Theoretical Aspects of Astroparticle Physics, Cosmology and Gravitation - 2026

Multi-messenger searches for dark matter: Exercises

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All quantities will be expressed in **natural units** (i.e. GeV units) except when stated otherwise. You can find all of the necessary Jupyter/Mathematica notebooks on the following GitHub repository:

https://github.com/JordanKoechler/GGI_School_DM_2026

1 The γ-ray Galactic Center Excess explained by dark matter annihilation

Since 2009, several groups have claimed the detection of a nearly spherically symmetric γ-ray excess around the Galactic Center (GC), extending out to 10° or 20°, ranging from 0.5 to 5 GeV in energy. Later, it was confirmed by the Fermi collaboration [1]. Today, it is believed that this excess can be explained by the two following equally compelling possibilities: i) emissions from a population of unresolved millisecond pulsars, and ii) annihilation of dark matter (DM). In this exercise, we will explore the second hypothesis and show how convincing it is.

The key quantity we must compute is the predicted flux of γ-rays coming from DM annihilation within a relevant region of interest (ROI). The expression of this flux is given by

$$\Phi_\gamma^{\rm DM} = \frac{1}{4\pi}\frac{\langle\sigma v\rangle}{2m_{\rm DM}^2} N_\gamma^{\rm DM}\mathcal{J} \, , \tag{1}$$

where $\langle\sigma v\rangle$ is the thermally averaged DM annihilation cross section, $m_{\rm DM}$ is the particle DM mass, $N_\gamma^{\rm DM}$ is the number of photons produced per DM annihilation in a specific channel and $\mathcal{J}$ is the so-called J-factor

$$\mathcal{J} = \int_{\Delta\Omega} d\Omega \int_{\rm l.o.s.} ds\; \rho_{\rm DM}^2(r(s,\Omega)) \, , \tag{2}$$

which integrates the spherically symmetric density profile of DM $\rho_{\rm DM}$ along the line of sight (l.o.s.) encoded by the coordinate s and within the ROI $\Delta\Omega$. Here r is the Galacto-centric distance, linked with the coordinate s by $r = \sqrt{r_\odot^2 + s^2 - 2sr_\odot\cos\theta}$, where θ is the angle between the l.o.s. and the GC-Sun axis.

1.1 Dark matter density profile and J-factor

The DM density profile of the Milky-Way has yet to be correctly measured. One thing we know about is that the local DM density is around 0.4 ± 0.1 GeV/cm^3. In indirect detection studies, it is known that the choice of the DM profile is the largest source of systematic uncertainty. However, by interpreting the GCE to be originated by DM annihilation, we can attempt to fix the profile. Let us consider, for instance, the gNFW profile, which is written as

$$\rho_{\rm gNFW}(r) = \frac{\rho_s}{(r/r_s)^\gamma\,(1+(r/r_s))^{3-\gamma}} \, , \tag{3}$$

where ρ_s and r_s are scale parameters and $\gamma > 0$ encodes the inner slope of the profile, as $\rho_{\rm gNFW}(r) \propto r^{-\gamma}$ when $r \to 0$. Due to this behavior, we say that the gNFW is a cusped profile.

Another choice for the profile is the following, known as the isothermal profile:

$$\rho_{\rm iso}(r) = \frac{\rho_s}{1 + (r/r_s)^2} \ , \tag{4}$$

which has the characteristic of being constant when $r \to 0$. This is a so-called cored profile.

The scale parameters of the DM profile ρ_s and r_s can be fixed using two somewhat reliable measurements:

- The local density of DM should be $\rho(r = r_\odot) = 0.4 \pm 0.1$ GeV/cm^3.
- The DM mass enclosed within the virial radius $r_{\rm vir} \sim 200$ kpc of the Milky-Way should be around $M_{\rm vir} \sim 10^{12} M_\odot$.

The notebook `Jfact.ipynb/nb` allows you to compute the J-factor of any user-input DM profile. It can use the two mentioned prescriptions to calculate the scale parameters before computing $\mathcal{J}$.

Question: Using this code, estimate the J-factor within 20° from the GC, for both the classic NFW profile ($\gamma = 1$) and for a cuspier version of if ($\gamma = 1.26$). Can you use the aforementioned prescriptions to fix the scale parameters of the isothermal profile? Taking $r_s = 4$ kpc, verify that the isothermal profile gives $\mathcal{J} \sim 10^{22}$ GeV^2/cm^5 in the same ROI.

