Search for Higgs boson pair-production in the $bb\tau\tau$ final state using proton-proton collisions at $\sqrt{s}$ = 13 TeV data with the ATLAS detector

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The Standard Model and beyond

Particle physics is in the heart of our understanding of the laws of nature. The subject is concerned with the fundamental constituents of the Universe, the *elementary particles*, and the interactions between them, the *forces*. The Standard Model (SM) embodies the current understanding of particle physics, provides a unified picture where the forces between the particles are themselves described by the exchange of particles. It provides a successful description of all current experimental data and represents on of the triumphs of mordern physics.

The last missing piece of the SM, the Higgs boson has been observed with 5 σ significance by the ATLAS and CMS experimental at the Large Hadron Collider [1, 2]. The SM has therefore hailed by many as the most successful theory ever conceived. Despite its success, the SM is yet not the ultimate theory, as many unanswered questions remain. For example, why the SM has so many free parameters (26) that have to be input by hand; what is the particle content of the dark matter; what is the origin of the matter-antimatter assymmetry in the universe, and etc.

This chapter introduces the basic concepts of the SM, including the gauge theory and fundamental forces, the Higgs mechanism, and a hint of beyond the SM (BSM) theories that address some of the important and unanswered questions.

2.1 The Standard Model of Particle Physics

The core of the SM was outlined by Steven Weinberg just over a half-century ago, which he published the short but revolutionary paper titled “A Model of Leptons” in the journal Physical Review Letters [3]. The SM was developed in stages throughout the latter half of the 20th century, through the work of many scientists around the world, with the current formulation being finalized in the mid–1970s upon experimental confirmation of the existence of quarks [4, 5].

In the SM, most of the everyday phenomena we see in the physical world is just the low energy manifestation of the twelve elementary particles and three interactions: electromagnetic, weak and strong forces. For example, atoms, which were believed to be the most basic building blocks of the world, are in fact comprised of negatively charged electrons and positively charged neclues, bound by electromagnetic attraction. Such phenomenon is a low energy manifestation of the fundamental theory of electromagnetism, namely Quantum Electrodynamics (QED). While in the neclues, the protons and neutrons are bound together by the strong nuclear force (where the protons and neutrons are comprised of quarks bound by the strong force), which is again the low energy manifestation of the fundamental theory of strong interactions, namely Quantum Chromodynamics (QCD). The fundamental interactions of the particles are completed by the weak force, which controls particles decay, nuclear fission and ect. One must note that, although gravity is oftenly ignored in the context of particle physics due to its relatively much smaller strength compared to the other three fundamental interactions (around 10^{34} times weaker than the electromagnetic force), gravity must be integrated into the theory. The SM is widely considered to be incompatible with the most successful theory of gravity to date, general relativity. Yet no clear approach is available for combining the two behemoths, but huge effort has been put into physics beyond SM, such as the Supersymmetry (SUSY), Loop quantum gravity or String theory, which may shed some light on the ultimate theory of everything.

2.2 Particle Content

In the SM, the electron, the electron neutrino, the up-quark and the down-quark are known collectively as the first generation. As far as we know, they are elementary particles, instead of being composite, and represent the basic building blocks of the low-engery universe. For each of the first generation particles, there are exactly two copies which differ only in their masses. These additional eight particles are known as the second and the third generations. For example, the second generation muon is essentially a heavier version of the electron with mass $m_\mu \approx 200 m_e$, and the third generation τ-lepton is an even heavier copy with $m_\tau \approx 3500 m_e$. The three generations of particles, are called collectively as *fermions*. Fermions have intrinsic spin $s = \frac{1}{2}$ and obey the Pauli exclusion principle, which states that no two fermions can occupy the same quantum state.

The dynamics of each of the twelve fundamental fermions are governed by the Dirac equation of relativistic quantum mechanics [6],

$$(i\gamma^\mu \partial_\mu - m)\psi = 0,$$

where the γ^μ are the gamma matrices, and ψ is the Dirac spinor. An important consequence is that for each of the twelve fermions there exists an antiparticle state with exactly the same mass, but opposite charge and intrinsic spin. The antiparticle are denoted either by their charge or by a bar over the corresponding particle symbol, for example, the anti-electron (positron) is denoted by e^+, and the anti-up-quark is written as $\bar{u}$.

In oppose to fermions, *bosons* are defined as particles that has integer intrinsic spin, and does not obey the Pauli exclusion principle. In particular, bosons with spin 1 are called *gauge bosons*. In modern particle physics, the three fundamental forces are described by a Quantum Field Theory (QFT), each gauge boson can be seen as the excitations of the quantum field of each forces. For example, the familiar photon is the gauge boson of the Quantum Electrodynamics (QED), and for the strong interaction, the force-carrying partilce is called *gluon*. While the photon and gluon are massless, the weak charged-current interaction, which is responsible for nuclear β-decay, is mediated by the W^- or the W^+ bosons with masses of 80.4 GeV. In addition, the neutral-current interaction is mediated by the chargeless Z boson, with a mass of 91.2 GeV. Due to the large mass of the mediator, the weak force is as its name suggests, much weaker than the electromagnetic force and the strong force: about 10^5 times weaker than the electromagnetic force; while the strong force is intrinsicly much stronger than the other two: about 1000 times stronger than the electromagnetic force (note well that the strength of interaction depends greatly on the distance and energy scale being considered). Another consequence is, the weak force has an extremely short effective range of around 10^{-18} m, while the massless photon enables the electromagnetic to apply at infinite distance. The gluon is also massless, but has an effective range of around 10^{-15} m, due to the *colour confinement* (which will be discussed in more details in section 2.4). Lastly, all fermions can 'feel' the weak force, while the electromagnetic force only applies to particles with charge and the strong force only applies to quraks.

The final element of the SM is the Higgs bosons. In short, the Higgs boson differs from all other SM particles that have spin of either 1 or $\frac{1}{2}$, it has a spin of 0. This is the only fundamental scalar discovered to date. Its importance role in the SM comes from the mechanism that provides mass for all other particles: without the Higgs boson, the universe will be very different, that all particles will be massless! More specifically, unlike other fields associated with the fundamental fermions and bosons, the Higgs field has a non-zero vacuum expectation value; the interaction of the particles with the Higgs field is what provides them masses. This mechanism is called Higgs mechanism and is discussed in section 2.6.

The 12 elementary fermions and the 4 elementary bosons (5, if counting the hypothetical graviton) are illustrated in Figure 2.1. All particles in the SM are assumed to be point-like.

