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Optimizing Trajectory-Based Vorticity Computation For Wave-Driven Circulations Around Headlands

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Motivation

When waves approach a coastline like Cape Falcon (Fig. 1a), they are refracted by protruding headlands (Visual 1) into embayments (Fig. 1b-d), driving two alongshore inflows and one or more central outflows. These inflows and outflows together can be seen as a rotating cell with vorticity.

$$\omega = \nabla \times V \quad (1)$$

Vorticity is the curl of the velocity field.

Vorticity is typically calculated from a gridded velocity field using Eq. 1. Using trajectory data from drifters in the water, however, may allow for a more accurate vorticity computation. Lagrangian Gradient Regression (LGR) is a recently developed algorithm that uses this approach (Harms et al. 2023). In a simulated flow field of wave-driven circulation on headlands, this project attempts to minimize the error of LGR vorticity computation by optimizing drifter seeding strategies.

The Idealized Headland/Embayment System

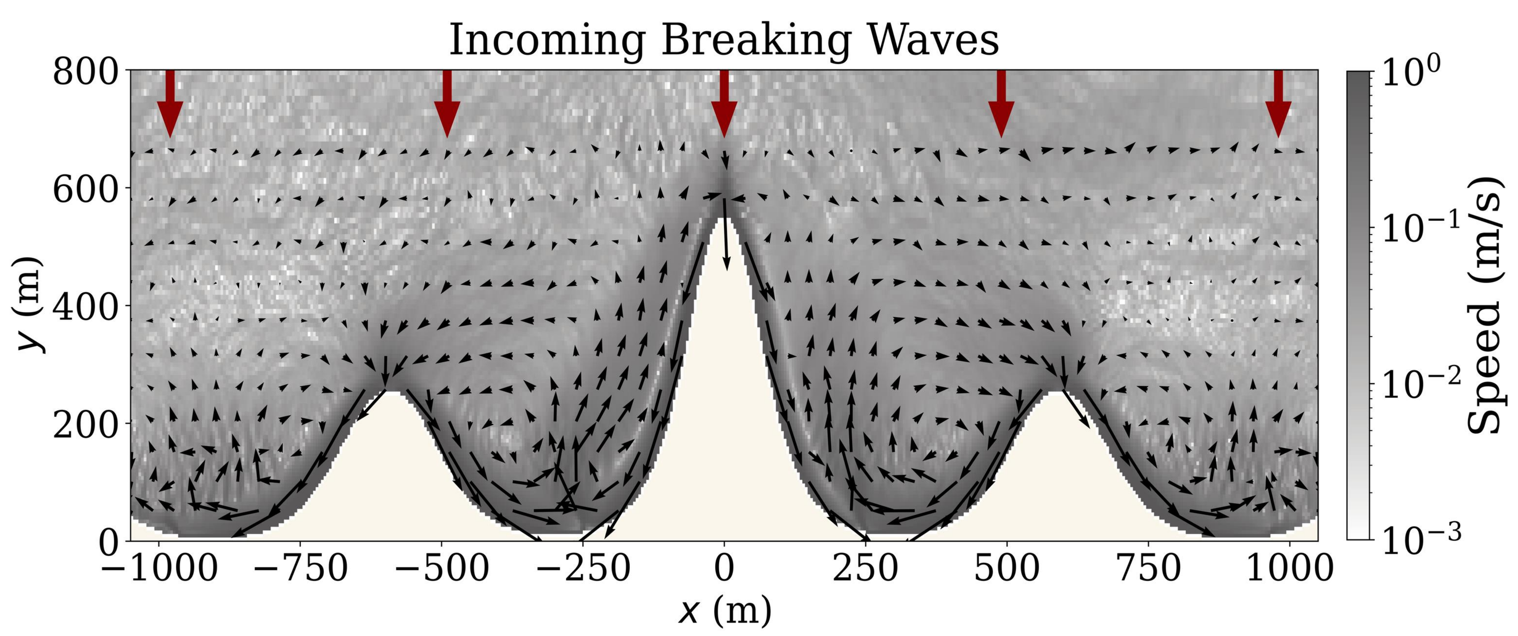

