

# Maximum Restorable Load for Substation during Power System Restoration

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**Abstract**—In power system reconfiguration after successful black start, the maximum restorable load that substation can pick up at one time should be given. Presently, researches about the problem usually concentrate on frequency constraint and steady-state voltage constraint. In this paper, a method that can give the maximum restorable load is proposed which considers transient voltage-dip acceptability (TVDA) as a constraint. Firstly, a two-element table describing the requirements on TVDA for each bus is determined. Secondly, the maximum load restoration model is established which can give the maximum load amount for substation. Some system components, such as generator, inductive motor and network, are simplified with certain precision in order to improve the calculation speed. For improving the convergence speed, the modified dichotomy theorem which includes the improved margin for TVDA is used in the iteration. The proposed method is realized by using Borland C++ Builder and practical Shandong power system is used to verify its validity and effectiveness.

**Index Terms**—Dichotomy theorem; Load restoration; Power system restoration; Transient voltage-dip acceptability; Two-element table

## I. INTRODUCTION

POWER system restoration after a major disturbance may span over three stages: black start, system reconfiguration and load restoration [1]-[2]. The last two stages are alternative. System skeleton should be rebuilt step by step after successful black start and many constraints should be considered [3]. In this process, certain important load must be restored in order to stabilize system frequency and bus voltage. Because of the weak grid with small capacity in these early stages of restoration, load pickup in large increments may causes excessive under-frequency deviation and severe voltage-dip, which induces system outage again and set back the restoration process. With small increments, the restoration duration will be prolonged. Then the maximum load amount that substation can be pick up at one time should be determined in the practical system restoration.

Presently, researches about the problem usually concentrate on frequency constraints only. Or the problem is changed to an optimal problem and the steady state voltage is included as a constraint [4]-[6]. But during load restoration, especially when load that needs to be picked up for certain substation includes

a lot of motors, large amount of reactive power is needed and severe voltage dip may happen. Further, due to the rapid dynamic response of inductive motor, voltage usually decays faster than frequency does. So one situation may happen during load restoration, i.e., transient voltage-dip acceptability (TVDA) limit is violated while system frequency constraint is not. Thus, in this situation, the TVDA should be considered as a constraint.

In this paper, a method that can give the maximum restorable load for substation at one time is proposed which also considers TVDA as a constraint. Two-element table describing the requirements on TVDA for each bus is introduced. Also, the maximum load restoration model is established. For improving the convergence speed, the modified dichotomy theorem which includes the improved margin for TVDA is used in the iteration.

## II. TWO-ELEMENT TABLE FOR TVDA

TVDA is one aspect of transient voltage security. It against the condition that some users ask for the guarantee that the duration for voltage dip at load buses to exceed a certain level is not longer than a predefined time period. Two-element table describing the TVDA consists of two aspects: a given voltage dip and the maximum duration acceptable [7]-[8]. A practical usage of the two-element table is taken in Shandong power system. The two-element table for TVDA of Shandong power system is given according to national criteria. Then the setting for under-voltage load shedding is given as (0.75p.u., 0.2s) [9].

Besides, for different systems and areas the requirements of transient voltage dip are quite different. Table I shows the survey of transient voltage dip/sag criteria for some systems or areas [10].

TABLE I  
SURVEY OF TRANSIENT VOLTAGE DIP/SAG CRITERIA FOR DIFFERENT SYSTEMS OR AREAS

Transient voltage dip/sag criteria	
WECC	Voltage-dip of 0.8p.u. for 20 cycles(N-1 contingencies)
China	Voltage-dip of 0.75p.u. for 1s
ITIC/CBEMA(related to equipment performance)	Voltage-dip of 0.7p.u. for 1.2 to 30 cycles

Based on the mentioned above, two-element table describing TVDA for load restoration is conservatively given as (0.8p.u., 0.5s) in this paper.

