1. Write the language determined by the regex /a*b*/

$\{ a^m b^n \mid m \geq 0, n \geq 0\}$

2. Write a **regular** grammar to generate the language determined by the regex a*b*/

G=({S,T},{a,b, ε },S, with following production rules:

S ::= ε | a | aS | bT
T ::= b | bT
)

3. Write the language determined by the regex /(ab)*/

$\{ab^n \mid n \geq 0 \}$

4. Write a regular grammar to generate the language determined by the regex /(ab)*/

G=({S,T},{a,b, ε },S, with following production rules:

S ::= ε | aT
T ::= b | bS
)

5. Write the language determined by /Whiske?y/. The alphabet is {W,h,i,s,k,e,y} (W is a terminal symbol here).

L = {Whisky, Whiskey}

6. Write a regular grammar to generate the language matched by /Whiske?y/

G=({S,A,B,C,D,E,Y},{W,h,I,s,k,e,y},S, with following production rules:

S ::= WA
A ::= hB:
B ::= iC
C ::= sD
D ::= kE
E ::= y | eF
F ::= y
)

7. Write a regular grammar to generate decimal numbers; the relevant regex is /[1-9][0-9]*(\.[0-9]*[1-9])?/.
   You may find it useful to use notation resembling D ::= 0 | 1 | … | 9 to denote an evident set of ten production rules.

G=({D,T,C},{0,1,2,3,4,5,6,7,8,9, . },D, with following production rules:

D ::= 1 | … | 9
D ::= 1D | … | 9D
D ::= 0T | … | 9T
T ::= .C
C ::= 1 | … | 9
C ::= 0C | … | 9C

)

8. Give a **context free** grammar for the language L = { a^n b^n | n ∈ ℕ}.

G=({S},{a,b, ε },S, with following production rules:

S ::= ab | aSb | ε

)

9. Give a context free grammar for the set of properly bracketed sentences over alphabet { 0 , (,) }. So 0 and ((0)) are fine, and 00 and 0) are not fine.

G=({S},{0 , (,) },S, with following production rules:

S ::= 0 | (S)

)

10. Write a context free grammar to generate possibly bracketed expressions of arithmetic with + and * and single digit numbers.
    So 0 and (0) and 0+1+2 and (0+9)are fine, and (0+)1 and 12 are not fine.

G=({S,A},{0,1,2,3,4,5,6,7,8,9,+, },S, with following production rules:*

S ::= 0 | … | 9
S ::= (S)
S ::= 0A | … | 9A
*A ::= +S| *S*

)

11. Give a grammar for palindromes over the alphabet { a , b }.

G=({P},{a,b, ε },P, with following production rules:

P ::= a | b | aPa | bPb | ε

)

12. A parity-sequence is a sequence consisting of 0s and 1s that has an even number of ones. Give a grammar for parity-sequences.

G=({S,A},{0,1, ε },S, with following production rules:

S ::= 0S | 1A | ε
A ::= 0A | 1S

)

13. Write a grammar for the set of numbers divisible by 4 (base 10; so the alphabet is [0-9]). You might like to read this page on divisibility testing, first. You may use dots notation to represent an evident sequence, as in for example the sequence $Z3,Z6,Z9,\dots,Z999$ to mean “Z followed by some number divisible by three and strictly between 0 and 1000”.

G=({S,A,B},{0,1,2,3,4,5,6,7,8,9 },S, with following production rules:

S ::= 0 | 4 | 8
S ::= 1A | 3A | 5A | 7A | 9A
S ::= 0B | 2B | 4B | 6B | 8B
A ::= 2 | 6 | 0S | 1S | 2S | 3S | 5S | 6S | 7S | 9S
B ::= 0 | 4 | 8 | 0S | 4S | 8S

)

14. Write a grammar for the set of numbers divisible by 3 (base 10; so the alphabet is $[0\text{-}9]$). So for example 0, 003, and 120 should be in your language, and 1, 2, and 5should not.

G=({S,A,B,C,D},{01,2,3,4,5,6,7,8,9},S, with following production rules:

S ::= 0 | 3 | 6 | 9 | 0S | 3S | 6S | 9S
S ::= 1A | 4A | 7A
S ::= 2B | 5B | 8B
A ::= 2 | 5 | 8 | 2S | 5S | 8S | 0A | 3A | 6A | 9A | 1B | 4B | 7B
B ::= 1 | 4 | 7 | 1S | 4S | 7S | 2A | 5A | 8A | 0B | 3B | 6B | 9B

)