Instead of annihilating, we can also assume that DM decays. In this case, the flux of photons can be written in a similar fashion as Eq. 1:

$$\Phi_\gamma^{\rm DM} = \frac{1}{4\pi} \frac{\Gamma}{m_{\rm DM}} N_\gamma^{\rm DM} \mathcal{D} \ , \tag{5}$$

where Γ is the DM decay rate and $\mathcal{D}$ is the so-called D-factor:

$$\mathcal{D} = \int_{\Delta\Omega} d\Omega \int_{\rm l.o.s.} ds \ \rho_{\rm DM}(r(s, \Omega)) \ . \tag{6}$$

Question: Plot both a dimensionless version of the J-factor and of the D-factor on the same figure with respect to θ (do not integrate over the ROI $\Delta\Omega$ for this). Use the classic NFW profile for this.

1.2 Number of photons per annihilation

The number of photons per annihilation depends on two things: $m_{\rm DM}$ and the annihilation channel considered. In order to obtain a spectrum of DM-produced γ-rays that is broad enough to explain the Galactic Center excess (GCE), it is common to assume that DM annihilates into heavy particles, prone to decay, hadronize, ... To compute N_γ, numerical tools such as `PPPC4DMID` [2, 3] or `CosmiXs` [4, 5] can be used.

The notebook `Ngamma.ipynb/nb` allows you to extract from `CosmiXs/PPPCDMID` the spectrum of photons produced by DM annihilation for a given DM mass and channel.

Question: Using this code, compute the number of photons N_γ produced per DM annihilation for $m_{\rm DM} = 100$ GeV and the e^+e^-, $\mu^+\mu^-$, and $b\bar{b}$ channels. Why does the $b\bar{b}$ channel overall produce more photons than the leptonic channels? Also, can you come up with a simple relationship that links N_γ for annihilating DM and N_γ for decaying DM?

1.3 Annihilation cross-section

If we are agnostic about the origin of DM particles, we can consider that $m_{\rm DM}$ and $\langle \sigma v \rangle$ are free parameters. However, in order to have a DM scenario in agreement with the measurement of the DM relic density by Planck ($\Omega_{\rm DM} h^2 = 0.120 \pm 0.001$). We can therefore consider the WIMP paradigm, in which $m_{\rm DM} \sim 50\,$GeV and $\langle \sigma v \rangle \sim 3 \times 10^{-26}$ cm^3/s is the correct configuration to retrieve the measured DM relic density.

dSph	D [kpc]	$\log_{10}(\rho_s)$ $[M_\odot/\mathrm{kpc}^3]$	r_s [kpc]	r_c [kpc]	n	δ	r_t [kpc]
Leo I	254	7.355	1.483	0.4084	0.5209	4.257	1.374
Draco	76	7.341	1.678	0.1943	0.5389	4.159	1.114
Ursa Major II	35	7.425	1.115	-	-	-	9.910

Table 1: Distance from the Earth and parameters of the ρ_{cNFWt} profile of a sample of dSphs [7]. For Ursa Major II, the profile is a NFW one truncated at r_t.

Question: Knowing that the flux of γ-rays measured by FERMI-LAT between 0.5 and 5 GeV, and within 20° from the GC is $\Phi_\gamma^{\mathrm{obs}} \sim 1.6 \times 10^{-6}$ photons/cm^2/s, find possible parameterizations of DM that would explain both the GCE and the measured DM relic density. In particular:

- Is DM annihilating in leptons a viable scenario?
- Do you think the GCE can be explained by a cored or cusped profile? If the latter, what is the best-fit value of the inner slope?

2 Dark matter signals from dwarf spheroidal galaxies

Dwarf spheroidal galaxies (dSphs) are among the most favored target for DM indirect searches, as they are known to be rich in DM as well as poor in gas, dust and stars. Their mass-to-light ratio M/L can be several tens of $M_\odot/L_\odot$ (e.g. Sculptor) to even a few thousands $M_\odot/L_\odot$ (e.g. Segue 1). But so far, there is still no detection of indirect signal of DM in dSphs. In this exercise, we will compute a upper bound on $\langle\sigma v\rangle$ for a few m_{DM} from the sensitivity of the γ-ray observatory FERMI-LAT.