Figure 2 – The depth-averaged velocity field of the idealized headland/embayment system from particle trajectories was generated. The off-white color represents land. The velocity field is computed with the ocean circulation model ROMS, which solves the hydrostatic primitive Reynolds-averaged Navier Stokes equations. The Coupled Ocean-Atmosphere-Wave-Sediment Transport (COAWST) modeling system allows surface wave forcing (see Fig. 4) to impart momentum to the circulation field via the SWAN wave model (Warner et al. 2008). The red arrows show the direction of incident wave forcing ($H_s = 3.0\ m$).

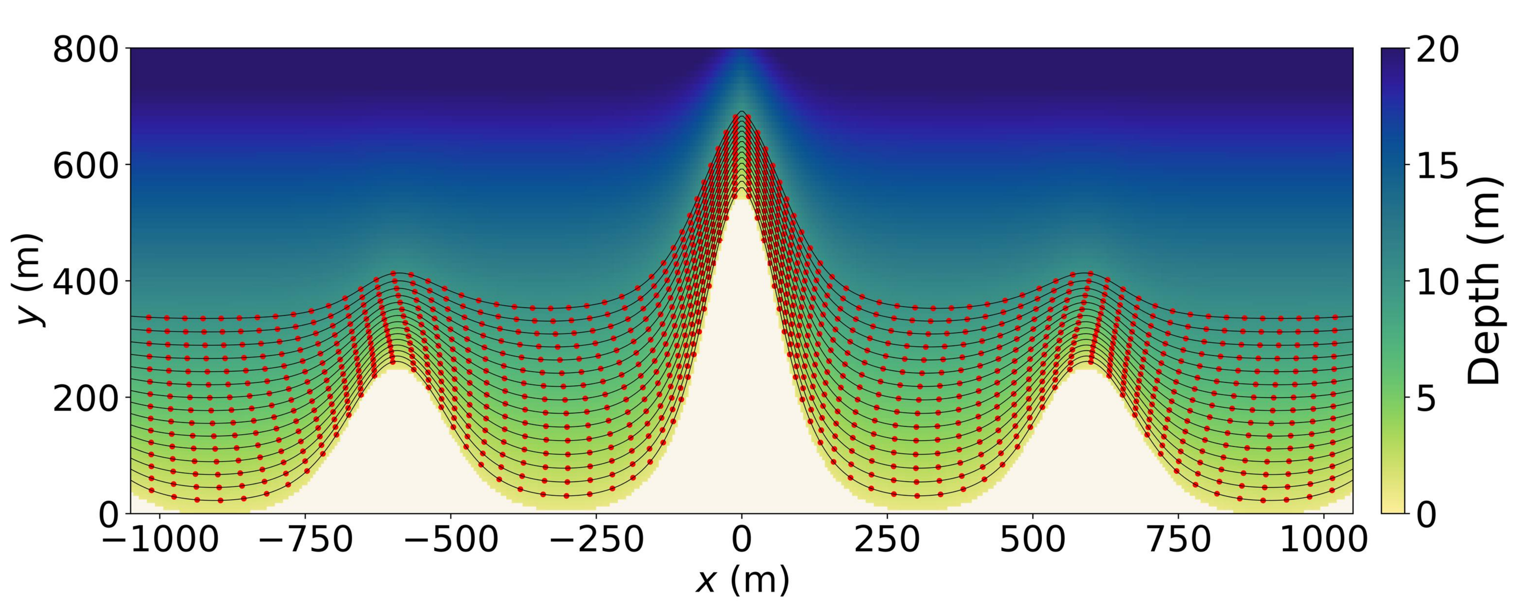

Figure 3 – Bathymetric map of the idealized domain. Red dots are used to represent the initial positions of 1200 seeded particles. They were placed on 15 evenly spaced isobaths, lines connecting points of equal depth beneath the surface, between 1 and 10 meters (a 0.6 m isobath spacing). 80 particles are then spaced evenly along each contour's path. In this project, the isobath spacing (and therefore the number of particles per contour) is changed, while the 1200 total particle count is kept constant.

