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Abstract: Scheduling is becoming much more important in every industry. However, the standard RCSP usually does not cover all the characteristics of real world problems. In this work, we present two new Event Based MILP formulations for a Multimode Resource Constraint Problem of direct application for some industries, as aeronautical assembly lines. Its main contributions are the treatment of general precedence constraints, the possibility of having activities with multiple modes and the use of a time constraint approach instead of a resource constraint one. Along with the new formulations, we present a new set of instances that extends the previously existing ones. This data sets have been used for a computational study and comparison of both models.

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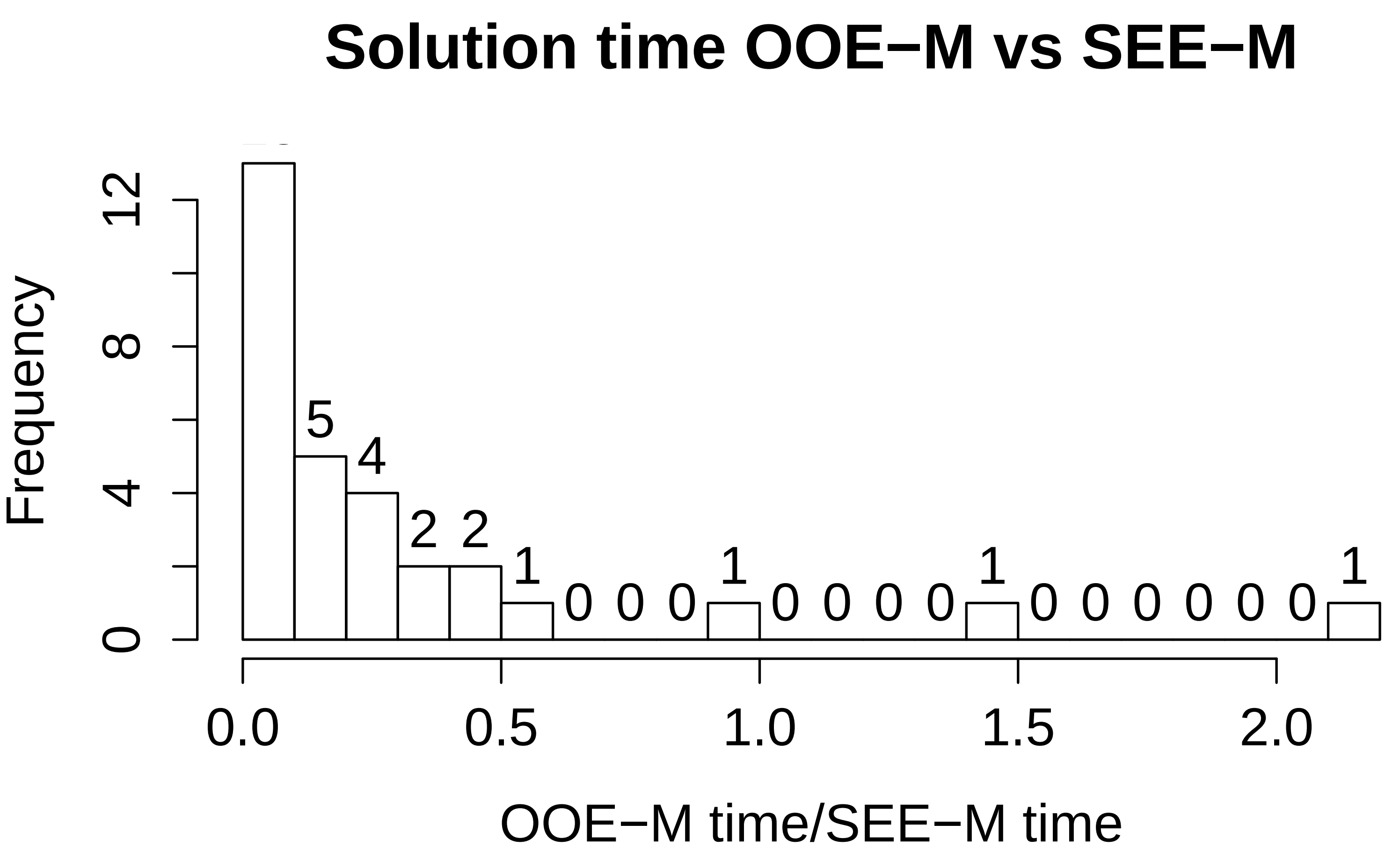