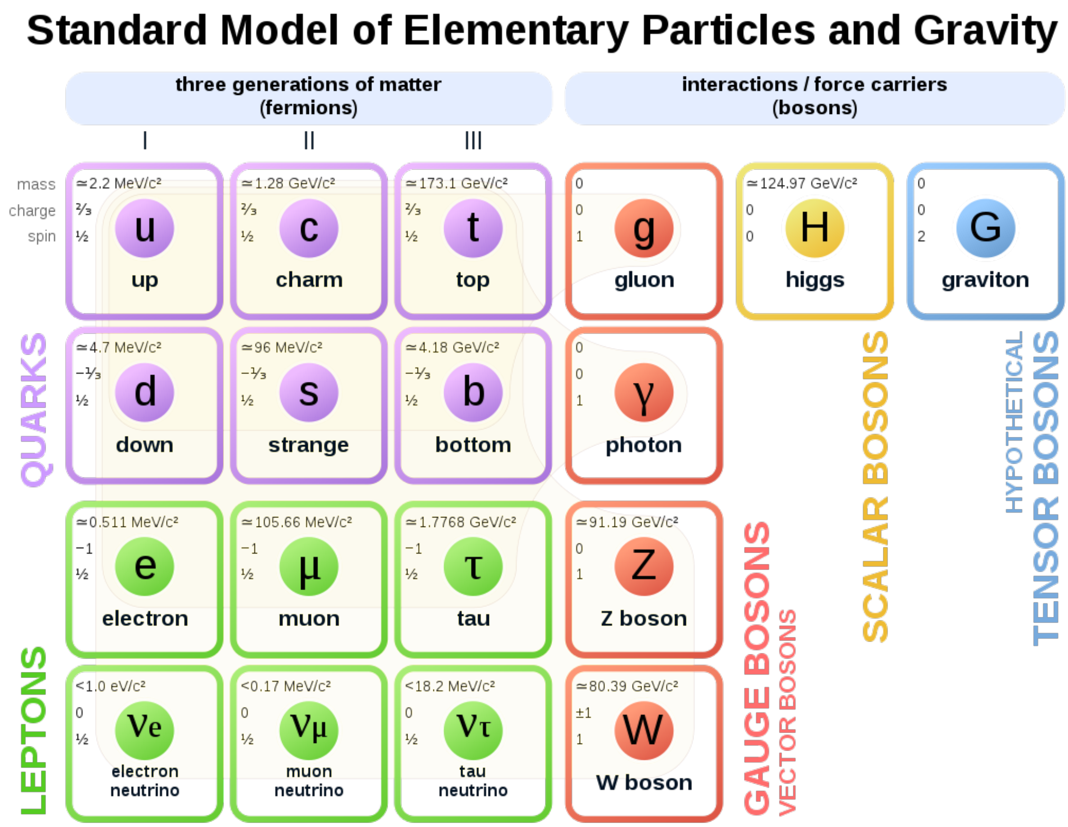


Figure 2.1: SM of elementary particles: the 12 fundamental fermions and 5 fundamental bosons (along with the hypothetical Graviton). The mass, charge and spin of each particle are given inside the particle boxes [7].

2.3 Symmetries in SM

Symmetry plays a crucial role in modern physics, particularly in the SM. SM is a relativistic quantum quantum field gauge theory containing the internal symmetries of the unitary product group $\mathrm{SU}_C(3) \times \mathrm{SU}_L(2) \times \mathrm{U}_Y(1)$. The $\mathrm{SU}_C(3)$ is the symmetry group of the strong interaction and the letter C refers to the color charge which is the corresponding conserved quantity; the $\mathrm{SU}_L(2) \times \mathrm{U}_Y(1)$ is the symmetry group of the electroweak interaction that unifies the weak and electromagnetic interactions, the letter L stands for left and indicates that the symmetry only involves left-handed particles and the letter Y stands for the weak hypercharge which is the conserved quantity corresponding to the $\mathrm{U}_Y(1)$ group, and it is related to the electric charge (Q) whose conservation arises from the global gauge invariance of the electromagnetic field and the weak isospin (T) given as $\mathrm{Q} = T_3 + Y/2$, with T_3 being the third component of the weak isospin and is conserved due to the $\mathrm{SU}(2)_L$ symmetry.

To understand the relation between symmetries and conserved charges, one can consider this simple example: suppose the dynamics is determined by an action S written in terms of a Lagrangian density $\mathcal{L}(x)$ that contains the free Lagrangian of the fields ($\psi(x)$), which accounts for their free propagation, and additional terms that respect the above symmetries and account for their interactions:

$$S = \int d^4x \mathcal{L}(x).$$

The Euler-Lagrangian equations can be derived assuming that the action is stationary, i.e. $\delta S = 0$:

$$\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi)} \right) - \frac{\partial \mathcal{L}}{\partial \psi} = 0.$$

In gauge theory, the Lagrangian is invariant under gauge transformation:

$$\psi(x) \longrightarrow \psi'(x) = \psi(x) + \delta\psi.$$

The Noether's theorem states (informally) that if a system has a continuous symmetry property, then there are corresponding quantities whose values are conserved. In this simple example, the Noether's theorem follows as:

$$\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi)} \delta\psi \right) = \partial_\mu J^\mu = 0,$$

where J^μ is the conserved current and

$$Q = \int dx J^0 = \text{constant},$$

where Q is the conserved charge associated to the symmetry.

2.4 Quantum Chromodynamics

Before going into QCD, it is useful to introduce the concept of local gauge invariance, which is a familiar idea from electromagnetism. The physical electric file **E** and the magnetic field **B** is unchanged under this transformation:

$$\psi \longrightarrow \psi' = \psi - \partial\chi/\partial t \text{ and } \mathbf{A} \longrightarrow \mathbf{A}' = \mathbf{A} + \nabla\chi,$$

and in covariance form, this can be written as

$$A_\mu \to A'_\mu = A_\mu - \partial_\mu \chi,$$

where $A_\mu = (\psi, -\mathbf{A})$ and $\partial_\mu = (\partial_0, \nabla)$. For the U(1) transformation on the wave function ψ, $\psi(x) \to \psi'(x) = \hat{U}(x)\psi(x) = e^{iq\chi(x)}\psi(x)$ where $\hat{U}(x)$ is the *generator* of the U(1) group, the Dirac equation for a free particle,

$$i\gamma^\mu \partial_\mu \psi = m\psi,$$

becomes:

$$i\gamma^\mu \partial_\mu (e^{iq\chi(x)}\psi) = i\gamma^\mu (\partial_\mu + iq\partial_\mu \chi)\psi = m\psi.$$

