

# Big-O Notation

Time Complexity Analysis  
Algorithmic Efficiency

How your algorithm's performance is affected with a change in the size of an input.

# Algorithimically



- Execution Speed
- Resource optimization (e.g. memory usage)

# Big-O Notation



**$O(n)$**



Size of an input (ex: size of an array)

Also called Landau's symbol, is a symbolism used in complexity theory, computer science, and mathematics to describe the asymptotic behavior of functions. Basically, it tells you how fast a function grows or declines.

# Big-O Notation



## Common Complexities

$O(1)$	Constant Time
$O(\log n)$	Logarithmic Time
$O(n)$	Linear Time
$O(n \log n)$	Log-Linear Time
$O(n^2)$	Quadratic Time
$O(2^n)$	Exponential Time



**Faster Algorithm**

**Slower Algorithm**

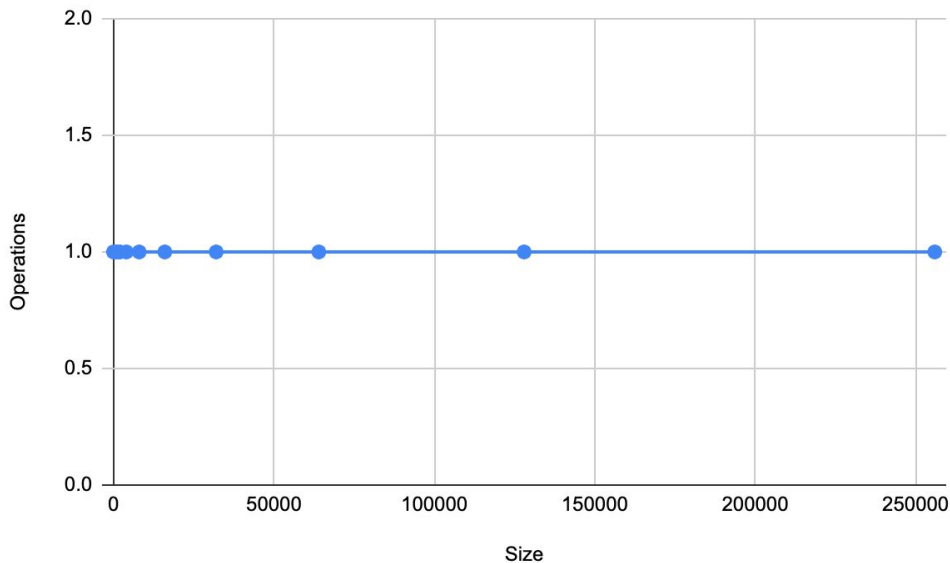
# Big-O Notation (constant time)



## $O(1)$

```
arr = [1,2,3,4,5,6,7,8,9,10]  
arr[5]
```

**Ex: Accessing single  
element in an array**



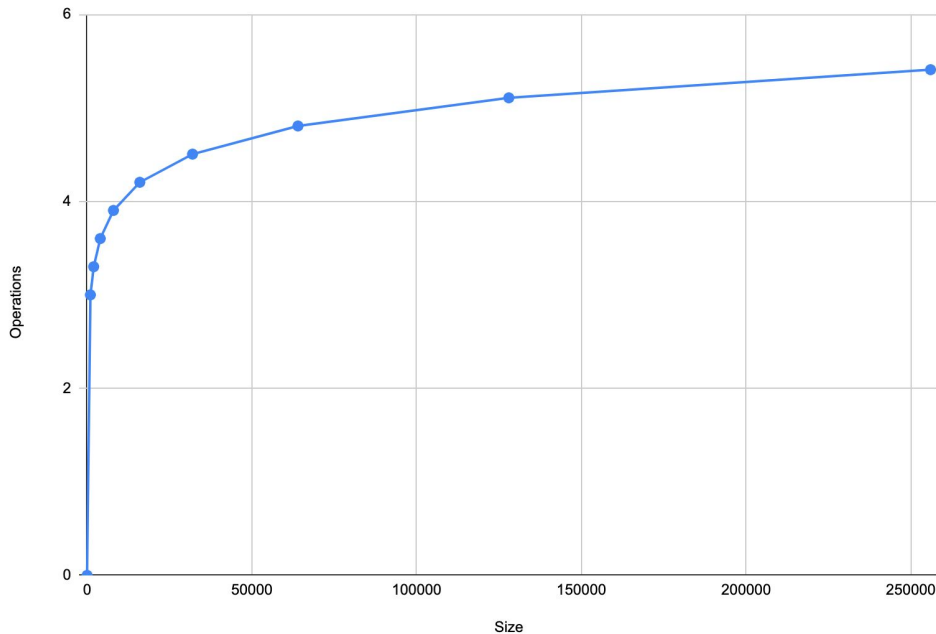
# Big-O Notation (logarithmic time)



