

# Evaluating U.S. Public Pension Policy <sup>\*</sup>

Jordan Pandolfo <sup>†</sup> and Kurt Winkelmann <sup>‡</sup>

PRELIMINARY

May 2021

## Abstract

Recent estimates suggest that U.S. public pension plans have 71 cents in assets for every dollar of future pension benefits. Due to the poor health of these funds, public pension reform has come to the forefront of debate in many states. Proper analysis of public pension reform should account for all constituents involved (directly and indirectly) as well as the funding of current deficits. We develop a lifecycle model to evaluate the welfare impact of particular public pension reforms on relevant constituencies within a state environment. The state environment is populated by a distribution of (i) public and private workers and (ii) different age cohorts within the lifecycle. We define a pension policy as a set of rules governing taxes (contributions), investments, benefits and the funding of existing deficits. The model is tractable for state-by-state analysis in that a subset of model parameters are specific to a particular state's age demographics and fiscal policy. As an example, we calibrate the model and evaluate a reform similar to that which was implemented in the state of Oklahoma in 2015. We find large welfare losses in the public sector which are mostly attributed to cost-of-living adjustment (COLA) freezes while the private sector experienced slight welfare gains due to a reduction in the level and volatility of taxes. In both sectors, young age cohorts fare better due to their ability to self-insure and benefit from lower long run tax rates.

---

<sup>\*</sup>We are especially grateful for excellent research assistance provided by Wesley Janson, Matthew Murphy, Cameron Brummund and Joey DiSpirito. We would like to acknowledge financial support from Arnold Ventures, The Sarah Scaife Foundation and the Heller-Hurwicz Economics Institute. We are grateful for comments and participation at the HHEI Public Pension Conference and Minnesota Macro & Public workshops, as well as helpful comments from V.V. Chari, Mariacristina DeNardi, Ellen McGrattan, Anmol Bhandari, Chris Phelan and Greg Mennis.

<sup>†</sup>University of Minnesota, HHEI, email: pando004@umn.edu

<sup>‡</sup>University of Minnesota, HHEI, Navega Strategies, email: kurt.winkelmann@navegastrategies.com

---

# Contents

<b>1</b>	<b>Introduction</b>	<b>2</b>
<b>2</b>	<b>Related Literature</b>	<b>4</b>
<b>3</b>	<b>Model</b>	<b>6</b>
<b>4</b>	<b>Calibration</b>	<b>11</b>
<b>5</b>	<b>Evaluating Actual Reforms</b>	<b>14</b>
<b>6</b>	<b>Conclusion</b>	<b>21</b>
<b>7</b>	<b>Appendix</b>	<b>25</b>
7.1	Welfare Analysis . . . . .	25
7.2	Computing Age Cohort Distributions . . . . .	26
7.3	Calculating Real Value of Pension Benefit with COLAs and Inflation . . . . .	27
7.4	Additional Figures and Tables . . . . .	28

---

# 1 Introduction

U.S. public pension plans fund retirement income for public employees. These systems take pension contributions from taxpayers and aggregate them into a large pool of investable capital, and this capital is invested on the behalf of public employees. The returns from the invested capital are used to pay retirement income for public employees where the level of retirement income is usually guaranteed, and is determined as a fraction of late career salary.

It has been well-documented that in the aggregate, U.S. public pension funds do not have sufficient funds to finance the value of future retirement benefits. For example, Public Plans Data documents that as of 2020, public pensions were operating with a 71.5% funded ratio such that the aggregate underfunding gap was in excess of \$1.5 trillion. The burden of financing underfunded pension systems falls squarely on taxpayers, as they are ultimately responsible for public employee wages and benefits. For the most part, public employee pension systems provide a defined benefit, i.e. a guaranteed retirement income. There are two main risks to this income- the first is an actuarial risk, as measured by mortality tables. The second risk is the risk of prolonged periods of low investment returns. Taxpayers are underwriting both of these risks on behalf of public employees.

Several states have considered reforms to their defined benefit pension systems.<sup>1</sup> These reforms usually include replacing some part of the defined benefit system (i.e. a guaranteed retirement income) with some version of a defined contribution system (i.e. a guaranteed contribution that the beneficiary invests). While reforms of this type reduce the amount of risk underwritten by taxpayers, they are also always contentious. Reform discussions are contentious for predictable reasons- public employees are being asked to give up a certain benefit in exchange for taking on extra risk. The question public employees ask is what they are receiving in exchange for taking on the extra risk. In this paper, we develop a quantitative model framework which is able to evaluate the impact of pension reform on relevant constituencies within a state.

Specifically, the model framework consists of a distribution of workers, by age and sector of employment, who solve an individual lifecycle problem subject to the fiscal policy of the state and other demographic trends over time. Individual agents vary by sector of employment (public or private) and public employees are enrolled in a state-run defined benefit pension plan. Each agent solves a lifecycle portfolio choice model similar to [Cocco et al. \[2005\]](#) in that they live through

---

<sup>1</sup>See [Brainard and Brown \[2018\]](#) for an exhaustive list of recent state pension reforms.

---

two life stages (working and retirement) and make decisions over consumption/savings and asset portfolio shares each period. Further, agents are exposed to three sources of risk: mortality, labor income and market risk. In addition, both public and private employees are subject to a labor income tax which varies with the management and health of the state-run pension system.

In terms of fiscal policy, the government operates a balanced budget each period which is used to finance public sector workers and pension benefits. The government is responsible for managing the pension fund which involves the investment of pension assets (portfolio management) as well as the raising of contributions (i.e. savings) for funding current and future benefits. The state is unable to fully commit to funding the pension and this model feature generates the type of chronic underfundedness that is observed in the data and can lead to higher taxes in the long run.

Further, the calibration of the quantitative model is broken up into two sets: universal and state-specific parameters. State-specific parameters mostly relate to demographic features and fiscal characteristics that we observe at the state level. This allows considerable tractability in parameterizing the model to a specific state and/or point in time for our analysis. Our model framework is novel in the sense that the quantitative model can easily be calibrated at the state-level for policy analysis. Moreover, we are able to capture the positive and normative effects of pension reform on the key constituencies within the state: public and private sector workers as well as current and retired workers. This feature provides additional insight into the varied effects of reform across different types of agents.

As an example of the type of policy analysis this framework can be used for, we evaluate a reform similar to that which was implemented in Oklahoma in 2015. The two main features of this reform were freezing cost-of-living-adjustments (COLAs) for pension recipients as well as closing the pension plan to new entrants who become auto-enrolled in a defined contribution plan. We document that this style of reform led to large welfare losses within the public sector (current and retired workers) while it provided slight welfare gains to private sector taxpayers. Welfare losses within the public sector are driven entirely by the freeze on COLAs which erodes the real value of their pension in retirement. Alternatively, by freezing the pension to new entrants, the aggregate pension liabilities drop significantly over time leading to a reduction in the level and volatility of taxes, which benefits public and private sector workers alike. For both sectors, younger cohorts fare better. For private workers this is due largely to the long run reduction in tax rates, while for public sector workers, they are better able to self-insure against the reform's COLA freeze relative

---

to older workers. In an alternative reform, without the COLA freeze, we observe that both the private and public sector experience slight welfare gains due to the long-run closure of the public pension plan.

The next section of this paper provides additional background with respect to U.S. public pension plans as well as a review of the existing literature on pension policy. Section 3 provides details of the model. Section 4 provides an overview of the model calibration as well as relevant data sources. Section 5 provides policy analysis for the Oklahoma-style reform, providing both positive and normative analysis. Section 6 concludes the paper.

## 2 Related Literature

The topic of public pension plans in the United States lies at the intersection of many cross-disciplines: political economy, accounting, empirical finance as well as a large body of economic modeling related to retirement systems and the lifecycle. Constructive analysis of pension reform must take account of all these factors as they jointly affect both the state fiscal environment and household welfare.