This differs from the free particle Dirac equation by the term $-q\gamma^\mu \partial_\mu \chi \psi$. It can be seen that the free particle Dirac equation cannot be local gauge invariant due to this additional term. The solution here is to introduce a field, A_μ which transforms as

$$A_\mu \to A'_\mu = A_\mu - \partial_\mu \chi,$$

such that the original form of the Dirac equation becomes

$$i\gamma^\mu (\partial_\mu \psi + iqA_\mu) = m\psi.$$

This idea can be applied to the QCD, which obeys the SU(3) group. Suppose a SU(3) transformation is applied on the wave function, i.e.

$$\psi(x) \to \psi(x)' = \exp\left[ig_S \alpha(x) \cdot \hat{\mathbf{T}}\right] \psi(x),$$

where g_S is some coupling constant, the $\hat{\mathbf{T}} = \{T^a\}$ are the eight generators of the SU(3), and are related to the *Gell-Mann matrices* by $T^a = \frac{1}{2}\lambda^a$ [8], and $\alpha(x)$ are eight functions of the space-time coordinate x, corresponding to each of the eight SU(3) generators. Due to the fact that the SU(3) group is represented by 3 by 3 matrices, the additional degrees of freedom is accounted by a vector of three components, namely red, green and blue. Hence, the idea of color charge comes naturally from requiring the local gauge invariance. Finally, the concept of gluon also comes out when reusing the local gauge invariance, which is the quanta of the eight introded fields. The Dirac equation with interactions with the eight type of gluons becomes:

$$i\gamma_\mu \left[\partial_\mu + ig_S G^a_\mu T^a\right] \psi - m\psi = 0,$$

which is invariant under local SU(3) transformation if the new fields transform as:

$$G_\mu^k \to G_\mu^k = G_\mu^k - \partial_\mu \alpha_k - g_S f_{ijk} \alpha_i G_\mu^j,$$

where the f_{ijk} is the structure constant to account for the fact that the SU(3) generators do not commute (and therefore, QCD is a non-Abelian theory).

An important result of the extra $g_S f_{ijk} \alpha_i G_\mu^j$ term is that, gluon can interact with itself, which is the origin of the colour confinement. So far, there is no free quark observed in the nature, and the reason might well possibly be the colour confinement. The qualitative explaination of this hypothesis is as follows: consider two quarks are in a bound state, in order to create a free quark, one would need to pull the two quarks far away from each other until they become 'free'. However, as gluons can interact with themselves (as attraction), and the interaction between the two quarks can be think of as exchanging gluons, the exchanged gluons actually attract themselves! The effect is that the gluon field is 'squeezed' in the shape of a tube, which has an energy density approximately constant over the distance. Therefore, the energy stored in the field is proportional to the separation of the quarks, giving a term in the potential of the form: $V(\mathbf{r} \sim \kappa r)$, where experimentally $\kappa \sim$ 1GeV/fm. This corresponds to the a force of the oder of 10^5 N, and consequently, the gluon field can store enough energy that could create new pairs of quarks. This is shown in Figure 2.2.

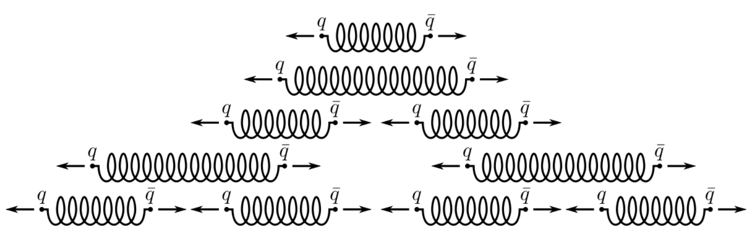


Figure 2.2: An illustration of a pair of quarks being pulled away: new pair of quarks are created and become new bound states with the two quarks being pulled.

Another consequence of the colour confinement is that the coloured gluons are confined to the colourless objects, which is the reason why gluons are massless but the strong force range is not macroscopic like photons and electromagnetic force. In addition, in short distances (or equivalently, high energy scale), the coupling strength of the strong force $\alpha_S = \frac{g_s^2}{4\pi}$ is small, and in bound states quarks behave like free particles. This is reffered to as *asymptotic freedom*. For example, for momentum transfer at the scale of the mass of the Z boson, α_S has a value of around 0.12. In modern particle detectors, the α_S value is sufficiently small for the pertubation theory to be used. Finally, the Lagrangian of QCD describing the quarks intercations via gluons and the gluon self-intercation in a compact form is:

$$\mathcal{L}_{QCD} = \bar{\psi}(i\gamma^{\mu}D_{\mu} - m)\psi - \frac{1}{4}G^{a}_{\mu\nu}G^{\mu\nu}_{a},$$

where the with the covariant derivatives D_{μ} given by:

$$D_{\mu} = \partial_{\mu} + ig_{s}T^{a}G^{a}_{\mu}.$$

2.5 Electroweak unification

Following a similar approach described in the previous section, consider the SU(2) transformation, i.e.

$$\psi(x) \to \psi(x)' = \exp\left[ig_{W}\alpha(x)\cdot\mathbf{T}\right]\psi(x),$$

with $\mathbf{T}$ being the three generators of the SU(2) group, which is related to the Pauli spin matrices by $\mathbf{T} = \frac{1}{2}\sigma$, and $\alpha(x)$ are three functions which specify the local phase at each point in space-time. To satisfy the local gauge invariance, three gauge fields are needed to be introduced, which are W^{k}_{μ} with $k = 1, 2, 3$, corresponding to three gauge bosons $W^{(1)}, W^{(2)}, W^{(3)}$. Since the weak force only interact with left-handed (LH) chiral particles or right-handed (RH) chiral antiparticles, and the the generators are 2×2 spin matrices, the LH particles and RH antiparticles states can be expressed as a weak isospin doublet, i.e.

$$\psi_{L}^{\ell=e,\mu,\tau} = \begin{pmatrix} \nu_{\ell} \\ \ell \end{pmatrix}_{L}, \psi_{L}^{q=1,2,3} = \begin{pmatrix} u_{q} \\ d'_{q} \end{pmatrix}_{L},$$

where the d'_{q} are the flavour states representing the three generations of the up-type quarks and the d'_{q} are the down-type quarks. Notice the flavour eigenstates d'_{q} differ from the mass eigenstates d_{q}, where the former is a mixture of the later using the Cabibbo-Kobayashi-Maskawa (CKM) matrix [9]. The RH chiral particles and LH antiparticles are represented by a weak isospin singlets, with

$$\psi_{R}^{\ell=e,\mu,\tau} = \ell_{R} \text{ and } \psi_{R}^{q=u,c,t,d,s,b} = q_{R}.$$