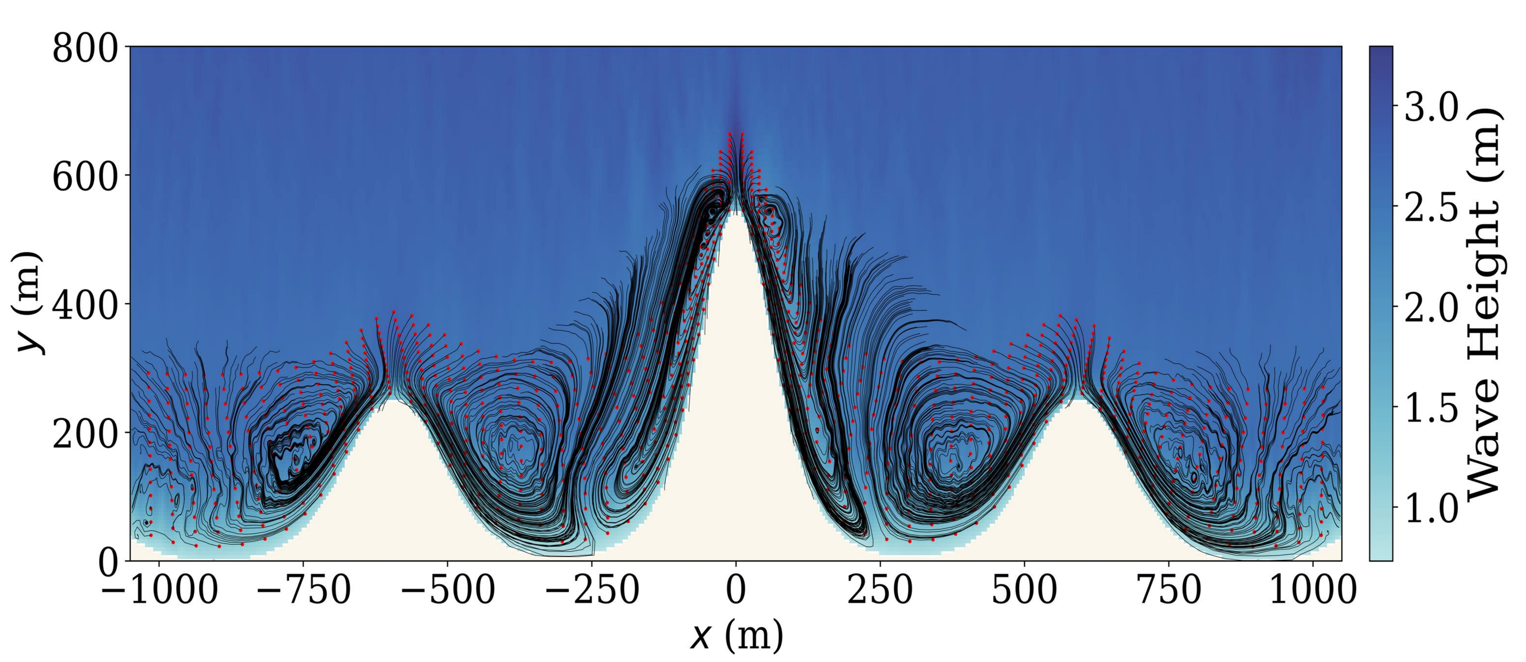

Figure 4 – Wave heights across the domain caused by incoming waves (see the red arrows in Fig. 2) and 1200 seeded particles as red dots (Fig. 3). The trajectories of the particles over one hour in the velocity field were integrated using the Parcels python package (Delandmeter and van Sebille 2020).