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### III. MAXIMUM RESTORABLE LOAD FOR SUBSTATION

During power system reconfiguration, the maximum restorable load for substation at one time needs to be given based on the practical operation-state of system. In this process, the constraint for TVDA also needs to be considered as well as that of system frequency and steady-state voltage.

#### A. Voltage Constraint

During load restoration, voltage constraint consists of two aspects: steady-state voltage limit and transient voltage security. In this paper, transient voltage security mainly concentrates on requirements for TVDA.

The constraint about steady-state voltage can be easily realized by power flow for given load amount. The constraint reflecting transient voltage variation should be realized by solving the differential-algebraic equations. In order to improve the calculation speed with certain precision, proper simplifications are taken for system components, such as generator, load and network.

Dynamic load model which consists of constant impedance and inductive motor is employed in this paper. For inductive motor, the electro-mechanical transient model is better than mechanical transient model. And the third order model can be represented as

$$\begin{cases} \dot{U} = \dot{E}' + (r_s + jX')\dot{I} \\ \frac{d\dot{E}'}{dt} = -js\dot{E}' - [\dot{E}' - j(X - X')\dot{I}]/T_{d0}' \\ 2H\frac{ds}{dt} = T_m - T_e \end{cases} \quad (1)$$

where  $\dot{U}$  is the phasor of stator terminal voltage;  $\dot{E}'$  is the phasor of voltage behind transient impedance;  $r_s$  is stator resistance;  $X'$  is transient reactance;  $X$  is stator leakage reactance;  $T_{d0}'$  is the transient open-circuit time constant;  $H$  is the combined inertia constant;  $T_m$  and  $T_e$  are separately load torque and electromagnetic torque.

With analyzing transient voltage variation, excitation system including AVR needs to be considered in detail. Thus the first order of simplification to the synchronous machine model with amortisseurs neglected is a proper choice.

For network model, its equations in terms of the node admittance matrix can be expressed as

$$YU = I \quad (2)$$

where  $Y$  is admittance matrix;  $U$  is the vector of node voltages;  $I$  is the vector of node currents flowing into the network.

During the numerical calculation, generator and load are linked with network equations by modifying the admittance and current of corresponding nodes. Therefore, equation (2) must be written in form as

$$\sum_{j=1}^N \begin{bmatrix} G_{ij} & -B_{ij} \\ B_{ij} & G_{ij} \end{bmatrix} \begin{bmatrix} U_{xj} \\ U_{yj} \end{bmatrix} = \begin{bmatrix} I_{xi} \\ I_{yi} \end{bmatrix} \quad (i=1,2,\dots,N) \quad (3)$$

where  $G_{ij}$  and  $B_{ij}$  are separately real and imaginary part of admittance corresponding node  $i$  and  $j$ .  $N$  is the total node number of system.

When the simplified model for generator mentioned above is used in dynamic simulations, the current flowing into the

network for the generator can be written as

$$\begin{bmatrix} I_{xi} \\ I_{yi} \end{bmatrix} = \begin{bmatrix} b_{xi} \\ g_{yi} \end{bmatrix} E' - \begin{bmatrix} G_{xi} & B_{xi} \\ B_{yi} & G_{yi} \end{bmatrix} \begin{bmatrix} U_{xi} \\ U_{yi} \end{bmatrix} \quad (4)$$

where  $b_{xi}$ ,  $g_{yi}$ ,  $G_{xi}$ ,  $B_{xi}$ ,  $G_{yi}$ ,  $B_{yi}$  are coefficients related to the referred generator itself constant parameters and power angle corresponding to the system.

Equation (3) at nodes linked with generators can be modified according to (4). Because of the rapid dynamic response of inductive motor, the deviation of generator power angle is ignored in this paper. Then the coefficient matrix of modified network equations is constant in iterative processes which is different from transient stability calculation. So the factorization of coefficient matrix can be solved only one time before iteration, and then calculation speed can be improved.

