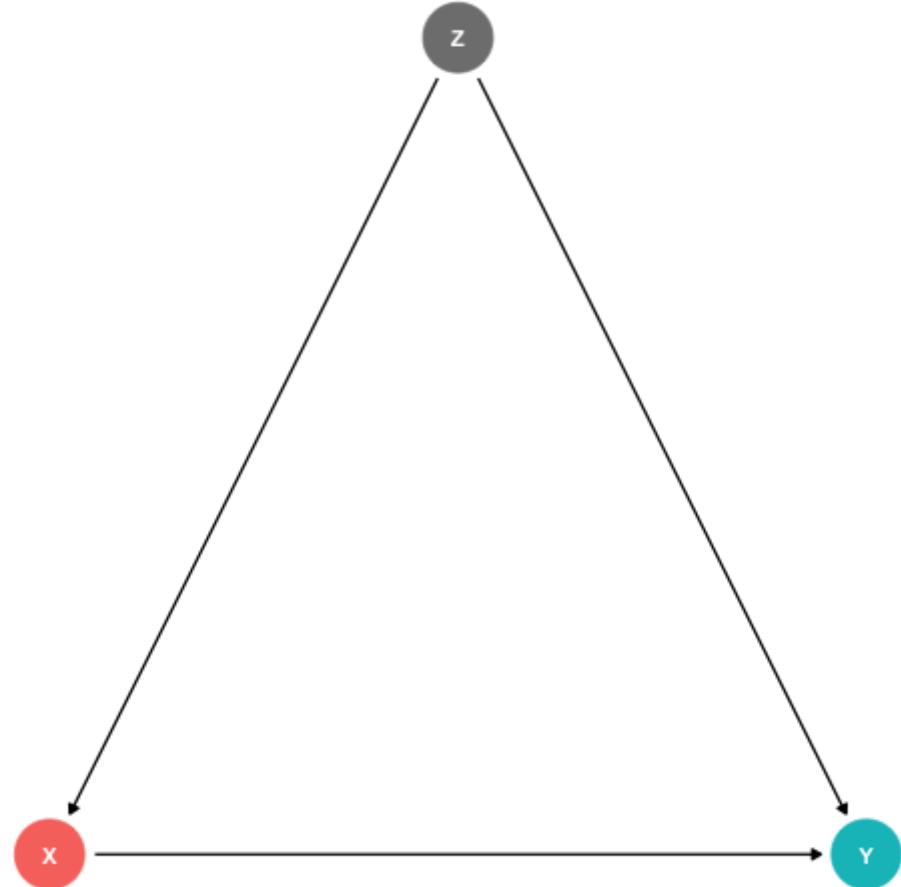
1. A **causal “front-door” path**: $X \rightarrow Y$

- 👉 what we want to measure

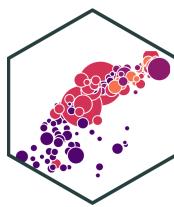
2. A **non-causal “back-door” path**: $X \leftarrow Z \rightarrow Y$

- At least one causal arrow runs in the opposite direction
- 👎 adds a confounding bias

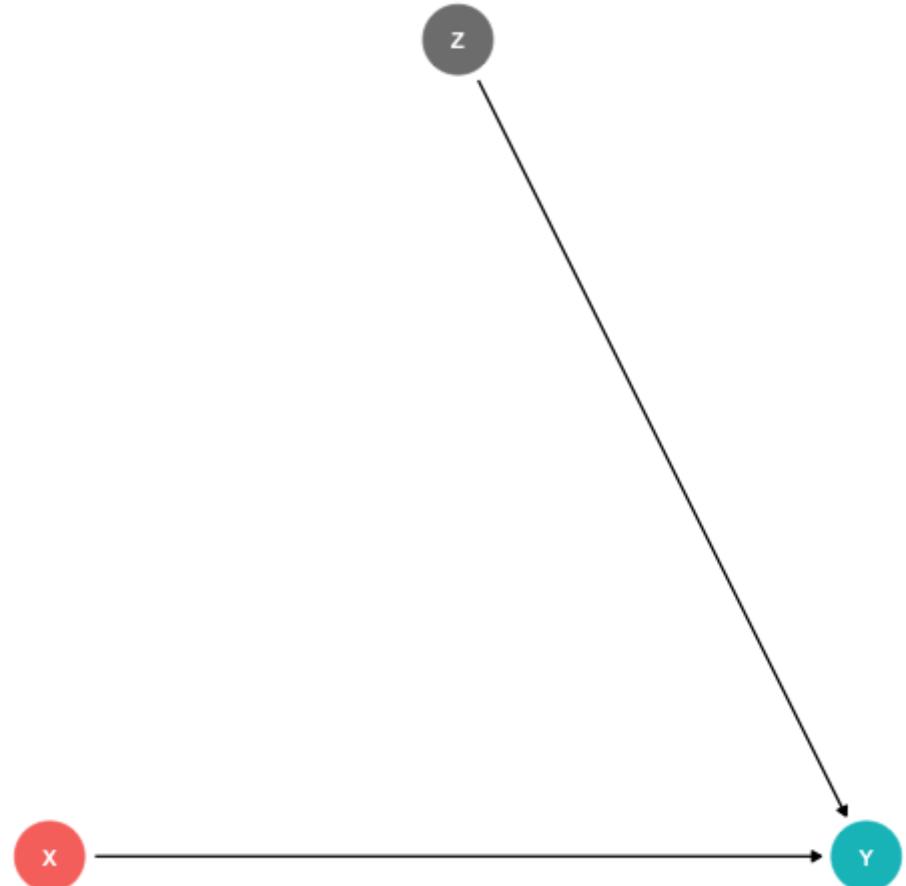
[†] Regardless of the *directions* of the arrows!



