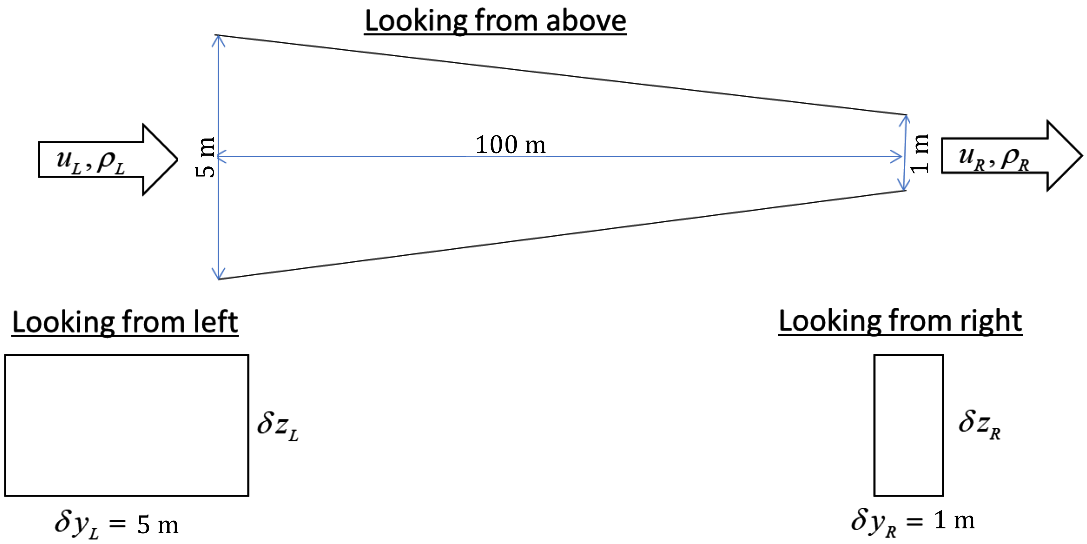
ATOC30004 Dynamical Meteorology and Oceanography.

Assignment 1

(20 possible marks – but final grade out of 10 because it contributes 10% to your final grade)

1. [7 marks]. A fluid flows through a 100 m long channel with a rectangular cross-section (see figure below). The width of the channel at one end (the left side) is 5 metres and at the other end (the right side) is 1 metre. To begin, assume the fluid has constant density and that the water is flowing from the wide end of the channel to the narrow end of the channel.



General constraints on this problem:

The set up is assumed to be in steady state in the sense that the partial time derivatives $\frac{\partial}{\partial t}$ of all of the variables is assumed to be equal to zero.

Mass flux out of channel on right side, m_{out}, over time δt is given by

$m_{out} = \rho_R \left(u_R \delta t\right) \delta y_R \delta z_R$

In order to achieve the steady state condition $\frac{\partial \rho}{\partial t} = 0,$ the mass flux m_{in} into the channel on the left hand side of the channel, over time $\delta t,$ given by $m_{in} = \rho_L \left(\delta t\right) u_L \delta y_L \delta z_L$ must be equal to the mass flux out $m_{out} = \rho_R \left(u_R \delta t\right) \delta y_R \delta z_R$

In other words, we must have

$\rho_R \left(u_R \delta t\right) \delta y_R \delta z_R = \rho_L \left(u_L \delta t\right) \delta y_L \delta z_L$

which implies that

$\rho_L u_L \delta y_L \delta z_L = \rho_R u_R \delta y_R \delta z_R$ (1.1)

We are also told that $\delta y_L = 5\ m$ and $\delta y_R = 1\ m$ so (1.1) simplifies to

$5 \rho_L u_L \delta z_L = 1 \rho_R u_R \delta z_R$ (1.2)

a) (3 marks) Assuming that the fluid depth is constant along the channel and that the along-channel fluid speed is equal to 10.0 m/s at the narrower end (right side) of the channel, what is the fluid velocity at the wide end (left side) of the channel?

Answer: We are told that

$(i)\rho_L = \rho_R = \rho$ because the fluid is incompressible, and

(ii) $\delta z_L = \delta z_R = h$ because the depth of the fluid is constant

(iii) $u_R = 10\ ms^{-1}$

Using (i) - (iii) in equation (1.2) gives

$$5\rho u_L h = \rho u_R h = 10\rho h$$

$$\Rightarrow u_L = \frac{10}{5}\ \mathrm{ms}^{-1} = 2\ \mathrm{ms}^{-1}$$

b) (2 marks) Use your answer from (a) for the fluid flow at the wide end (left side) of the channel; further assume that the fluid depth varies along the channel and that the fluid velocity at the narrow end of the channel is 5 m/s (instead of the 10 m/s used in part(a)). Assuming that the fluid depth at the wide end of the channel is 1 metres, what is the depth of fluid at the narrow end of the channel?

Answer: The question makes it clear that

$(i)\rho_L = \rho_R = \rho$ because the fluid is incompressible, and

(ii) $u_L = \frac{10}{5} = 2\ ms^{-1}$ (our answer from part a)

(iii) $u_R = 5\ ms^{-1}$

$(iv)\ \delta z_L = 1\ m$

Using (i) - (iv) in equation (1.2) gives

$$\rho_L u_L \delta y_L \delta z_L = \rho_R u_R \delta y_R \delta z_R$$

$$\rho\left(\frac{10}{5}\right)5(1) = \rho 5(1)\delta z_R$$

$$\Rightarrow \delta z_R = \frac{10}{5} = 2\ \mathrm{m}$$

c) (2 marks) Now assume (i) the fluid is compressible (ii) the fluid depth is 2 metres along the length of the channel (iii) the velocity is 4.0 m/s at the wide end and 10 m/s at the narrow end, and (iv) the density is 1000 kg m-3 at the wide end of the channel. Given conservation of mass, what must be the density at the narrow end of the channel?

