METEO 300
FUNDAMENTALS OF ATMOSPHERIC SCIENCE

10.8 A Closer Look at the Four Force Balances

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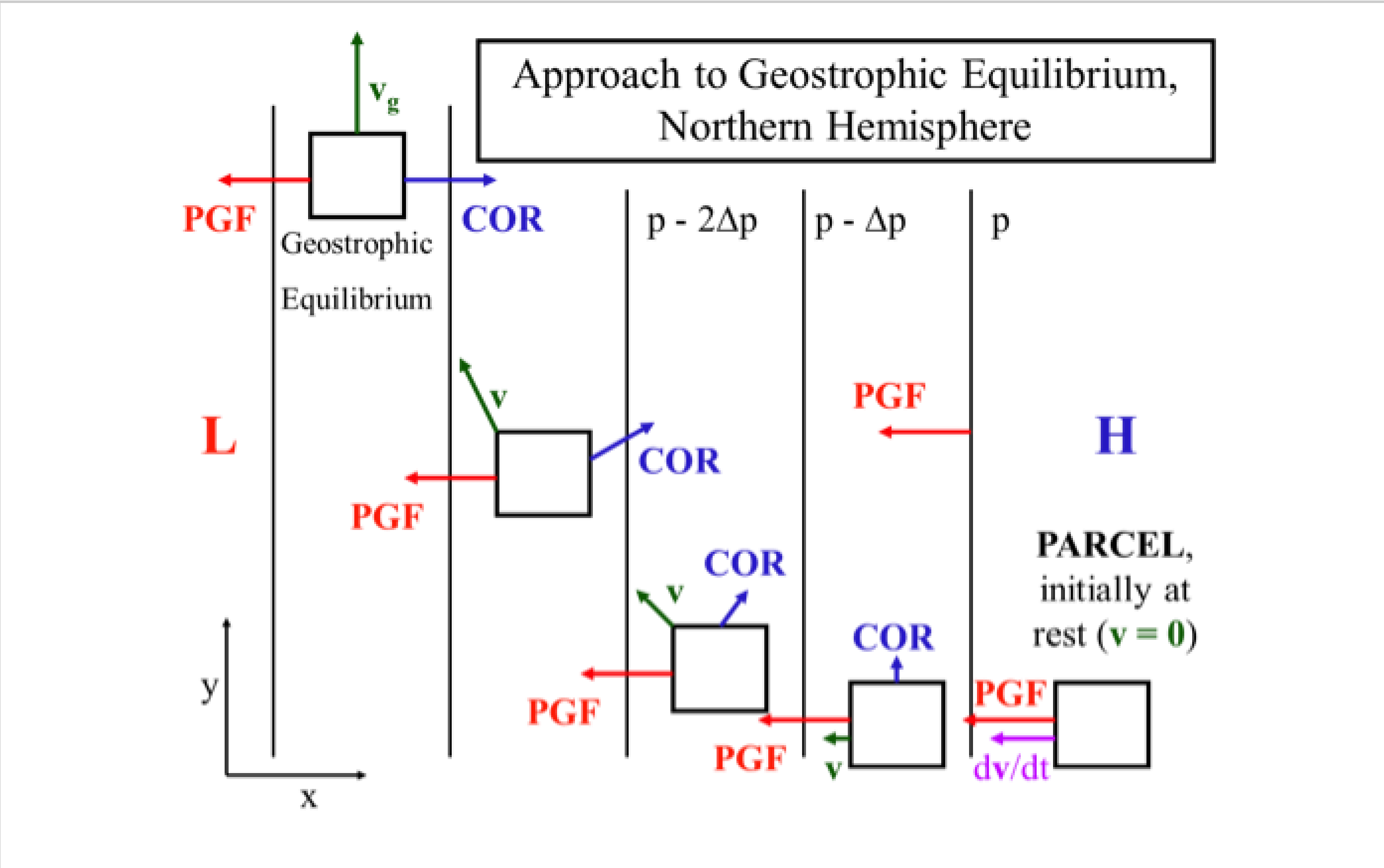
Geostrophic Balance

This balance occurs often in atmospheric flow that is a straight line ($R = \pm\infty$) well above Earth's surface, so that friction does not matter. The Rossby number, R_o, is much less than 1.