Just like the previous exercice, we use Eq. 1 in order to evaluate the flux of photons from DM annihilation. And again, a crucial quantity to set is the J-factor. Observations of dSphs of the Local Group show us that their DM profile can be both cored and cusped. We adopt here a very flexible choice for the profile, that can both describe cored and cusped systems, as well as take into account their tidal stripping (i.e. the pulling of matter from a larger galaxy, here the Milky-Way) [6]. This profile is built upon the NFW profile (Eq. 3 with $\gamma = 1$):

$$\rho_{\mathrm{cNFWt}}(r) = \begin{cases} \rho_{\mathrm{cNFW}}(r) & r \le r_t \\ \rho_{\mathrm{cNFW}}(r_t)(r/r_t)^{-\delta} & r > r_t \end{cases}, \tag{7}$$

where r_t is the tidal radius of the dSph, and

$$\rho_{\mathrm{cNFW}}(r) = f^n \rho_{\mathrm{NFW}}(r) + \frac{n f^{n-1}(1-f^2)}{4\pi r^2 r_c} M_{\mathrm{NFW}}(r)\,, \tag{8}$$

with $f = \tanh(r/r_c)$, r_c is the core radius, and $M_{\mathrm{NFW}}(r)$ is the total DM mass within r. This profile is already implemented in `Jfact.ipynb/nb`.

Question: Compute the J-factor within 0.5° of the three sample dSphs in Tab. 1. Which of the three dSph can produce the most stringent upper bound on $\langle\sigma v\rangle$? Compute this upper bound for $m_{\mathrm{DM}} = 50$ GeV and DM DM $\to b\bar{b}$ using the sensitivity of FERMI-LAT between 0.1 and 100 GeV which is $\Phi_{\mathrm{sens}} \sim 10^{-12}$ erg/cm^2/s. Does this upper bound exclude the DM interpretation of the GCE?

3 Secondary emissions from dark matter annihilation

3.1 Estimation of the peak IC and synchrotron energies

Inverse-Compton scattering is the process in which high-energy $e^\pm$ up-scatter lower energy photons. In the context of indirect searches of DM, it can be an important contribution of photon emissions, especially in the case where DM annihilation can produce $e^\pm$, since they can up-scatter Galactic

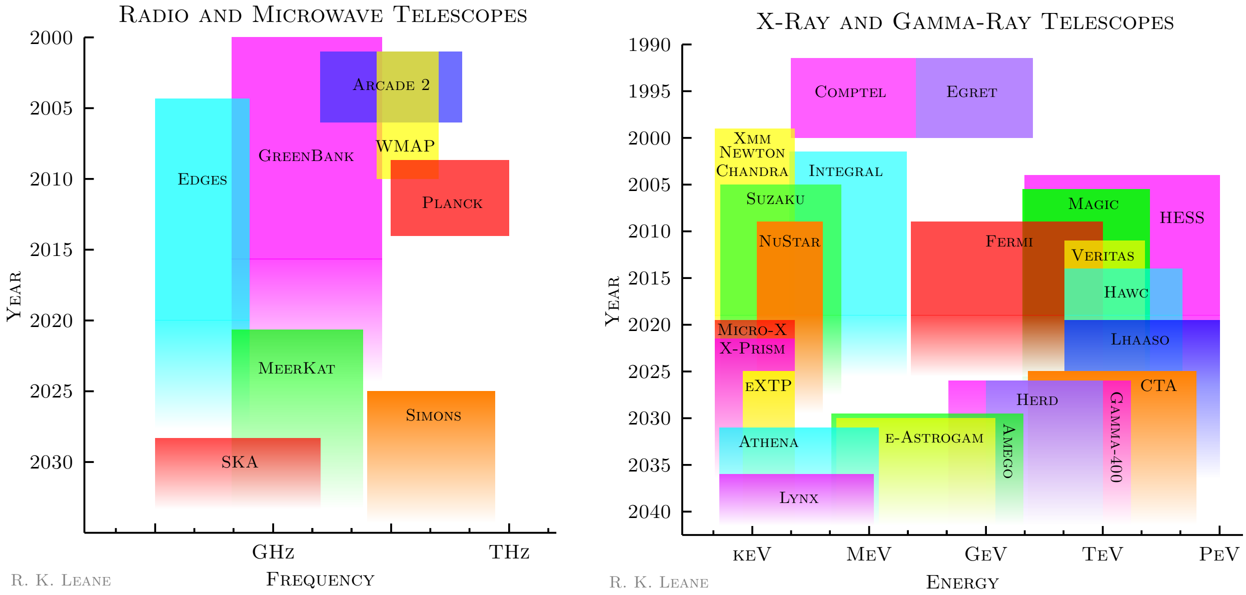


Figure 1: *Left:* Representation of the energy range (x-axis) and operation dates (y-axis) of a selection of radio, microwave telescopes. *Right:* Same as in the left pannel, but for X- and γ-ray observatories. Taken from [8].

ambient photons. These ambient photons are composed of three sources: the CMB (with typical photon energies of ~ 0.6 meV), optical starlight (1 eV), and infrared from the scattering of SL on Galactic dust (~ 10 meV).