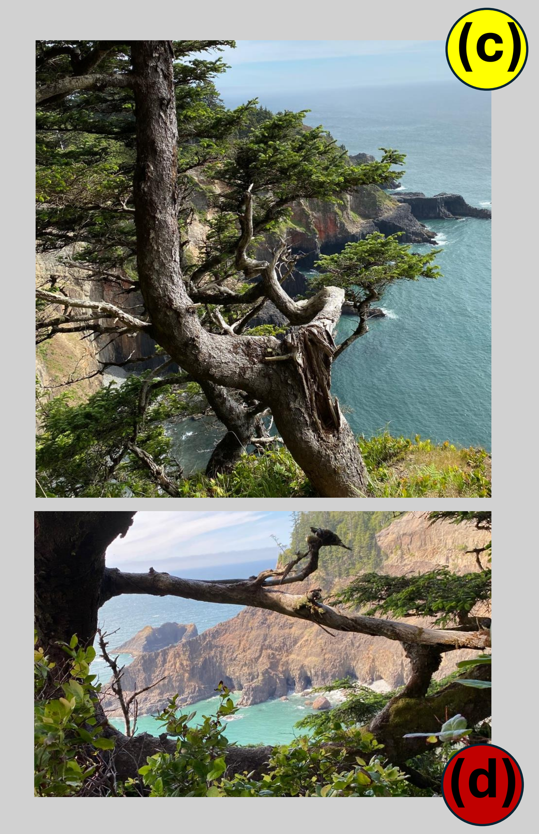

Figure 1 – (a) - Topological map of Cape Falcon OR (Google Maps) (b-d) - photographs of the embayments corresponding to (a).

LGR-Derived Vorticity

LGR uses the trajectories of passive particles in the flow, a *Lagrangian* perspective, to compute vorticity (ω) so there is no need to specify a velocity field. LGR first obtains a flow *gradient* for each particle over a short time window (t to τ) via Gaussian-weighted *regression* using the five nearest neighbors of the point (Eq. 2). This allows approximate computation of the velocity gradient (Eq. 3). ω can then be inferred using the skew-symmetric component of the velocity gradient.

$$\underbrace{\chi_\tau}_{\text{positions at later time } \tau} = \overbrace{\nabla F_t^\tau}^{\text{the flow gradient between times t and } \tau} \underbrace{\chi_t}_{\text{positions at earlier time } t} \quad (2)$$

$$\overbrace{\nabla V_t}^{\text{the velocity gradient at time t}} \approx \frac{\overbrace{\nabla F_t^\tau}^{\text{the flow gradient between times t and } \tau} - \overbrace{I}^{\text{the identity matrix}}}{\underbrace{\tau - t}_{\text{a small time difference}}} \quad (3)$$