Analogous to the QCD formulation, an extra interaction term arises, which is

$$ig_{W}T_{k}\gamma^{\mu}W^{k}_{\mu}\psi_{L} = ig_{W}\frac{1}{2}\sigma_{k}\gamma^{\mu}W^{k}_{\mu}\psi_{L},$$

where ψ_{L} is the LH weak isospin doublet, which leads to three gauge field W^{k}. The physical W bosons are in fact the linear combinations of these three:

$$W^{\pm}_{\mu} = \frac{1}{\sqrt{2}}(W^{(1)}_{\mu} \mp iW^{(2)}_{\mu}).$$

It is natural to think the physical Z boson corresponds to the third W^k_μ, as it implies a neutral current which can be related to the chargeless Z. However, experimentalally the Z boson does not only couple to the LH particles, but also to the RH particles, even though not equally. To solve this conflict, electromagnetism is introduced into the story, which was so far not in consideration. In the electroweak theory by Glashow, Salam and Weinberg [10–12], The $\mathrm{U}(1)$ gauge symmetry of electromagnetism is related by a new $\mathrm{U}(1)_Y$ local gauge symmetry, and it transforms as:

$$\psi(x) \to \psi(x)' = \hat{U}\psi(x) = \exp\left[ig'\frac{Y}{2}\zeta(x)\right]\psi(x),$$

and requiring the local gauge invariance necessitates the interaction term:

$$g'\frac{Y}{2}\gamma^\mu B_\mu \psi$$

(notice the same form as the simple example in section 2.4). Using the intercation term, one can now write the photon and Z boson in terms of linear combinations of the new B_μ field and the third W^k_μ:

$$A_\mu = +B_\mu cos\theta_W + W^{(3)}_\mu sin\theta_W, \text{ and}$$

$$Z_\mu = -B_\mu sin\theta_W + W^{(3)}_\mu cos\theta_W,$$

where θ_W is called the *weak mixing angle*.

One can deduce the $Y = 2(Q - I^3_W)$ using the following logic: the electroweak theory is invariant under $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ transformation, and the corresponding hyper charge Y is conserved. Using that, and assuming the relation between th charge, weak isospin and the hypercharge is linear, i.e.

$$Y = \alpha Q + \beta I_W,$$

and since the Y must be the same for a LH electron and a LH neutrino, i.e. $Y_{e_L} = Y_{\nu_L}$ (otherwise the $\mathrm{U}(1)$ transformation will break the symmetry of the isospin doublet), using their charge and weak isospin values respectively one can conclude that:

$$Y = 2(Q - I^3_W).$$

The full formulation might not be as important, but one can deduce the electromagnetic current j^μ_{em} has terms equal to:

$$\bar{e}_L\gamma^\mu e_L: \qquad Q_e e = \frac{1}{2}g'Y_{eL}cos\theta_W - \frac{1}{2}g_W sin\theta_W, \text{ and}$$
$$\bar{\nu}_L\gamma^\mu \nu_L: \qquad 0 = \frac{1}{2}g'Y_{\nu L}cos\theta_W - \frac{1}{2}g_W sin\theta_W,$$

and since $Y_{e_L} = Y_{\nu_L} = -1$ and $Y = 2(Q - I_W^3)$, the coupling constant g' follows this relation:

$$e = g'cos\theta_W = g_W sin\theta_W.$$

The expected ratio of the weak to electromagnetic coupling constant is

$$\frac{\alpha}{\alpha_W} = \frac{e^2}{g_W^2} = sin^2\theta_W \sim 0.23.$$

Finally, the Lagrangian of the electroweak theory is:

$$\mathcal{L}_{\text{electroweak}} = \bar{\psi}_L\gamma^\mu D_\mu^L\psi_L + \bar{\psi}_R\gamma^\mu D_\mu^R - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}\vec{W}_{\mu\nu}\vec{W}^{\mu\nu},$$

with

$$D_\mu^L = i\partial_\mu - \frac{g}{2}\vec{\sigma}\cdot\vec{W}_\mu - \frac{g'}{2}YB_\mu, \text{ and } D_\mu^R = i\partial_\mu - \frac{g'}{2}YB_\mu,$$

where $\vec{\sigma}$ are the three Pauli matrices.

2.6 The Higgs mechanism

The local gauge invariance in preserved in $\mathrm{SU}_L(2)$ group only if the bosons are massless. This can be shown in the following example: consider if the photon were massive, the Lagrangian in QED becomes:

$$\mathcal{L} \to \bar{\psi}(i\gamma^\mu\partial_\mu - m_e)\psi + e\bar{\psi}\gamma^\mu A_\mu\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_\gamma^2 A_\mu A^\mu$$

where the new term $\frac{1}{2}m_\gamma^2 A_\mu A^\mu$ arises assuming a massive photon. It is clear that this new term is not gauge invariant under the $\mathrm{U}(1)$ group transformation. This simple example can be applied to the $\mathrm{SU}_L(2)$, and to solve the conflict that experimental observations show the weak bosons are massive while the local gauge invariance requires the weak bosons to be massless, the Higgs mechanism is proposed.

Now consider a complex scalar field:

$$\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2),$$

with the Lagrangian of the form:

$$\mathcal{L} = (\partial_\mu \phi)^*(\partial^\mu \phi) - V(\phi) \text{ with } V(\phi) = \mu^2(\phi^*\phi) + \lambda(\phi^*\phi)^2,$$

for the potential $V(\phi)$ to have a finite minimum, $\lambda > 0$. However, the coefficient μ^2 can be either positive or negative. When $\mu^2 < 0$, the potential has inifinite minima defined by

$$\phi_1^2 + \phi_2^2 = \frac{-\mu^2}{\lambda} = v^2,$$

which is a circle on the $\phi_1 - \phi_2$ plane, as shown in Figure 2.3. The physical vacuum state

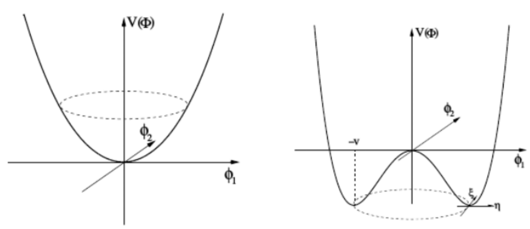


Figure 2.3: The potential $V(\phi)$ for a complex scalar field for $\mu^2 > 0$ (left) and $\mu^2 < 0$ (right). Image taken from Ref.[13].

corresponds to a particular point on the circle, where the U(1) symmetry is *spontaneously broken*.