From a political economy standpoint, it is essential to understand the legal & fiscal constraints which state legislatures operate under, as well as their incentive structure. [Monahan \[2015\]](#) and [Beermann \[2013\]](#) describe the current, chronic underfundedness of U.S. public pension plans as a symptom of underlying fiscal constraints and limited commitment on the part of state legislatures. While most states face balanced budget constraints, pension contributions are not an explicit component of their liabilities. Thus, lowering pension contributions is a means of relaxing fiscal pressures. This is particularly appealing as the costs of underfundedness (e.g. insolvency risk) materialize long after the current political cycle. This is echoed by the qualitative study of [Fitzpatrick and Monahan \[2015\]](#) which finds that despite governance provisions for maintaining a well-funded pension, most public plans lack the proper enforcement mechanisms to maintain their provisions, resulting in continued underfunding. [Monahan \[2010\]](#) addresses the legal framework by which pension benefits are protected under state-specific constitutions, and therefore the extent to which various reforms are legally feasible. Ultimately, it is states (and not the federal government) which are responsible for the protection of public employee retirement benefits. Differences in the legal designation of retirement benefits, as well as idiosyncrasies in state constitutions, affect

---

whether reforms can alter the level of accrued benefits and/or *future* accrued benefits for public workers. [Monahan \[2017\]](#) also casts doubt on the degree to which pension benefits are treated as risk-free guarantees, citing mixed legal precedent to ensure the enforcement of state promises. We incorporate this work into our own modeling framework by positing a state government which manages fiscal policy as well as the investment & contributions strategy of a public pension plan. The state government faces a balanced budget constraint as well as limited commitment to properly funding the pension plan.

A large body of empirical work has also documented differences in lifetime earnings, savings and wealth across various sectors (public or private) and retirement savings vehicles (defined contribution or defined benefit). [Keefe \[2012\]](#) asks whether public employees (state and local) are overpaid and documents characteristic differences in education, gender, race, age and tenure across sectors. The paper finds, on average, there is a 3.7% penalty in total compensation for full-time public employees when compared to private-sector counterparts. [Poterba et al. \[2007\]](#) consider wealth accumulation with both defined benefit and defined contribution plans, under a variety of simulation scenarios. They find average wealth accruals under DC plans exceed that of DB plans. In our model, we incorporate fundamental differences in the work compensation structure for both public and private employees and employ welfare analysis as our primary metric for evaluating reforms. The benefit of this metric is that it captures worker preferences for the timing, risk and level of work compensation, and not just averages.

In this paper, the economic environment is a state in which agents are distributed across working sectors (public and private), age cohorts, through time and are connected via the fiscal policy of the state government. At its core agents solve an individual lifecycle problem. We most closely follow the work of [Cocco et al. \[2005\]](#) and [Campbell et al. \[2001\]](#) where agents solve a portfolio-choice lifecycle problem with fixed retirement and idiosyncratic risk from three sources: market return, mortality and labor income. Similar to our approach, [Myers \[2019\]](#) models a public pension system within a municipal environment with detailed focus to the government problem and fiscal policy. Distortions arise due to an agency problem and under the model calibration, the author finds that governments are highly vulnerable to another stock market bust. [Chai et al. \[2011\]](#) substantially builds on the portfolio-choice lifecycle framework by incorporating flexible work and endogenous retirement and uses this to fit important empirical facts such as age-conditional retirement rates, the hump-shaped pattern of work hours and the low annuity take-up for older households.

---

In the past decade, a large number of states have implemented reforms to their pension systems, with varying degrees of significance and impact. Proper reform analysis starts with a quantification of existing problems followed with a decision over the feasible policy toolkit and then complemented with some metric to evaluate reform effects. [Healey et al. \[2012\]](#) document existing national and state-level funding deficits for public pensions. They divide potential policy solutions into two groups: benefit design changes and financing changes. The former group deals with worker-level changes such as eligibility requirements, benefit accruals and employee contributions. The latter group includes changes such as discount rate reform, changes in the amortization of deficits and legally-binding commitments to funding strategies to ensure long-run solvency. [Novy-Marx and Rauh \[2011a\]](#) evaluate projected pension cash flows to consider the impact of changes in actuarial methods, reductions in cost-of-living adjustments (COLAs) and extending the retirement age. They find adjusting to typical actuarial methods significantly increases total pension deficits (from \$4.4 trillion to \$5.2 trillion) and a 1% point reduction in COLAs lowers total liabilities by 9-11%; increasing retirement age by 1 year reduces liabilities by 2-4%. [Novy-Marx and Rauh \[2011b\]](#) also address the issue of discount rate reform. Due to certain accounting exemptions, public pensions are allowed to discount liabilities with the target rate of return on assets. This deviation from normal accounting practices masks the severity of the funding deficit. Using zero-coupon Treasury yields as a discount factor leads to a 63% increase in the total value of pension liabilities. In our framework, we primarily focus on policy reforms related to (i) discount rate adjustments, (ii) the introduction of a hybrid DB/DC plan for current workers, (iii) plan closures for future public workers and (iv) various financing strategies to eliminate current deficits.

### 3 Model

Model agents solve a lifecycle problem with a fixed working and retirement period. During the lifecycle, agents are exposed to three sources of risk: wage risk, market risk and mortality risk.<sup>2</sup> Further, agents can differ by their sector of work, belonging to either the public or private sector. Sector of employment determines the compensation the worker receives in terms of wages and retirement benefits. We assume that all public sector workers receive a state-sponsored pension plan. Within the state environment, there is a deterministic joint distribution of age cohorts, which

---

<sup>2</sup>Wage and mortality risk is idiosyncratic whereas market risk is common across agents in time.

varies by (i) age group (ii) employment sector and (iii) model time.

Lastly, there exists a state fiscal authority which follows a set of rules to fund public sector wages and pension contributions. The fiscal authority is limited in committing sufficient contributions to ensure the long run health of the pension system, and this lack of commitment creates chronic underfundedness for the pension plan.

**Lifecycle Problem.** Agents are born at age  $t = 1$  and can live to age  $t = 80$  but are subject to mortality risk, given by the conditional survival probabilities  $p(t + 1|t)$  for each age. Agents have CRRA preferences over consumption

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma} \quad (1)$$

and discount the future at a rate  $\beta \in (0, 1)$ . Each period, agents solve a consumption/savings problem along with a portfolio problem. Specifically, given beginning-of-period wealth  $x$ , agents choose consumption  $c$  and savings  $a$  to satisfy the budget constraint

$$c + a = x \quad (2)$$

and choose a risky asset portfolio share  $\alpha \in [0, 1]$  where  $R' \sim N(\mu, \sigma)$  is the risky asset gross return next period and  $R^f$  is the risk-free rate.<sup>3</sup> These decisions also induce a law of motion on next-period available wealth, which will be derived after introducing agent wage processes and tax.

Agents work for the first 45 years and then enter retirement. During the working period, agents receive a wage  $w(s, t, \epsilon, \eta)$  which is a function of the job sector  $s \in \{pub, priv\}$ , age and idiosyncratic shocks  $(\epsilon, \eta)$  where  $\epsilon$  is a transitory shock to wages while  $\eta$  is a persistent shock. More specifically,  $\epsilon$  is a mean-zero, normally distributed random variable with variances  $\sigma_\epsilon^2$  while  $\eta$  is a random walk

$$\eta' = \eta + \omega' \quad (3)$$

where  $\omega'$  is a mean-zero, normally distributed random variable with variance  $\sigma_\omega^2$ . Upon reaching retirement, agents receive a sector-specific annuity  $b^s$ . All public sector workers are enrolled in a state-funded pension plan and receive a pension  $\bar{b}$  as a part of their retirement annuity  $b^{pub}$ . Lastly, during the working period, agents are subject to a wage tax  $\tau(\chi, T)$  which depends upon

---

<sup>3</sup>Given the recursive structure of the problem, prime notation  $'$  is used to denote objects realized in the following period.



the pre-contribution funded ratio  $\chi$  of the pension, as well as model time  $T$ .<sup>4</sup> Given both the wage and tax process, the law of motion on next-period agent wealth, during the working period, can be expressed as

$$x' = \underbrace{[\alpha R' + (1 - \alpha)R^f]a}_{\text{portfolio return}} + \underbrace{(1 - \tau(\chi', T + 1))w(s, t + 1, \epsilon', \eta')}_{\text{after-tax wages}} \quad (4)$$

In solving the recursive worker problem, we define an agent's individual state by the vector  $\varsigma = (x, t, s, \eta, \chi, T)$ . Thus, for a given individual state, the agent solves

$$\begin{aligned} V(\varsigma) = \max_{c, a, \alpha} \quad & u(c) + \beta p(t + 1|t)E[V(\varsigma')|\varsigma] \\ \text{s.t.} \quad & c + a = x \\ \text{s.t.} \quad & x' = [\alpha R' + (1 - \alpha)R^f]a(1 - \tau(\chi', T + 1))w(s, t + 1, \epsilon', \eta') \\ \text{s.t.} \quad & \chi' = \Gamma(\chi, R', T) \\ \text{s.t.} \quad & c \geq 0, \quad \alpha \in [0, 1] \\ \text{s.t.} \quad & \text{exogenous processes } \{R', \eta', \epsilon'\} \end{aligned} \quad (5)$$

where the agent faces short-selling portfolio constraints and conditions upon the law of motion for the pension funded ratio which affects the wage tax.<sup>5</sup> In the retirement period of the lifecycle, the state space reduces to  $\varsigma^r = (x, t, s)$  as the agent is no longer subject to wage risk or wage taxes. Further, the wealth law of motion reduces to

$$x' = [\alpha R' + (1 - \alpha)R^f]a + b^s \quad (6)$$

where the annuity benefit depends upon the agent's sector of employment.

