H02e Lab Report

Lab Group 6: Jordan Grey([redacted]) *(50%)*, [redacted] *(50%)*

November 5, 2024

Abstract

This study examines the kinematic processes of an object in free fall and a rolling wheel on an inclined plane, utilizing slow-motion video analysis. The analysis revealed that the viscous drag model provided the most accurate representation of free fall dynamics, with a drag constant $\alpha = (3.2209 \pm 0.0028)s^-1$, demonstrating a high correlation with measured data $r^2 = 0.9996$. In the rolling wheel experiment, the motion was influenced by frictional forces, fitting the friction model for rolling on the inclined plane without slipping. The measured friction parameter is $k = 8.518$, with standard deviation of $k_x = (8.518 \pm 0.0106)$ and $k_y = (8.518 \pm 0.0084)$ on the $\hat{x}$, $\hat{y}$ directions, and measured data $r_x^2 = 0.9993$ and $r_y^2 = 0.9961$. The results of this report pertain solely to the objects of similar design to the free fall and rolling wheel objects utilized. It predicted that objects of differing designs, differing models will be more appropriate in describing its motion.

1 Introduction

This experiment aims to analyze the kinematic processes captured in slow-motion videos. The report is divided in two main parts, corresponding to different processes. The first part presents the free fall of an object in air, under gravitational influence. By collecting experimental data and comparing with mathematical predictions, the aim is to explore the kinematics of free fall. An kinematic analysis is performed to measure the observed displacements at various time intervals and determine the frictional parameters through use of non-linear curve fitting. Additionally, comparisons will be made between the non-linear fitting and computed values, as well as fitting the numerical data to an appropriate model. The second part of the report is designed to study the kinematic process of a rolling wheel on an inclined plane. Moreover, by observing the wheel's kinematic behavior as it rolls down the incline, the report aims to capture the effects of gravitational force components along the plane and the impact frictional resistance has on the motion of the rolling wheel. The investigation considered the impact the object of inertia has on the frictional resistance of the rolling wheel.

2 Free Fall - Task I

2.1 Theoretical Basis

The main goal of this section is to describe the kinematical processes in theoretical form, in order to provide the ideal cases of this phenomenon and compare them with reality. Free fall represents an idealized situation, where an object experiences uniform acceleration due to gravity, without interference from air resistance, viscous drag or turbulent friction. The understanding of each interference is essential for further analysis.

2.1.1 Free fall without friction (without air resistance):

Free fall describes the motion of an object falling in a gravitational field. In an ideal case, we assume the body is falling in a straight line perpendicular to the surface, having one-dimensional movement. To describe this type of motion, we use following formulas [2]:

$$v = v_0 - gt \tag{1}$$

$$y = y_0 + v_0 t + \frac{1}{2} g t^2 \tag{2}$$

$$v^2 = v_0^2 - 2g(y - y_0) \tag{3}$$

Where:

$$y_0 = \text{Initial Position}(m)$$
$$v_0 = \text{Initial Velocity}(m/s)$$
$$g = \text{Gravitational Acceleration}(m/s^2)$$

2.1.2 Free fall with viscous drag:

If the object falls through the atmosphere, there is an additional drag force acting on the object. The drag force is proportional to object's velocity and acts in opposite direction, given by [6]:

$$\mathbf{F}_d = -m\alpha\mathbf{v} \tag{4}$$

The constant α depends on the proprieties of the medium and object's geometry, but in most cases, the force changes proportionally with velocity. With this additional external force, the Newton's Second law would imply that:

$$\mathbf{F}_g - \mathbf{F}_d = m\mathbf{g} - \alpha\mathbf{v} \tag{5}$$

Where:

$$\mathbf{F}_g = \text{Gravitational Force(N)}$$
$$m = \text{Mass of the Object}(kg)$$
$$\alpha = \text{Drag Constant}(1/s)$$