#### B. Frequency Constraint

During load restoration, magnitude of frequency deviation is determined by the ratio of the amount of the load/generation mismatch to the level of load and generation already restored, and by distribution of generation reserve among prime movers [2]. The frequency response rates of different prime movers are quite different. Based on the typical data given in [2] and [6], the approximate frequency response rates of some typical prime movers are given in Table II.

TABLE II  
FREQUENCY RESPONSE RATES OF TYPICAL PRIME MOVERS

Prime movers	$\Delta f/\Delta L(\text{Hz/p.u.})$
Steam-electric(SE)	-10
Combustion turbine(CT)	-4.8
Hydro-electric(HE)	-5

Then frequency deviation of given load can be estimated as

$$\Delta f = \frac{L_{fix}}{S_{SE}/r_{SE} + S_{CT}/r_{CT} + S_{HE}/r_{HE}} \quad (5)$$

where  $L_{fix}$  is the fix load amount that needs to be picked up;  $S$  and  $r$  are separately the capacity of generator already restored and its approximate frequency response rate; the subscripts stand for the types of prime movers.

Furthermore, frequency response is also inversely proportional to the initial load amount. The rates in Table II are conservatively given at minimum initial load.

#### C. Maximum Load Restoration Model

Considering all the constraints above, the model that can give the maximum restorable load for substation at one time is expressed as

$$\begin{aligned} \max \quad & L \\ \text{s.t.} \quad & \dot{x} = f(x, y, L) \\ & g(x, y) = 0 \\ & T_{(V_i \leq V_{set})} \leq T_{set} \quad (i=1,2,\dots,N) \\ & f_{min} \leq f(L) \leq f_{max} \\ & V_{min} \leq V_{i,st} \leq V_{max} \quad (i=1,2,\dots,N) \end{aligned} \quad (6)$$

where  $L$  is the load amount restored at one time for certain substation; the differential equations represent the dynamic characteristics of system and the algebraic equations mainly consist of network equations and stator voltage equations for

generators or inductive motors.  $T$  stands for the duration in which node voltage  $V_i$  is lower than the threshold  $V_{set}$ , and then constraint about TVDA can be expressed by  $T \leq T_{set}$ ;  $f$  is the system frequency, which is related to the load amount  $L$  according to (5);  $V_{i,st}$  is the steady-state voltage of node  $i$ , which can be given by power flow calculation.

#### IV. SOLVING FOR MATHEMATICAL MODEL

The mathematical model given in (6) belongs to one-dimension-optimization problem. In order to improve the convergence speed, the modified dichotomy method is used for calculating the mathematical model, which considers constraint margin based on the conventional dichotomy theorem.

Constraint margin for one-dimension-threshold problem can be easily acquired because of its good smoothness and linearity. Constraints for frequency and steady-state voltage are classified as this problem. But constraint for TVDA belongs to two-dimension-threshold problem. Its strong nonlinearity may take difficulties into iteration for the maximum restorable load. For improving this situation, a method based on curve-fitting technique is proposed in [7]. In this paper, the proposed TVDA margin in [7] is introduced to improve the dichotomy theorem for searching the maximum load. Thus the step size given by modified dichotomy theorem based on TVDA constraint can be written as

$$\Delta L = k \cdot \eta \cdot (L_{max} - L_{min}) / 2 \quad (7)$$

where  $\eta$  is the improved TVDA margin;  $k$  is the constant coefficient;  $L_{min}$  and  $L_{max}$  separately stand for lower and upper load limit for current iteration;  $\Delta L$  is step size for modifying load amount.