Without loss of generality, one can pick the vacuum state to be

$$(\phi_1, \phi_2) = (v, 0),$$

and the complex scalar field can be expanded about the vacuum state, where

$$(\phi_1, \phi_2) = (v + \eta(x), \zeta(x)),$$

where $\eta(x)$ and $\zeta(x)$ are pertubations of real fields in the ϕ_1 and ϕ_2 direction. The Lagrangian can now be written in the form of:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) + \frac{1}{2}(\partial_\mu \zeta)(\partial^\mu \zeta) - V(\eta, \zeta),$$

with $V(\eta, \zeta)$ given by:

$$V(\eta, \zeta) = \mu^2\phi^2 + \lambda\phi^4, \text{ and } \phi^2 = \phi\phi^* = \frac{1}{2}[(\mu + \eta)^2 + \zeta^2],$$

and expanding $V(\eta,\zeta)$ gives:

$$V(\eta,\zeta) = -\frac{1}{4}\lambda v^4 + \lambda v^2\eta^2 + \lambda v\eta^3 + \frac{1}{4}\lambda\eta^4 + \frac{1}{4}\lambda\zeta^4 + \lambda v\eta\zeta^2 + \frac{1}{2}\lambda\eta^2\zeta^2.$$

The second term $\lambda v^2\eta^2$ can be seen as the mass term of field η, i.e. $\frac{1}{2}m_\eta^2\eta^2 = \lambda v^2\eta^2$, while the rest can be seen as interaction term. Notice that the field ζ along the ϕ_2 direction (the direction that the potential does not change) does not have a mass term, and therefore it is massless. The massless particle corresponding to this field is called *Goldstone boson*.

The full formulation of the Higgs mechanism is rather long and not appropriate in the context of this thesis. The general idea is that, by requiring symmetry in a particular group, one can use the vacuum state function with pertubations in the gauge invariant Lagrangian and derive the kinematic terms of the massive field η and massless ζ, and the massive gauge field B (which was massless originally). In this process, the massless field B has acquired mass, and by choosing the gauge carefully (known as the *Unitary gauge*), the massless ζ field can be absorbed into the now massive gauge field B.

In the SM, the Higgs mechanism is embedded in the U(1) and SU$(2)_L$ group, and to account for the three degrees of freedom of the $W^\pm$, Z bosons, three Goldstone bosons are required. Therefore, the simplest method would be to have two complex fields, and since one of the electroweak bosons is neutral, so is the field, which would be denoted by ϕ^0. The second must be charged to account for the $W^\pm$, one denote the scalar field as ϕ^+ and such that $(\phi^+)^* = \phi^-$. The scalar field can now be written as:

$$\phi = \begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}\phi_1 + i\phi_2\\ \phi_3 + i\phi_4\end{pmatrix}.$$

For the Higgs potential with the form:

$$V(\phi) = \mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2,$$

the vacuum state satisfies:

$$\phi^\dagger\phi = \frac{1}{2}\sum_{i=1,2,3,4}\phi_i^2 = \frac{v^2}{2} = -\frac{\mu^2}{2\lambda}.$$

Because the photon is required to remain massless, the minimum of the potential must correspond to a non-zero vacuum expectation value only of the neutral scalar field ϕ^0. Now writting the doublet in unitary gauge:

$$\phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v + h(x)\end{pmatrix}.$$

The resulting Lagrangian is known as the Salam-Weinberg model. To preserve the $\mathrm{SU}(2)_L\times$ $\mathrm{U}(1)$ symmetry, the derivatives needs to be replaced by appropriate covariant derivatives:

$$\partial_\mu \rightarrow D_\mu = \partial_\mu + ig_W \mathbf{T}\cdot\mathbf{W}_\mu + ig'\frac{Y}{2}B_\mu .$$

The mass term can then be extracted by subsituting the $\phi(x)$ in the kinematic term $(D_\mu\phi)^\dagger(D^\mu\phi)$, i.e.

$$\frac{1}{8}v^2 g_W^2 (W_\mu^{(1)}W^{(1)\mu} + W_\mu^{(2)}W^{(2)\mu}) + \frac{1}{8}v^2(g_W W_\mu^{(3)} - g'B_\mu)(g_W W^{\mu(3)} - g'B^\mu).$$

Therefore the mass of the W boson is determined by the weak coupling constant and the vacuum expectation value:

$$m_W = \frac{1}{2}g_W v,$$

and the mass of the Z boson is given by:

$$m_Z = \frac{1}{2}v\sqrt{g_W^2 + g'^2} = \frac{m_W}{cos\theta_W}.$$

The mass of the Higgs boson is given by:

$$m_H = \sqrt{2\lambda}v.$$

Finally, the Lagrangian of the Higgs field is given by:

$$\begin{aligned}\mathcal{L} \ &= \frac{1}{2}\partial_\mu h\partial^\mu h + \frac{g^2}{4}(v+h)^2 W_\mu^+ W^{-\mu}) + \frac{1}{8}(g^2+g'^2)(v+h)^2 Z_\mu Z^\mu \\ &- \lambda v^2 h^2 - \lambda v h^3 - \frac{1}{4}h^4,\end{aligned}$$

which in addition to the mass terms, includes the interaction terms of $VVhh$, VVh (V for $W^\pm$ or Z) and the Higgs self-interaction h^3, h^4 terms (trilinear and quadlinear). The Lagrangian does not depend in A_μ, and therefore the $\mathrm{U}(1)$ symmetry is unbroken, and the photon remains massless. The vacuum expectation value of the Higgs field, $v \approx 246\ GeV$ [7].