The velocity is as a function of time. For a short period of time, the velocity is small, the drag is therefore negligible and the motion is just free fall. For longer periods of time, the velocity approaches infinity, where the drag forces equals gravitational force. When assuming small velocity, the equation of motion is expressed in the ideal case of one-dimensional motion:

$$\frac{dv}{dt} = g - \alpha v \text{ as } v \to 0 \implies \frac{dv}{dt} \approx g \tag{6}$$

For large velocity, it's expected that the acceleration to disappear, giving a balanced gravitational and drag forces:

$$\frac{dv}{dt} = g - \alpha v \text{ as } \frac{dv}{dt} \to 0 \implies v_t = \sqrt{\frac{g}{\alpha}} \tag{7}$$

The velocity and the position would be represented as functions of time as:

$$v(t) = v_t(1 - e^{-\alpha t}) \tag{8}$$

$$z(t) = v_t - \frac{v_t^2}{g}(1 - e^{-\alpha t}) \tag{9}$$

2.1.3 Free fall with turbulent friction:

In free fall with turbulent friction (or quadratic drag), the drag force depends on the squared value of the velocity. This type of friction often applies to objects moving at higher speeds through a fluid, where the drag force is given by [6]:

$$F = -\beta m v^2 \implies m\frac{dv}{dt} = mg - \beta m v^2 \tag{10}$$

In the same way as for viscous drag, the terminal velocity would be:

$$v_t = \sqrt{\frac{g}{\beta}} \tag{11}$$

Solving the integration for Eq. 11, the velocity and position are determined by:

$$\textbf{for } v_0 = 0 \implies z(t) = -\frac{1}{\beta}[\ln(\cosh(\frac{gt}{v_t}))] \tag{12}$$

2.2 Experiment Setup

The experiment consisted in measuring the parameters of an object in a free fall movement through air, from a provided video. The video analysis software is Tracker application, as it provides the values for parameters such as position, velocity and acceleration.

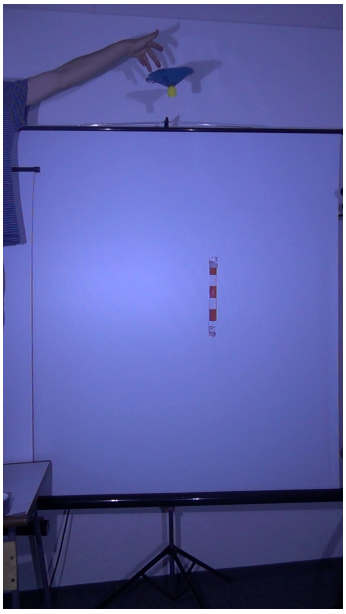

Figure 1: setup of free fall scenario

2.2.1 Initial Conditions

The only known information is linked to a measuring frame placed on the movement axis of the object, with a length of l = *30 cm* Fig. 1 and the video's frame rate of *500 frames/second*. The mass of the object is unknown and it's not necessary in the following computations. The chosen system of coordinates was cartesian *[x,y]*, with the origin at the point where the object leaves the image.

2.2.2 Data Gathering

The data were gathered manually, frame by frame. The object was considered a mass point and all effects caused by the object's geometry are contained in the constants specific to the system. The center of mass is assumed to be at the lowest point of the object, to be more precised about the evolution of displacement in time than the actual location of the center of mass. With each frame, the functions that describe object's motion become more accurate. With a total of 552 points, the graphs describing motion would be:

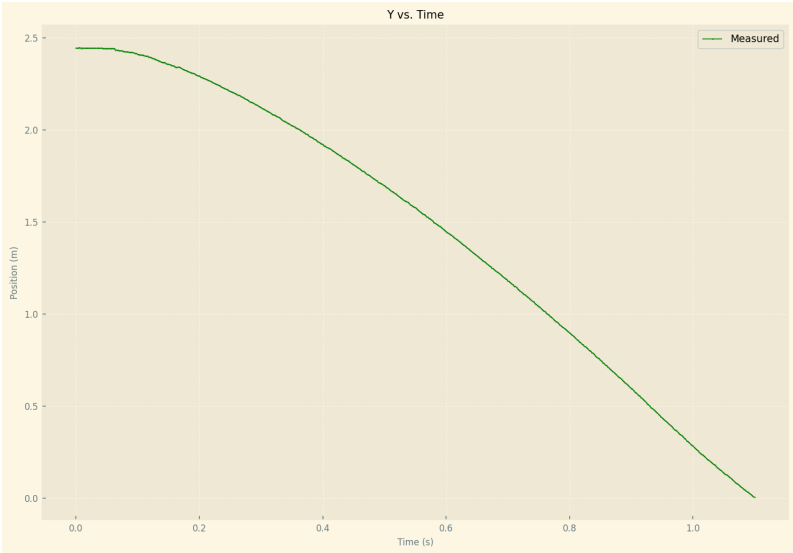

Figure 2: Displacement on Y-axis

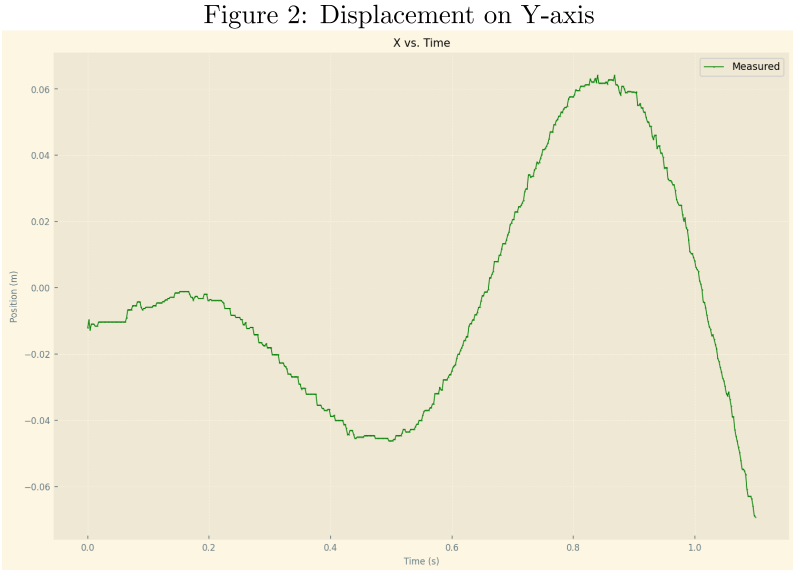

Figure 3: Displacement on X-axis

2.3 Results and Analysis

2.3.1 Free fall without friction

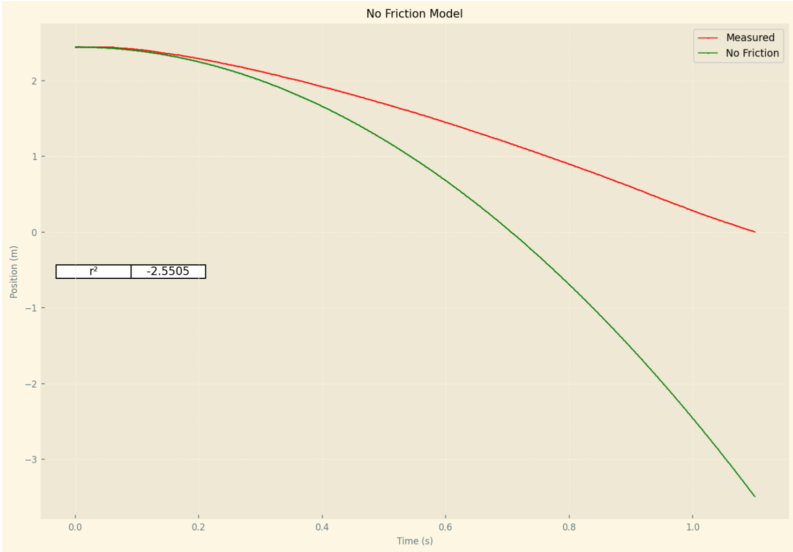

Figure 4: No friction free fall model

Fig. 4 compares the measured results against the model of free fall movement with no friction Eq. 2, as it's apparent this ideal model does not accurately fit the object's change in displacement on y-path throughout time. This is predictable as the object has a large surface area that generates a significant drag force as it falls through air. The no friction model does not account for this and as is shown in Fig. 4, predicting the object's falling significantly further than it's measured. The $r^2 = -2.5505$ is initially peculiar. However, when considering that there is no fitting being done here, the r^2 indicates the model significantly deviates from the measured results in a manner where the total sum of squares (SS_{tot}) is significantly larger than the residual sum of squares (SS_{res}).

2.3.2 Viscous drag model

Figure 5: Viscous drag free fall model

Fig. 5 compares the measured results with the viscous drag model Eq. 10. In comparison to the previous model is significantly more accurate, with an $r^2 = 0.9996$ against the $r^2 = -2.5505$. This result have a corresponding drag constant of $\alpha = (3.2209 \pm 0.0028)s^-1$.

2.3.3 Turbulent friction model

Figure 6: Turbulent drag free fall model

The results of the turbulent friction model Fig. 6 show a predicted $\beta = (1.5032 \pm 0.0080)s^{-}1$ and a predicted path. This is similar to the measured path, with mention of $t \approx 0.5s$ and $t \approx 1.1s$, where the measured path deviates considerably, with a high $r^2 = 0.9944$. This result is more accurate than the no friction model ($r^2 = -2.5505$) but not as accurate as the viscous drag model ($r^2 = 0.9996$).

2.3.4 Appropriate model

Across all the types of free fall movement, the viscous drag model is the most accurate and precise model with a $r^2 = 0.9996$ and a predicted $\alpha = (3.2209 \pm 0.0028)s^{-}1$. The largest acting force on the object in free fall, aside from gravity, is the viscous drag generated as a result of the object's motion through its medium (in this case, air).