**State Fiscal Authority.** We assume the state is populated by a continuum of agents of mass  $M$  and is distributed according to the joint distribution  $\Phi : s \times t \times T \rightarrow [0, 1]$  which is a function of employment sector, agent age and model time. Further, cohort growth rates  $\phi(t, T)$  induce a law of motion for  $\Phi(\cdot)$  over time. There exists a state fiscal authority which follows a set of rules to fund public sector wages and the state pension plan. Specifically, the authority sets a tax  $\tau = \tau^w + \tau^p$

<sup>4</sup>The funded ratio  $\chi$  and tax formula will be explicitly derived in the *State Fiscal Authority* section of the paper.

<sup>5</sup>Detailed in the following section.

which can be decomposed into a component towards public sector wages and one towards the public sector pension. For any given time period  $T$  and pension funded ratio  $\chi$ , the authority sets  $\tau$  to balance the budget

$$\tau \sum_s \sum_t \Phi(s, t, T) \int \int w(s, t, s_1, s_2) dF(s_1, s_2) = C(\chi, T) + \sum_t \Phi(pub, t, T) \int \int w(pub, t, s_1, s_2) dF(s_1, s_2) \quad (7)$$

where the left-hand side represents the tax revenues from setting  $\tau$  against the state tax base, and the right-hand side consists of pension contributions  $C(\chi, T)$  and total expense on public sector wages. Given that workers are subject to idiosyncratic wage risk via  $(\epsilon, \eta)$ , the cross-section of worker wages are integrated over with the joint distribution  $F(s_1, s_2)$ .<sup>6</sup>

Aggregate pension contributions are a function of the funded ratio and model time, and can be summarized as

$$C(\chi, T) = \theta [NC(T) + AUFL(\chi, T)] \quad (8)$$

where  $NC(T)$  represents normal cost expenditures and  $AUFL(\chi, T)$  represents the cost of repaying the amortized unfunded liability of the pension. Essentially, pension contributions account for newly accrued benefits (the *normal costs*) and old deficits (the *amortized unfunded liability*). Importantly, the parameter  $\theta \leq 1$  represents the state's willingness to pay contributions such that a value of  $\theta$  less than one results in insufficient contributions to properly fund the pension.

The normal cost for the pension fund is computed as

$$\begin{aligned} NC(T) &= \sum_{k=1}^{45} \Phi(pub, k, T) \sum_{m=1}^{35} \frac{p(45 + m|k)}{(1 + r^p)^{45-k+m}} \frac{w_k}{\sum_{l=1}^{45} w_l} \bar{b} \\ &= \underbrace{\sum_{k=1}^{45} \Phi(pub, k, T)}_{\text{aggregation}} \underbrace{\tilde{\beta}(k)}_{\text{discount factor}} \underbrace{\tilde{b}(k)}_{\text{accrued benefit}} \end{aligned} \quad (9)$$

where  $r^p$  is a state-specific rate to discount the measure. The outer summation accounts for the size of each age cohort currently working in the public sector, and thus is an aggregation term for determining the cost. For a given age cohort, the inner summation computes the present value of newly accrued pension benefits where the benefit is discounted by mortality risk and the rate

---

<sup>6</sup>These are assumed independent shocks; thus,  $F(s_1, s_2) = F_1(s_1)F_2(s_2)$ .

$r^p$ . The ratio  $\frac{w_k}{\sum_{l=1}^{45} w_l}$  is an accrual factor that represents the agent being vested in the pension benefit  $\bar{b}$  over the working period of their lifecycle. To understand the remaining cost concept (the amortized unfunded liability) in equation (8), some additional notation and accounting must first be introduced.

Each period, the fiscal authority manages the investment portfolio of the pension, as well as contributions to the fund.<sup>7</sup> To calculate required contributions the state must compute the discounted present value of pension liabilities  $PVL(T)$  at model time  $T$ . In the baseline model, we assume the state uses the Entry Age Normal (EAN) method to value liabilities, as this is the most prominent valuation formula used by U.S. public plans. Thus, at model time  $T$ , the present value of liabilities is

$$PVL(T) = \sum_{j=0}^{\infty} \frac{1}{(1+r^p)^j} \sum_{k=45}^{80} \Phi(pub, k, T+j) \alpha(T+j, k) \bar{b} \quad (10)$$

where  $\alpha(T+j, k)$  is the EAN accrual factor for a worker of age  $j$  and time  $T+j$ , written

$$\alpha(T+j, k) = \begin{cases} 1, & \text{if } k-j \geq 45 \\ \frac{\sum_{k=1}^{k-j} w_k}{\sum_{k=1}^{45} w_k}, & \text{if } k-j < 45 \end{cases} \quad (11)$$

As equation (10) makes clear, pension benefits accrue fractionally over the working period of a specific agent; that is, even though benefits  $\bar{b}$  are guaranteed at retirement, they accrue slowly as a liability of the state. In addition, the state discounts pension liabilities at particular rate  $r^p$ .<sup>8</sup> Define the post-contribution funded ratio  $\tilde{\chi}(\chi, T)$  as a function of the pre-contribution funded ratio and model time  $T$ . Then, given the pairing  $(\chi, T)$ , the next-period pre-contribution funded ratio is defined as

$$\begin{aligned} \chi' &= \frac{\text{Return on Assets} - \text{Distributions}}{PVL(T+1)} \\ &= \frac{[\alpha^p R' + (1 - \alpha^p) R^f] \tilde{\chi} PVL(T) - \bar{b} \sum_{k=45}^{80} \Phi(pub, k, T+1)}{PVL(T+1)} \end{aligned} \quad (12)$$

which is a function of the pre-contribution funded ratio  $\chi$ , model time  $T$  and the market return

<sup>7</sup>It is assumed the state follows a fixed investment strategy  $\alpha^p$  for its risky asset share.

<sup>8</sup>In the United States, public pension plans are allowed considerable flexibility in determining the discount rate  $r^p$  in comparison to corporate pension plans.

shock  $R'$ . Given this, the amortized unfunded liability  $AUFL(\chi, T)$  is computed as

$$AUFL(\chi, T) = (1 - \chi)PVL(T) \frac{r^p}{1 - (1 + r^p)^{-\bar{T}}} \quad (13)$$

where  $\bar{T}$  is the amortization window and the state amortizes its deficits using the same pension discount factor. Thus, in a given period, the state realizes an unfunded liability  $(1 - \chi)PVL(T)$  which is the stock of liabilities not currently covered by pension assets. The state then applies an amortization rate to this stock such that the amortized unfunded liability and normal cost, together, show up as a required contribution for the state to guarantee that the pension is fully funded in the long run, on average. Despite this required contribution, the state will contribute the fraction  $\theta$  of this number which allows for the possibility of chronic underfundedness of the pension.