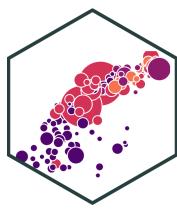
Controlling I



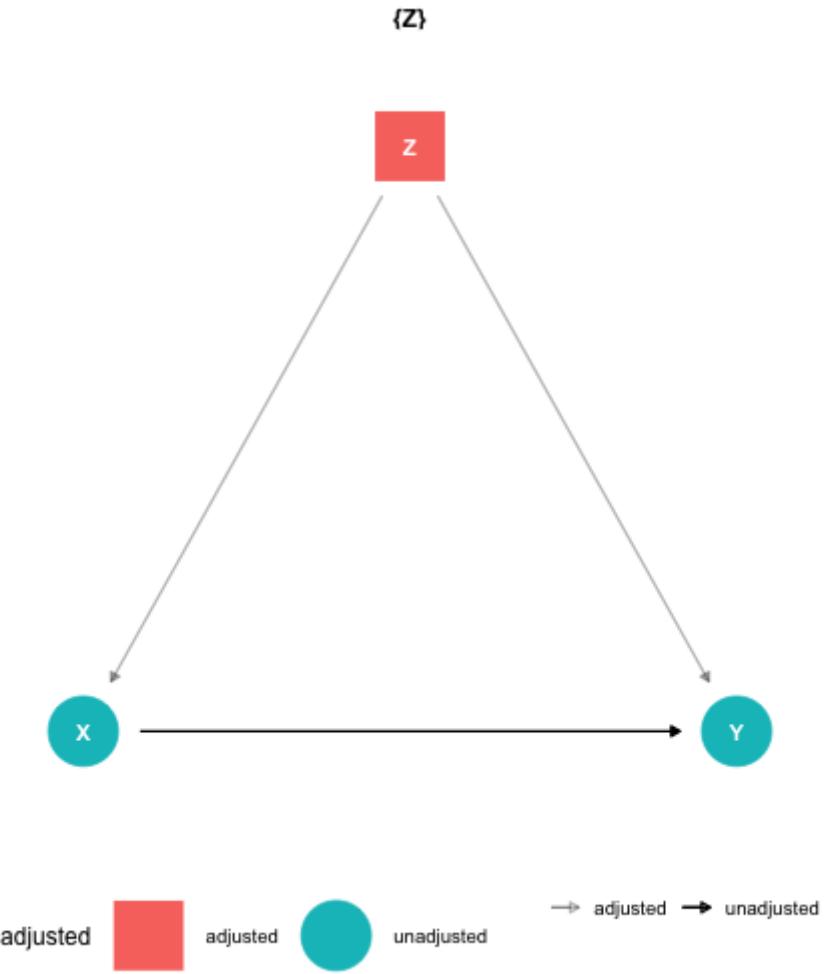
- Ideally, if we ran a **randomized control trial** and randomly assigned different values of X to different individuals, this would delete the arrow between Z and X
 - Individuals' values of Z do not affect whether or not they are treated ($\$X\$$)
- This would only leave the front-door,
 $X \rightarrow Y$
- But we can rarely run an ideal RCT



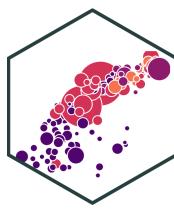
Controlling I



- Instead of an RCT, if we can just “adjust for” or “control for” Z , we can *block* the back-door path $X \leftarrow Z \rightarrow Y$
- This would only leave the front-door path open, $X \rightarrow Y$
- “As good as” an RCT!

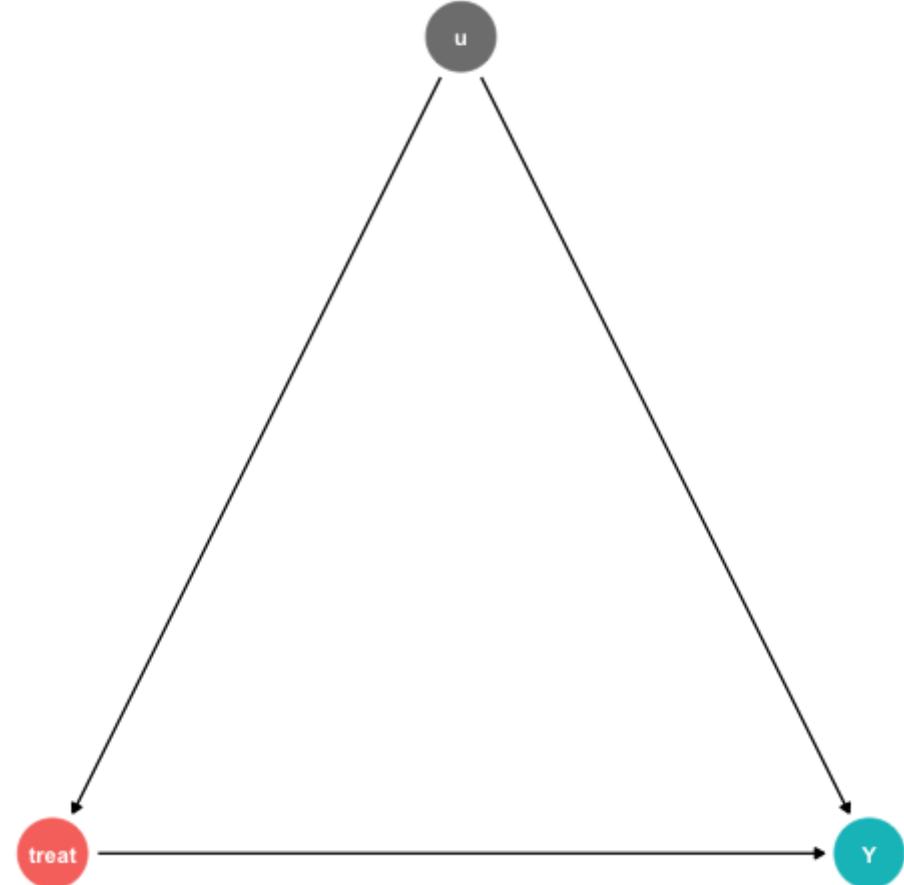


Controlling I

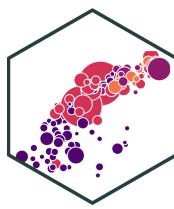


- Using our terminology from last class, we have an outcome (Y), and some treatment
- But there are **unobserved factors** (u)

$$Y_i = \beta_0 + \beta_1 Treatment + u_i$$



Controlling I

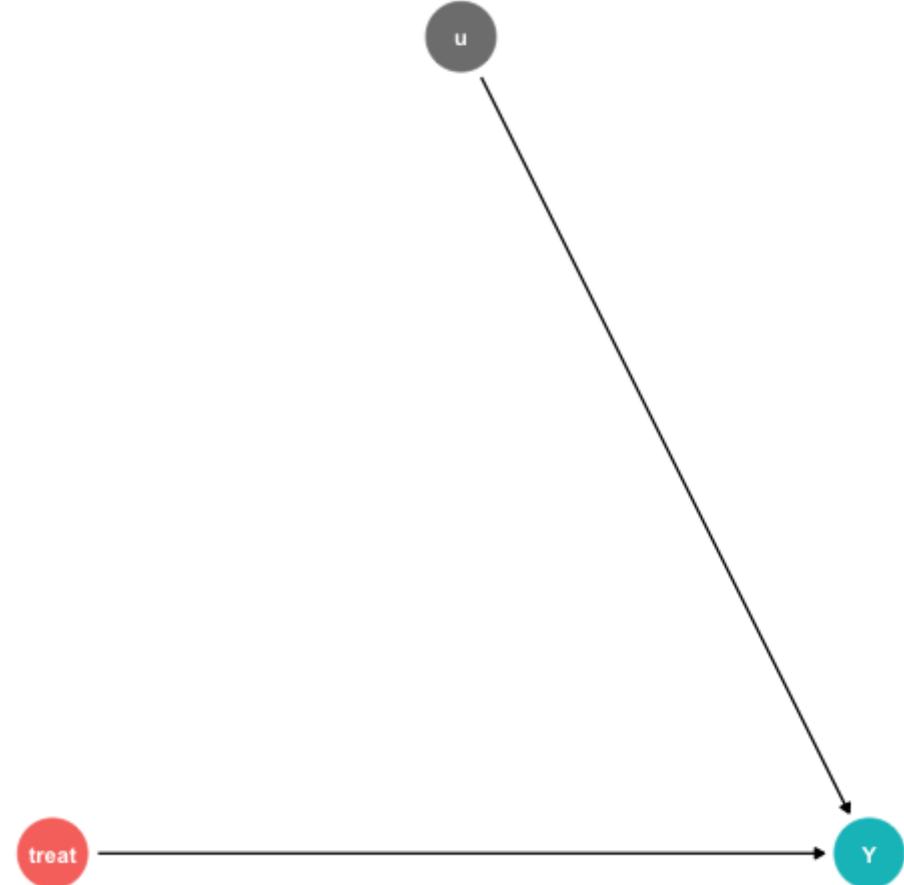


- Using our terminology from last class, we have an outcome (Y), and some treatment
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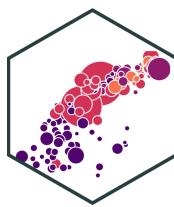
$$Y_i = \beta_0 + \beta_1 Treatment + u_i$$