## $O(\log n)$

```
arr = [1,2,...256000]
```

### Ex: Binary Search



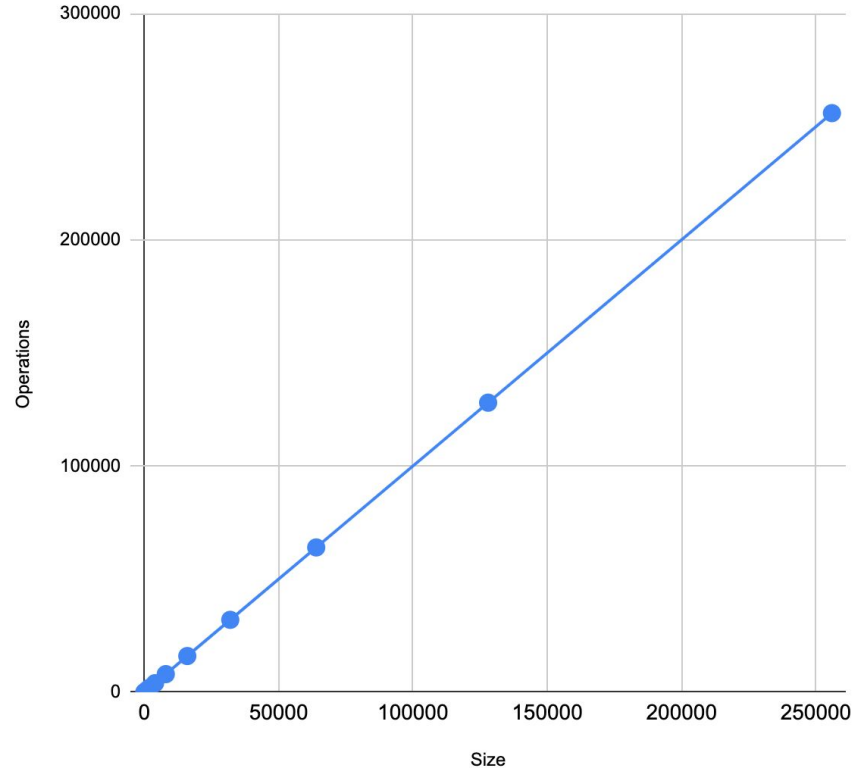
# Big-O Notation (linear time)



## $O(n)$

```
arr = [1,2,3,4,5,6,7,8,9,10]
```

### Ex: Linear Search





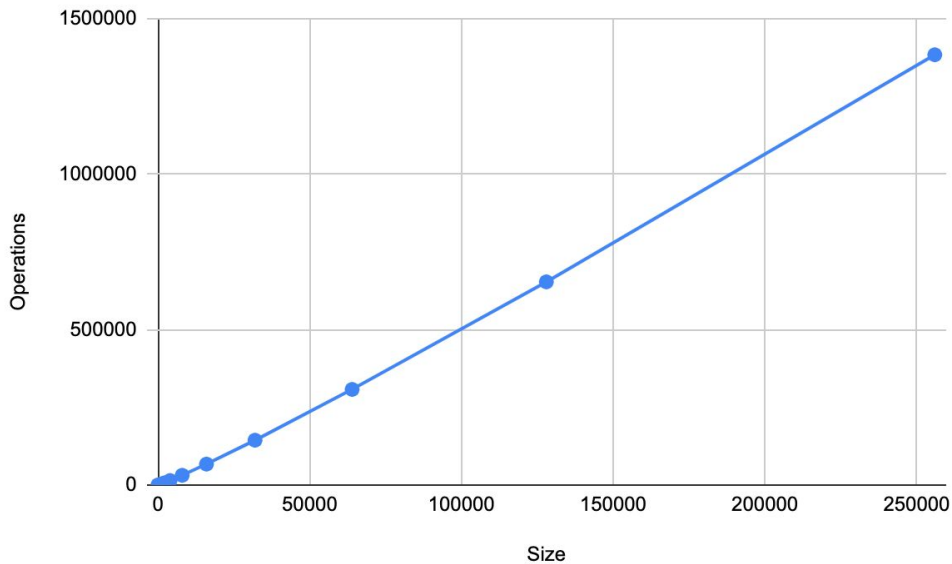
# Big-O Notation (log linear time)