**Equilibrium.** An equilibrium is defined as a set of stochastic processes  $\{R', \epsilon', \eta'\}$  and deterministic aggregates  $\{\Phi(s, t, T)\}$  such that for periods  $T = 1, 2, \dots$

1. Agents solve their lifecycle problem in Equation (5)
2. Fiscal authority sets  $\tau$  to balance the budget in Equation (7)
3. Pension policy rules induce the law of motion

$$\chi' = \Gamma(\chi, R', T)$$

## 4 Calibration

The modeling framework has been developed with the intent of capturing a demographic and fiscal environment at the state level for the purpose of evaluating and testing the impact of reforms to the pension system. Thus, in calibrating the model to a particular state environment, we identify two sets of parameters: *universal* parameters which are common across states and *state-specific* parameters which are unique to the given state of focus.

**Universal Parameters.** Table 1 details the set of universal parameters. While we account for price level differences in wages and benefits across states, we employ the same functional form for

the wage process. Specifically, we use the the form and estimates from [Cocco et al. \[2005\]](#) which utilizes PSID data to estimate wages as a function of age as well as the error processes, using the method of [Carroll and Samwick \[1997\]](#).<sup>9</sup> Specifically, worker wages are defined as

$$\begin{aligned} w(s, t, \epsilon, \eta) &= \lambda(s) e^{f(t) + \epsilon + \eta} \\ &= \lambda(s) e^{b_0 + b_1 t + b_2 \frac{t^2}{10} + b_3 \frac{t^2}{100} + \epsilon + \eta} \end{aligned} \quad (14)$$

where

$$\lambda(s) = \begin{cases} 0.91, & \text{if } s = \textit{pub} \\ 1, & \text{if } s = \textit{priv} \end{cases} \quad (15)$$

and parameter values  $\{b_0, b_1, b_2, b_3\} = \{-1.9317, .3194, -.0577, .0033\}$ .<sup>10</sup> Using data from the BEA, we set the wage gap between public and private sector workers at 91%, respectively. Conditional survival probabilities were obtained from the National Center for Health Statistics (NCHS).

Table 1: Universal Parameters

Parameter	Label	Value	Source/Target
$\beta$	Discount Factor	0.96	Standard
$\gamma$	Risk Aversion	10	CGM [2005]
$r_f$	Risk-free Rate	.02	Navega
$\mu_r$	Equity Premium	.04	Navega
$\sigma_r$	Equity Vol	.157	Navega
$\{f(t)\}$	age wage trend	—	CGM [2005], PSID
$\sigma^\epsilon$	Transitory Vol	.074	CGM [2005], PSID
$\sigma^\nu$	Persistent Vol	.011	CGM [2005], PSID
$\{p(t+1 t)\}$	Mortality Risk	—	NCHS
$\lambda(\textit{pub})$	Sector Wage Gap	.91	BEA

<sup>9</sup>Wages are defined broadly as to include total reported labor income plus unemployment compensations, workers compensations, social security, supplemental social security, other welfare, child support and total transfers for the head of household.

<sup>10</sup>[Cocco et al. \[2005\]](#) wage estimates were based upon dollar estimates using 1992 as a base year. We use the CPI to update measures to 2018 dollars so as to match the nominal value of pension benefits, detailed in the State-Specific Parameters section.

---

**State Parameters.** State-specific parameters relate to details of the fiscal and demographic environment. The majority of pension-relevant data comes from comprehensive annual financial reports that are publicly released by individual plans.<sup>11</sup> Table 2 details the key parameters of interest. While we assume a constant pension benefit, which is known at the beginning of employment, most pensions accrue based upon formulas which factor in a worker’s wages, wage growth and years of service. Thus, in determining pension benefit  $\bar{b}$  we compute the average benefit, given aggregate data on benefit distributions and total number of annuitants. Further, model agents receive sector-specific annuities  $b^s$  meant to replicate additional retirement benefits, such as social security. Information on state-level social security distributions are taken from the Social Security Administration congressional report and sectoral wage gaps  $\lambda(s)$  are applied to these, as well, given social security is a function of past wages/contributions.<sup>12</sup>

Details such as the current funded ratio of a state’s pension plan, the discount factor used for valuing liabilities and the pension portfolio strategy are also taken from annual financial reports.<sup>13</sup> We internally set the population parameter  $M$  such that the computed total value of pension liabilities is equal to the nominal level observed in the data.

We use state-level measures and projections for age cohorts, provided by the Weldon Cooper Center for Public Service at the University of Virginia. While this data pertains to total state populations, we are interested in the current workforce, as well as retirees who were previously in the workforce. To make this transformation, we employ labor force participation rates from the BEA, as well as state information about the ratio of pension annuitants to active workers. Refer to the appendix for a more detailed description.

Lastly, we assume, in the model, that the state fiscal authority follows an explicit rule for setting contributions to the pension fund which is based upon the Annual Required Contributions

---

<sup>11</sup>When multiple plans exist in a state, data is aggregated up or weighted by the market value of assets, across individual plans.

<sup>12</sup>Average social security distributions are taken from

[https://www.ssa.gov/policy/docs/factsheets/cong\\_stats/2018/index.html](https://www.ssa.gov/policy/docs/factsheets/cong_stats/2018/index.html)

and Social security coverage information from

[https://www.everycrsreport.com/reports/R41936.html\\_Toc299368664](https://www.everycrsreport.com/reports/R41936.html_Toc299368664)

<sup>13</sup>Much of this data can be found at the National Association of Retirement Administrators (NASRA) website as well as the Center for Retirement Research (CRR) at Boston College’s Public Plans data website.

(ARC) formula. While the majority of states do not follow such a rule, it is a metric most states utilize and make publicly available. Thus, for a given state, we compute the average ratio of total pension contributions to their ARC as an estimate for  $\theta$ .

Table 2: State-Specific Parameters

Parameter	Label	Source/Target
$\bar{b}$	Pension Benefit	State Reports
$\{b^s\}$	Social Security	SSA
$r^p$	State Discount Factor	State Reports
$\chi^0$	Initial Funded Ratio	State Reports
$\theta$	Contribution Commitment	NASRA
$\alpha^p$	Pension Portfolio	State Reports
$\Phi$	Agent Distribution	UVA Weldon Cooper, NASRA, BEA
$M$	Total Population	2019 PVL
$\bar{T}$	Amortization Window	State Reports

## 5 Evaluating Actual Reforms

In the last decade, a considerable number of reforms have been implemented or proposed across numerous states. While the range of policy instruments is varied, they can be generally differentiated into two groups: *fiscal* and *plan* reforms. *Fiscal* reforms largely pertain to plan funding policy, contribution rates, tax & bond-financing as well as accounting standards which affect the value of liabilities. Alternatively, *plan* reforms directly alter the compensation package for current, future and retired public sector workers and therefore the value of liabilities, as well. These reforms can include cost-of-living-adjustment (COLA) modifications, reductions in benefits, the introduction of hybrid DB/DC plans and more. In what follows, we examine the welfare impact of a reform similar to that which was implemented in the state of Oklahoma in 2015. Our analysis evaluates the impact of this reform on both public and private workers, as well as current and retired workers.

**Oklahoma Reform [2015].** We focus on the two most significant features of the Oklahoma reform:

---

freezing cost-of-living-adjustments (COLAs) and closing the plan to new entrants. COLAs are upward adjustments in the value of an annuitant’s pension which are meant to track growth in the price level, thereby maintaining the pension’s real value. By freezing or reducing a pension’s COLA, the state is implicitly reducing the pension benefit and thereby the aggregate value of pension liabilities. In addition, by closing the plan to new entrants the state is still responsible for the pension benefits of current and retired workers, but future generations of public workers must accumulate their own retirement savings (as in a defined contribution plan). In this way, the state slowly reduces the aggregate stock of pension liabilities until the last current worker ends their retirement period (which is fixed at 80 years in our model framework). Lastly, new hire workers are automatically enrolled in a defined contribution plan at which the state offers a 6% match. This analysis is not meant to be a direct evaluation of the Oklahoma reform as we do not include other significant features of the actual reform.<sup>14</sup>

Our objective is to evaluate the impact of a state’s pension reform on current and retired workers, and in both sectors. Our main criterion is consumption-equivalent welfare for each age cohort, and a more detailed explanation of this metric is provided in the Appendix. We attain these measurements in the following way:

1. For all working cohorts at model time  $T = 0$ 
  - (a) solve baseline (no-reform) model
  - (b) solve reform model
2. simulate baseline model to generate wealth distribution of workers
3. Compute welfare

Further, for the analysis we calibrate the state-specific parameters to Oklahoma at the time of reform. The main parameters are summarized in Table 3. In terms of the baseline pension plan, public workers receive a \$20,400 pension benefit in retirement. This number is based upon the size of aggregate annual pension distributions relative to the number of active annuitants in the state. The state discounts its pension liabilities at the rate 7.5%. Further, the state at the time of reform has a 79% funded ratio, or a 21% unfunded liability relative to total liabilities.<sup>15</sup>

---

<sup>14</sup>Most importantly, teachers and public safety officers, which are a significant proportion of defined benefit plan participants, were exempted from the plan change and allowed to continue participation in the defined benefit plan.