- If we can *randomly* assign treatment, this makes treatment exogenous:

$$\text{cor(treatment, } u) = 0$$



Controlling I

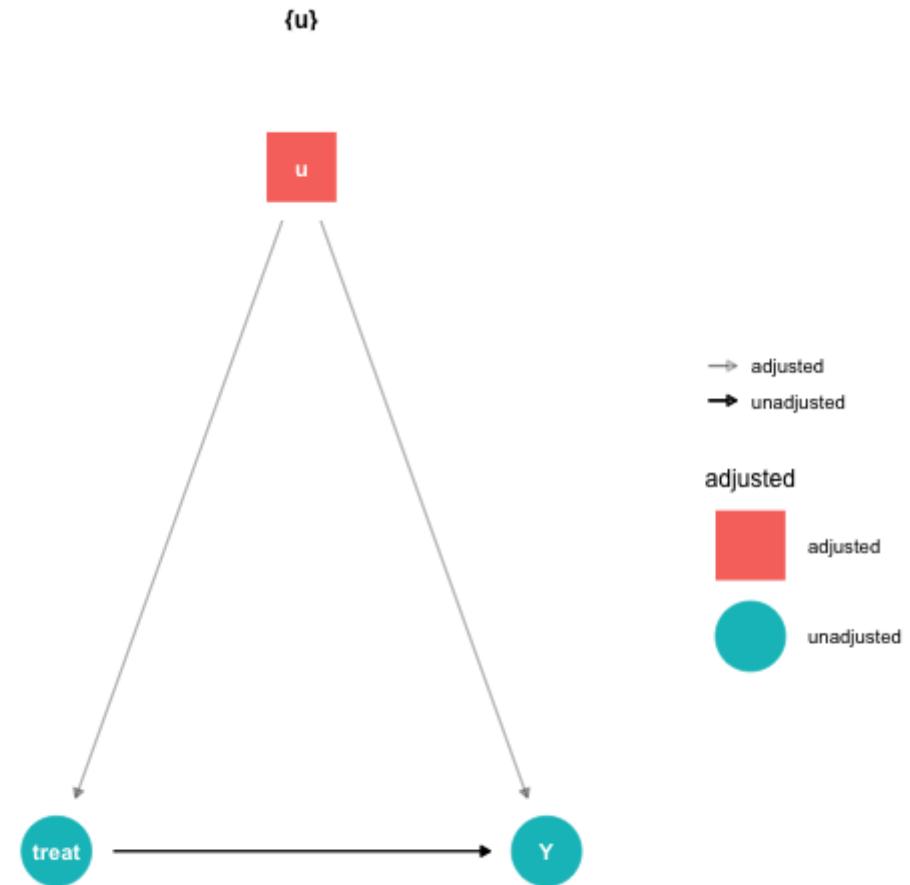


- Using our terminology from last class, we have an outcome (Y), and some treatment

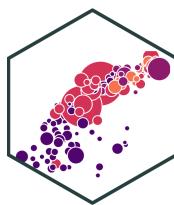
- But there are other **unobserved factors** (u)

$$Y_i = \beta_0 + \beta_1 Treatment + u_i$$

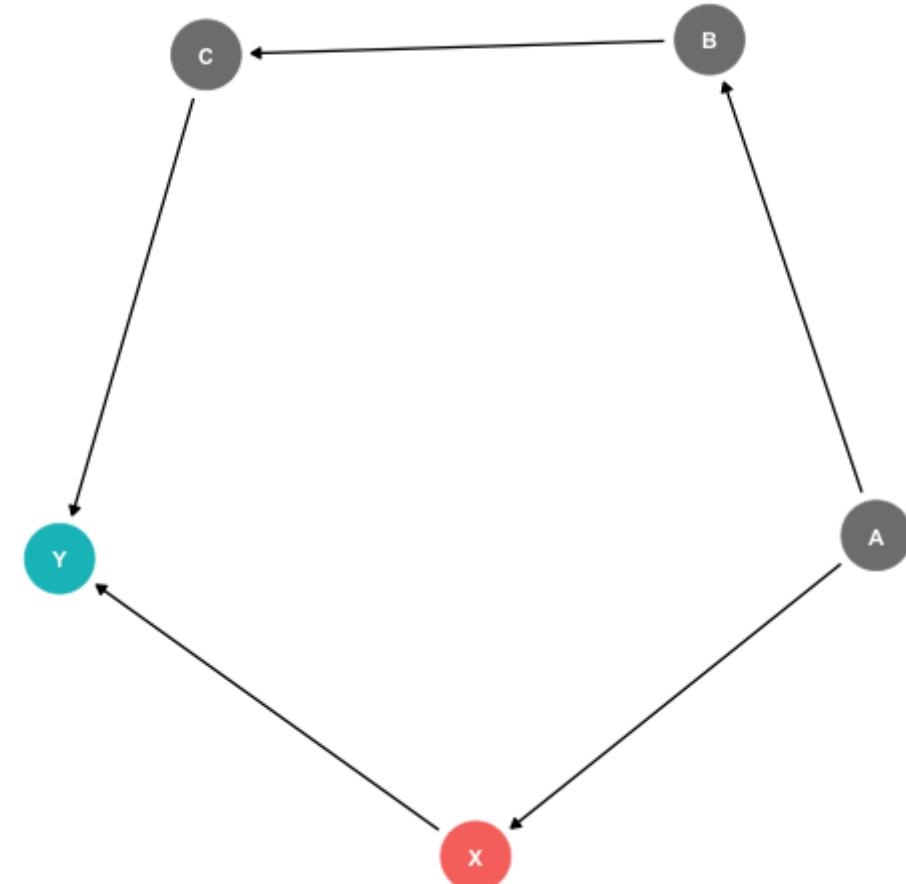
- When we (often) can't randomly assign treatment, we have to find another way to control for measurable things in u