Answer: In this situation,

(i) $\delta z_L = \delta z_R$=2 m

(ii) $u_L = 4\ ms^{-1}, \rho_L = 1000\ kg\ \ m^{-3}$

(ii) $u_R = 10\ ms^{-1}$

Using (i) - (iii) in equation (1.2) gives

$u_L \rho_L \delta z_L \delta y_L = u_R \rho_R \delta z_R \delta y_R$

$4(1000)2(5) = 10\rho_R(2)1$

=> $\rho_R = \frac{40000}{20}$ =2000 $kg\ \ m^{-3}$

2. [7 marks].

A weather observer from Tornadoville calls his observer friends in Tornadoville East and Tornadoville West and they determine that the spatial gradient in temperature is 0.02°C per kilometre (warmer to the east, cooler to the west) and that there is a steady westerly wind (u>0) equal to 10 m/s.

(a) (3 marks) The observer at Tornadoville wants to determine how the temperature would be changing in time at Tornadoville if the assumption $\frac{DT}{Dt} = 0$ was correct. You discuss the situation and you agree that if $\frac{DT}{Dt} = 0$ the temperature in Tornadoville should be getting ….

(i) Colder, or

(ii) Warmer.

(Choose which of the above you think is the right answer)

Now show your observer friend how to calculate what the rate of temperature change in the town of Tornadoville would be. Be quantitative. Give your answer in °C per hour.

(b) (4 marks) Having made your calculations of the local rate of temperature change based on the $\frac{DT}{Dt} = 0$ assumption, you go outside to check his temperature recording device. You notice that it's quite sunny and you both wonder whether the $\frac{DT}{Dt} = 0$ assumption was justified. On reading the instruments you find that the local temperature tendency is, in fact, 1°C per hour (getting warner with time). Your observer friend then wonders what that implies about the heating rate Q in the equation $\frac{DT}{Dt} = Q$. Show your observer friend how to compute Q. Be quantitative. Give your answer in °C per hour.

Answer: Assuming horizontal flow at constant pressure, the rate of change of temperature following a fluid parcel is equal to the heating rate Q; i.e.

$$\frac{DT}{Dt} = Q. \qquad (2.0)$$

In the situation described, $v = w = 0$ but $u = 10ms^{-1}$. Since

$$\frac{km}{hr} = \frac{1000m}{3600s} \Rightarrow 1\ ms^{-1} = \frac{3600km}{1000hr} = 3.6\frac{km}{hr} \Rightarrow u = 10ms^{-1} = 36\text{ km }\left(\text{hr}\right)^{-1}.$$

Part (a):

For part (a) we assume that $\frac{DT}{Dt} = \frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} = 0$. Hence, with $\frac{\partial T}{\partial x} = 0.02\ C\ \left(km\right)^{-1}$

$\frac{\partial T}{\partial t} = -u\frac{\partial T}{\partial x} = -36(0.02) = -0.72\ C\ \left(\text{hr}\right)^{-1}$. Hence, with $Q = 0$, it would be cooling at $-0.72\ C\ \left(\text{hr}\right)^{-1}$ in Tornadoville.

Part (b):

We observed that, in fact, $\frac{\partial T}{\partial t} = 1\ C\ \left(\text{hr}\right)^{-1}$

Using all this information gives the heating rate following a parcel of air as

$$Q = \frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} = 1 + 36(0.02) = 1 + 0.72 = 1.72\ C\ \left(\text{hr}\right)^{-1}$$

3. [7 marks]. Locally, the horizontal velocity field of an incompressible flow is:

$$u = 0.00000005x^2 + 0.0003y + 14$$
$$v = 0.00010x + 0.00000005y^2$$

where, u is the zonal wind, v is the meridional wind, x is the distance in the zonal direction and y is the distance in the meridional direction; x and y are in units of metres, and u and v are in units of m/s. If u and v do not vary with height, calculate the vertical velocity at 1 km above the ground at a point that is 3 km to the West and 1 km to the South of the origin (x=0, y=0).

Answer: *We* know that $w = 0 = w\left(x, y, 0, t\right)$ at the ground and we know that in an incompressible flow

$$\frac{\partial w}{\partial z} = -\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$$

$$\Rightarrow \int_{z=0}^{z=1000} \frac{\partial w}{\partial z} dz = w\left(x, y, 1000\right) - w\left(x, y, 0\right) = w\left(x, y, 1000\right) = -\int_{z=0}^{z=1000} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) dz \quad (3.1)$$

but

$$u = 0.00000005x^2 + 0.0003y + 14$$

$$v = 0.00010x + 0.00000005y^2$$

Hence,

$$\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)=0.0000001x+0.0000001y \qquad (3.2)$$

Using (3.2) in (3.1) gives

$$w(x,y,1000)=-\int_{z=0}^{z=1000}(0.0000001x+0.0000001y)\,dz=-10^{3}(0.0000001x+0.0000001y)$$

$$=-0.0001y-0.0001x \qquad (3.3)$$

The question asks for eq (3.3) to be evaluated at the point $(x,y)=(-3000m,-1000)m.$

Doing so gives

$$w(-3000,-1000,1000)=0.3+0.1=0.4ms^{-1} \qquad (3.4)$$

End of worksheet.