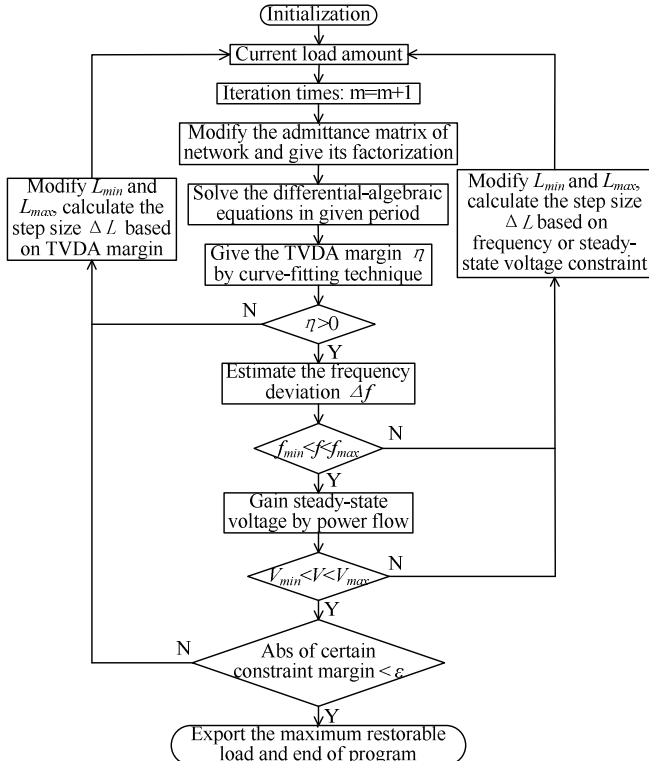


Fig. 1. Flow Chart for calculating maximum restorable load

The model expressed by (6) is realized by using Borland C++ Builder. Fig. 1 shows the flow chart of calculating program. And the main stages are listed as follows:

Stage-1. Calculate the equations based on the current load amount and plot the transient-voltage curve of certain substation in given period. Give the TVDA margin  $\eta$  by curve-fitting technique, and judge whether the TVDA constraint is satisfied or not.

Stage-2. If  $\eta$  is lesser than zero, calculate the step size by the improved dichotomy theorem based on (7) and go to Stage-1.

Stage-3. If  $\eta$  is greater than zero, analyze the frequency constraint according to (5) or the steady-state voltage constraint by power flow. With either constraint limit violated, the step should be calculated and go to Stage-1.

Stage-4. If all the constraints are satisfied, check whether the constraint margin exists whose absolute value is small enough. With this situation happened, the program ends and exports the maximum restorable load which satisfies all the constraints; otherwise, the step should be calculated according to TVDA constraint and go to Stage-1.

#### V. CASE STUDIES

Take Shandong power system as an example, and Fig. 2 shows the schematic diagram of sample system. After a diffuse blackout, Shihengyi#5 is successfully restarted with supporting of water pumped power generator Taichou#1, which can be able to self restarting with no external voltage provided. With the subsystem formed, the priority load needs to be restored as soon as possible.

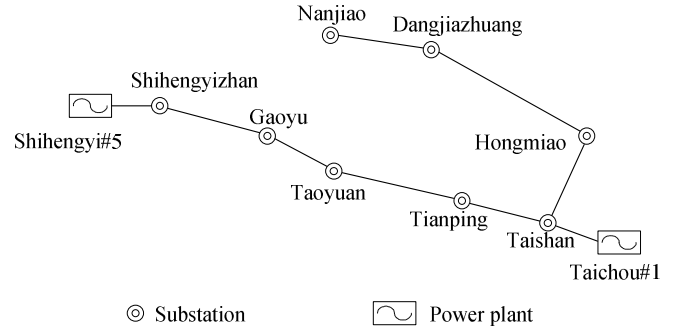


Fig. 2. Schematic Diagram of Sample System

Assume that the generator Taichou#1, with its capacity of 295MVA, operates at about minimum stable operation power. And the capacity of Shihengyi#5 is about 350MVA.