## $O(n \log n)$

```
arr = [1,2,3,4,5,6,7,8,9,10]
```

### Ex: Merge Sort



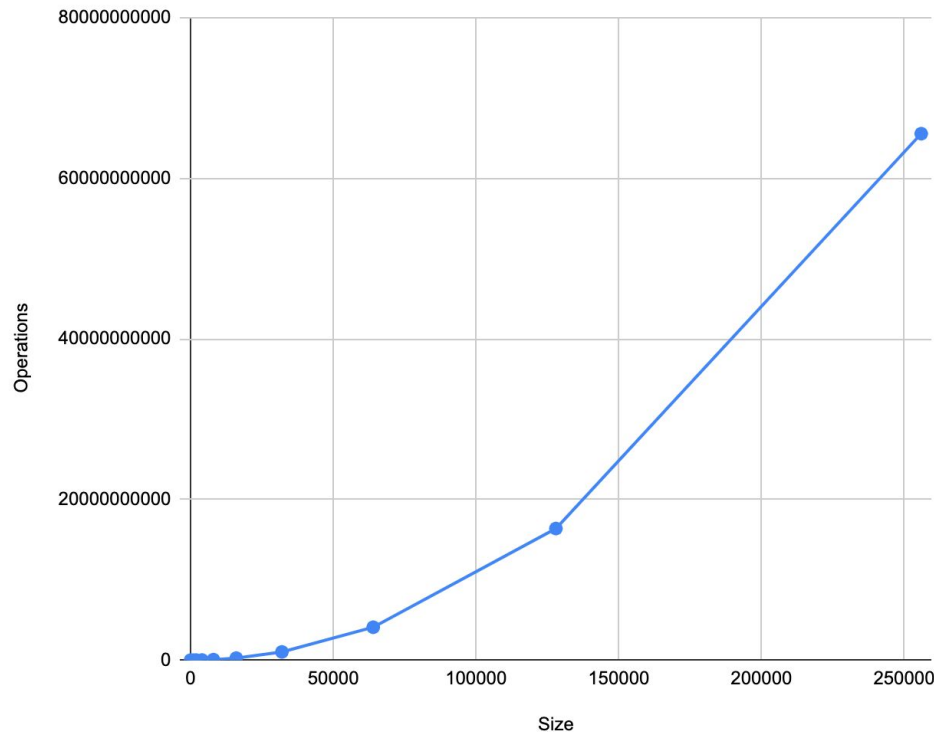
# Big-O Notation (quadratic time)