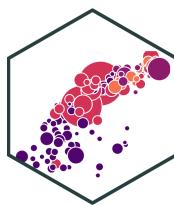
Controlling II



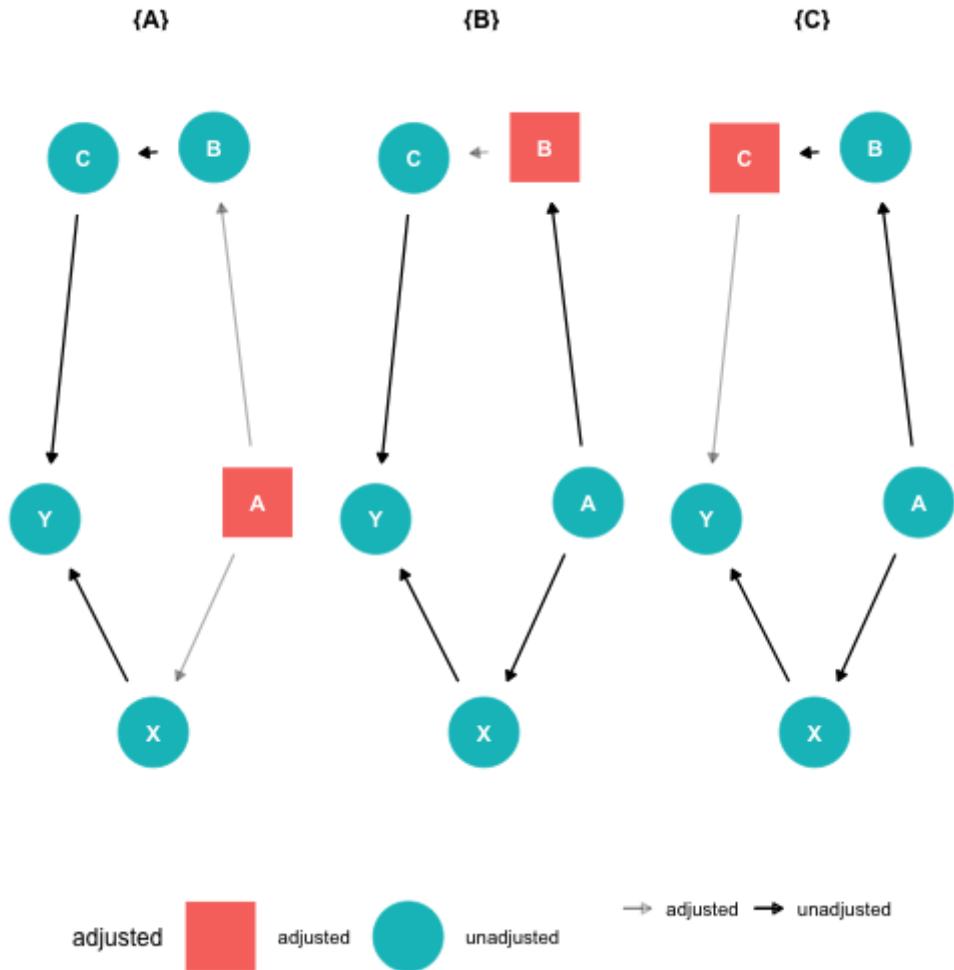
- Controlling for a single variable along a long causal path is sufficient to block that path!
- Causal path: $X \rightarrow Y$
- Backdoor path:
 $X \leftarrow A \rightarrow B \rightarrow C \rightarrow Y$
- It is sufficient to block this backdoor by controlling **either A or B or C!**



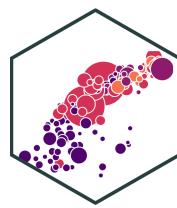
Controlling II



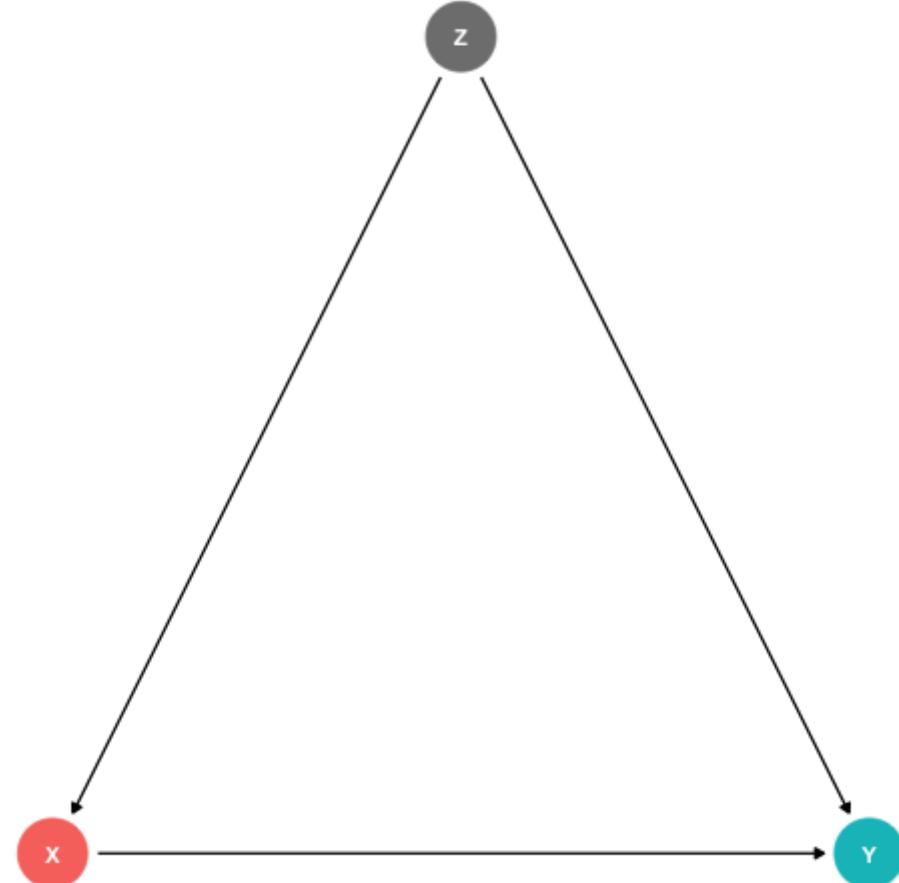
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 $X \leftarrow A \rightarrow B \rightarrow C \rightarrow Y$
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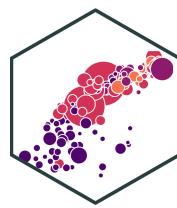
The Back Door Criterion



- To **identify** the causal effect of $X \rightarrow Y$:
- “**Back-door criterion**”: control for the minimal amount of variables sufficient to ensure that **no open back-door exists** between X and Y
- **Example:** in this DAG, control for Z



