GETTING TO KNOW THE CARDINALS

NOTES ON AKIHIRO KANAMORI'S *The Higher Inifite*

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0 Preliminaries

Definition 0.1. For $X \subseteq \mathrm{On}$, γ is a *limit point of* X iff $\bigcup(X \cap \gamma) = \gamma > 0$

Notice that a limit point is necessarily a limit ordinal: if $\gamma = \alpha + 1$ then $\bigcup \gamma = \alpha$ and we have $\bigcup(X \cap \gamma) \subseteq \bigcup \gamma = \alpha < \gamma$.

Definition 0.2. C is *closed unbounded in* δ iff C is an unbounded subset of δ containing all its limit points less than δ.

These sets are also called *club* sets. Here C is unbounded iff for all $x \in \delta$ there exists $y \in C$ such that $x < y$ (the alternative would have $x \leq y$). This definition of unbounded implies that δ must be infinite for club sets to exist.

Definition 0.3. For regular $\nu < \delta$, C is *ν-closed unbounded in* δ iff C is an unbounded subset of δ containing all its limit points less than δ of cofinality ν.

Definition 0.4. For limit ordinals δ, S is *stationary in* δ iff $S \subseteq \delta$ and $S \cap C \neq \emptyset$ for any C closed unbounded in δ.

Definition 0.5. If $\langle X_\alpha \mid \alpha < \delta \rangle \in {}^{\delta}\mathcal{P}(\delta)$, then its diagonal intersection is $\{\xi < \delta \mid \xi \in \bigcap_{\alpha < \xi} X_\alpha\}$, denoted $\triangle_{\alpha < \delta} X_\alpha$.

Definition 0.6. For $X \subseteq \mathrm{On}$ and $f : X \to \mathrm{On}$, f is *regressive* iff $f(\alpha) < \alpha$ for every $\alpha \in X - \{\emptyset\}$.

Proposition 0.7. *Suppose that $\lambda > \omega$ is regular.*
(a) If something
(b) fkflkadsl;