Let's think about how this balance might occur. Assume that an air parcel is placed in the midst of a fixed horizontal pressure gradient in the Northern Hemisphere and is initially at rest ($\vec{V} = 0$), as shown in the figure below. The Coriolis force is thus zero and the parcel begins to move from high pressure toward low pressure. However, as the parcel accelerates and attains a velocity parallel to the pressure gradient, the Coriolis force develops perpendicular and to the right of the velocity vector and the PGF. The resulting acceleration is now the vector sum of the PGF and the Coriolis force and turns the parcel to the right. As the velocity continues to increase, the Coriolis force increases but always stays perpendicular and to the right of the velocity vector while the PGF always stays parallel to the pressure gradient. Eventually, the PGF and Coriolis force become equal and opposite and the air parcel will move perpendicular to the horizontal pressure gradient. This condition is called **geostrophic balance**. We have simplified somewhat the approach to geostrophic equilibrium because, in reality, air parcels would overshoot and undergo inertial oscillations (discussed below) and because the pressure field would evolve in response to the motion.

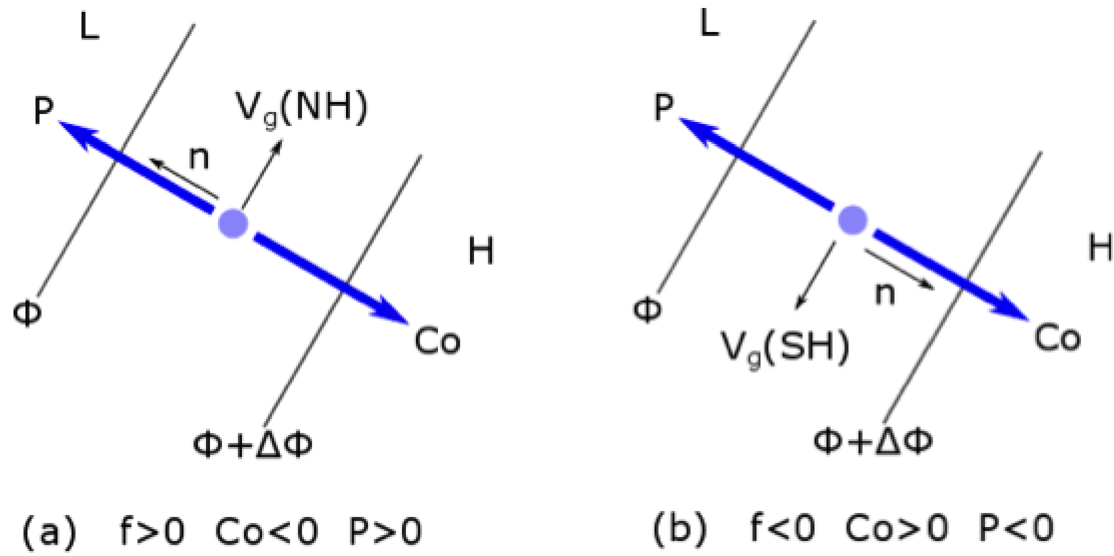


How geostrophic balance is achieved for an air parcel starting at rest. The PGF is always there, but the Coriolis force is zero until the air parcel acquires some velocity. In the figure, v_g is used to represent the geostrophic velocity.

Credit: H.N. Shirer

$$-fV - \frac{\partial \Phi}{\partial n} = 0 \qquad \text{geostrophic wind balance} \tag{10.36}$$