The Back Door Criterion

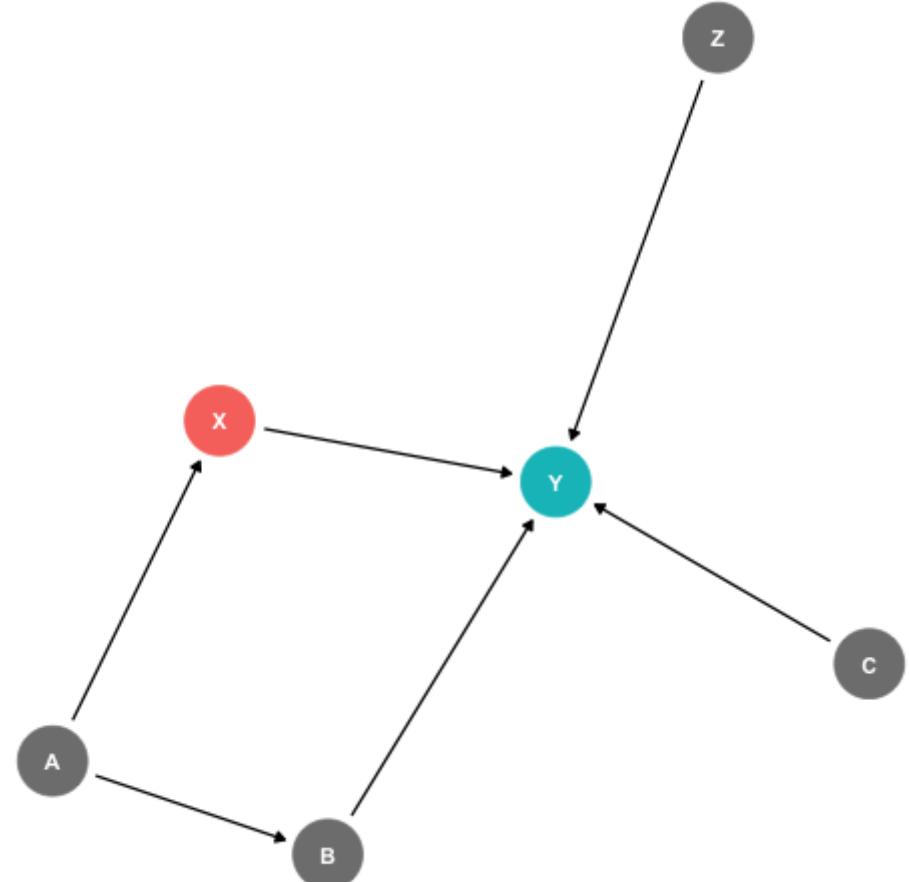


- Implications of the Back-door criterion:

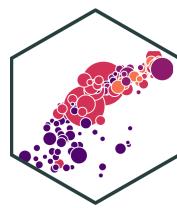
1) You *only* need to control for the variables that keep a back-door open, *not all other variables!*

Example:

- $X \rightarrow Y$ (front-door)
- $X \leftarrow A \rightarrow B \rightarrow Y$ (back-door)



The Back Door Criterion



- Implications of the Back-door criterion:

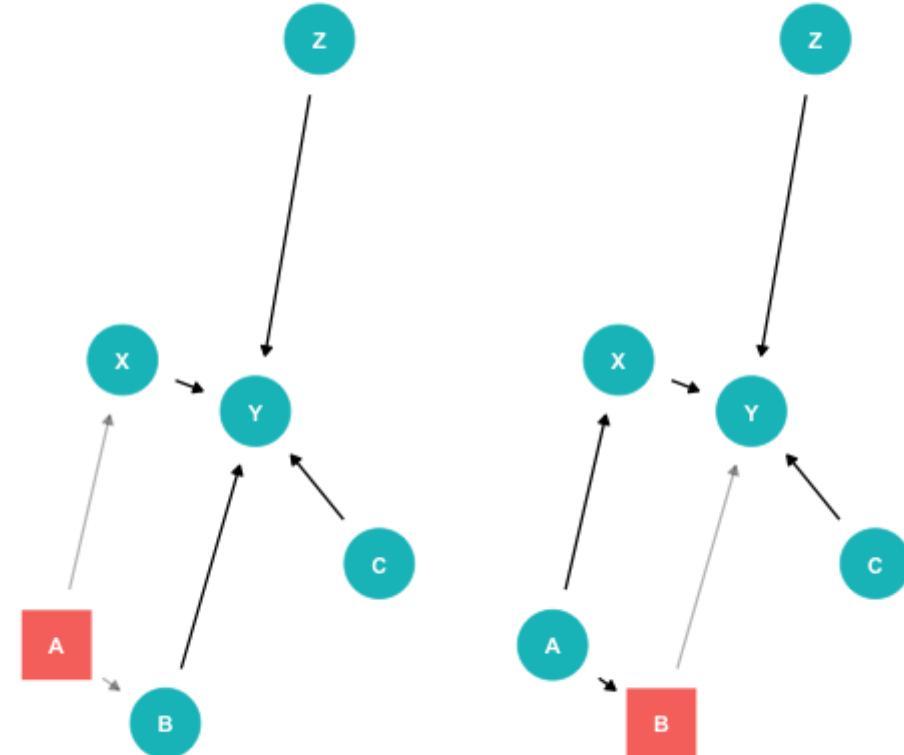
1) You *only* need to control for the variables that keep a back-door open, *not all other variables!*

Example:

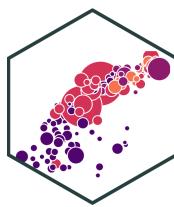
- $X \rightarrow Y$ (front-door)
- $X \leftarrow A \rightarrow B \rightarrow Y$ (back-door)
- Need only control for A *or* B to block the back-door path
- C and Z have no effect on X , and therefore we don't need to control for them!

{A}

{B}



The Back Door Criterion: Colliders

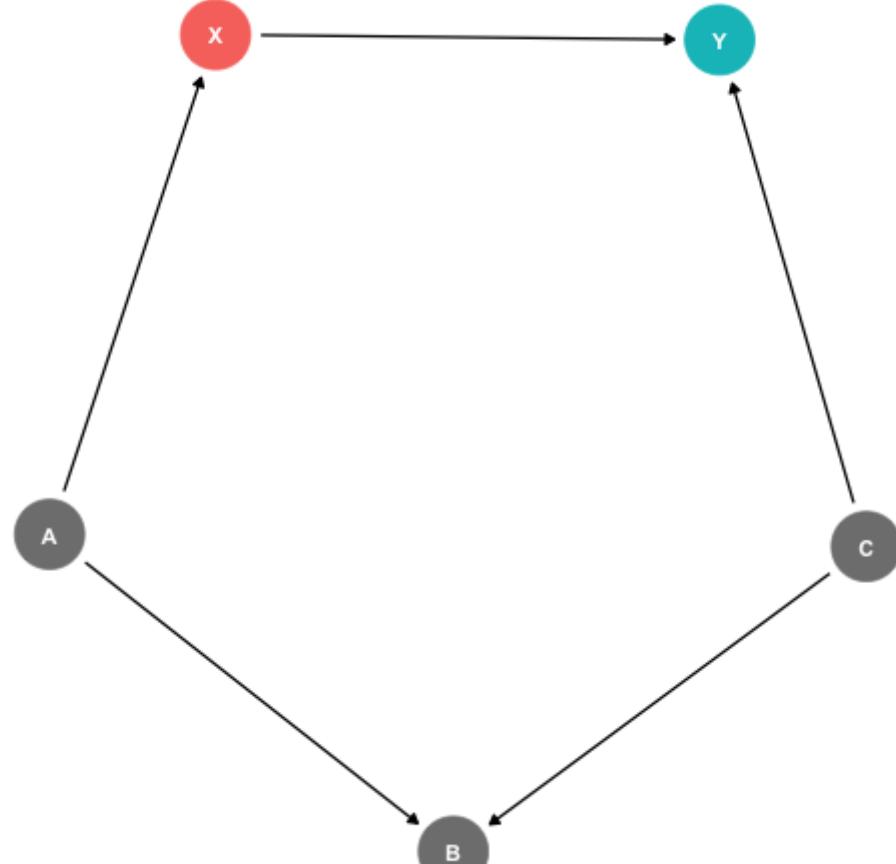


2) Exception: the case of a “**collider**”

