Two-dimensional geometric transformations

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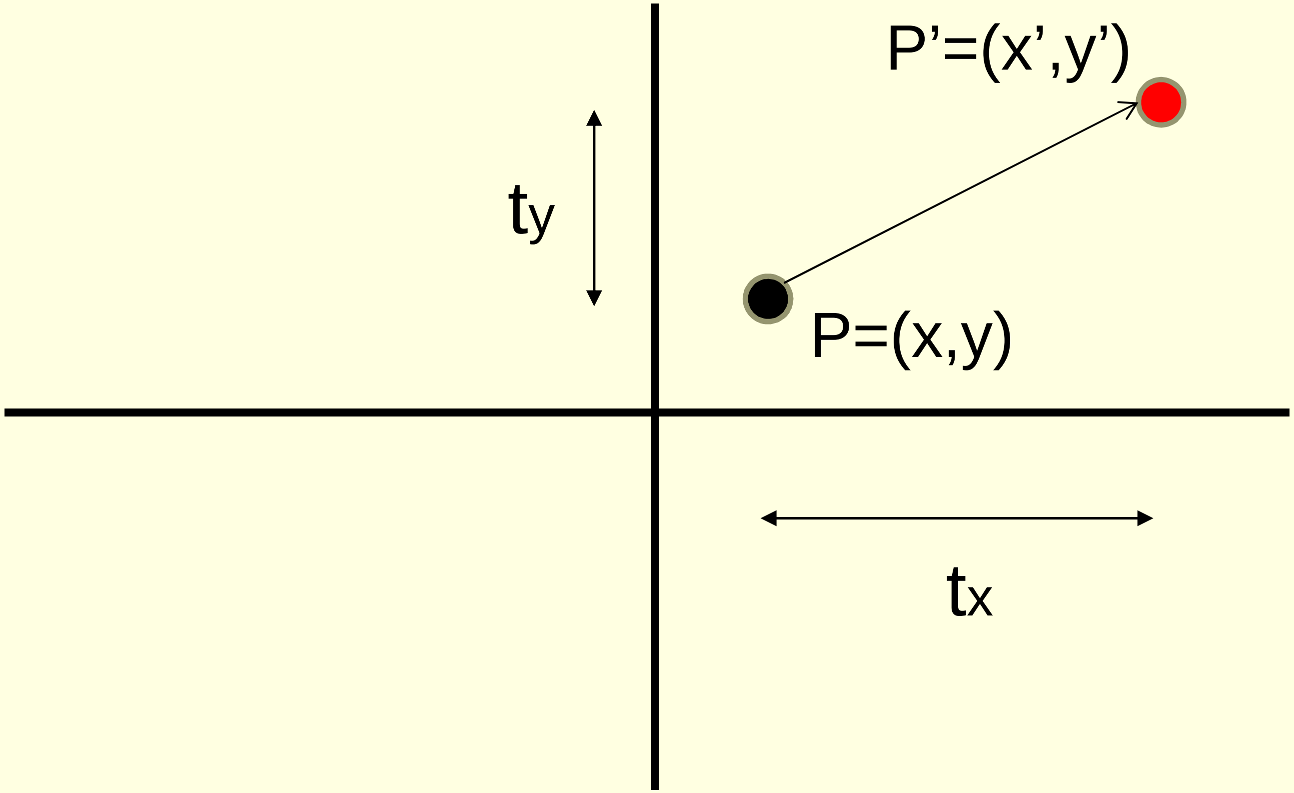
Geometric transformations

- We have some geometric object given by its coordinates.
- Our objective is to compute its coordinates after transforming it.
  - Translation
  - Rotation
  - (⋯)

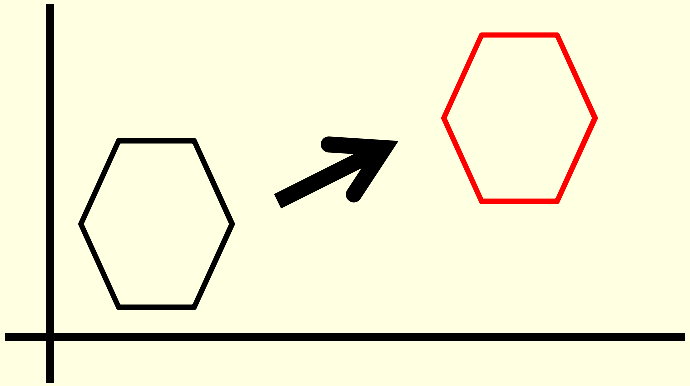
Translation

- Specified by translation distances (t_x, t_y).