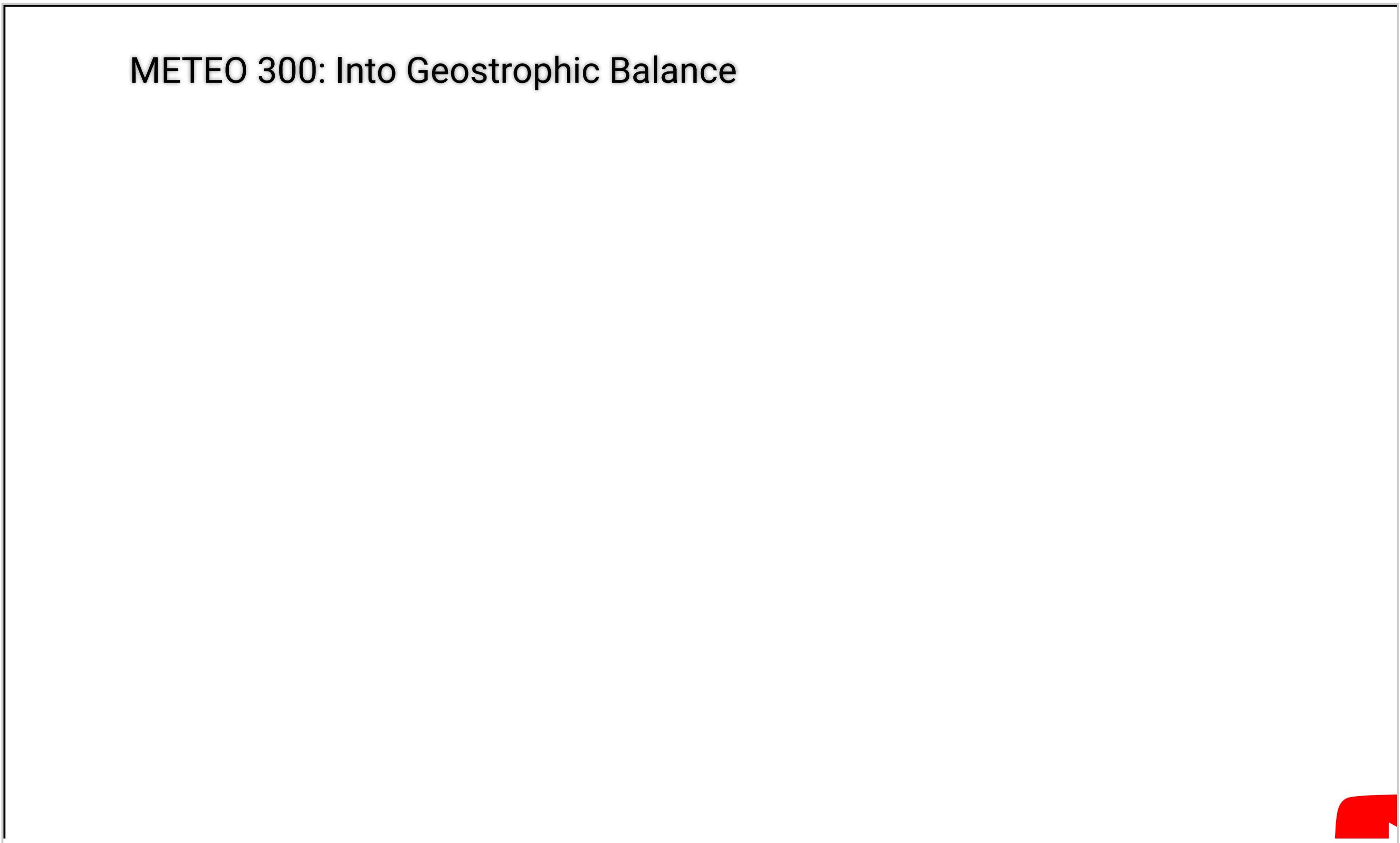
Let the Coriolis force per unit mass be designated as $Co = -fV$ and the pressure gradient force per unit mass as $P = -\frac{\partial \Phi}{\partial n}$. Then the force balances are shown in the figure below.



Geostrophic force balance in (a) the Northern Hemisphere and (b) the Southern Hemisphere shown in natural coordinates. Note that the *n* direction is always to the left of the velocity when looking downwind. In the figure, V_g is used to represent the geostrophic velocity.