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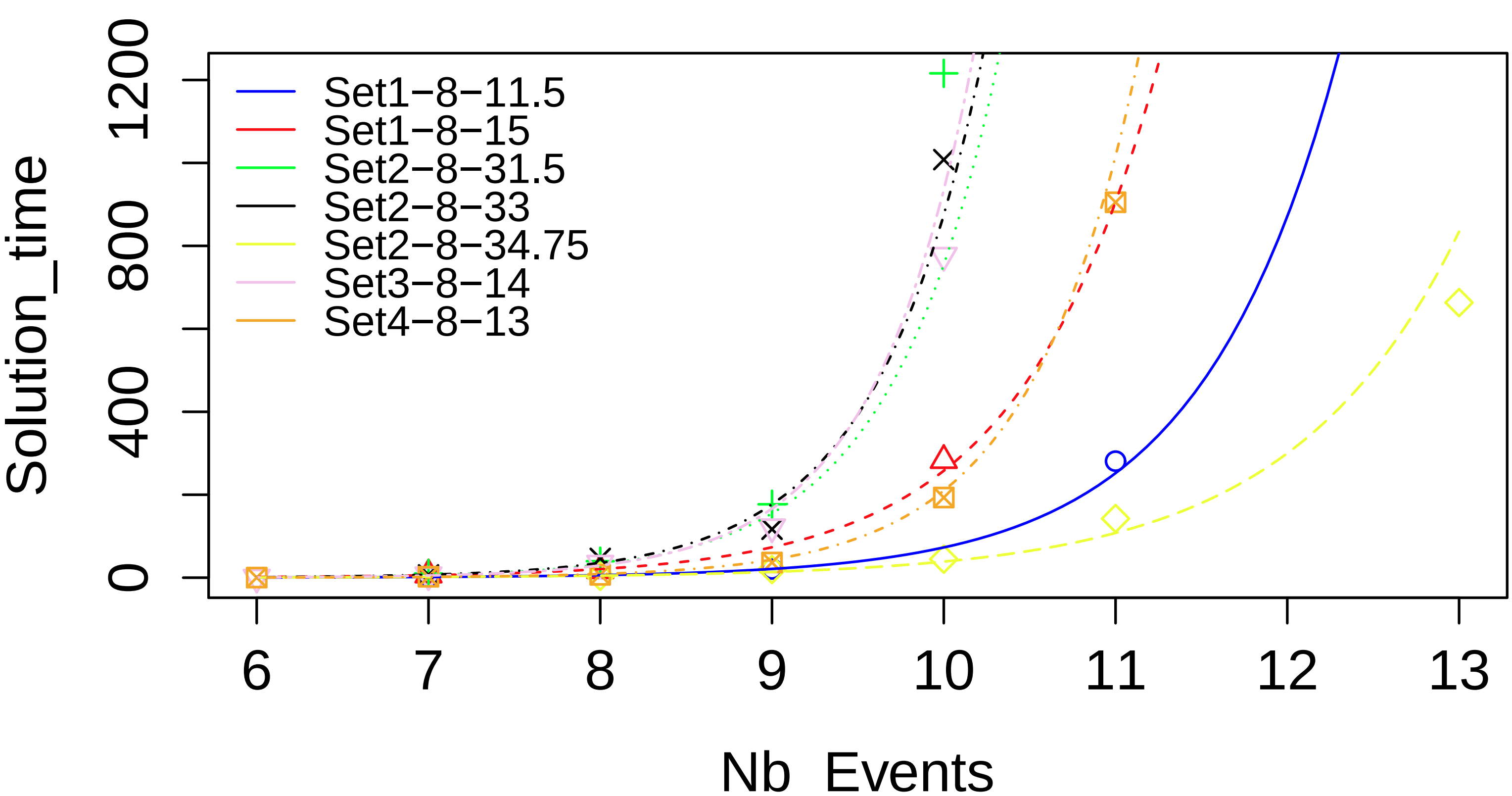

Madrid, 29/04/2014

Dear Editor,

Attached is a manuscript for your consideration for the Elsevier journal Computer & Operations Research. The paper "*Two MILP Event Based Formulations for a Multimode TCSP with Generalized Temporal Constraints, applied to the Aeronautical Industry*" presents two MILP formulations that address a scheduling case that includes most of the elements that previous authors have already considered but also some additional features not present in the literature so far.