2.7 Yukawa coupling

The last missing piece of the mystery of mass is of fermions. Naively the mass term in the Lagrangian would look like $-m\bar{\psi}\psi$, however, this term is not invariant under $\mathrm{SU}(2)$ transformations obviously. Instead, one can construct a term: $\bar{L}\phi$ which is invariant under $\mathrm{SU}(2)$, because ϕ transforms as: $\phi \rightarrow \phi' = (I + ig_W\epsilon(x)\cdot\mathbf{T})\phi$, and $\bar{L} \equiv L^\dagger\gamma^0$ transforms as: $\bar{L} \rightarrow \bar{L}' = L(I - ig_W\epsilon(x) \cdot \mathbf{T})$. When combined with the RH doublet, the $\bar{L}\phi R$ term is invariant under SU$(2)_L$ transformation, and so as its Hermitian conjugate: $\bar{R}\phi^\dagger L$. For the Higgs field of the form:

$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix},$$

and taking the example of an electron, the Lagrangian is:

$$\mathcal{L} = -g(\bar{L}\phi R + \bar{R}\phi^\dagger L) = -\frac{g_e}{\sqrt{2}}v(\bar{e}_L e_R + \bar{e}_R e_L) - \frac{g_e}{\sqrt{2}}h(\bar{e}_L e_R + \bar{e}_R e_L),$$

where the first term has the form required for the fermion masses. The g_e, which is the *Yukawa coupling constant*, takes the form of:

$$g_e = \sqrt{2}\frac{m_e}{v}.$$

Rewriting the Lagrangian:

$$\mathcal{L} = -m_e\bar{e}e - \frac{m_e}{v}\bar{e}eh,$$

one can see the first term is again the mass term, which originates from the interaction of the massless electron with the vacuum expectation of the Higgs field, and the second term of the electron and the Higgs boson.

One may notice that, the mass term is only acquired through the interaction of the lower part of the weak doublets and of the complex scalar field, which means in this process, only leptons and the down-type quark obtain masses. What about the up-type quark and the neutrinos? Ignoring the neutrinos for now, the up-type quark can acquired by writing the scalar field in its conjugate form of:

$$\phi_c = -i\sigma_2\phi^* = \frac{1}{\sqrt{2}}\begin{pmatrix} -\phi^{0*} \\ \phi^- \end{pmatrix}.$$

And with the same Lagrangian, just by replacing ϕ by ϕ_c, the up-type quark can also acquire mass. In conclusion, the Yukawa couplings of the fermions to the Higgs field are given by:

$$g_f = \sqrt{2}\frac{m_f}{v}.$$

$g_f = \sqrt{2}\frac{m_f}{v}$. Interestingly, for the top quark with mass $\sim 173.5GeV$, the coupling strength of the top quark to the Higgs field very close to unity. While the neutrinos have such small mass that they are oftenly considered as massless, they might be acquiring their masses in a different way. A possbiliy is the *seesaw mechanism*, but it is outside the scope of this chapter.

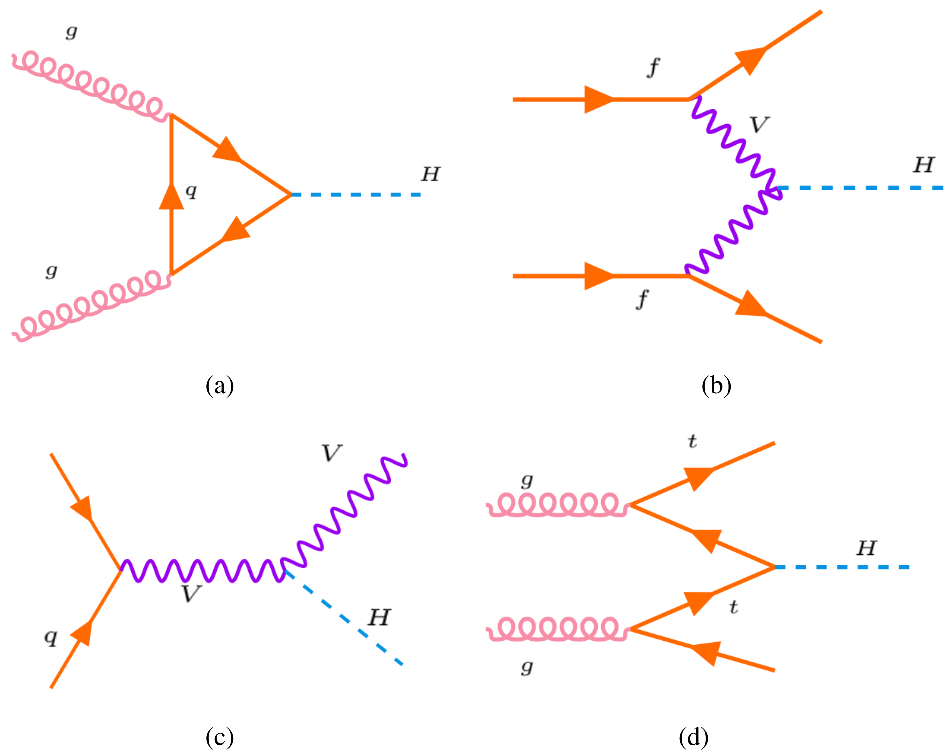


Figure 2.4: Feynman diagrams of the four production mechanism: (a) ggF, (b) VBF, (c) VBFH and (d) ttH.

2.8 Higgs boson production

As discussed in the above sections, the couplings of the Higgs boson to the fermions and bosons are proportional to the mass of the particles, i.e. for fermions: $\alpha \propto m_f \frac{g_W}{2m_W}$ and for W and Z bosons: $\alpha \propto m_W g_W$ and $\alpha \propto m_Z \frac{g_W}{cos\theta}$ respectively. Given the *branching ratio* is the fraction of all decays that result in a particular final state, $\mathrm{BR}(h \to x) = \frac{\Gamma(h \to x)}{\Gamma}$, the largest decay branching ratio predicted by the SM of the Higgs boson is to bottom quark (for the Higgs boson mass of 125 GeV), of 58.2%; the next largest decay branching ratio is 21.4% of the decay to a pair of W bosons, where one of them is off-shell [14]. The branching ratios of the other final states are listed in Table 2.1.

To change the subjet completely, in the proton-proton collisions, Higgs bosons are produced via four main mechanism: gluon-gluon fusion (ggF), vector boson fusion (VBF), production associated with a vector boson (VBFH) and production associated with a top or bottom quark-antiquark pair (ttH), as shown in the four Feynman diagrams in Figure 2.4. All these production modes have been observed with cross-sections compatible with the SM prediction, as shown in Figure 2.5.

The dominant production mode is the ggF, which is an order of magnitude greater than the next largest production mode VBF. The production cross-sections are shown in

Decay mode	Branching ratio
$H \to b\bar{b}$	58.24%
$H \to WW^*$	21.37%
$H \to gg$	8.19%
$H \to \tau^+\tau^-$	6.27%
$H \to c\bar{c}$	2.89%
$H \to ZZ^*$	2.62%
$H \to \gamma\gamma$	0.23%
$H \to Z\gamma$	0.15%
$H \to \mu^+\mu^-$	0.02%

Table 2.1: The SM predicted branching ratios in descending order of the Higgs boson for m_H = 125 GeV. Values taken from Ref.[14].