<sup>15</sup>Reported state funded ratio measures are based upon an asset-weighted average of all the public plans within in a state.



Based upon the historical amount of contributions, relative to the annual required amount, we set  $\theta = 0.79$  which implies the state contributes/saves only 79% of the contributions required to make the plan fully funded in the long run. Further, we assume a 1.6% annual rate of inflation and that the state’s pre-reform COLA matches this rate such that the real value of the pension remains constant throughout a worker’s retirement.

Table 3: OK State-Specific Parameters

Parameter	Label	Value	Units
$\bar{b}$	Pension Benefit	20.4	\$ Thousand
$\{b^s\}$	Social Security	$\{16.5, 18.3\}$	\$ Thousand
$r^p$	State Discount Factor	7.5	Percent
$\chi^0$	Initial Funded Ratio	79	Percent
$\theta$	Contribution Commitment	0.79	Fraction
$\alpha^p$	Pension Portfolio	75	Percent
$\bar{T}$	Amortization Window	30	Years

In what follows, we first document the impact of the reform on aggregates within the fiscal environment of the state such as taxes and pension funded ratio. From there, we document the normative effects of the reform by computing welfare effects for each age cohort and working sector. The reform has an immediate impact on relevant fiscal aggregates, as documented in Figure 1 and Figure 2. Figure 1 plots the average simulated value of pension assets and liabilities for the 45 years following the reform. In the baseline No Reform scenario (left panel) we observe the assets decline in the long run relative to liabilities. This is largely due to the state’s inability to properly contribute funds to the pension as exemplified by a contribution rate  $\theta \leq 1$ . In the Reform scenario (right panel) pension liabilities are immediately reduced and continue to decline in the long run. The immediate reduction in pension liabilities is a function of the COLA freeze which lowers the real value of promised pension benefits and therefore the size of the liability. Further, in this scenario, liabilities continue to decline because of the plan’s closure to new entrants: with each year, new workers enter the public sector but are not enrolled in the defined benefit plan. Therefore, liabilities do not accrue throughout their working lifecycle as they normally would, illustrated by

the PVL formula in Equation (10).

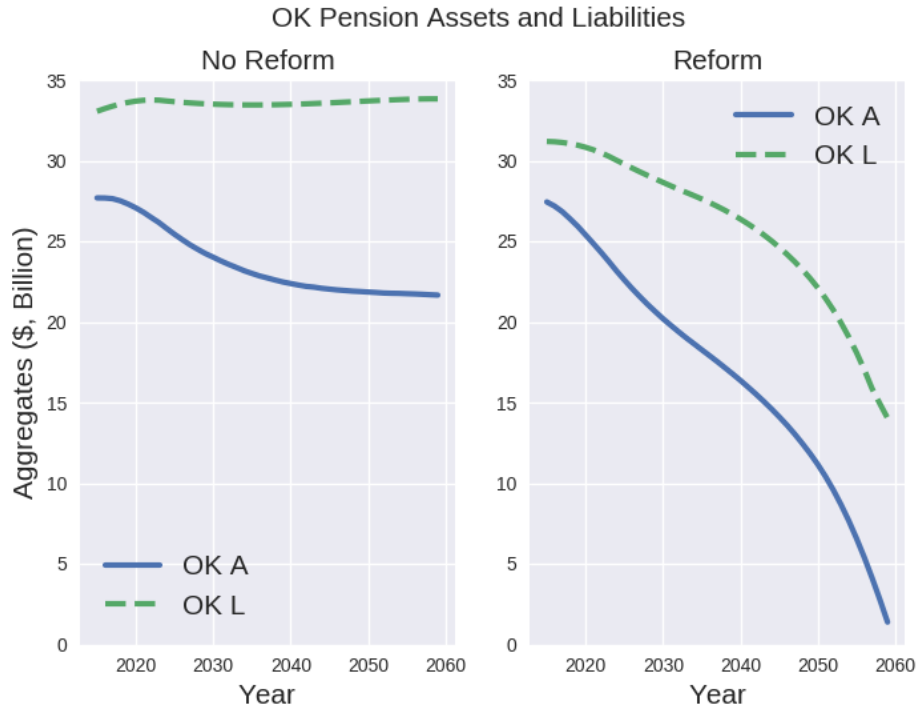


Figure 1: Simulated Pension Assets & Liabilities

Figure 2 plots the simulated funded ratio for both scenarios which is simply the ratio of pension assets to liabilities. Again, under the baseline No Reform plan, the funded ratio continues to deteriorate from 80% to approximately 60%. Alternatively, while the funded ratio declines further in the Reform scenario, the effects are negligible (in terms of aggregate contributions and therefore taxes) as the stock of liabilities is much smaller. Further, note how the funded ratio in the Reform scenario experiences an immediate increase due to the reduction in pension liabilities from the COLA freeze.

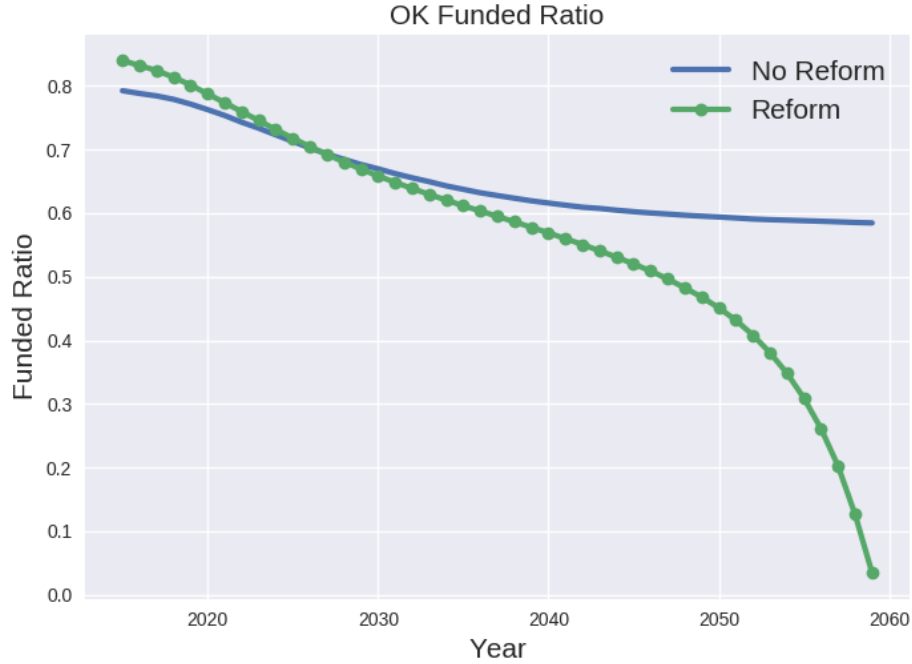


Figure 2: Simulated Funded Ratios

Figure 3 plots the average tax level and volatility bands for both scenarios. The tax on worker wages is used to fund public sector wages, pension benefits as well as contributions to the defined contribution plan (in the Reform scenario). In the model framework, taxes related to the pension are the only source of volatility because the pension funded ratio fluctuates with the investment returns of the pension assets, which are exposed to aggregate market risk. As the Figure makes clear, both the tax level and volatility are reduced in the Reform scenario and this is due to the large reduction (presently and into the future) of the state's pension liabilities. Further, under the baseline No Reform scenario, taxes actually tend to rise over time as the pension becomes more under-funded (Figure 2) and liabilities slightly grow (Figure 1).