2.4 Errors consideration for curve fitting

The experimental data collected was analyzed using a non-linear curve-fitting approach making use of the Numpy Python module and the *curve_fit* function[5] along with a calculated coefficient of determination with use of three different free fall models. To account for the uncertainty in time that was predicted to arise as a result of a discrete increase in time intervals, a weighted residual value for time that represented the uncertainties was introduced into the least squares fitting calculation [1]:

$$S = \sum_{i=1}^{n} \frac{1}{\sigma_i^t}^2 (y_i - f(t,c)_i)^2 \tag{13}$$

Where:

$$\sigma_i^t = \text{std of the } i^{th} \text{ element of t}$$

This approach was also applied to the subsequent experiment, rolling on an inclined plane.

3 Rolling on an Inclined Plane - Task II

3.1 Theoretical Basis

When regarding the motion of an object rolling, without slipping, the object rolls down the incline plane due to force of gravity ($\hat{g}$), acting upon it. However, in practice, there are additional considerations that contribute to the motion rolling object down the incline. When modeling motion of the such an object the accuracy of the predicted path by the model is determined by the forces the model considers. For the purpose of this experiment two models were used, one that only considers the force of gravity, and the other considers the force of gravity and frictional forces acting on the object.

3.1.1 Rolling on an incline without friction

A model that only considers gravity acting on the object will have an acceleration **down the incline**[4] of:

$$a_{inc} = g \sin\theta \tag{14}$$

Then through use of Newton's Equations of motion [4] the displacement of the object down the incline is:

$$s = \frac{1}{2} a_{inc} t^2 = \frac{1}{2} g \sin\theta t^2 \tag{15}$$

with considering initial conditions $\hat{x}$,$\hat{y}$ components are:

$$x(t) = x_0 + \frac{1}{2} g \sin\theta \cos\theta t^2 \tag{16}$$

$$y(t) = y_0 + \frac{1}{2} g \sin\theta^2 t^2 \tag{17}$$

Eqs. 16, 17 were used as the no friction model to predict the path of the rolling object overtime.