## $O(n^2)$

```
arr = [1,2,3,4,5,6,7,8,9,10]
```

**Ex: Finding  
Duplicate Values  
(nested for loop of  
same array)**

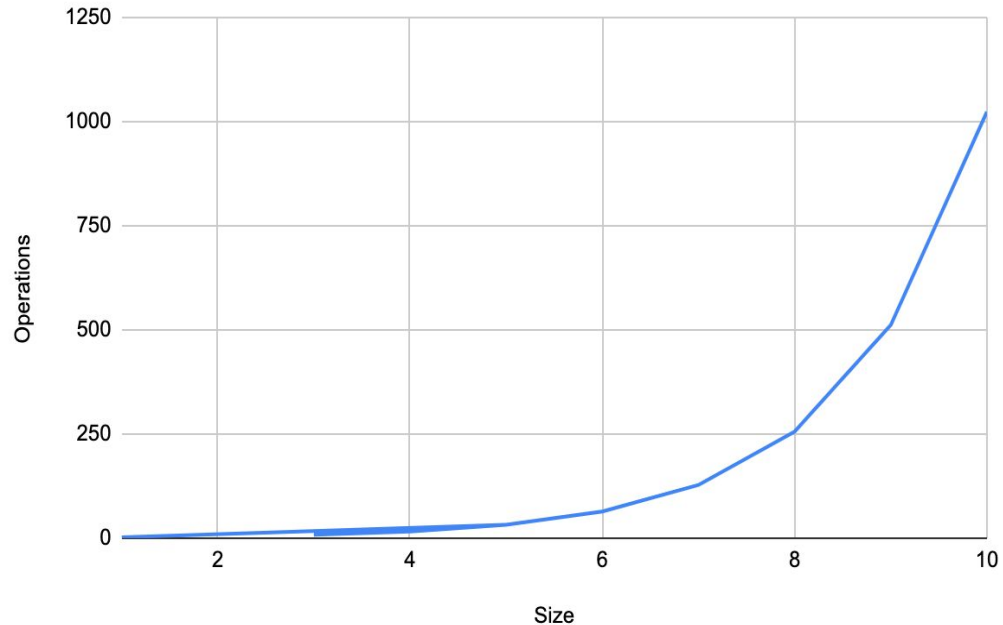


# Big-O Notation (exponential time)



$$O(2^n)$$

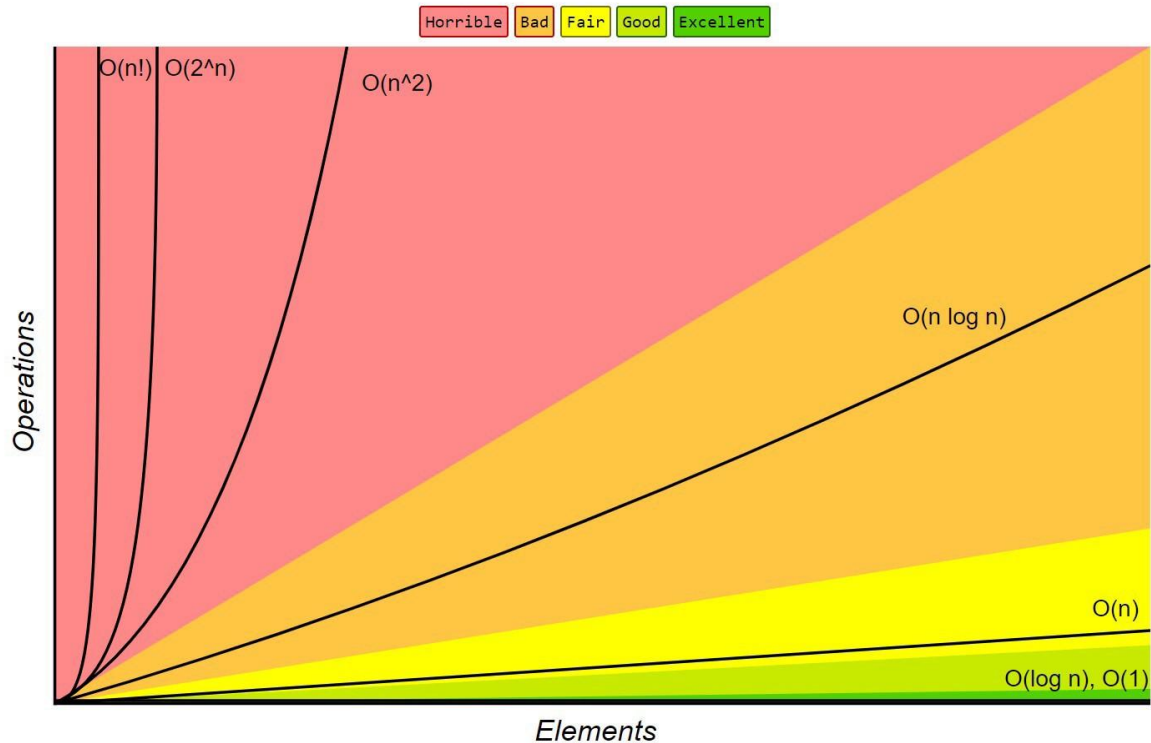
**Ex: Finding the nth value in the fibonacci sequence**