When the load level of system is 175MW, the maximum restorable load that Nanjiao substation can pick up at one time for different composite load models is listed as follows:

TABLE III RESULTS FOR DIFFERENT COMPOSITE LOAD MODELS			
Composite load model (inductive motor + constant impedance)	Maximum restorable load (MW)	Frequency dip (Hz)	Steady-state voltage (p.u.)
Model_1:85%+15%	20.73	-0.2198	0.9253
Model_2:75%+25%	22.99	-0.2438	0.9221
Model_3:50%+50%	31.02	-0.3290	0.9081

Curves in Fig. 3 show the voltage variation of Nanjiao

substation corresponding to the results given in Table III for different load models.

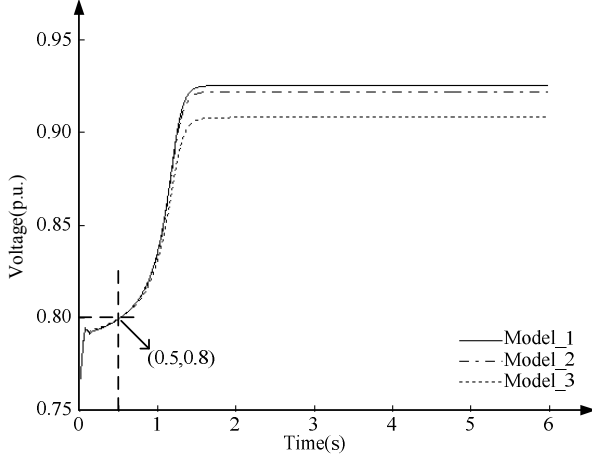


Fig. 3. Voltage Curves for Different Composite Load Models

The results both in Table III and Fig. 3 demonstrate that in these situations both frequency and steady-state voltage constraints are satisfied, and the maximum load amount for Nanjiao substation is determined by TVDA constraint. And the maximum restorable load amount is inversely proportional to the proportion of inductive motor in composite load model.

Table IV shows the comparison between the results of improved dichotomy theorem and that of conventional dichotomy theorem. It obviously illustrates that the improved method is better than the conventional dichotomy theorem in calculation speed and the maximum restorable load is similar in both methods.

TABLE IV  
COMPARISON BETWEEN TWO METHODS

Composite load model	The improved dichotomy theorem		The conventional dichotomy theorem	
	Maximum restorable load (MW)	Iteration times	Maximum restorable load (MW)	Iteration times
Model_1	20.73	5	20.73	10
Model_2	22.99	5	22.96	10
Model_3	31.02	6	31	8

When the composite load model is used as Model\_1, the maximum restorable load amount of Nanjiao substation for different load levels is listed as follows:

TABLE V  
RESULTS FOR DIFFERENT LOAD LEVELS

Load level (MW)	Maximum restorable load (MW)	Frequency dip (Hz)	Steady-state voltage (p.u.)
Load_1:175	20.73	-0.2198	0.9253
Load_2:200	20.13	-0.2135	0.9215
Load_3:225	19.59	-0.2077	0.9179

The results in Table V also illustrate that TVDA constraint is the main factor for determining the maximum restorable load during restoration in the three load levels. Also, the maximum restorable load amount is inversely proportional to load level of system.

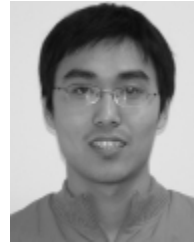
## VI. CONCLUSIONS

The results obtained in sample system show that the maximum restorable load for certain substation is mainly affected by the transient voltage constraint in some situations and is also related to many other factors, such as load model and load level of system. In this paper, the TVDA constraint is considered as well as frequency and steady-state voltage constraint for establishing the maximum load restoration model. The proposed model is also proved to be valid and effective on calculating the maximum load for substation picking up at one time and the simplification used in the model can improve the calculation speed. The modification of dichotomy theorem for solving mathematical model includes all constraint margins and can improve the iteration speed.

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## VIII. BIOGRAPHIES



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