3.1.2 Rolling on an incline with friction

Assuming no slipping is occurring, an energy approach is first taken to consider which forces are doing work on the object [3] and which doesn't. Total energy of the system is conserved as the static frictional force does not contribute any work to the system. There is also gravitational potential energy and, for the object used in this experiment, it has translational and rotational kinetic energy. The conservation of energy states, the gravitational potential energy is equivalent to the kinetic energies [3]:

$$mgy(t) = \frac{1}{2} m v_{inc}^2 + \frac{1}{2} I \omega^2 \tag{18}$$

Where:

$$I = \text{moment of inertia of the object}(kgm^2)$$

$$\omega = \text{angular velocity}(rad/s) = \frac{v_{inc}}{r}$$

Rearranging:

$$v_{inc}^2 = \frac{2mgy(t)}{m + \frac{I}{r^2}} \tag{19}$$

Where:

$$r = \text{radius of the object}(m)$$

$$y(t) = \text{height at time}(t) in (m) = s \sin(\theta)$$

s is substituted from Eq. 15 and considering $v^2 = a^2 t^2$, a_{inc} is found as:

$$a_{inc} = \frac{g \sin\theta}{1 + \frac{I}{mr^2}} \tag{20}$$

Substituting Eq. 20 into 15, considering initial conditions and $\hat{x}$, $\hat{y}$ components are:

$$x(t) = x_0 + \frac{g \sin\theta \cos\theta}{2(1+k)} t^2 \tag{21}$$

$$y(t) = y_0 - \frac{g \sin\theta^2}{2(1+k)} t^2 \tag{22}$$

Where:

$$k = \text{Inertial constant} = \frac{I}{mr^2}$$

3.2 Experiment Setup

The setup of this experiment utilizes an inclined plane and the rolling of a wheel object down the inclined plane. As with the first experiment, the video analysis software is Tracker application.

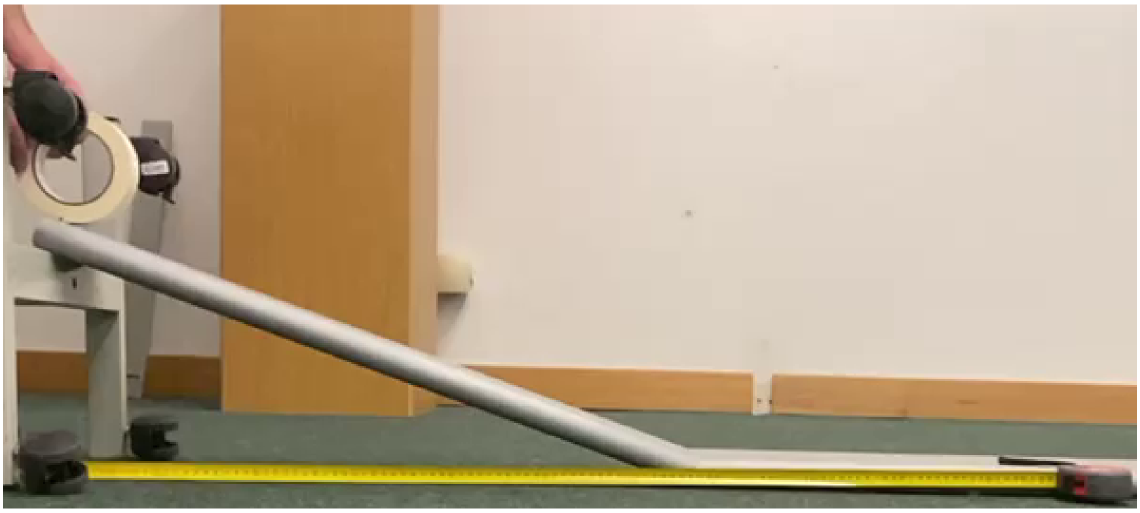

Figure 7: Setup of inclined plane scenario

3.2.1 Initial Conditions

For the second experiment, the setup was built with two metal surfaces, a roll of tape, a measuring tape and a phone with *"slow motion"* functionality. The video was taken with a frame rate of *120 frames/second*. The chosen coordinate system was cartesian *[x,y]*, with the origin at the start point of the object, on Y-axis, and at the level of the flat surface, on X-axis. The inclined plane and the two coordinate axis formed a rectangular triangle, with lengths of $hyp = 72cm$, $opp = 27.1cm$, and $adj = 66.7cm$ and a corresponding angle of $22.11°$.