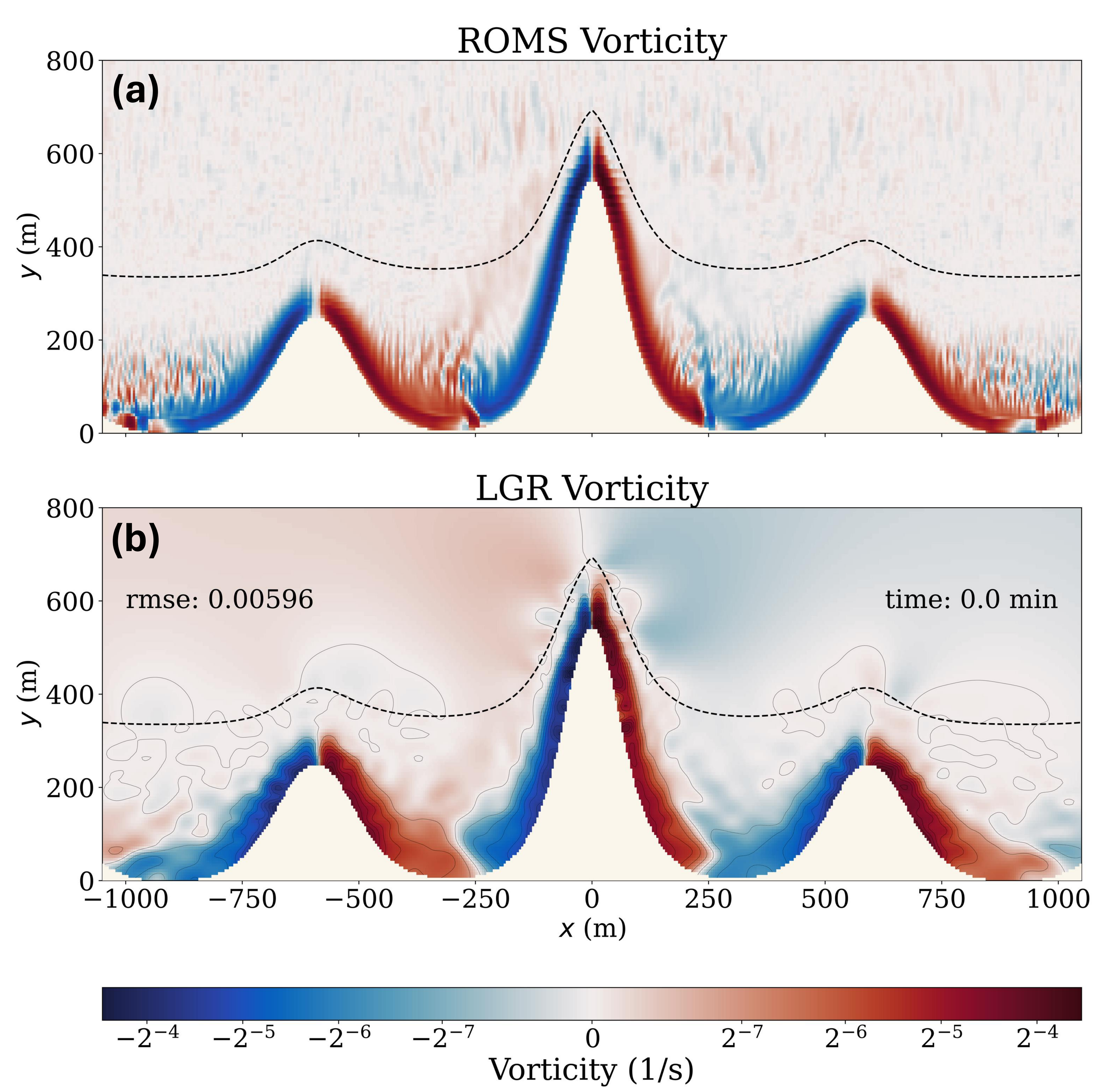

Figure 5 – This figure compares LGR and ROMS vorticities. The ROMS vorticity (a) was exactly computed from the modeled vector field described in Fig. 2 using Eq. 1. It was smoothed by a five-point moving spatial average. The LGR vorticity was computed at the first timestep of the trajectory data from Fig. 4 using the computation described in the section above. The result was then interpolated to the same grid as the ROMS vorticity to facilitate numerical comparison.

Error Comparison of LGR With ROMS

$$RMSE = \sum_{i=0}^{n} \sqrt{\frac{(LGR[i] - ROMS[i])^2}{n}} \quad (4)$$

We compare LGR vorticity to ROMS using root mean squared error (RMSE) (Eq. 4). The error is computed over their n shared interpolated grid points.