Credit: W. Brune (after R. Najjar and An Introduction to Dynamic Meteorology, Fifth Edition, J. R. Holton and G. J. Hakim, 2013)

Note that the Coriolis force is always to the right of the velocity vector in the Northern Hemisphere. It is always to the left of the velocity vector in the Southern Hemisphere. When the pressure gradient force and Coriolis force are in balance, the PGF is to the left of the velocity vector and the Coriolis force is to the right in the Northern Hemisphere. Watch the video below (1:10) for further explanation:



Into Geostrophic Balance

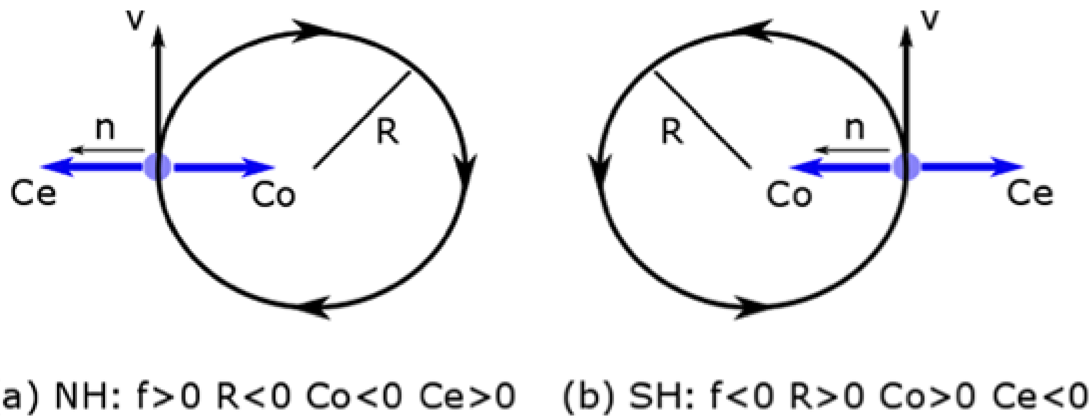
Click here for transcript of the into geostrophic balance video.

Inertial Balance

In this case, the pressure gradient force is minimal and the centrifugal and Coriolis forces are in balance.

$$-\frac{V^2}{R} - fV = 0 \qquad \text{inertial balance} \tag{10.37}$$

Let the centrifugal force be designated by $Ce = -\frac{V^2}{R}$.



Inertial balance in (a) the Northern Hemisphere and (b) the Southern Hemisphere shown in natural coordinates. Note that the *n* direction is always to the left of the velocity when looking downwind.

Credit: W. Brune (after R. Najjar)

We can manipulate Equation [10.37] to find the radius of the circle:

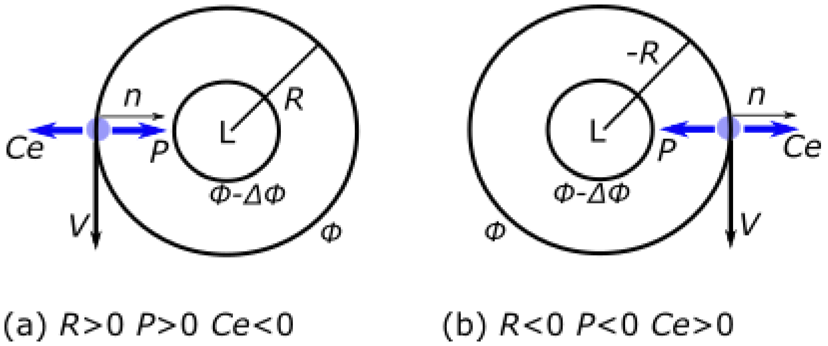
$$R = -\frac{V}{f}$$

For $f = 10^{-4}$ s^{-1} and $V = 10$ m s^{-1}, $R = -100$ km. Inertial balance is not a major balance in the atmosphere because there is almost always a significant pressure gradient, but it can be important in oceans.

Cyclostrophic Balance

The balance in this case is between the pressure gradient force and the centrifugal force.

$$-\frac{V^2}{R} - \frac{\partial \Phi}{\partial n} = 0 \qquad [10.38]$$



Cyclostrophic balance for (a) cyclonic flow and (b) anticyclonic flow in the Northern Hemisphere shown in natural coordinates. Note that the *n* direction is always to the left of the velocity when looking downwind. In the Southern Hemisphere, (b) is cyclonic and (a) is anticyclonic.