3.2.2 Data Gathering

The data were gathered manually, frame by frame for 8 seconds. The observations were made considering the object a point mass, measuring the evolution of displacement of the contact point between the tape and the surface. The video was recorded from a parallel plane to the motion of the rolling object in the $\hat{x} - \hat{y}$ plane, to minimize parallax error, as the goal of the experiment lays on the values collected from $\hat{x} - \hat{y}$ plane. With a total of 244 points, the graphs of motion are:

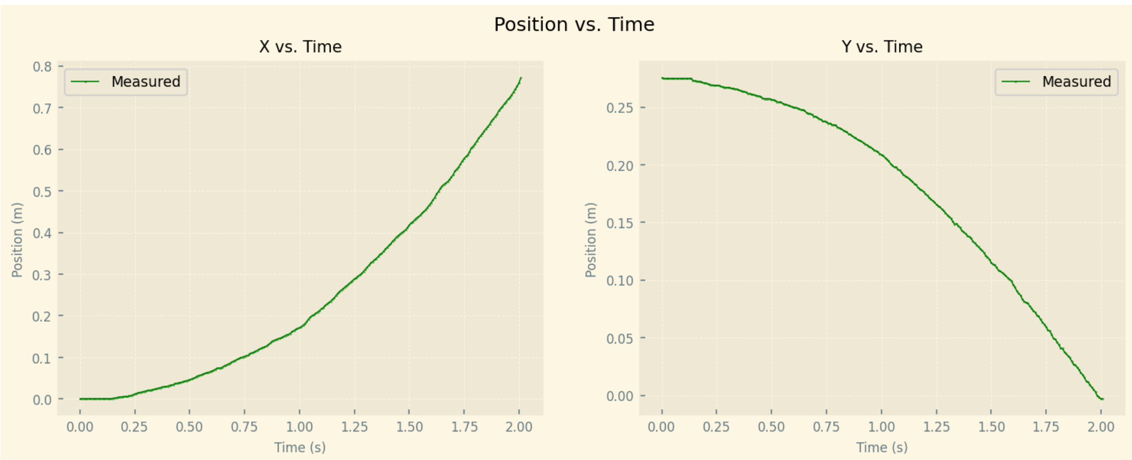

Figure 8: Displacement on X-axis and Y-Axis

3.3 Results and Analysis

3.3.1 No Friction Model

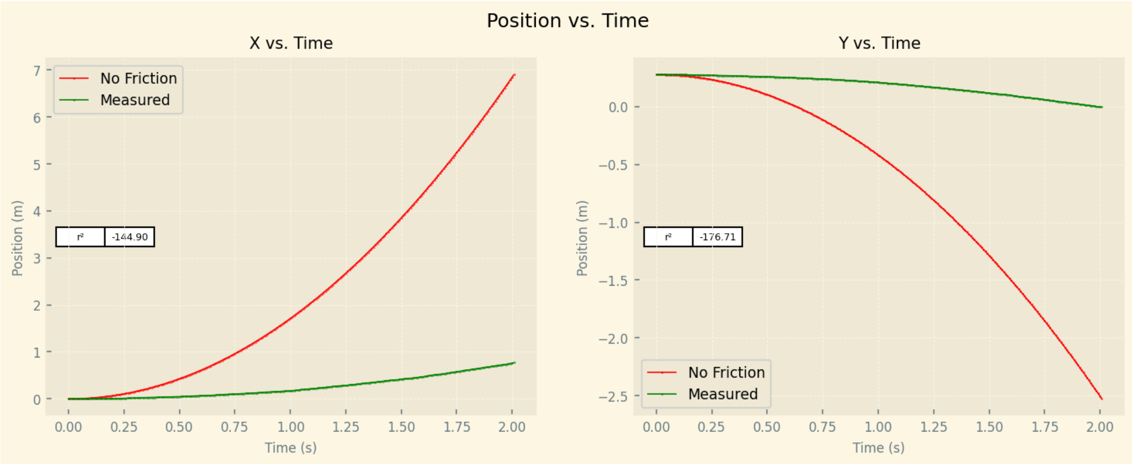

Figure 9: Inclined plane: no friction model

Fig. 9 shows the motion of the $\hat{x}$, $\hat{y}$ paths of the rolling object from measured data and the predicted $\hat{x}$, $\hat{y}$ paths based on the model of no friction. Both the $\hat{x}$, $\hat{y}$ paths are not in strong disagreement with the predicted no friction model, with $\hat{x}$, $\hat{y}$ r^2 values of r_x^2 = -144.901 and