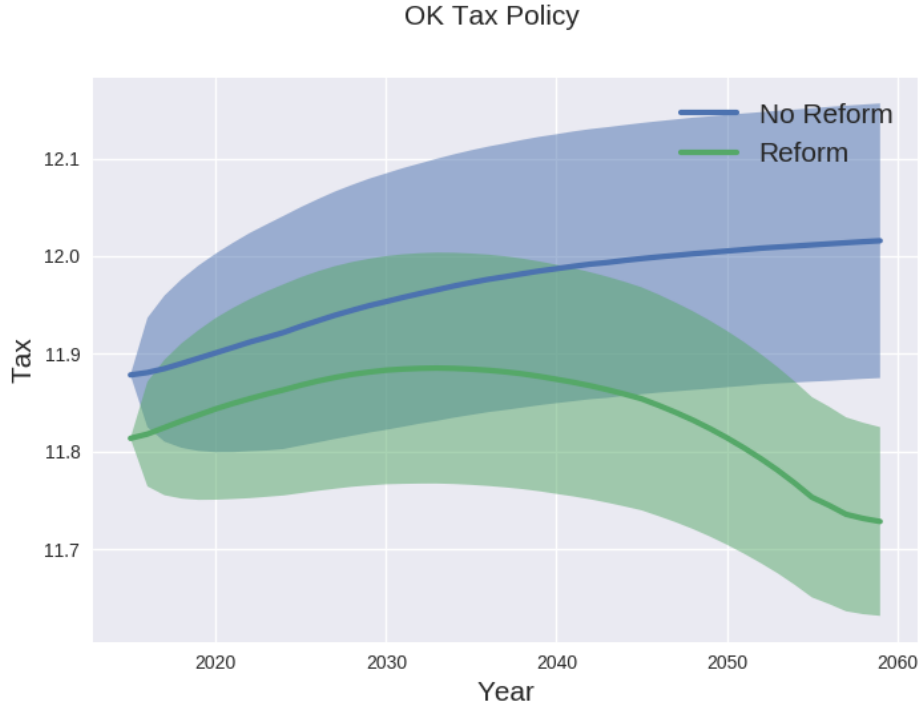


Figure 3: Simulated Tax Policy

Figure 4 plots the welfare effects of pension reform on current workers in both the private and public sectors. Since private sector workers do not benefit in any way from pension benefits, welfare effects are solely driven by the reform's impact on the level and volatility of wage taxes, currently and in the future. Alternatively, while public sector workers are affected through the state's tax code they are also directly affected by changes to the pension plan. Private sector workers (left panel) are uniformly made better off by the pension reform which is due to the reduction in level and volatility of taxes. Further, young cohorts benefit the most because they capture larger tax reductions in the future as evidenced in Figure 3. On the other hand, current public workers are made uniformly worse off due to the COLA freeze: while they remain enrolled in the same nominal pension plan, they observe a large deterioration in the real value of future pension benefits. As was the case for private sector workers, young public workers also fare relatively better. This is due to the ability of lifecycle agents to self-insure: while public workers near retirement are unable to adjust for the large reduction in retirement benefits, young public workers can respond to the reform by saving a larger percentage of wage income and benefit from compounded investment returns.



Figure 4: Welfare Effects by Age Cohort

While this analysis focuses on current workers and retirees, it is worth mentioning another relevant constituency within the state environment: future generations of new hires. Under the Oklahoma-style reform, new hire public workers are auto-enrolled into a defined contribution plan with a 6% matching rate but often workers may require additional wage compensation to be made indifferent between the old and new retirement plan (see [Pandolfo and Winkelmann \[2021a\]](#)). In addition, future new hire workers enter the work force subject to a different tax environment because pension reforms can alter the level of legacy debt (or unfunded liability) that future generations must pay. [Pandolfo and Winkelmann \[2021b\]](#) have examined the impact of various pension reforms on long run legacy debt and therefore tax levels.

Table 4 provides welfare summary statistics for the baseline Oklahoma reform as well as a version of the reform without the COLA freeze and therefore just the closure of the DB plan to new entrants. Welfare measurements for both sectors are a weighted average using the relative proportion of each age cohort within the state population. As can be seen from the baseline reform (first row), the private sector experiences a slight welfare gain while the public sector suffers large welfare losses due to the suspension of COLAs. In the aggregate, the slight gain for private sector workers (which make up roughly 90% of the state population) is offset by the large losses in the public sector. Alternatively, when the reform only consists of closing the plan to new entrants,

both private and public sector workers experience a slight welfare gain. Essentially, by closing the plan, the state is able to greatly reduce pension liabilities and therefore the size and volatility of pension-related taxes while offering new public workers a defined contribution plan with matching.

Table 4: OK CE Welfare Measures

	<b>Public Sector</b>	<b>Private Sector</b>	<b>Statewide</b>
Baseline $\Delta$ Welfare	-2.46%	0.04%	-0.27%
No COLA Freeze $\Delta$ Welfare	.01%	.01%	.01%

## 6 Conclusion

U.S. public pension plans offer guaranteed retirement income as a form of compensation to public employees and are tasked with the accrual of savings and management of assets. On the whole, they have been unable to maintain well-funded plans in recent decades and this has resulted in additional taxes for private sector workers and/or reductions in pension benefits for public workers. Given the evidenced inability of states to properly manage these funds, significant reforms have been proposed and implemented in the last ten years. To aid in this policy analysis, we develop a quantitative lifecycle model which can be tractably calibrated at the state-level and used for both positive and normative analysis. Specifically, our framework accounts for both public and private sector workers who must pay taxes to fund deficiencies in the pension system over time. In the model, state's lack the ability to fully commit to funding the pension in the long run, given parameter values  $\theta < 1$ .

As an example of the relevant policy analysis this framework provides, we evaluate a reform similar to that which was implemented in the state of Oklahoma in 2015. The two main features of the reform were the suspension of COLAs and the closure of the defined benefit plan to new entrants, where new entrants are auto-enrolled in a defined contribution plan with matching. We document that the reform leads to an immediate and persistent reduction in pension liabilities for the state, and this translates into a lower level and volatility of taxes. From a welfare perspective,

---

private sector workers are made uniformly better off due to the reduction in taxes while current public sector workers are made uniformly worse off due to the suspension of their COLAs. In both sectors, younger workers fare better because of their ability to self-insure and lower taxes in the long run. In an alternative reform with just the closing of the defined benefit plan, all workers fare slightly better.

There are several promising areas for future research that can be supported by our modeling structure. One natural extension would be to apply our existing modeling framework to public pension reforms done by other states (e.g., Rhode Island, South Dakota, Wisconsin). The benefit of this exercise is to isolate what lessons can be learned for other would-be pension reformers.

Other promising research comes in the area of model extensions. For instance, this paper focused on the welfare tradeoffs between current public workers and taxpayers. It would be of interest to include the impact of any policy changes on future generations. A second promising area is to introduce labor/leisure choice into our model. Adding labor/leisure choice means that we can consider the impact of public policy pension choices in the context of equilibrium in the labor market. Third, our model does not consider the impact of aggregate shocks on public sector finances or private and public sector wages and employment. Finally, our model assumes a very crude decision-making process on the part of policymakers. However, it may be worth endogenizing the public pension policy decisions. All of these issues strike us as interesting extensions of the modeling framework and useful contributions to better policymaking.

---

## References

- Beermann, J. M. (2013). The public pension crisis. *Washington and Lee Law Review*, 70.
- Brainard, K. and Brown, A. (December 2018). Significant reforms to state retirement systems. *National Association of State Retirement Administrators (NASRA) Policy Brief*.
- Campbell, John Y Cocco, J., Gomes, F., and Maenhout, P. (2001). Investing retirement wealth: A life-cycle model. *Risk Aspects of Investment-Based Social Security Reform*.
- Carroll, C. and Samwick, A. (1997). The nature of precautionary wealth. *Journal of Monetary Economics*.
- Chai, J., Horneff, W., Maurer, R., and Mitchell, O. S. (2011). Optimal portfolio choice over the life cycle with flexible work, endogenous retirement, and lifetime payouts. *Review of Finance*, 15.
- Cocco, J., Gomes, F., and Maenhout, P. (2005). Consumption and portfolio choice over the life cycle. *Review of Financial Studies*.
- Fitzpatrick, T. J. and Monahan, A. B. (2015). Who's afraid of good governance? state fiscal crises, public pension underfunding, and the resistance to governance reform. *Florida Law Review*, 66.
- Healey, T. J., Hess, C., and Nicholson, K. (2012). Underfunded public pensions in the united states: The size of the problem, the obstacles to reform and the path forward. *Harvard M-RCBG Working Paper No. 2012-08*.
- Keefe, J. H. (2012). Are public employees overpaid? *Labor Studies Journal*, 37(1).
- Monahan, A. B. (2010). Public pension plan reform: The legal framework. *Education Finance and Policy*, 5(4).
- Monahan, A. B. (2015). State fiscal constitutions and the law and politics of public pensions. *University of Illinois Law Review*.
- Monahan, A. B. (2017). When a promise is not a promise: Chicago-style pensions. *UCLA Law Review*.
- Myers, S. (2019). Public employee pensions and municipal insolvency. *working paper*.