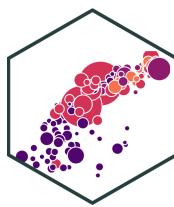
- If arrows “collide” at a node, **that node is automatically blocking the pathway, do not control for it!**
- Controlling for a collider would *open* the path and **add bias!**

Example:

- $X \rightarrow Y$ (front-door)
- $X \leftarrow A \rightarrow B \leftarrow C \rightarrow Y$ (back-door, but **blocked by B!**)



The Back Door Criterion: Colliders



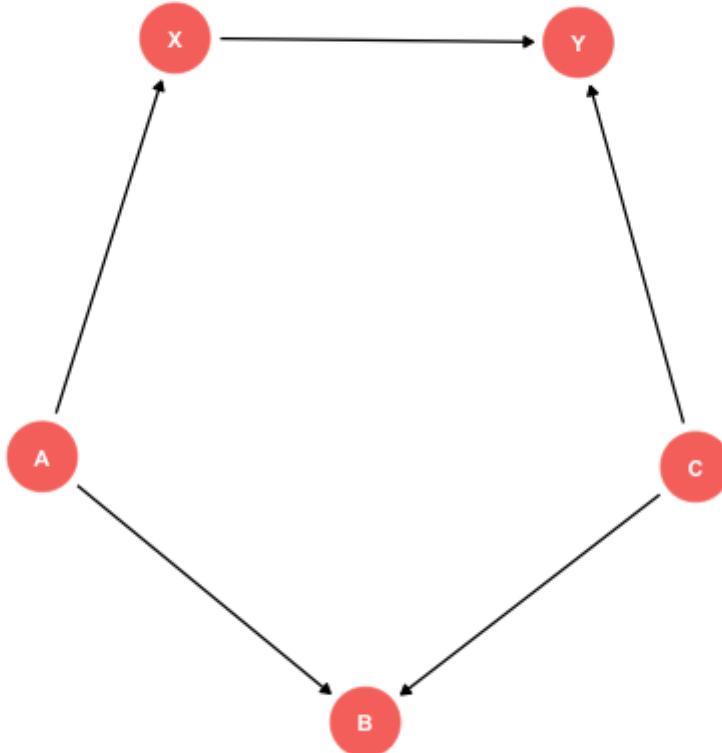
2) Exception: the case of a “collider”

- If arrows “collide” at a node, **that node is automatically blocking the pathway, do not control for it!**
- Controlling for a collider would *open* the path and **add bias!**

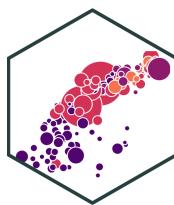
Example:

- $X \rightarrow Y$ (front-door)
- $X \leftarrow A \rightarrow B \leftarrow C \rightarrow Y$ (back-door, but **blocked by B!**)
- Don’t need to control for anything here!

{(Backdoor Paths Unconditionally Closed)}

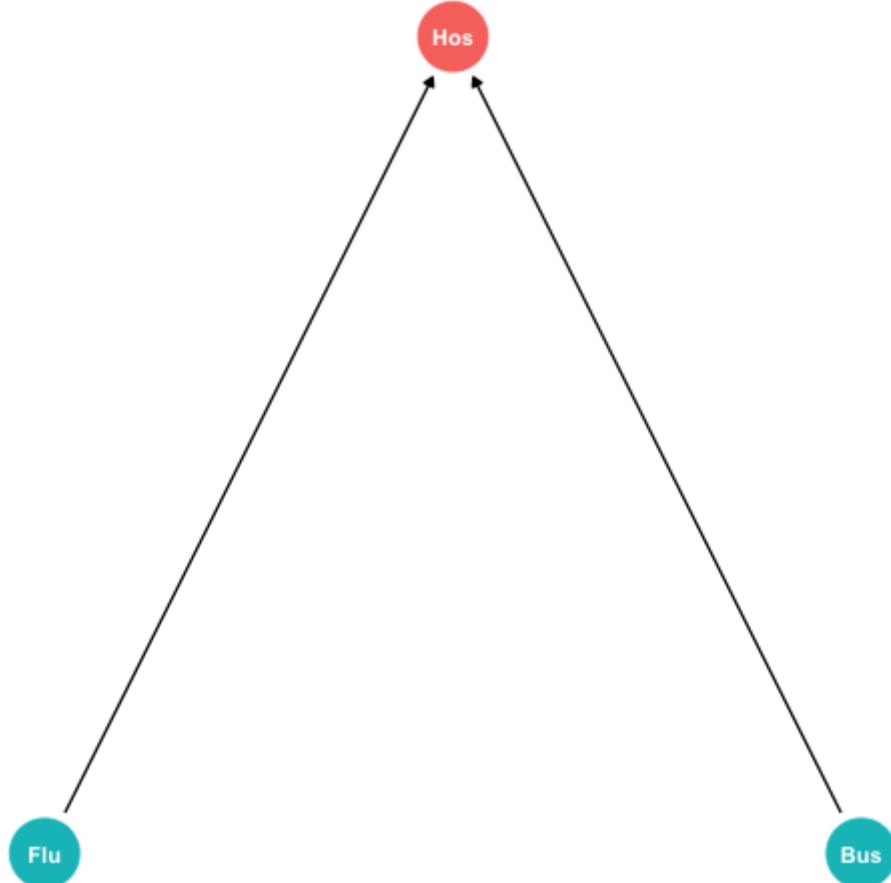


The Back Door Criterion: Colliders

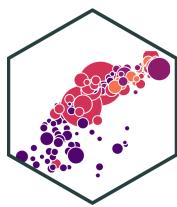


Example: Are you less likely to get the flu if you are hit by a bus?