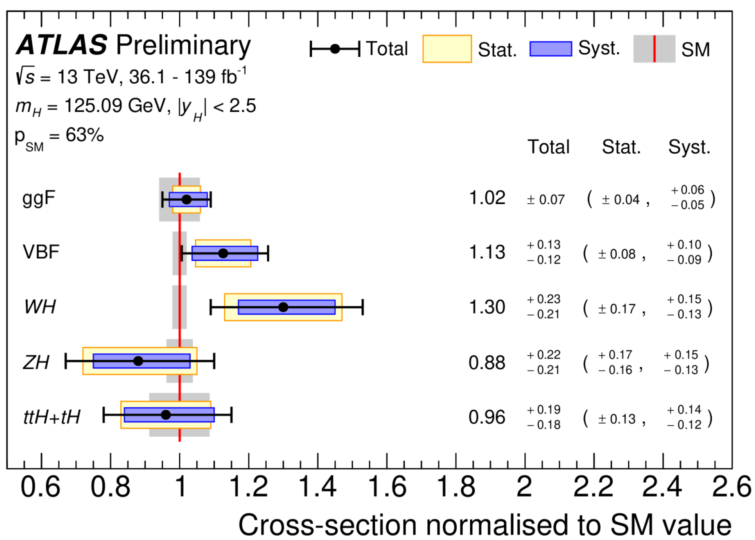


Figure 2.5: Cross sections for ggF, VBF, WH, ZH and ttH+tH production modes. The cross sections are normalised to their SM predictions, measured assuming SM values for the decay branching fractions. The black error bars, blue boxes and yellow boxes show the total, systematic, and statistical uncertainties in the measurements, respectively. The gray bands indicate the theory uncertainties on the SM cross-section predictions. The level of compatibility between the measurement and the SM prediction corresponds to a p-value (p-value is discussed in more details in section 3) of 63%. Image taken from Ref.[15].

Figure 2.6 as a function of the center-of-mass energy of proton-proton collisions $\sqrt{s}$. At $\sqrt{s} \sim 8$ TeV, the production cross-section of Higgs boson with mass of 125 GeV is approximately 20 pb. The first observations of the Higgs boson were based on approximately 20fb^{-1}of data (ATLAS and CMS combined) collected from 2011 to 2012, corresponding to a total of approximately 400000 Higgs bosons produced. While this number may seem large, only a very small fraction is picked up by the detector, and even worse, most of the decays involve QCD production of multi-jets final states. Hence it is difficult to distinguish the decays of the Higgs boson to the large QCD background in proton-proton collisions. For this reason, physicists focused on the more distinitive decay channels, such as $H \rightarrow \gamma\gamma$ and $H \rightarrow WW^*/ZZ^*$, despite the small branching ratio. The results show statistically compelling evidence for the discovery of a new particle with the expected properties of the Higgs boson, which has a significance of 5.9(5.0) σ of the ATLAS (CMS) observations. [1, 2].

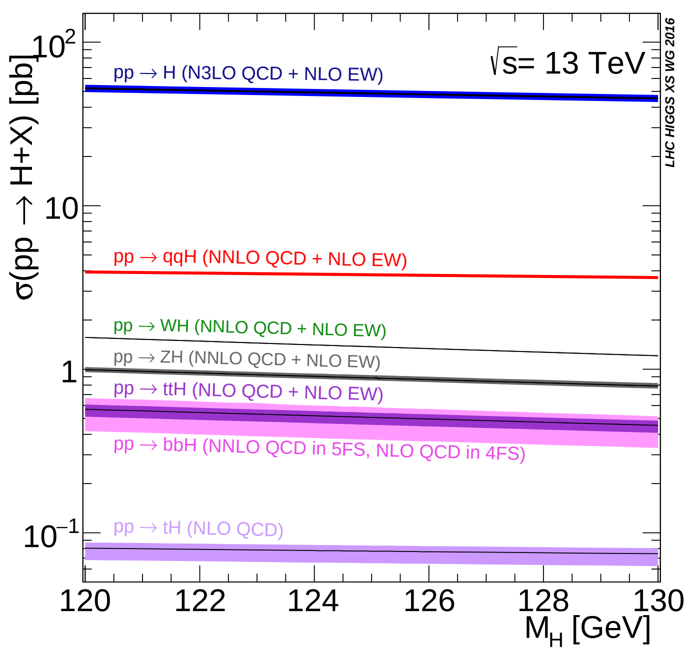


Figure 2.6: The production cross-sections of the SM Higgs boson at $\sqrt{s}$ = 13 TeV. The $pp \rightarrow H$ corresponds to the ggF production and the $pp \rightarrow qqH$ corresponds to the VBF production. The $pp \rightarrow WH, ZH, ttH$ corresponds to the VBFH, ttH. Image taken from Ref. [16].

The interactions between the Higgs boson and fermions were first established by the observations of the Higgs decaying to a pair of τ leptons with a combined significance of 5.5 σ [17, 18]. While the interaction with bottom quarks were observed a bit later, the Higgs bosons were observed to decay to two bottom quarks in 2018 by ATLAS and CMS [19, 20]. Even though this decay channel should account for nearly 60% of all Higgs decays at the LHC, it had proven extremely difficult to spot it amongst the vast number of background QCD particles that are produced by proton-proton collisions at the collider. The reduced coupling-strength modifiers $\gamma_F = \kappa_F \frac{g_F}{\sqrt{2}} = \kappa_F \frac{m_F}{v}$ for fermions ($F = t, b, \tau, \mu$) and $\gamma_v = \sqrt{\kappa_V \frac{g_V}{2v}} = \sqrt{\kappa_V} \frac{m_V}{v}$ for weak gauge bosons ($V = W, Z$) are shown as a function of their masses m_F and m_V, respectively in Figure 2.7 with the vacuum expectation value of the Higgs field $v = 246$ GeV.