Translation

- Given $P = \begin{pmatrix} x \\ y \end{pmatrix}$, we compute $P' = \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \end{pmatrix}$.
- Applying this operation to all coordinates, we translate the whole object.



Translation

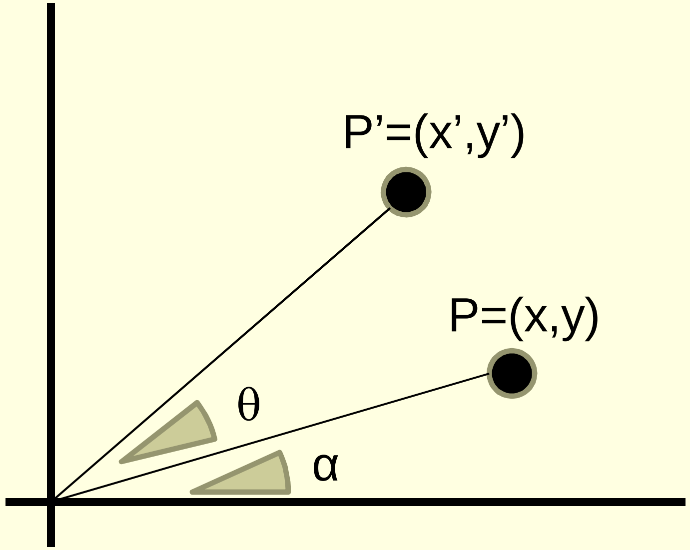
- We employ **homogeneous coordinates** to implement transformations as matrix operations.

$$P = \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

$$P' = \underbrace{\begin{pmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{pmatrix}}_{\text{Translation matrix}} \cdot \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

Rotation

- Specified by the rotation angle θ.



- P=(x,y)= (r·cos α, r·sen α)
- P’=(x’,y’)= (r·cos (α+θ), r·sen (α+θ))

Rotation

- **Exercise:** Give the rotation matrix in homogeneous coordinates, taking into account that:

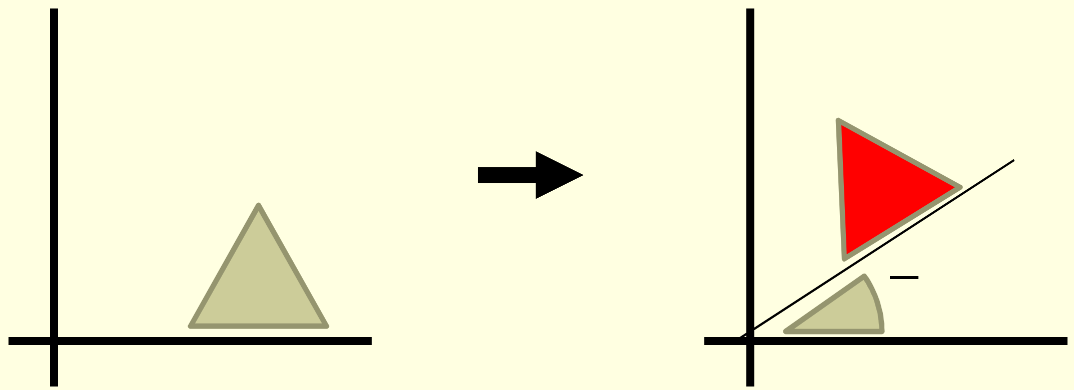
  - $\cos(\alpha+\theta) = \cos\alpha \cos\theta - \sin\alpha \sin\theta$
  - $\sin(\alpha+\theta) = \sin\alpha \cos\theta + \cos\alpha \sin\theta$

Rotation

- Solution:

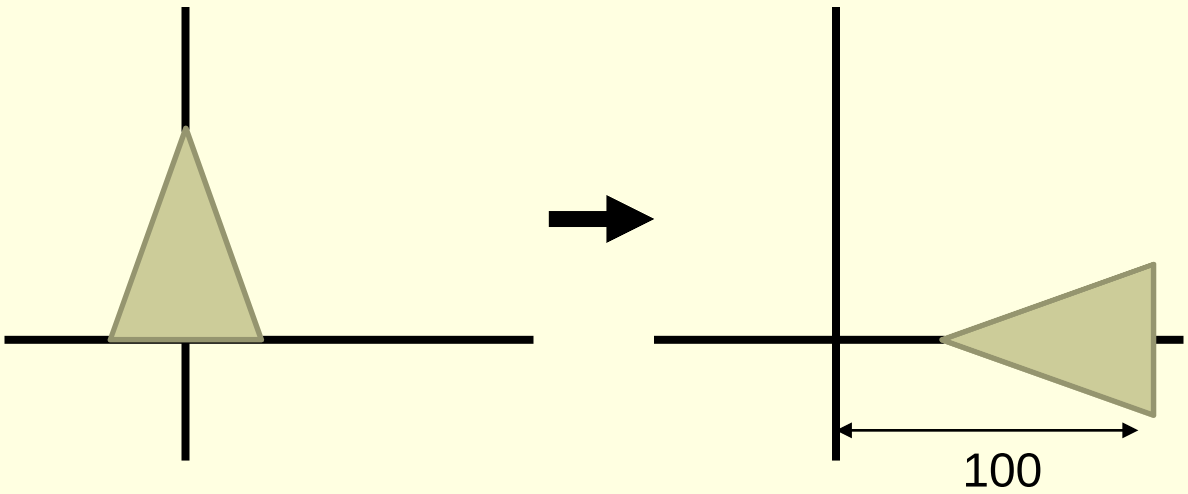
$$M = \begin{pmatrix} \cos\text{¤} & -\sin\text{¤} & 0 \\ \sin\text{¤} & \cos\text{¤} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- Then, P' = M·P



Composing transformations

- Compute the transformation matrix for:



Composing transformations

- We first rotate 90º and next translate 100 pixels to the right

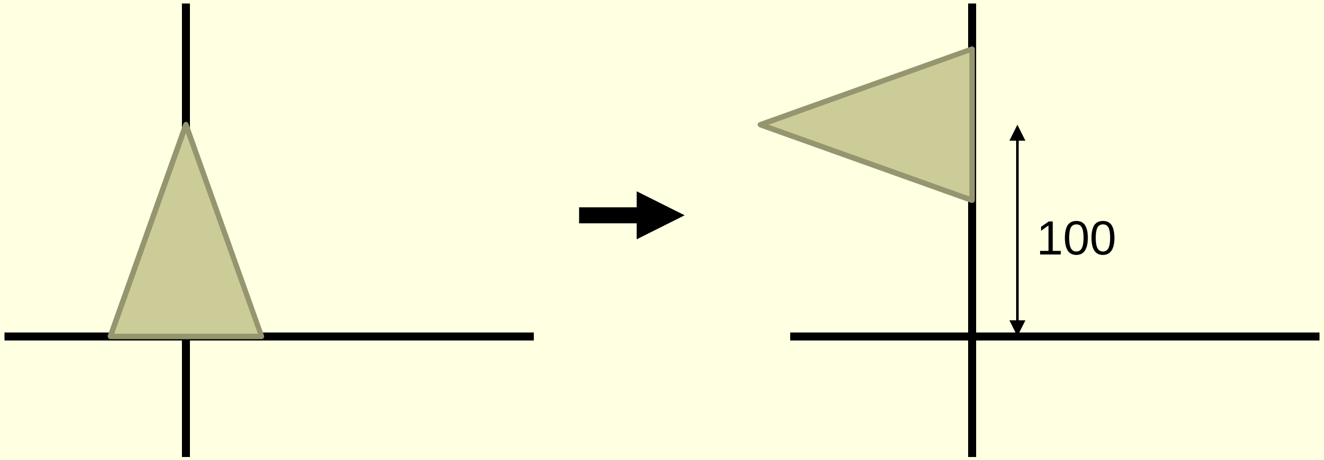
$$M_1 = \begin{pmatrix} \cos 90^o & -\sin 90^o & 0 \\ \sin 90^o & \cos 90^o & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad M_2 = \begin{pmatrix} 1 & 0 & 100 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- Then $P_1=M_1 \cdot P$, and next $P_2=M_2 \cdot P_1$
- So that $P_2=M \cdot P$, with $M=M_2 \cdot M_1$

$$M = \begin{pmatrix} \cos 90^o & -\sin 90^o & 100 \\ \sin 90^o & \cos 90^o & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Composing transformations

