Esperanza condicional

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1 Esperanza condicional

Demuestre que $\mathrm{E}[\mathrm{E}[Y_1|Y_2]] = E[Y_1]$.

Caso discreto. D! Sea Y_1 y Y_2 variables aleatorias, entonces:

$$\begin{aligned}
\mathrm{E}[\mathrm{E}[Y_1|Y_2]] &= \sum_{y_2} \underbrace{\mathrm{E}[Y_1|Y_2]}_{\sum_{y_1} y_1 \cdot p(y_1|y_2)} \cdot p_2(y_2) \\
&= \sum_{y_2} \left[\sum_{y_1} y_1 \cdot p(y_1|y_2) \right] \cdot p_2(y_2) \\
&= \sum_{y_2} \sum_{y_1} y_1 \cdot p(y_1|y_2) \cdot p_2(y_2) \\
&= \sum_{y_1} \sum_{y_2} y_1 \cdot \frac{p(y_1, y_2)}{\cancel{p_2(y_2)}} \cdot \cancel{p_2(y_2)} \\
&= \sum_{y_1} y_1 \cdot \sum_{y_2} p(y_1, y_2) \\
&= \sum_{y_1} y_1 \cdot p(y_1) \\
&= E[Y_1]
\end{aligned}$$

2 Ejercicio 5.139

2.1 Inciso a)

¿$\mathrm{E}[T|N = n]$?

$$\mathrm{E}[T|N = n] = \mathrm{E}\left[\sum_{k=1} \right.$$

2.2 Inciso b)

$\mathrm{E}[T]$, ¿El tiempo total esperado para completar todos los trabajos?

$$\begin{aligned}
\mathrm{E}[T] &= \mathrm{E}[\mathrm{E}[T|N]] \\
&= \mathrm{E}[N\frac{\alpha}{\beta}] \\
&= \lambda\frac{\alpha}{\beta}
\end{aligned}$$