Question: Knowing that the energy of the IC emission peak can be written $E^{\rm IC}_{\gamma,\rm peak} = 4\gamma^2 E^0_\gamma$ where E^0_γ is the initial energy of the photon prior to its up-scattering and $\gamma = E_e/m_e$ is the Lorentz factor of the $e^\pm$, compute this quantity for $m_{\rm DM} = 1$ GeV, 1 TeV, and 1 PeV. What wavelength bands do they cover? With the help of Fig. 1, can you cite some relevant observatories that could detect such emissions today?

Synchrotron radiation is the process in which $e^\pm$ emits photons while traveling through a magnetic field. It can be an important component of photon emissions to consider in the case where DM annihilation produces $e^\pm$, which travel in the Galactic magnetic field (which is of the order of 5 μG).

Question: The peak synchrotron energy is defined near the so-called 'critical energy' $E^{\rm syn}_{\gamma,\rm c} = 3eB\gamma^2/(2\pi m_e)$ where B is the strength of the magnetic field. This expression of $E^{\rm syn}_{\gamma,\rm c}$ is only valid if B and e are expressed in GeV units (1 G $= 1.95 \times 10^{-20}$ GeV^2 and $e = \sqrt{4\pi\alpha}$). With the critical energy defined as it is, the peak synchrotron is around $E^{\rm syn}_{\gamma,\rm peak} = 0.114 E^{\rm syn}_{\gamma,\rm c}$ (we will cover this later in the exercise). Keeping that in mind, compute the $E^{\rm peak}_{\gamma,\rm syn}$ for $m_{\rm DM} = 1$ GeV, 1 TeV, and 1 PeV. What wavelength bands do they cover? Can you cite some relevant observatories that could detect such emissions today?

3.2 Towards the flux of secondary emissions: Radiating power spectrum

The flux of secondary emissions are a bit more tricky to compute compared to the prompt emissions we discussed in the previous exercises. We can, however, write this flux for ICS, synchrotron and bremsstrahlung in a similar formalism:

$$\frac{d\Phi^{\rm sec}_\gamma}{dE_\gamma d\Omega} = \frac{1}{E_\gamma}\int_{\rm l.o.s} ds\, \frac{j_{\rm sec}(E_\gamma,\vec{x})}{4\pi}\,, \tag{9}$$

Photon field	T [K]	R_0 [kpc]	z_0 [kpc]	$u_{\gamma,\odot}$ [eV cm^{-3}]
CMB	2.725	-	-	0.26
IRc	40	2	1	0.20
IRw	400	3.5	2	0.05
Opt	5000	2.5	1.5	0.50

Table 2: Parameters for the simplified models of the Galactic photon fields.

where $j_{\rm sec}$ is the emissivity, which can in turn be written as a convolution between the radiating power spectrum $\mathcal{P}_{\rm sec}$ and the energy density of DM-produced $e^{\pm}$ n_e:

$$j_{\rm sec}(E_\gamma, \vec{x}) = 2 \int_{m_e}^{m_{\rm DM}(/2)} dE_e \; \mathcal{P}_{\rm sec}(E_\gamma, E_e, \vec{x}) \, n_e(E_e, \vec{x}) \; , \tag{10}$$

where the upper bound of integration is the maximum energy of the DM-produced $e^{\pm}$: $m_{\rm DM}$ for annihilating DM and $m_{\rm DM}/2$ for decaying DM. The factor 2 takes into account the sum the individual contribution of e^+ and e^-. We will deal with the $e^{\pm}$ energy density later.

