Solucionario de Estadística

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Contents

1 Demostraciones

1.1 Función binomial

1.1.1 Función generadora de momentos

D!

$$\begin{aligned} m(t) &:= E[e^{tX}] \\ &= \sum_{k=0}^{n} \binom{n}{k} p^k q^{n-k} e^{kt} \\ &= \sum_{k=0}^{n} \binom{n}{k} (pe^t)^k q^{n-k} \\ &= (q + pe^t)^n \end{aligned}$$

□

Recuerde que $q = (1 - p)$:

$$m(t) = (1 - p + pe^t)^n, \quad t \in \mathbb{R}$$

1.1.2 Esperanza

D!

$$
\begin{aligned}
E[X] &= \sum_{k=0}^{n} k \binom{n}{k} p^k q^{n-k} \\
&= \sum_{k=0}^{n} \frac{n!}{k!(n-k)!} p^k q^{n-k} \\
&= \sum_{k=1}^{n} \frac{n(n-1)!}{(k-1)!(n-k)!} p^k q^{n-k} \\
&= np \sum_{k=1}^{n} \binom{n-1}{k-1} p^{k-1} q^{n-k} \\
&= np \sum_{m=0}^{n-1} \binom{n-1}{m} p^m q^{n-1-m} \\
&= np(p+q)^{n-1} \\
&= np
\end{aligned}
$$

□

1.1.3 Varianza

$$
\begin{aligned}
E[X^2] &= \sum_{k=0}^{n} k^2 \binom{n}{k} p^k q^{n-k} \\
&= \sum_{k=0}^{n} \frac{n!}{k!(n-k)!} p^k q^{n-k} \\
&= \sum_{k=1}^{n} \frac{n(n-1)!}{(k-1)!(n-k)!} p^k q^{n-k} \\
&= np \sum_{k=1}^{n} \binom{n-1}{k-1} p^{k-1} q^{n-k} \\
&= np \sum_{m=0}^{n-1} \binom{n-1}{m} p^m q^{n-1-m} \\
&= np(p+q)^{n-1} \\
&= np
\end{aligned}
$$

$$
\begin{aligned}
Var(X) &:= E[X^2] - (E[X])^2 \\
&= n(n-1)p^2 + np - (np)^2 \\
&= npq
\end{aligned}
$$

1.2 Función poisson

1.2.1 Función generadora de momentos

$$
\begin{aligned}
m(t) &:= E[e^{tX}] \\
&= \\
&= \\
&= e^{\lambda(e^t - 1)}
\end{aligned}
$$