Credit: W. Brune (after R. Najjar and An Introduction to Dynamic Meteorology, Fifth Edition, J. R. Holton and G. J. Hakim, 2013)

In this case, the scale of the motion is so small that Coriolis acceleration is not important. The Rossby number, R_o = centrifugal acceleration / Coriolis >> 1.

Examples of motion in cyclostrophic balance are tornadoes, dust devils, water spouts, and other small atmospheric circulations, such as the vortex you sometimes see when leaves get swept off the ground. These can be either cyclonic or anticyclonic and, in fact, a few percent of tornadoes in the Northern Hemisphere are anticyclonic. Another common example of cyclostrophic balance is the vortex seen when a bathtub or sink is draining.

NOAA Ship NANCY FOSTER dwarfed by waterspout. Gulf of Mexico. Summer, 2007.

Credit: NOAA Photo Library via flickr

Gradient Balance

In gradient balance, the pressure gradient force, Coriolis force, and horizontal centrifugal force are all important. This balance occurs as wind in a pressure gradient field goes around a curve. There are many examples of this type of flow on any weather map—any synoptic-scale pressure gradient for which the isobars curve is an example of gradient flow.

$$-\frac{V^2}{R} - fV - \frac{\partial \Phi}{\partial n} = 0 \qquad [10.39]$$

To solve this equation for velocity, we can use the quadratic equation:

$$ax^2 + bx + c = 0 \quad \rightarrow \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{V^2}{R} + fV + \frac{\partial \Phi}{\partial n} = 0 \quad \rightarrow \quad V = \frac{-f \pm \sqrt{f^2 - 4\left(\frac{1}{R}\right)\frac{\partial \Phi}{\partial n}}}{\frac{2}{R}}$$

$$V = -\frac{fR}{2} \pm \sqrt{\frac{f^2R^2}{4} - R\frac{\partial \Phi}{\partial n}} \qquad [10.40]$$

$\frac{f^2R^2}{4}$ is always positive, so, for a given hemisphere (say, the Northern Hemisphere) there are eight possibilities because R can be either positive or negative, $\frac{\partial \Phi}{\partial n}$ can be positive or negative, and we have the ± sign in between the two terms on the right-hand side of Equation [10.40].

Gradient balance velocity solutions

Northern Hemisphere	$R > 0$	$R < 0$
$\frac{\partial \Phi}{\partial n} > 0$	no roots are physical	only positive root is physical
$\frac{\partial \Phi}{\partial n} < 0$	only positive root is physical	both roots are physical

The table above gives the results for the Northern Hemisphere ($f > 0$). We are looking for whether positive or negative values of R and $\frac{\partial \Phi}{\partial n}$ give non-negative and real values for V because only non-negative and real values for V are physically possible. The reason why real negative values of V are not possible is because the gradient wind balance has been written down in natural coordinates.

For $R > 0$ and $\frac{\partial \Phi}{\partial n} > 0$, V is always negative, so there are no physical solutions.

For $R > 0$ and $\frac{\partial \Phi}{\partial n} < 0$, only the plus sign gives a positive V and thus a physical solution.