The radiating power spectrum $\mathcal{P}_{sec}$ carry all of the physics of the secondary emissions. For IC scattering, it is written as follows in GeV s^{-1} GeV^{-1} (detailed calculation in Ref. [9]):

$$\mathcal{P}_{\rm IC}(E_\gamma, E_e, \vec{x}) = \frac{3\sigma_t}{4\gamma^2} \int_{1/(4\gamma^2)}^{1} dq \; \left(E_\gamma - E_\gamma^0(q)\right) \frac{n_\gamma\left(E_\gamma^0(q), \vec{x}\right)}{q} \left[2q \log q + q + 1 - 2q^2 + \frac{\epsilon^2(1-q)}{2(1-\epsilon)}\right] \; , \tag{11}$$

where $\sigma_t = 0.665 \times 10^{-24}$ cm^2 is the Thomson cross section, $q = E_\gamma / \left(4\gamma^2 E_\gamma^0 (1-\epsilon)\right)$ and $\epsilon = E_\gamma/E_e$. Finally, n_γ is the photon field number density (in GeV^{-1}cm^{-3}) that is up-scattered by DM-produced $e^{\pm}$. In our galaxy, we can model the three components (CMB, IR, Optical) as blackbodies with different temperatures. Moreover IR and Optical components are not isotropic and mostly lie in the Galactic disk. We model this by an exponential suppression in R and z:

$$n_{\rm IR/Opt}(E_\gamma, \vec{x}) \propto \exp\left(-\frac{R - r_\odot}{R_0} - \frac{|z|}{z_0}\right) \; . \tag{12}$$

The temperature, cutoff radius R_0 and height z_0, as well as the local energy density of the fields $u_{\gamma,\odot} = \int dE_\gamma \; E_\gamma n_\gamma(E_\gamma, \vec{x}_\odot)$ (important for normalization) are reported in Tab. 2.

Question: Using `Power.ipynb/nb`, plot the different components of the photon field vs E_γ^0 at $(R, z) = (5, 0)$ kpc. Plot also $\mathcal{P}_{\rm IC}$ vs E_γ, for $E_e = 10$ MeV, 100 MeV, 1 GeV, and 10 GeV, at Earth's position. Compare the results with Fig. 2 and conclude on the accuracy of the blackbody+exponential cutoff model for the photon fields.

Similarly to IC, we can write the radiating power spectrum for synchrotron emissions in GeV s^{-1} Hz^{-1} (detailed in [10] and valid for $\gamma \gtrsim 2$):

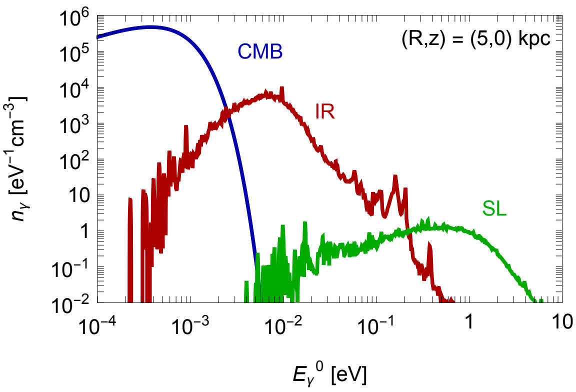
$$\mathcal{P}_{\rm syn}(\nu, E_e, \vec{x}) = \frac{\sqrt{3} e^3 B\left(\vec{x}\right)}{2\pi m_e} y^2 \left[K_{4/3}(y) K_{1/3}(y) - \frac{3}{5} y \left(K_{4/3}(y)^2 - K_{1/3}(y)^2\right)\right] \; , \tag{13}$$

where ν is the frequency of the radiated photon, e is the elementary charge, K_n are the modified Bessel functions of the second kind of order n, $y = \nu/\nu_c$, and finally $\nu_c = 3\gamma^2 eB/(2\pi m_e)$ is the critical frequency.

When it comes to the Galactic magnetic field, since it is mostly concentrated in the Galactic disk, we can adopt simplified models with an exponential cutoff in R and z, just like the case of the photon fields:

$$B(\vec{x}) = B_0 \exp\left(-\frac{R - r_\odot}{R_0} - \frac{|z|}{z_0}\right) \tag{14}$$

with the scale radius R_0, height z_0 and normalization B_0 for a three models [10] are reported in Tab. 3.



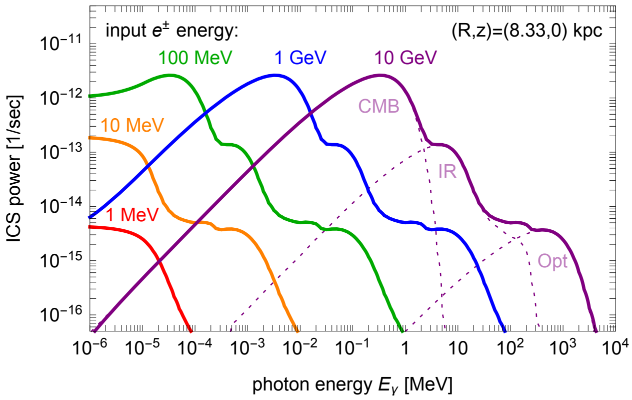


Figure 2: *Left:* Spectra of the photon number density per unit energy of ambient photons in the Galactic Plane. The IR and optical starlight (SL) components are extracted from `GALPROP` [11] and are based on observations from COBE/DIRBE. *Right:* IC radiating power spectrum at Earth's position using the photon fields described on the left panel.