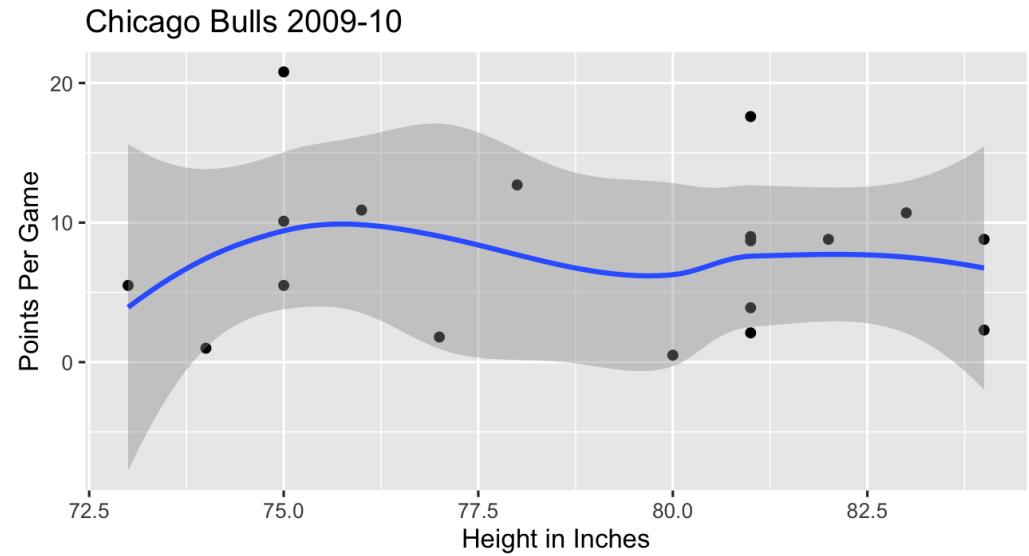
- *Flu*: getting the flu
- *Bus*: being hit by a bus
- *Hos*: being in the hospital
- Both *Flu* and *Bus* send you to *Hos* (arrows)
- Conditional on being in *Hos*, negative correlation between *Flu* and *Bus* (spurious!)



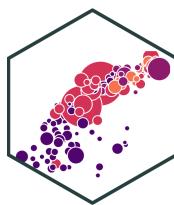
The Back Door Criterion: Colliders



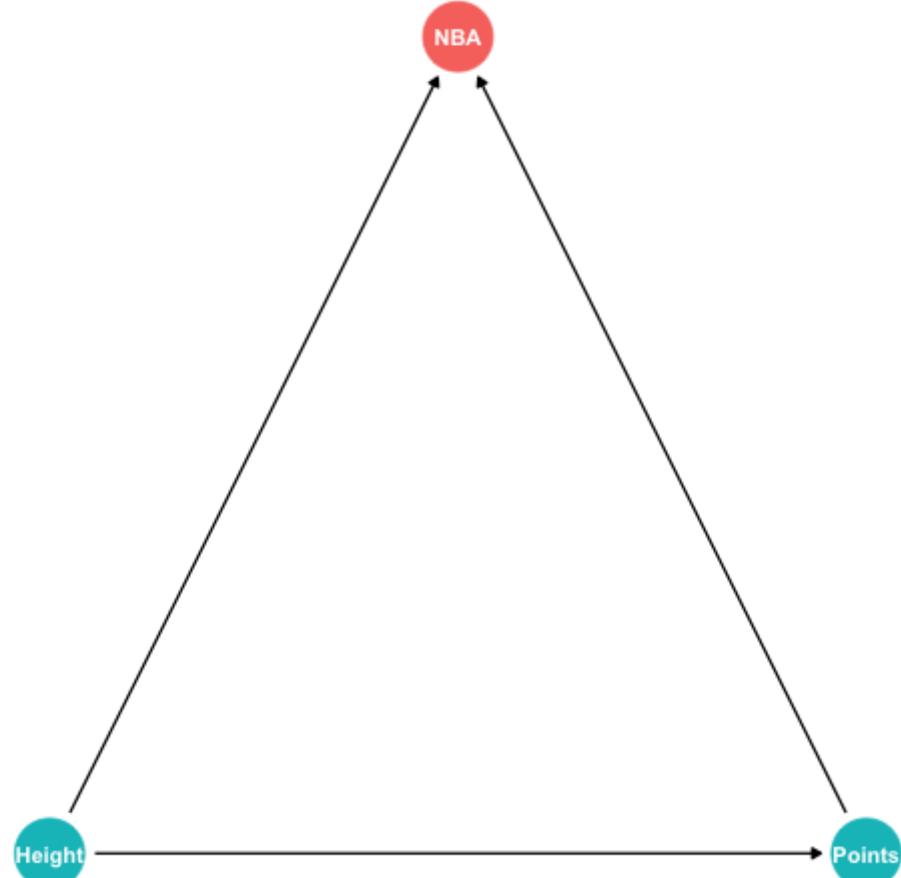
- In the NBA, apparently players' height has no relationship to points scored?



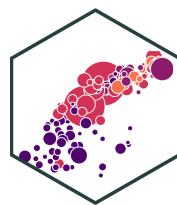
The Back Door Criterion: Colliders



- **In the NBA**, players' height has no relationship to points scored
- Naturally, taller people score more points in a basketball game, but if you *only* look at NBA players, that relationship goes away
- A person being in the NBA is a collider!
Colliders are another way to see **selection bias**



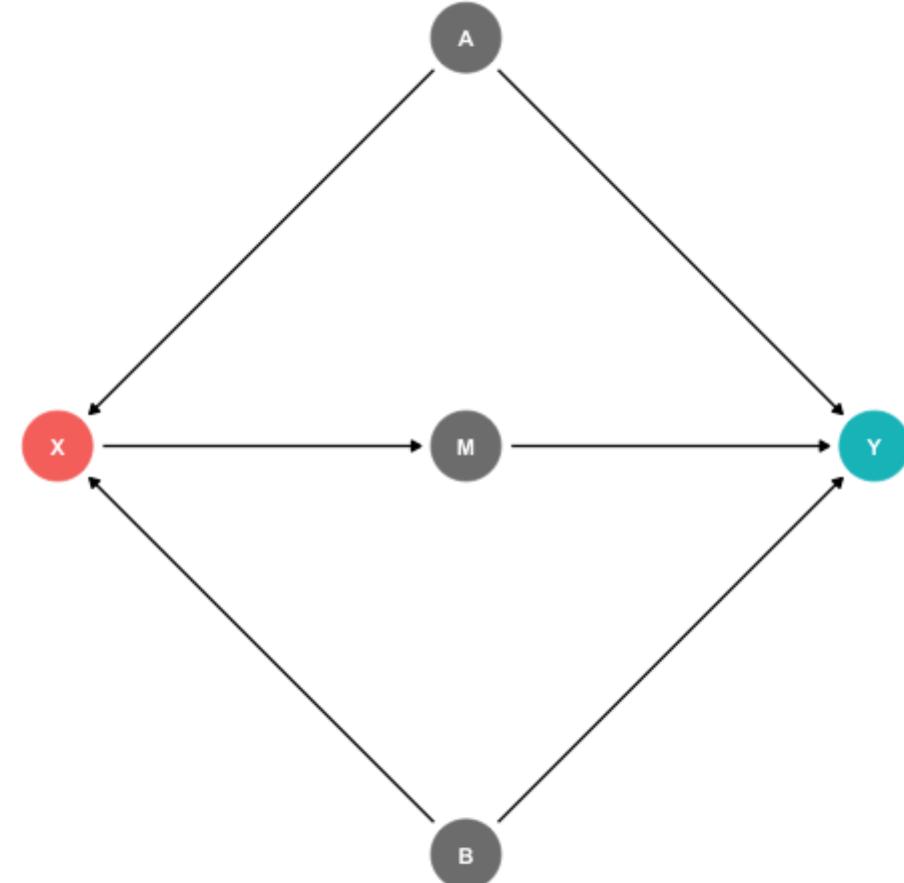
The Front Door Criterion: Mediators I



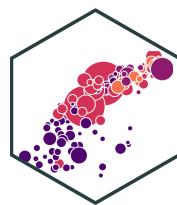
- Another case where controlling for a variable actually *adds bias* is if that variable is known as a “**mediator**”.