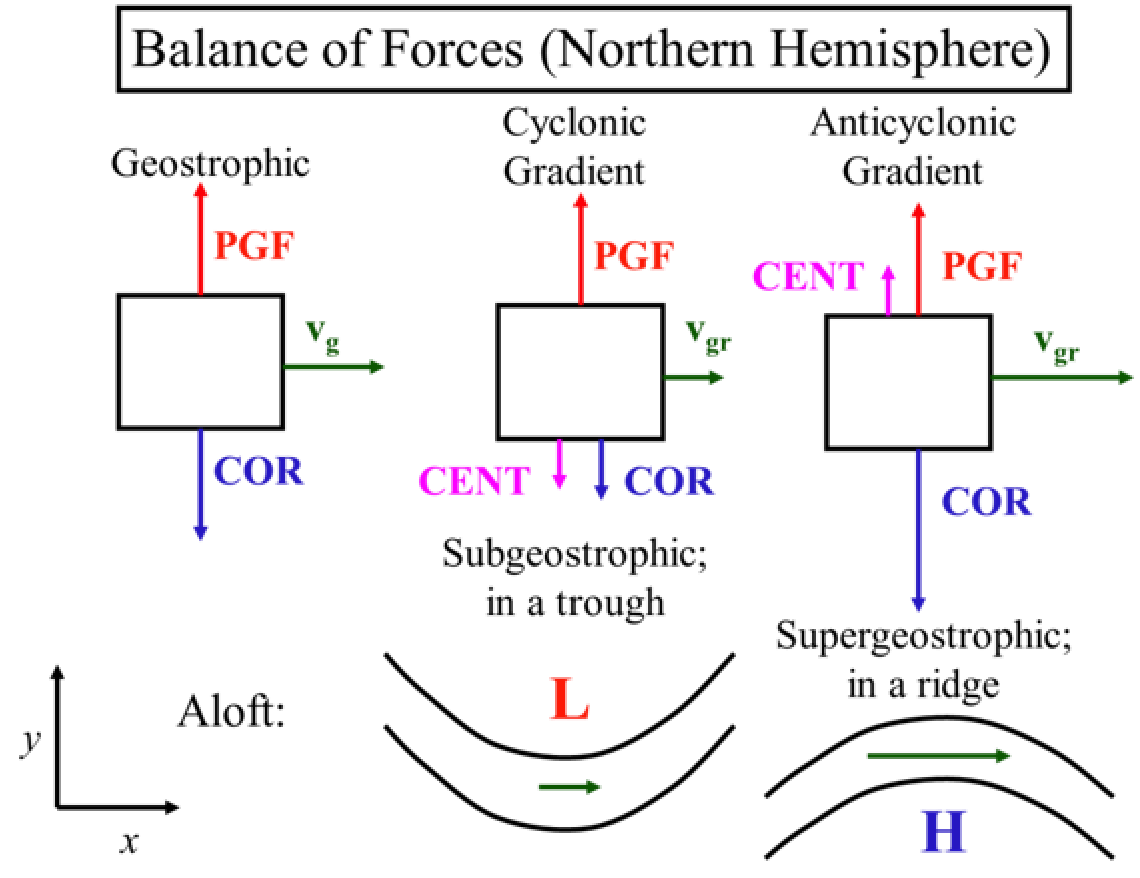
For $R < 0$ and $\frac{\partial \Phi}{\partial n} > 0$, only the plus sign gives a positive V and thus a physical solution.

For $R < 0$ and $\frac{\partial \Phi}{\partial n} < 0$, both roots give positive V and thus physical solutions.

So there are four physical solutions. However, there is one more constraint. This additional constraint is that the absolute angular momentum about the axis of rotation at the latitude of the air parcel should be positive in the Northern Hemisphere (and negative in the Southern Hemisphere). Without proof, we state that only two of the four physically possible cases meet this criterion of positive absolute angular momentum in the Northern Hemisphere. They are:

1. Regular low: $R > 0$ and $\frac{\partial \Phi}{\partial n} < 0$ and $V = -\frac{fR}{2} + \sqrt{\frac{f^2R^2}{4} - R\frac{\partial \Phi}{\partial n}}$
2. Regular high: $R < 0$ and $\frac{\partial \Phi}{\partial n} < 0$ and $V = -\frac{fR}{2} - \sqrt{\frac{f^2R^2}{4} - R\frac{\partial \Phi}{\partial n}}$