Model	B_0 [μG]	R_0 [kpc]	z_0 [kpc]
MF1	4.78	10	2
MF2	5.1	8.5	1
MF3	9.5	30	4

Table 3: Parameters for the simplified models of the Galactic magnetic fields.

Question: Using `Power.ipynb/nb`, plot the radiating synchrotron power spectrum $\mathcal{P}_{\rm syn}$ vs ν, for E_e = 10 MeV, 100 MeV, 1 GeV, and 10 GeV, at Earth's position and for the three magnetic field models. What is the magnetic field model that would produce the highest synchrotron flux? Find an approximate relationship between the peak frequency $\nu_{\rm peak}$ and the critical frequency ν_c.

Finally, for bremsstrahlung, i.e. the braking of a $e^\pm$ traveling close nuclei in the interstellar medium (ISM), the radiating power spectrum is written:

$$\mathcal{P}_{\rm brems}(E_\gamma, E_e, \vec{x}) = E_\gamma \sum_i n_i(\vec{x}) \frac{d\sigma_i}{dE_\gamma}(E_\gamma, E_e) \,, \tag{15}$$

where n_i is the ISM gas density of the species $i \in$\{HI, HII, H_2, He\} and $d\sigma_i/dE_\gamma$ is the differential bremsstrahlung cross-section:

$$\frac{d\sigma_i}{dE_\gamma}(E_\gamma, E_e) = \frac{3\alpha\sigma_T}{8\pi E_\gamma} \left\{ \left[1 + \left(1 - \frac{E_\gamma}{E_e}\right)^2\right] \phi_1^i(E_e, E_\gamma) - \frac{2}{3}\left(1 - \frac{E_\gamma}{E_e}\right) \phi_2^i(E_e, E_\gamma) \right\} \,, \tag{16}$$

where $\phi^i_{1,2}$ are scattering functions dependent on the properties of the scattering system. We essentially consider two limiting regimes. The first one is one where the impinging $e^\pm$ is ultra-relativistic ($\gamma \gtrsim 10^3$), where the electronic cloud around atoms in the gas screens their nucleus, effectively decreasing the bremsstrahlung emission. This is called the strong shielding regime. In this scenario the scattering functions are constants:

$$\phi^{\rm HI}_{1,\rm ss} = 45.79, \quad \phi^{\rm HI}_{2,\rm ss} = 44.46, \tag{17}$$
$$\phi^{\rm He}_{1,\rm ss} = 134.60, \quad \phi^{\rm He}_{1,\rm ss} = 131.40 \,, \tag{18}$$

and $\phi^{\rm H_2}_{(1,2),\rm ss} \simeq 2\phi^{\rm HI}_{(1,2),\rm ss}$. The second regime is the weak shielding regime, where $\gamma \lesssim 10^2$, in which electronic screening becomes inefficient, essentially corresponding to the case where the gas is ionized.

In this scenario the scattering functions are written:

$$\phi^i_{1,\mathrm{ion+ws}}(E_e, E_\gamma) = \phi^i_{2,\mathrm{ion+ws}}(E_e, E_\gamma) = 4Z_i(Z_i+1)\left\{\log\left[\frac{2E_e}{m_e}\left(\frac{E_e - E_\gamma}{E_\gamma}\right)\right] - \frac{1}{2}\right\} , \qquad (19)$$

where Z_i is the number of protons in a nucleus of gas species i. When the gas is ionized (relevant for HII), these scattering functions are valid for the whole impinging $e^\pm$ energy range, since there is no more electronic screening.

In order to study the intermediate regime between the strong- and weak-shielding ones (i.e. $10^2 \lesssim \gamma \lesssim 10^3$), we can perform an extrapolation.

Question: Again, the bremsstrahlung power spectrum and the gas densities (modeled as a disk with exponential cutoffs in R and z) are coded in `Power.ipynb/nb`. Plot $\mathcal{P}_{\mathrm{brems}}$ vs E_γ for $E_e = 10$ MeV, 100 MeV, 1 GeV, and 10 GeV at Earth's position. What are the most relevant observatories that could detect such emissions?