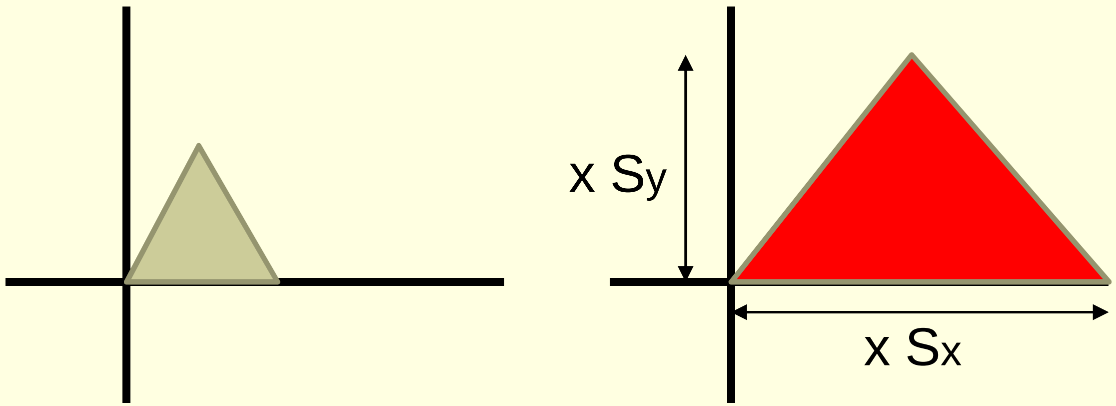
- If we first translate and next rotate, the resulting operation is different



$$M = \begin{pmatrix} \cos 90^{\circ} & -\sin 90^{\circ} & 100\cdot\cos 90^{\circ} \\ \sin 90^{\circ} & \cos 90^{\circ} & 100\cdot\sin 90^{\circ} \\ 0 & 0 & 1 \end{pmatrix}$$

Scaling

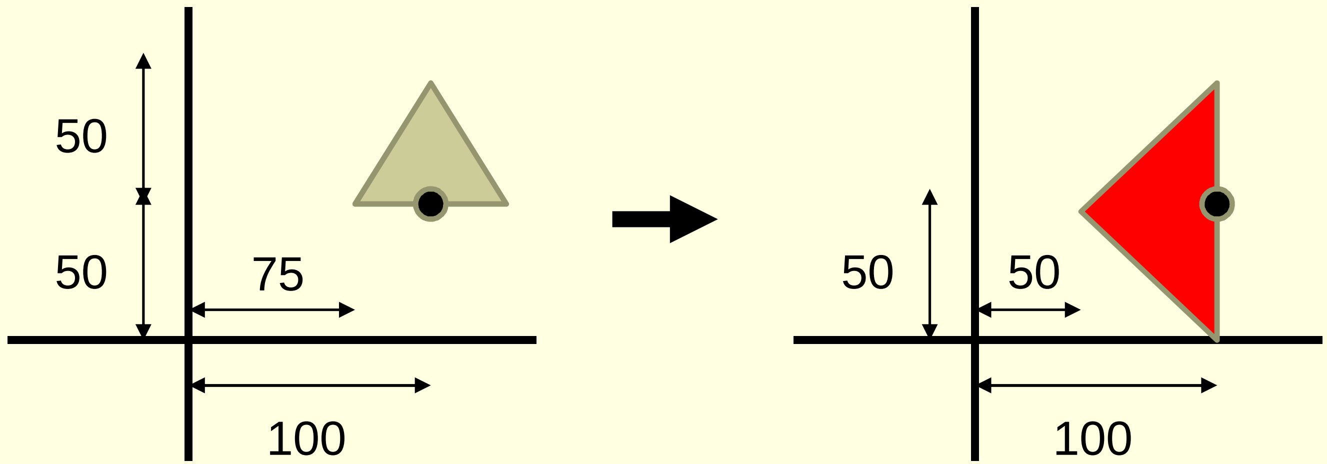
- x-coordinate is multiplied by S_x
- y-coordinate is multiplied by S_y



$$M = \begin{pmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Composing transformations