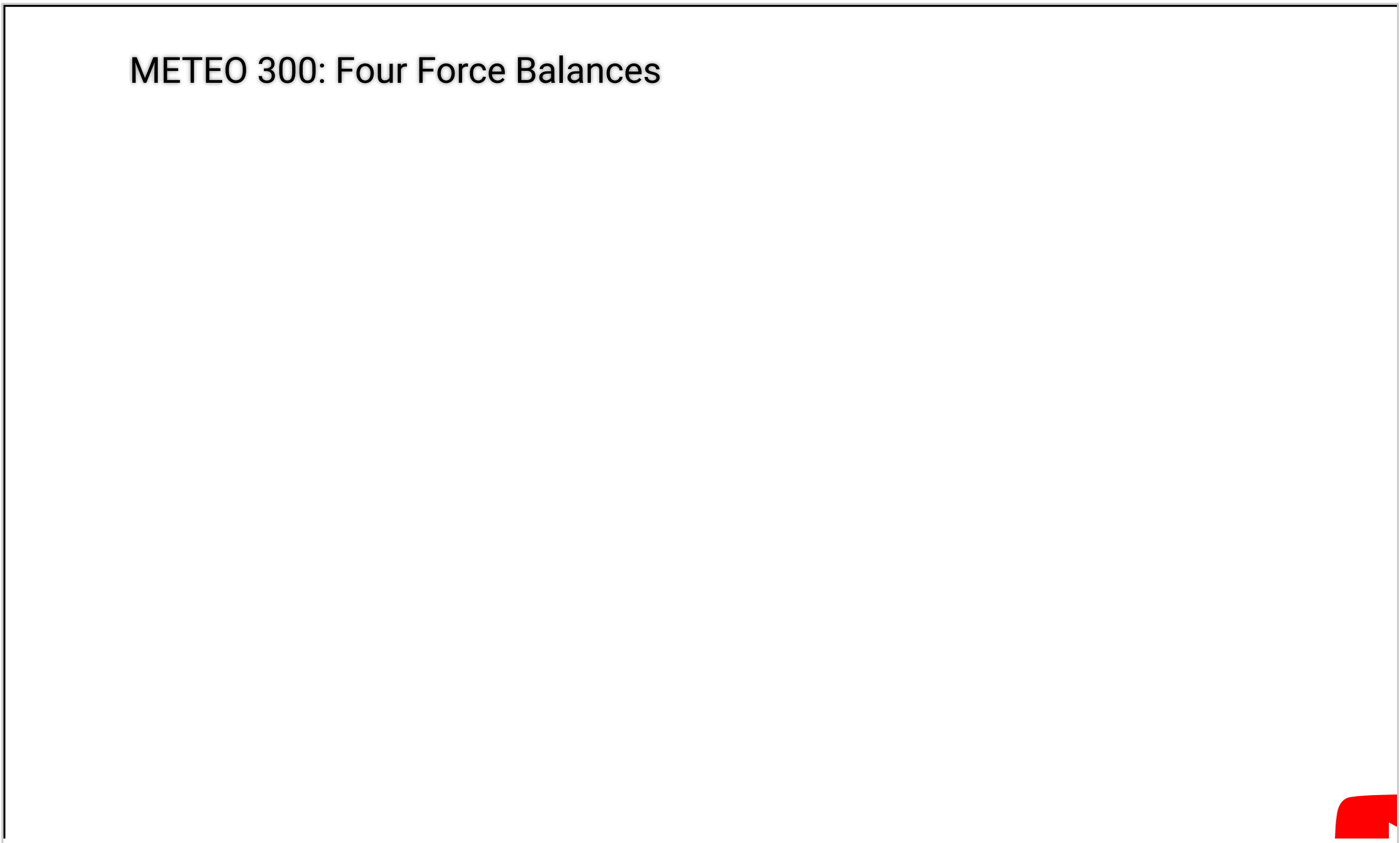
These two cases are depicted in the second and third panels of the figure below.



Gradient balance in Northern Hemisphere. left: Geostrophic balance; center: regular low balance; right: regular high balance. Note that the PGF is independent of velocity but both the Coriolis force and the centrifugal force are dependent on velocity. In the figure, v_g is used to represent the geostrophic velocity (only the PGF and Coriolis force are important) and v_{gr} is used to represent the gradient wind velocity (the PGF, Coriolis force, and centrifugal force are all important).

Credit: H.N. Shirer

The video below (3:22) explains these four force balances in more detail:



Four Force Balances

Click here for transcript of the Four Force Balances video.

‹ 10.7 Natural coordinates are better horizontal coordinates. up 10.9 See how the gradient wind has a role in weather. ›

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