3.3 DM-produced $e^\pm$ energy density

The final quantity needed to compute $d\Phi^{\mathrm{sec}}_\gamma/dE_\gamma d\Omega$ is the DM-produced $e^\pm$ energy density $n_e(E_e, \vec{x})$. As seen in the lecture, $e^\pm$ do no travel as straight as photons and neutrinos do, since they travel through the Galactic magnetic, photon and gas fields, which all in all contribute to their spatial diffusion and energy losses. The propagation of $e^\pm$ can be encoded in the diffusion-loss equation:

$$-\vec{\nabla}\cdot\left(D(E_e,\vec{x})\,\vec{\nabla}n_e\right) - \frac{\partial}{\partial E_e}\left(b_{\mathrm{tot}}(E_e,\vec{x})\,n_e\right) = Q_e^{\mathrm{DM}}(E_e,\vec{x}) , \qquad (20)$$

where D is the diffusion coefficient, $b_{\mathrm{tot}}(E_e,\vec{x}) = -dE_e/dt$ is the total energy-loss coefficient and Q_e^{DM} the injection rate of DM-produced $e^\pm$ (or source term):

$$Q_e^{\mathrm{DM}}(E_e,\vec{x}) = \begin{cases} \frac{1}{2}\langle\sigma v\rangle\left(\frac{\rho_{\mathrm{DM}}(\vec{x})}{m_{\mathrm{DM}}}\right)^2\frac{dN_e}{dE_e} & \text{annihilation} \\ \Gamma\left(\frac{\rho_{\mathrm{DM}}(\vec{x})}{m_{\mathrm{DM}}}\right)\frac{dN_e}{dE_e} & \text{decay}\end{cases} , \qquad (21)$$

where dN_e/dE_e is the $e^\pm$ spectrum from DM annihilation or decay.

Question: Solve Eq. 20 for $n_e(E_e,\vec{x})$ when diffusion is negligible compared to energy losses (relevant for ROIs close to the GC).

If diffusion is neglected, the last key quantity we have to cover is the total energy-loss coefficient $b_{\mathrm{tot}}(E_e,\vec{x})$, which encodes the energy lost by the propagating $e^\pm$ through radiative and non-radiative processes. For radiative processes (ICS, synchrotron, bremsstrahlung), this coefficient is simply written as:

$$b_{\mathrm{rad}}(E_e,\vec{x}) = \int_0^{E_e} dE_\gamma\,\mathcal{P}_{\mathrm{sec}}(E_\gamma, E_e, \vec{x}) . \qquad (22)$$

In addition, there are two important non-radiative energy loss processes for $e^\pm$: Coulomb scattering (i.e. scattering of $e^\pm$ on free e^- in an ionized plasma) and ionization of neutral gases. For Coulomb scattering, the energy loss coefficient is written:

$$b_{\mathrm{Coul}}(E_e,\vec{x}) = \frac{3\sigma_T}{4}m_e n_e^{\mathrm{free}}(\vec{x})\left[\log\left(\frac{E_e}{m_e}\right) + 2\log\left(\frac{m_e}{E_p(\vec{x})}\right)\right] , \qquad (23)$$

where n_e^{free} is the number density of free e^- (which basically follows n_{HII}) and $E_p = e\sqrt{m_e n_e}$ ($r_e = e^2/m_e$) is the plasma energy. For losses through ionization, we have the following:

$$b_{\mathrm{ion}}(E_e,\vec{x}) = \frac{9\sigma_T}{4}m_e\sum_i n_i(\vec{x})Z_i\left[\log\left(\frac{E_e}{m_e}\right) + \frac{2}{3}\log\left(\frac{m_e}{\Delta E_i}\right)\right] , \qquad (24)$$

where ΔE_i is the average excitation energy of the gas species i, which is 15.0 eV for HI and 41.5 eV for He.

Question: Using `Power.ipynb/nb` and Eqs. 22, 23, and 24, implement the different energy loss coefficients in a code. Then plot them vs E_e at Earth's position for $e^\pm$ energies between 1 MeV and 1 GeV. What are the most important contributions to the $e^\pm$ energy losses in this case?