Example:

- $X \rightarrow M \rightarrow Y$ (front-door)
- $X \leftarrow A \rightarrow Y$ (back-door)
- $X \leftarrow B \rightarrow Y$ (back-door)
- Should we control for M ?
- If we did, this would block the front-door!



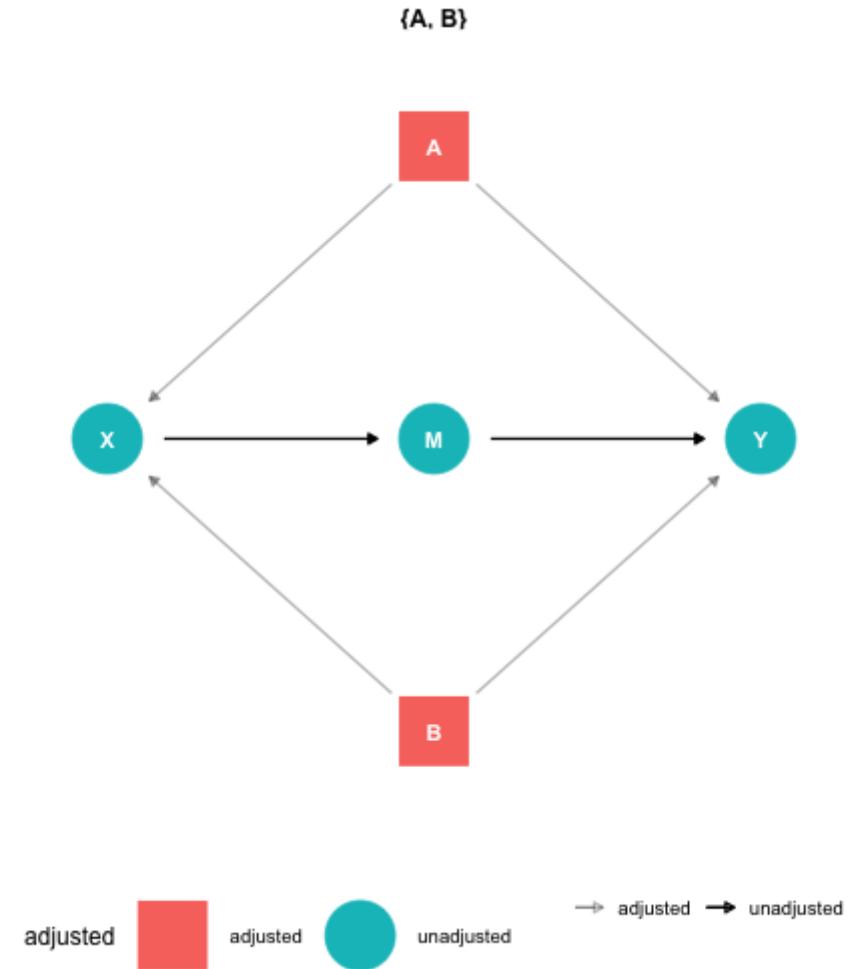
The Front Door Criterion: Mediators II



- Another case where controlling for a variable actually *adds bias* is if that variable is known as a “**mediator**”.

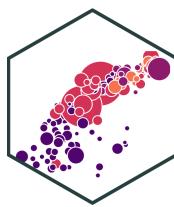
Example:

- If we control for M , would block the front-door!
- If we can estimate $X \rightarrow M$ and $M \rightarrow Y$ (note, no back-doors to either of these!), we can estimate $X \rightarrow Y$

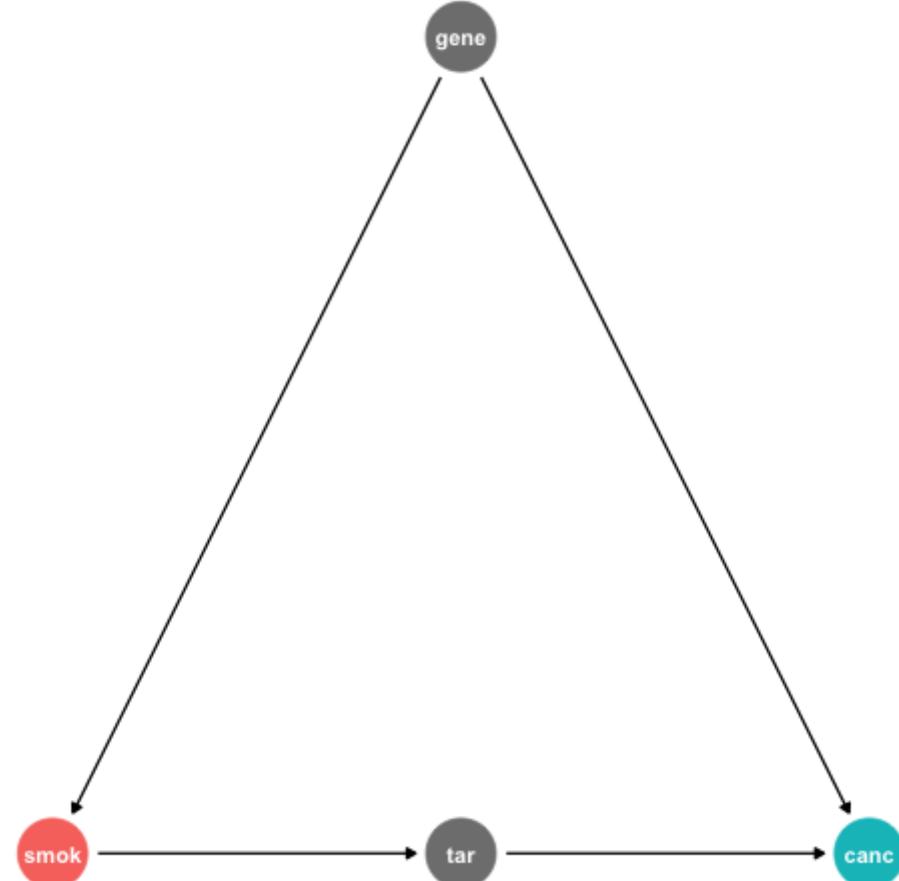


- This is the **front door method**

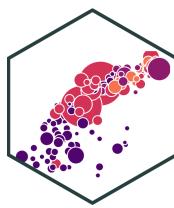
The Front Door Criterion: Mediators III



- Tobacco industry claimed that $\text{cor}(\text{smoking}, \text{cancer})$ could be spurious due to a confounding gene that affects both!
 - Smoking gene is unobservable
- Suppose smoking causes tar buildup in lungs, which cause cancer
- We should *not* control for tar, it's on the **front-door path**
 - This is how scientific studies can relate smoking to cancer



Summary: DAG Rules for Causal Identification



Thus, to achieve **causal identification**, control for the minimal amount of variables such that:

1. Ensure **no back-door path remains open**

- Close back-door paths by *controlling* for any one variable along that path
- Colliders along a path *automatically* close that path

2. Ensure **no front-door path is closed**

- Do not control for mediators

