ECE 485 Homework 4 - Jordan Carlson V00714886

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1(3.7). Given an unknown μ and Σ,

a) $\hat{\mu} = \Theta_1 = \frac{1}{n} \sum_{k=1}^{n} x_k = \frac{n}{n} \rightarrow \boxed{\hat{\mu} = 1}$

b) $p(x|\omega_1) \sim N(0,1)$, $p(x|\omega_2) \sim N(\hat{\mu}, 1)$

as $\hat{\mu} = 1$ from a, $\frac{1-0}{2} = \frac{1}{2} \rightarrow \boxed{x = 0.5}$

c) $p(x|\omega_1) = p(x|\omega_2)$

$\frac{1}{2\pi\sqrt{1}} e^{\left(-x^2/2\right)} = \frac{1}{2\pi\sqrt{10^6}} e^{\left(-\frac{(x-1)^2}{2(10^6)}\right)}$ $\boxed{x = \pm 3.72}$

d) $\frac{1}{2\pi} e^{\left(\frac{-3.72^2}{2}\right)} = \frac{1}{2\pi} e^{\left(-\frac{(\pm 3.72 - \mu)^2}{2}\right)}$

$\rightarrow 3.72^2 = -(\pm 3.72 - \mu)^2$

$\rightarrow -13.84 = \pm 13.84 - 7.44\mu + \mu^2$

$- \rightarrow 0 = \mu(\mu - 7.44)$

$+ \rightarrow 0 = \mu^2 - 7.44\mu + 27.68 \rightarrow$ irreducable

$\mu_1 = 0$, $\mu_2 = 7.44$, so $\boxed{x = 3.72}$

e) The value approximated in part c is a much more valuable parameter, demonstrating the effectiveness of the Bayes decision boundary calculation above poorly modelled distributions from a priori conjectures

2. (3.11.) Minimum D_{KL}, so $\frac{d}{d\mu} D_{KL}(p_1(x), p_2(x)) = 0$

$$= \frac{d}{d\mu} \int p_1(x) \ln \frac{p_1(x)}{p_2(x)} dx = 0 = -\int p(x) \Sigma^{-1} (x-\mu) dx$$

$$0 = \int p(x)(x-\mu) dx \rightarrow \mathcal{E}_2(x-\mu) = 0$$

so $\boxed{\mathcal{E}_2(x) = \mu}$

$$\frac{d}{d\Sigma} D_{KL}(p_1(x), p_2(x)) = 0 = \int p_1(x) \left(\Sigma^{-1} - (x-\mu)(x-\mu)^t\right) dx$$

$$p(x)((x-\mu)(x-\mu)^t) = \boxed{\Sigma = \mathcal{E}_2((x-\mu)(x-\mu^t))}$$

3. (3.17.)

a) $P(D|\theta) = \sum_{k=1}^{n} (s_i, \ldots s_d)$ for $\prod_k^d p(k|\theta)$

$$= \prod_{k=1}^{d} \theta_i^{\sum_{k=1}^{n}(s_i \ldots s_d)} (1-\theta_i)^{\sum_{k=1}^{n}(1-(s_i \ldots s_d))}$$

$$= \boxed{\prod_{i=1}^{d} \theta_i^{s_i} (1-\theta_i)^{n-s_i}}$$

b) $p(D) = \frac{p(D|\theta)(p(\theta))}{p(\theta|D)} = \int p(D|\theta)(p\theta)d\theta$

$$p(D) = \int_0^1 \prod_{i=1}^{d} \theta_i^{s_i}(1-\theta_i)^{n-s_i} \, d\theta_i$$

from identity $\rightarrow p(D) = \prod_{i=1}^{d} \frac{s_i!(n-s_i)!}{(n+1)!}$

$$p(\theta|D) = \frac{p(D|\theta)p(\theta)}{p(D)} = \frac{\prod_{i=1}^{d} \theta_i^{s_i}(1-\theta_i)^{n-s_i}}{\prod_{i=1}^{d} \frac{s_i!(n-s_i)!}{(n+1)!}}$$