r_y^2 = -176.71, which as previously analyzed indicates the model does not fit the model as $\hat{x}$, $\hat{y}$ paths measure. This strongly suggests that there are additional frictional forces acting on the rolling object that are not consider in this model.

3.3.2 Friction Model

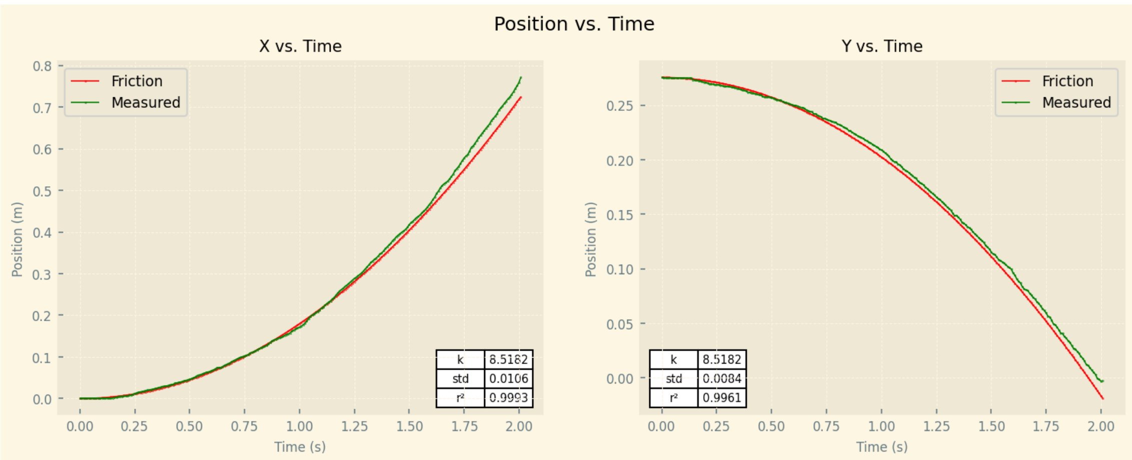

Figure 10: Inclined plane: friction model

The predicted motion of the $\hat{x}$, $\hat{y}$ paths strongly correlate with the one of the measured $\hat{x}$, $\hat{y}$ paths, $r_x^2 = 0.9993$ and $r_y^2 = 0.9961$. Through use of non-linear fitting a $k = 8.518$ was determined. When examining the friction parameter k for each the $\hat{x}$, $\hat{y}$ directions the standard deviation was $k_x = (8.518 \pm 0.0106)$ and $k_y = (8.518 \pm 0.0084)$, the precision the k_x is weaker than that of k_y, however both are within an acceptable margin of error.

3.3.3 Appropriate model

The strong correlation between the measured and predicted results of Fig. 10 suggest that this model is suitable for modeling the motion of a rolling object with no slipping down an inclined plane.