- Compute the transformation matrix for:

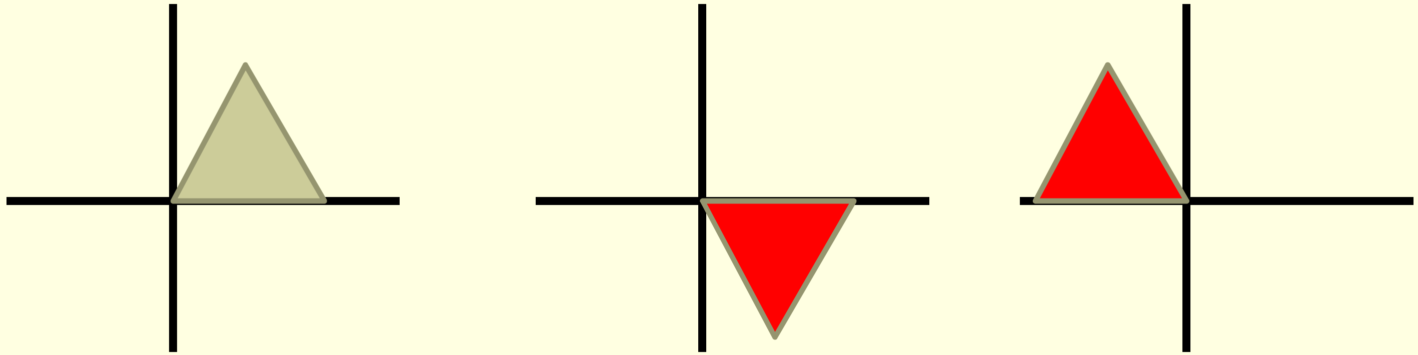


Composing transformations

- Solution:
  - First translate to the origin (M_1)
  - Next scale (M_2)
  - Next rotate (M_3)
  - Finally, apply the inverse of the translation of the first step (M_4)
- The result is $M = M_4 \cdot M_3 \cdot M_2 \cdot M_1$

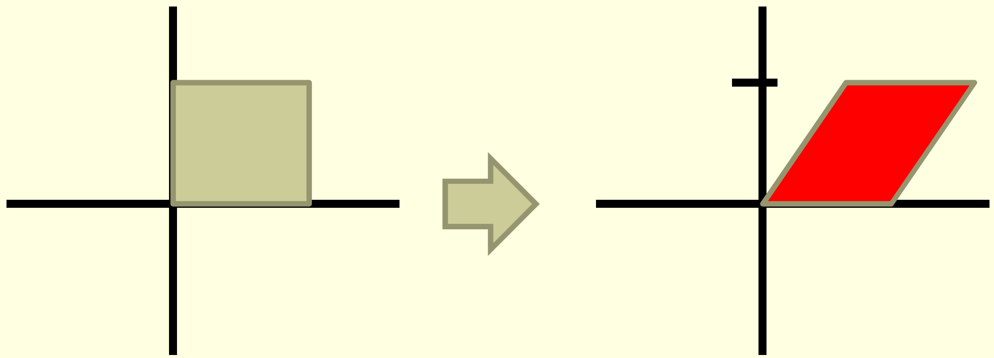
Reflection

- Produces a mirror image of our object.
  - Reflection about **x** or **y** axis.



$$M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad M = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Shear



- In this example:
  - x' = x+ $sh_x \cdot y$
  - y' = y

$$M = \begin{pmatrix} 1 & sh_x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Geometric transformations

- glMatrixMode(GL_MODELVIEW);
  - Mode for geometric transformations.

- glTranslatef(25.0, -10.0, 0.0);
  - **All** subsequent plotted figures will be translated 25 pixels over the x axis and -10 pixels over the y axis.
  - The z coordinate is 0.0 in two-dimensional applications

Geometric transformations

- glLoadIdentity();
  - Reset the matrix to identity

- glScalef(sx,sy,sz);

- glRotatef(90.0, 0.0, 0.0, 1.0);
  - Rotate 90 degrees about the z-axis ((0,0,1) vector)

Geometric transformations

- Geometric transformations are applied incrementally.
- OpenGL permits to save and restore matrices in a stack data structure
  - glPushMatrix();
  - glPopMatrix();