- 
- Novy-Marx, R. and Rauh, J. D. (2011a). Policy options for state pension systems and their impact on plan liabilities. *Journal of Pension Economics and Finance*, 10.
- Novy-Marx, R. and Rauh, J. D. (2011b). Public pension promises: How big are they and what are they worth? *Journal of Finance*, 66.
- Pandolfo, J. and Winkelmann, K. (2021a). Comparing db and dc plans. *Heller-Hurwicz Economics Institute Pension Policy Brief Series*.
- Pandolfo, J. and Winkelmann, K. (2021b). Public pensions and future generations. *Heller-Hurwicz Economics Institute Pension Policy Brief Series*.
- Poterba, J., Rauh, J., Venti, S., and Wise, D. (2007). Defined contribution plans, defined benefit plans and the accumulation of retirement wealth. *Journal of Public Economics*, 91(10).

## 7 Appendix

### 7.1 Welfare Analysis

One of the key contributions of this framework is the ability to analyze the welfare impact of particular pension reforms across various constituencies. Specifically, we focus on how proposed reforms affect workers (1) by sector (public or private) (2) by age cohort and (3) across states. To do this, we employ consumption-equivalent welfare as our metric. Thus, suppose a given workers optimal consumption policy function  $\{c_t^*\}_{t=1}^{80}$  yields ex-ante welfare  $V^*$  via

$$V^* = \sum_{t=1}^{80} \beta^t \left( \prod_{j=1}^t p_j \right) \frac{c_t^{*1-\gamma}}{1-\gamma}$$

where  $p_j$  is the conditional probability of surviving to date  $j + 1$ , given the worker is alive at age  $j$ . We define the consumption-equivalent (CE) welfare as a scalar  $\bar{c}^*$  such that

$$\begin{aligned} V^* &= \sum_{t=1}^{80} \beta^t \left( \prod_{j=1}^t p_j \right) \frac{c_t^{*1-\gamma}}{1-\gamma} \\ \Rightarrow \bar{c}^* &= \left[ \frac{V^*(1-\gamma)}{\sum_{t=1}^{80} \beta^t \left( \prod_{j=1}^t p_j \right)} \right]^{\frac{1}{1-\gamma}} \end{aligned}$$

Recall that the state space in solving a worker's dynamic program is given by  $\mathbf{s} = (t, I, X, \eta, T, \chi)$  for worker age, sector of employment, cash-in-hand, current persistent shock to labor income, time period and current funded status of the state pension plan, respectively. At any given time there can exist a cross-section of workers with respect to their available cash-in-hand. Thus, we rely upon model simulations to compute the an average CE welfare for each age cohort.<sup>16</sup> First, we fix model time  $T = 1$  and pension funded status  $\chi$  to the current level  $\chi^0$ . Then, we solve for the CE welfare of an agent of age  $t$  in sector  $I$ ; this yields a value  $\bar{c}^*(X, \eta|t, I)$  which varies in the agent's cash-in-hand and persistent income shock. From the model simulation, we obtain a conditional distribution  $F(X, \eta|t, I)$  and define age-cohort welfare as

$$\bar{c}(t, I) = \int \int \bar{c}^*(X, \eta|t, I) dF(X, \eta|t, I)$$

---

<sup>16</sup>A further complication arises in that to properly simulate the model and obtain the cross-section of wealth for current (i.e. time  $T = 1$ ) workers and retirees we require knowledge of the past history of price shocks, pension/fiscal policy decisions, as well as the evolution of age demographics. Instead, we simulate the model given current time  $T = 1$  data objects and do not allow for demographic change.

Thus, under the baseline calibration we compute  $\bar{c}^{base}(t, I)$  and evaluate all potential reforms via the % deviation

$$100 * \frac{\bar{c}^{ref}(t, I) - \bar{c}^{base}(t, I)}{\bar{c}^{base}(t, I)} \quad \forall t, I$$

and for each state where  $\bar{c}^{ref}$  is CE welfare under the policy reform.

## 7.2 Computing Age Cohort Distributions

We assume age cohort distributions are invariant to job sector; that is,

$$\Phi(s, t, T) = prob(pub)\tilde{\Phi}(t, T) + (1 - prob(pub))\tilde{\Phi}(t, T)$$

Define data cohort distributions as  $G(t, T)$  for age group  $t$  and time  $T$ , as well as labor force participation rates  $\gamma(t)$ , the ratio of annuitants to workers  $\phi$  and cohort growth rates  $\phi(t, T)$ .<sup>17</sup> To construct  $\tilde{\Phi}$  as the model analogue to  $G$ , we apply the following procedure

1. For  $T = 1$

- for working cohorts, apply labor force participation rates

$$\tilde{\Phi}(t, 1) = \gamma(t)G(t, 1) \quad \forall t = 1, \dots, 45$$

- For retired cohorts, set

$$\tilde{\Phi}(t, 1) = \bar{\kappa}G(t, 1) \quad \forall t = 46, \dots, 80$$

where the  $\kappa$  is recovered from the model-implied ratio of annuitants to workers  $\phi$ , given by

$$\phi = \frac{\bar{\kappa} \sum_{t=46}^{80} G(t, 1)}{\sum_{t=1}^{45} \tilde{\Phi}(t, 1)}$$

- Normalize joint distribution  $\tilde{\Phi}(\cdot, 1)$

2. For  $T = 2, \dots$

---

<sup>17</sup>The Weldon Cooper data is organized in 5-year age bins, while our model accounts for individual ages. To account for this, we take the proportions within each age bucket and evenly apply it to individual age buckets.

- 
- For each cohort, apply growth rates

$$\tilde{\Phi}(t, T+1) = \phi(t, T)\tilde{\Phi}(t, T) \quad \forall t = 1, \dots, 80$$

- Normalize joint distribution  $\tilde{\Phi}(\cdot, T)$

### 7.3 Calculating Real Value of Pension Benefit with COLAs and Inflation

Assume constant rate of inflation  $\pi$ . Suppose COLA rates are  $\omega$ . Then, given a date-0 nominal pension  $b_0$ , future nominal pensions are

$$b_t = (1 + \omega)^t b_0$$

and the real value of date- $t$  pension benefits is simply

$$\tilde{b}_t = \left(\frac{1 + \omega}{1 + \pi}\right)^t b_0$$

In our model, we assume the nominal (and real) pension benefit  $b$  is constant over time. In the reform, if COLAs are frozen then, simply adjust pension benefits at date  $t$  as

$$b_t = \left(\frac{1}{1 + \pi}\right)^t b$$

where  $\pi$  is the long-run constant rate of inflation. If COLAs are reduced, but not frozen, to a rate  $\omega$ , then simply