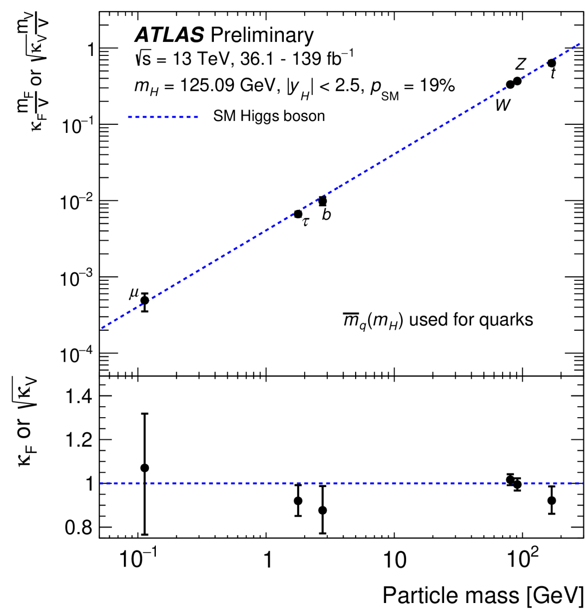


Figure 2.7: The reduced coupling-strength modifiers $\kappa_F \frac{m_F}{v}$ and $\sqrt{\kappa_V} \frac{m_V}{v}$ as a function of their masses m_F and m_V, for vacuum expectation value $v = 246$ GeV. The SM prediction for both cases is also shown (dashed line). The black error bars represent 68% CL intervals for the measured parameters. The lower panel shows the ratios of the values to their SM predictions. Image taken fromRef. [15].

2.9 Higgs boson pair production

The main goal of this thesis is to search for Higgs boson pair production and to probe the Higgs self-coupling. As defined in section 2.6, the Higgs potential is given by:

$$V(h) = \frac{1}{2} m_h^2 h^2 + \lambda v h^3 + \frac{\lambda}{4} h^4,$$

and to be explicit, the second term is the trilinear self-interaction of the Higgs boson, responsible for the di-Higgs production and the third term is the quadlinear term, responsible for the triple-Higgs production. The leading order Feynman diagrams are shown in Figure 2.8, where the left is referred to as *box diagram* and the right is called *triangle* *diagram*. While the pair production of the Higgs boson occurs at a very small rate due

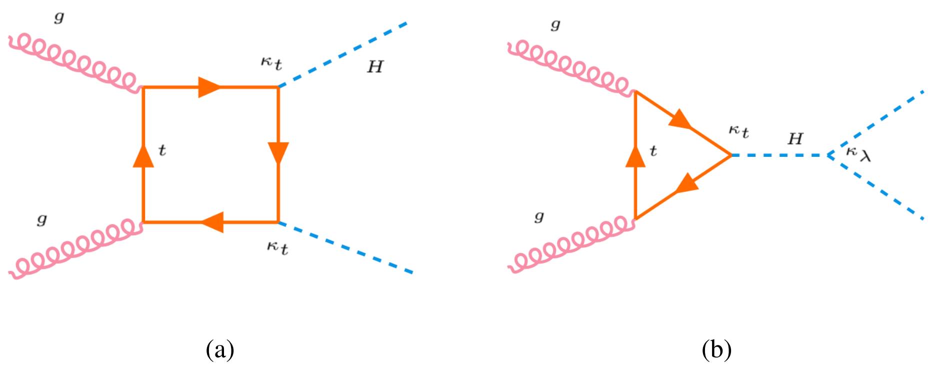


Figure 2.8: Leading order Feynman diagrams of (a) Box diagram and (b) triangle diagram of the di-Higgs production. The Box diagram amplitude is proportional to the square of the top yukawa coupling, $g_t^2 = \kappa_t^2$ and the triangle amplitude is proportional to the square of $g_t g_H HH = \kappa_t \kappa_\lambda$.

to the small phase space of decaying to two off-shell Higgs, these two diagrams interferes destructively, and therefore make the pair production cross-section even smaller. The dominant production mode is via ggF, and the cross-section ggF HH production is calculated at next-to-next-to-leading order (NNLO) FTApprox [21], taking into account the finite top-quark mass assumption. The cross-section is given by:

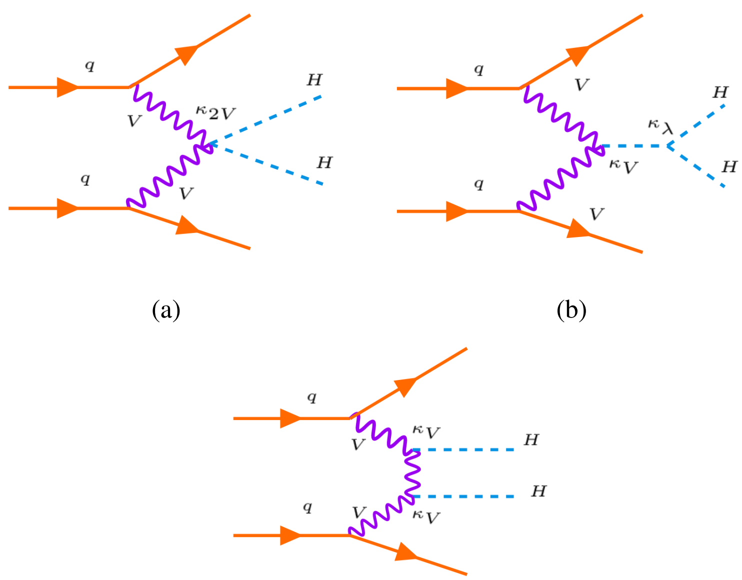
$$31.05^{+2.2\%}_{-5.0\%}(\text{scale}) \pm 2.1\%(\alpha_S) \pm 2.1\%(\text{PDF}) \pm 2.6\%(\text{m}_{\text{top}})\ \text{fb},$$

at $\sqrt{s} = 13$ TeV and $\text{m}_H = 125$ GeV [16]. The scale uncertainty is due to the finite order of quantum chromodynamics (QCD) calculations, the α_s and PDF terms account for the uncertainties on the strong coupling constant and parton distribution functions respectively, and the m_{top} uncertainty is related to the top-quark mass scheme.

The VBF di-Higgs production is also considered in this thesis. The Leading order Feynman diagrams are shown in Figure 2.9. The cross-section of the VBF non-resonant HH production is calculated at next-to-next-to-next-to-leading order (N3LO) in QCD in the limit in which there is no partonic exchange between the two protons [22]. It is given by:

$$1.726^{+0.03\%}_{-0.04\%}(\text{scale}) \pm 2.1\%(\text{PDF} + \alpha_S)\ \text{fb},$$

at $\sqrt{s} = 13$ TeV and $\text{m}_H = 125$ GeV [16].



(a) (b)

(c)

Figure 2.9: Leading order Feynman diagrams of the VBF HH production. The vertices denoted by κ_{2V}, κ_V and κ_λ represent the $VVHH$, VVH and HHH couplings, respectively.

Chapter 3

Multivariate techniques and Statistical interpretation

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