# Big-O Notation



Big-O Complexity Chart



# Data Structures



## Common Data Structure Operations

Data Structure	Time Complexity								Space Complexity
	Average				Worst				Worst
	Access	Search	Insertion	Deletion	Access	Search	Insertion	Deletion	
<u>Array</u>	$\theta(1)$	$\theta(n)$	$\theta(n)$	$\theta(n)$	$\theta(1)$	$\theta(n)$	$\theta(n)$	$\theta(n)$	$\theta(n)$
<u>Stack</u>	$\theta(n)$	$\theta(n)$	$\theta(1)$	$\theta(1)$	$\theta(n)$	$\theta(n)$	$\theta(1)$	$\theta(1)$	$\theta(n)$
<u>Queue</u>	$\theta(n)$	$\theta(n)$	$\theta(1)$	$\theta(1)$	$\theta(n)$	$\theta(n)$	$\theta(1)$	$\theta(1)$	$\theta(n)$
<u>Singly-Linked List</u>	$\theta(n)$	$\theta(n)$	$\theta(1)$	$\theta(1)$	$\theta(n)$	$\theta(n)$	$\theta(1)$	$\theta(1)$	$\theta(n)$
<u>Doubly-Linked List</u>	$\theta(n)$	$\theta(n)$	$\theta(1)$	$\theta(1)$	$\theta(n)$	$\theta(n)$	$\theta(1)$	$\theta(1)$	$\theta(n)$
<u>Skip List</u>	$\theta(\log(n))$	$\theta(\log(n))$	$\theta(\log(n))$	$\theta(\log(n))$	$\theta(n)$	$\theta(n)$	$\theta(n)$	$\theta(n)$	$\theta(n \log(n))$
<u>Hash Table</u>	N/A	$\theta(1)$	$\theta(1)$	$\theta(1)$	N/A	$\theta(n)$	$\theta(n)$	$\theta(n)$	$\theta(n)$
<u>Binary Search Tree</u>	$\theta(\log(n))$	$\theta(\log(n))$	$\theta(\log(n))$	$\theta(\log(n))$	$\theta(n)$	$\theta(n)$	$\theta(n)$	$\theta(n)$	$\theta(n)$
<u>Cartesian Tree</u>	N/A	$\theta(\log(n))$	$\theta(\log(n))$	$\theta(\log(n))$	N/A	$\theta(n)$	$\theta(n)$	$\theta(n)$	$\theta(n)$
<u>B-Tree</u>	$\theta(\log(n))$	$\theta(\log(n))$	$\theta(\log(n))$	$\theta(\log(n))$	$\theta(\log(n))$	$\theta(\log(n))$	$\theta(\log(n))$	$\theta(\log(n))$	$\theta(n)$
<u>Red-Black Tree</u>	$\theta(\log(n))$	$\theta(\log(n))$	$\theta(\log(n))$	$\theta(\log(n))$	$\theta(\log(n))$	$\theta(\log(n))$	$\theta(\log(n))$	$\theta(\log(n))$	$\theta(n)$
<u>Splay Tree</u>	N/A	$\theta(\log(n))$	$\theta(\log(n))$	$\theta(\log(n))$	N/A	$\theta(\log(n))$	$\theta(\log(n))$	$\theta(\log(n))$	$\theta(n)$
<u>AVL Tree</u>	$\theta(\log(n))$	$\theta(\log(n))$	$\theta(\log(n))$	$\theta(\log(n))$	$\theta(\log(n))$	$\theta(\log(n))$	$\theta(\log(n))$	$\theta(\log(n))$	$\theta(n)$
<u>KD Tree</u>	$\theta(\log(n))$	$\theta(\log(n))$	$\theta(\log(n))$	$\theta(\log(n))$	$\theta(n)$	$\theta(n)$	$\theta(n)$	$\theta(n)$	$\theta(n)$

# Things to remember



- Drop Constants (two separate for loops of same array)
  - $O(n + n) = O(2n) \Rightarrow O(n)$
- Use Worst Case Scenario
  - 1 for loop  $O(n)$
  - 1 nested for loop  $O(n^2)$  <- worst case
  - $O(n^2)$
- Searching through dictionaries or objects are faster than searching through lists or arrays because objects don't keep track of indices
- Wherever possible, avoid nested loops
- There's usually a way to avoid poor Big O