$$b_t = \left(\frac{1 + \omega}{1 + \pi}\right)^t b$$

## 7.4 Additional Figures and Tables

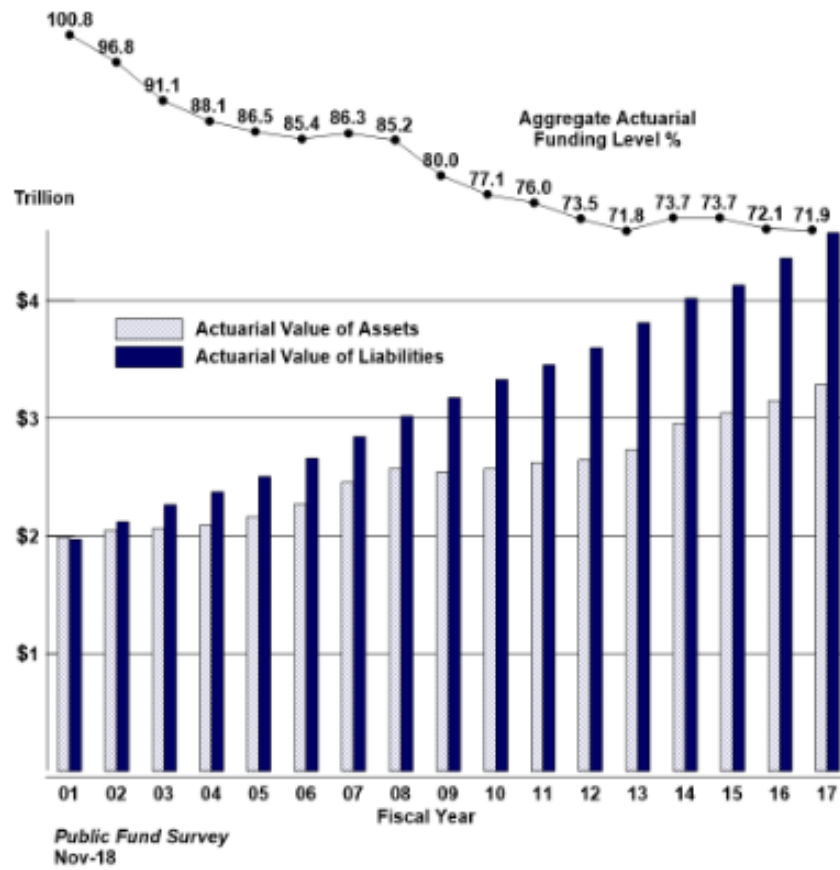


Figure 5: Simulated OK Pension Assets & Liabilities

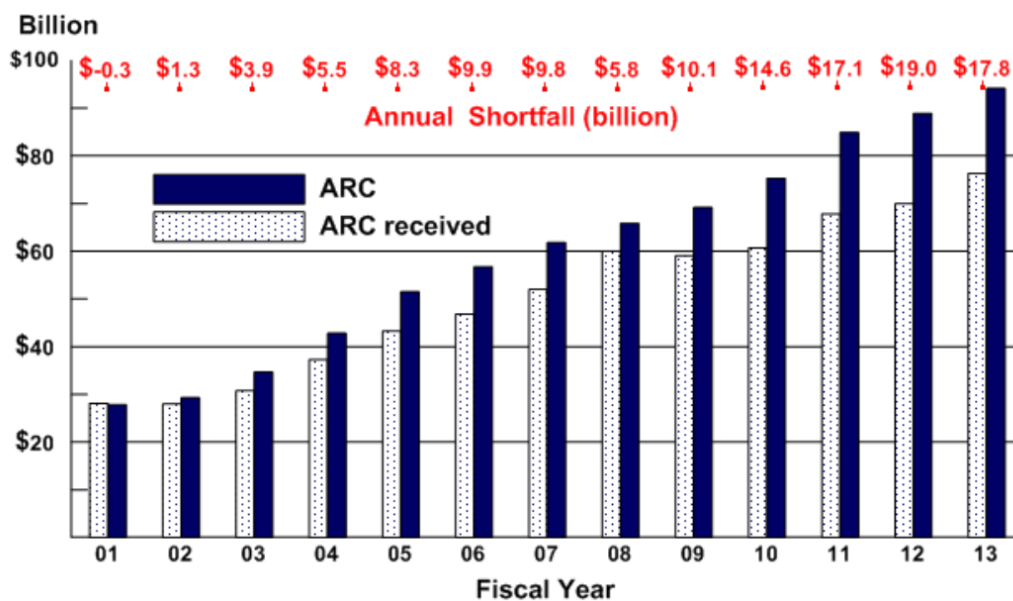


Figure 6: Simulated OK Pension Assets & Liabilities

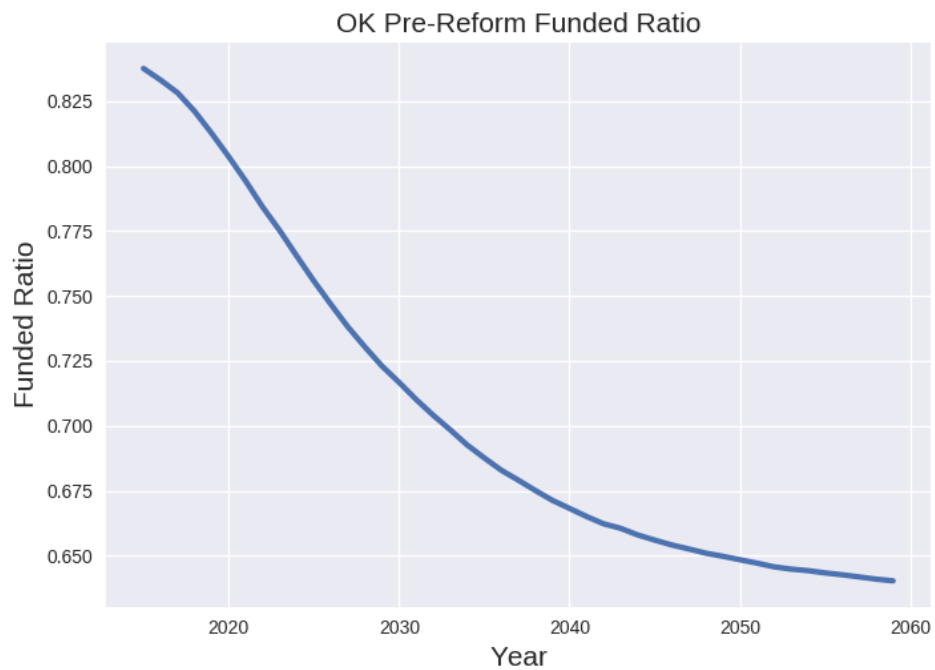


Figure 7: Simulated OK Pension Assets & Liabilities

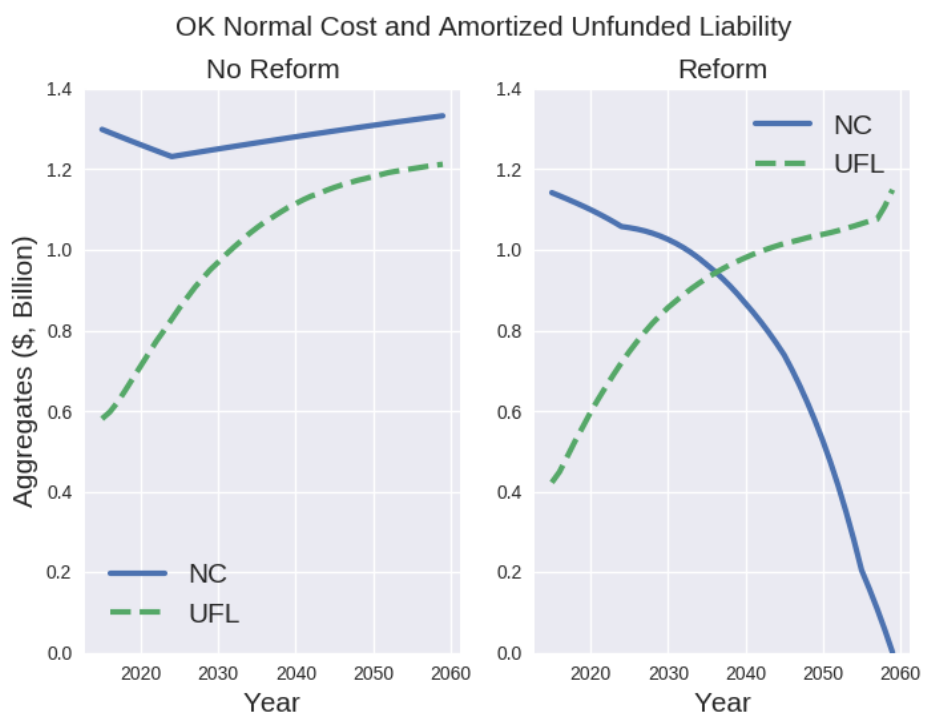


Figure 8: Simulated OK Pension Assets & Liabilities