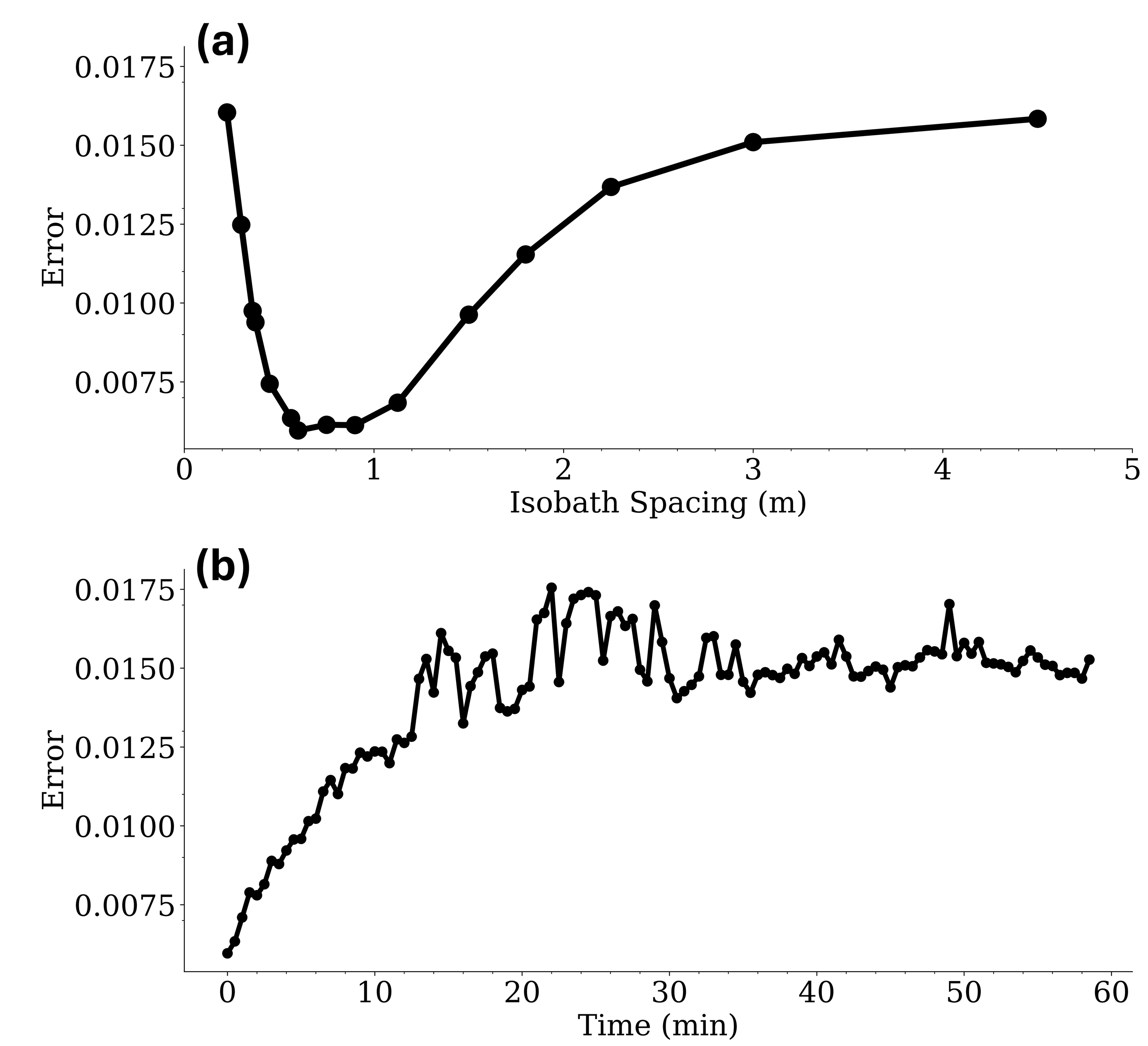

Figure 6 – This figure features results of root mean squared error (RMSE) computations (Eq. 4) comparing LGR vorticities with the ROMS vorticity shown in Fig. 5a inside the 10 m isobath (the dashed lines in Fig. 5). In (a), the isobath spacing used to distribute 1200 particles was altered while time was kept constant. In (b), 1200 particles are distributed with an isobath spacing of 0.6 m and error was computed over time.

Results and Implications

- The minimum RMSE occurs at an isobath spacing of 0.6 m immediately upon release of the particles as shown in Fig. 5b.
- RMSE increases over a 15 minute timescale before asymptoting.
  - Visual 2 (accessible via the QR code) shows that this timescale corresponds to the time required for seeded particles to leave the embayments' alongshore inflows.
    - Fig. 2 shows that these narrow inflows feature the simulated system's highest velocities.

This analysis has informed Dr. Torres' proposal for future oceanographic modeling and field study at Cape Falcon. The overall success of LGR justifies its application to in situ drifter measurements and optically derived trajectory observations.

Acknowledgements

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