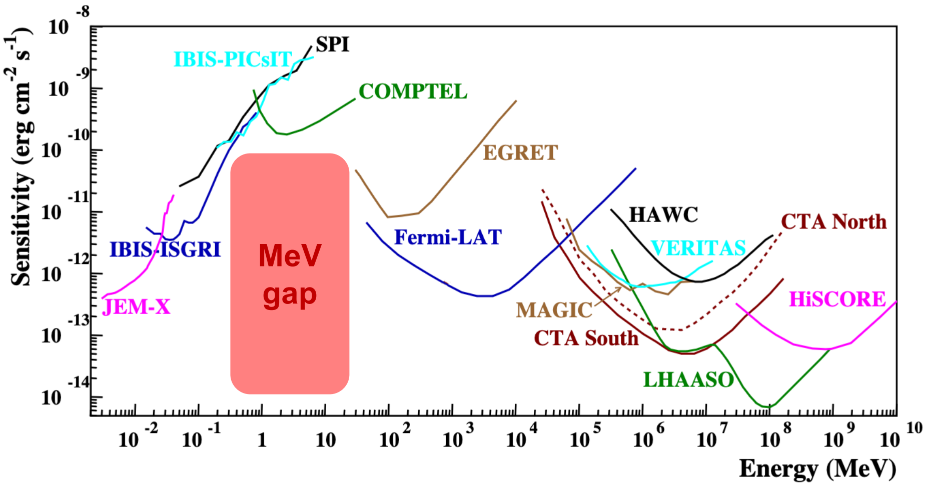


Figure 3: Sensitivities of a selection of X- and γ-ray observatories, illustrating the MeV gap. Adapted from [12].

4 Sub-GeV DM and the MeV gap

Sub-GeV DM is a well motivated alternative to the WIMP paradigm, as the latter fails more and more to provide us with experimental signals. In the following exercises, we will explore potential indirect signals coming from their annihilation, by investigating a few channels.

Question: For DM DM $\to \gamma\gamma$, what is the energy range of the emitted photons for $m_{\rm DM}$ between 1 MeV and 1 GeV? Using Figs. 1 and 3 can you say about the observatories that can detect such photons?

4.1 Prompt emissions

Let us focus on DM DM $\to e^+e^-$. In order to compute the flux of photons, we can use again Eq. 1 ... except that now the numerical implementation in `Ngamma.ipynb/nb` does not work, as the DM masses studied in this exercise are outside the allowed range for `CosmiXs` and `PPPC4DMID` (which is 5 GeV to 100 TeV). However, photons emitted during the DM annihilation into e^+e^- come from final state radiation, and their energy spectrum as a well defined analytical form:

$$\frac{dN_{\gamma,\rm FSR}}{dE_\gamma} = \frac{\alpha}{\pi\beta(3-\beta^2)m_{\rm DM}}\left[\mathcal{A}\log\frac{1+R(\nu)}{1-R(\nu)} - 2\mathcal{B}R(\nu)\right] , \tag{25}$$

where

$$\mathcal{A} = \frac{\left(1+\beta^2\right)\left(3-\beta^2\right)}{\nu} - 2\left(3-\beta^2\right) + 2\nu \quad \text{and} \quad \mathcal{B} = \frac{3-\beta^2}{\nu}(1-\nu)+\nu\ , \tag{26}$$

where we define: $\nu = E_\gamma/m_{\rm DM}$, $\beta^2 = 1-4\mu^2$ with $\mu = m_e/(2m_{\rm DM})$ and $R(\nu) = \sqrt{1-4\mu^2/(1-\nu)}$.

Question: For $m_{\rm DM} = 10$ MeV and 1 GeV, what is the most relevant telescope that could observe such emissions? Then, compute $N_\gamma = \int dE_\gamma\, dN_{\gamma,\rm FSR}/dE_\gamma$ in the energy range observed by INTEGRAL (20 keV – 2 MeV). Finally, derive the upper limits on $\langle\sigma v\rangle$ associated to these emissions, given that the flux measured by INTEGRAL within 30° in this energy range is around 3×10^{-2} cm^{-2}s^{-1}MeV^{-1}.

4.2 ICS emissions

The previous exercise gave you all of the necessary ingredients to compute the flux of photons from ICS of DM-produced $e^\pm$ on the Galactic photon fields.

Question: For DM DM $\to e^+e^-$ what is the expression of the $e^\pm$ spectrum dN_e/dE_e necessary for the injection rate Q_e? Again, derive the upper limits of $\langle\sigma v\rangle$ associated to these ICS emissions using the flux measured by INTEGRAL, for $m_{\rm DM} = 10$ MeV and 1 GeV. What is the most important contribution of the total photon flux from DM annihilation, for both of these masses?

Bonus: Do this exercise again for decaying DM!

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