4 Discussion and Conclusions

4.1 Free fall

The kinematic processes investigated in this experiment have a significant importance for the understanding and predicting the motion of the object under gravitational influence. From the analysis performed in this report and the models used it was found that; the no friction model was inaccurate $r^2 = -2.5505$ and unable to predict the $\hat{y}$ motion of the object beyond a initial time period $t \approx 0.1s$ as shown in Fig 3. $r^2 = -2.5505$ results from a poor model that is not correlated with measured data in a manner such that the SS_{tot} is much larger than the SS_{res} when r^2 was calculated. The quick deviation of the no friction model prediction from measured results is a result of a poor model and the unique shape of the object, that of a wide surface area atop the center of mass. The large surface area creates a significant drag force on the object that slows the object's descent greatly, deviating from the no friction model prediction in a short amount of time. The turbulent friction (or quadratic friction) model Fig. 6 was found in high correlation between predicted path and the measured data ($r^2 = 0.9944$) when paired with a determined $\beta = (1.5032 \pm 0.0080)s^-1$. this results is significantly more accurate than that of the the no friction model as it incorporates a drag force, dependent on the square of the objects velocity Eq. 10 . However the viscous drag model was shown to strongly correlate with measured results, with $r^2 = 0.9996$ and determined $\alpha = (3.2209 \pm 0.0028)s^-1$, Fig. 5 accurately and precisely models the descent of the object. Thus it is concluded that for the object used in this experiment (or of a similar design) the model appropriate is that of the viscous drag model with use of Eq. 9 It is important to outline that the significance of the results in this experiment and its conclusions remain limited to the design of the object used, that of a large surface area atop the objects center of mass. It is predicted that for other object designs differing models may be more appropriate.

4.2 Inclined plane

Similar to the free fall investigation, the study of a rolling wheel down an inclined plane is important in understanding and predicting the motion of wheel over time across the $\hat{x}$ and $\hat{y}$ axes. Across the two models that were investigated the no friction model, Fig. 10, was found to be wildly inaccurate with $r_x^2 = -144.901$ and $r_y^2 = -176.71$. similar to the free fall experiment, an $r^2 << 0$ conveys the model largely fails to predict the motion rolling wheel across both axes. In contrast the friction model, that uses Eqs. 16, 17, accurately models both the $\hat{x}$ and $\hat{y}$ with $r_x^2 = 0.9993$ and $r_x^2 = -176.71$ with a determined common inertial constant of $k_x = (8.518 \pm 0.0106)$ and $k_y = (8.518 \pm 0.0084)$. Demonstrating that of the two models investigated in this lab report the friction model was most appropriate for predicting the motion of the rolling wheel down the inclined plane. It should also be considered that the measured data points were recorded from the contact point of the rolling wheel, this was in a effort to improve the precision of the data collected. However this diminished the accuracy and correlation between the mathematical models and measured data as the models consider the motion of the rolling wheel from its center of mass. It is predicted that the accuracy was not significantly diminished however it was also not considered in the experimental approach nor results collection.

References

[1] A. J. Canty. Weighted least squares. https://ms.mcmaster.ca/canty/teaching/stat3a03/Lectures7.pdf, 2017. Accessed: 2024-11-02.

[2] N. Hall. Free fall without air resistance. https://www1.grc.nasa.gov/beginners-guide-to-aeronautics/free-fall-without-air-resistance/, 2024. Accessed: 2024-11-02.

[3] Y. D. Hugh and R. A. Freedman. Work and kinetic energy. In *University with Modern Physics*, pages 199–220. Pearson, 15th edition, 2020.

[4] Y. D. Hugh and R. A. Freedman. Applying newton's laws. In *University with Modern Physics*, pages 157–186. Pearson, 15th edition, 2020.

[5] G. S. Vallverdu. Non linear curve fitting with python. https://gsalvatovallverdu.gitlab.io/python/curve_fit/#uncertainties-on-both-x-and-y, 2019. Accessed: 2024-11-02.

[6] M. Ziese. *H02e Video Analysis of Kinematical Processes*. Universität Leipzig, October 2024.