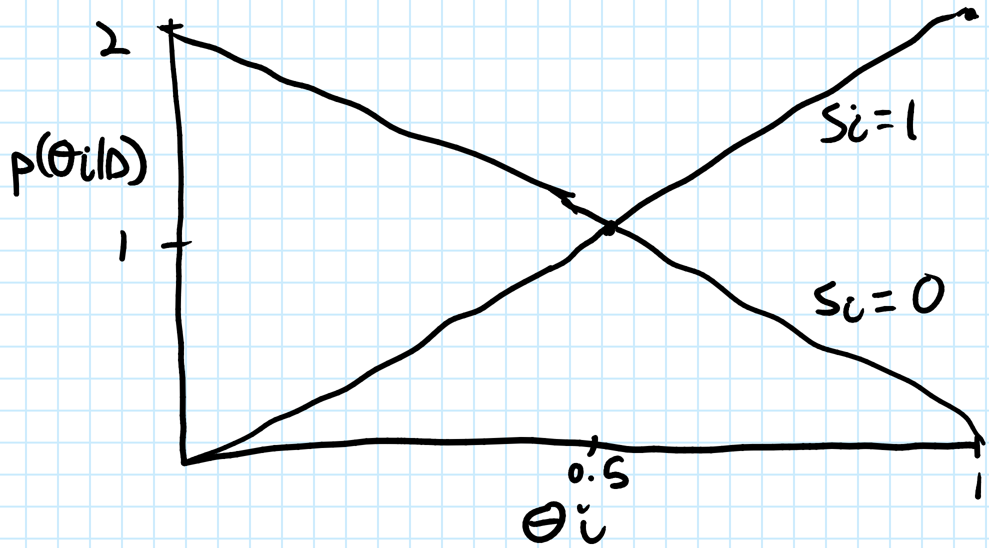
$$\rightarrow \boxed{P(\theta|D) = \prod_{i=1}^{d} \frac{(n+1)!}{s_i!(n-s_i)!} \theta_i^{s_i}(1-\theta_i)^{n-s_i}}$$

c)

$$P(\theta|D) = \frac{2}{s_i!(1-s_i)!} \theta_i^{s_i}(1-\theta_i)^{1-s_i}$$

when $s_i = 0$, $p(\theta_i|D) = 2 - 2\theta_i$

when $s_i = 1$, $p(\theta_i|D) = 2\theta_i$



d)
$$\int p(x|\theta)\,p(\theta|D)\,d\theta$$

$$= \int \left(\prod_{i=1}^{d} \theta_i^{x_i}(1-\theta_i)^{1-x_i}\right)\left(\prod_{i=1}^{d} \frac{(n+1)!}{s_i!(n-s_i)!}\theta_i^{s_i}(1-\theta)^{n-s_i}\right)d\theta$$

$$= \prod_{i=1}^{d} \int_0^1 \left(\theta_i^{x_i}(1-\theta_i)^{1-x_i}\right)\left(\frac{(n+1)!}{s_i!(n-s_i)!}\theta_i^{s_i}(1-\theta)^{n-s_i}\right)d\theta$$

$$= \prod_{i=1}^{d} \int_0^1 \frac{(\theta_i^{x_i}(1-\theta_i)^{1-x_i})(n+1)!\,\theta_i^{s_i}(1-\theta)^{n-s_i}}{s_i!(n-s_i)!}\,d\theta$$

$$\boxed{p(x|D) = \prod_{i=1}^{d}\left(\left(\frac{s_i+1}{n+2}\right)^{x_i}\left(1-\frac{s_i+1}{n+2}\right)^{1-x_i}\right)}$$

e) Using $P(x|D)$, $P(x|\theta)$, and $P(\theta|D)$, the estimate for θ will include both conditional probability functions and thus will be inclusive of both conditional implications, giving valuable information if both conditions are present in the system.

4. (3.22.)

$$p(\theta|s,D)\,p(D,s) = p(\theta|s)(p(s))(p(D|s,\theta))$$

$p(\theta|s,D) = p(\theta|s)$, so

$$p(\theta|s)\,p(D|s,\theta) = p(\theta|s,D)\,p(D|s)$$

$$p(D|s,\theta) = p(D|s)$$

$$\boxed{p(D|s) \not\simeq P(\theta)}$$

5. (3.29.)

a) $a^{n+1} = a^n a^1 \propto a^n a x = O(a^n)$ $\boxed{\underline{\text{Yes}}}$

b) $a^{bn} = (a^b)a^n >> x a^n = O(a^n)$ $\boxed{\text{No}}$

c) $a^{n+b} = a^b a^n \propto x a^n = O(a^n)$ $\boxed{\text{Yes}}$

d) $x f(n) \propto O(f(n))$

for any $x \geq 0$, so $\boxed{f(n) = O(f(n))}$

6.(3.43)

Eq 85. $= p(\omega_k | x) \propto \prod_{i=1}^{d} p(x_i | \omega_k)$