While most of the research on scheduling focuses on the minimisation of the Lead Time given a resource capacity (RCSP), we have studied the case where the Lead Time is known and the objective is to minimize the resource consumption (TCSP). This problem is not only applicable to the aeronautical industry, but also to other such as ships or mills manufacturing. Moreover, the use of multiple modes and generalized precedence constraints covers a gap between most of the research on scheduling and real case studies.

We the undersigned declare that this manuscript is original, has not been published before and is not currently being considered for publication elsewhere. We would like to draw the attention of the Editor to an ongoing publication from the same authors that has been the outcome of a congress presentation. This work is the natural evolution of that publication and it contributes further, by means of providing a new formulation, the comparison between the two formulations, and the theoretical validation of the models.

We wish to confirm that there are no known conflicts of interest associated with this publication and there has been no significant financial support for this work that could have influenced its outcome. We confirm that the manuscript has been read and approved by all named authors and that there are no other persons who satisfied the criteria for authorship but are not listed. We further confirm that the order of authors listed in the manuscript has been approved by all of us.

We confirm that we have given due consideration to the protection of intellectual property associated with this work and that there are no impediments to publication, including the timing of publication, with respect to intellectual property. In so doing we confirm that we have followed the regulations of our institutions concerning intellectual property.

Looking forward to your favorable consideration.

Most sincerely,

Tamara Borreguero Sanchidrián	Álvaro García Sánchez	Miguel Ortega Mier
29/04/2014	29/04/2014	29/04/2014

Two MILP Event Based Formulations for a Multimode TCSP with Generalized Temporal Constraints and Labor Skills, applied to the Aeronautical Industry

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Abstract

Scheduling is becoming much more important in every industry. However, the standard RCSP usually does not cover all the characteristics of real world problems. In this work, we present two new Event Based MILP formulations for a Multimode Resource Constraint Problem of direct application for some industries, as aeronautical assembly lines. Its main contributions are the treatment of general precedence constraints, the possibility of having activities with multiple modes and the use of a time constraint approach instead of a resource constraint one. Along with the new formulations, we present a new set of instances that extends the previously existing ones. This data sets have been used for a computational study and comparison of both models.

Keywords:
Scheduling, Multimode, Event based formulation, Temporal constraints, MILP

1. Introduction

Over the last years, the continuous changes on every industry have forced enterprises to explore new manufacturing methods in order to comply with the *On time, On Quality, On Cost* (OTOQOC) paradigm. Production systems based on the Toyota Production System have spread worldwide as a means of reducing waste and optimizing manufacturing processes.

The aeronautical industry, since the 1990's has been including Lean Manufacturing techniques into its production systems. In terms of Boeing, in a Lean production system the right resources and the right tools must be applied to achieve three key Lean principles: Takt Paced Production, One Piece Flow and Pull Production [1]. Scheduling and line balancing have therefore become two main enablers for Lean implementation.

Aeronautical Final Assembly Lines consist in different platforms or stations. Each platform has a fixed team of workers with different skills. The line balancing enables the distribution of the work tasks among the different platforms. Afterwards, the work tasks from each platform must be scheduled in order to complete them within the required Takt Time using the minimum number of operators. We will refer to the scheduling of the tasks as the Aeronautical Platform Scheduling Problem (APSP).

This APSP can be classified as a scheduling problem, which deal with the definition of the activities to be performed at a particular time and the resources allocated to each activity. Scheduling problems have been the subject of continuous research since the early days of operations research [2]. They are NP-hard optimization problems and, in practice, among the most intractable classical ones.

Most of the research on scheduling problems has focused on the Resource Constrained Scheduling Problem (RCSP), which consists in the scheduling of several tasks subject to resource and precedence constraints. Brucker [3] provided a classification for this kind of problems together with an overview on existing solution methods. There have been a wide range of studies on both heuristic and metaheuristic methods for solving the RCSP, as well as different MILP models [4], [5], [6], [7], [8]. Recently, Kone [9] proposed the use of Event Based Formulations. He provided a benchmark of different methods (including MILP exact methods and an heuristic) and concluded that event based formulations

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outperformed the previous MILP models and performed even better than the heuristic for some instances. However, Kone's Event Based Formulations are suitable for the standard RCSP, which includes some assumptions that are too restrictive for many applications [6]. Therefore, it is of great interest to improve this kind of formulations so that they can be used on more practical contexts.

Furthermore, the RCSP focuses on minimising the project makespam given a set of available resources. Although this is appropiate for many real problems there are other industries, such as aircraft or ships manufacturing, where the total makespam is usually fixed by the expected production rate or the client demand [10]. In the case of the APSP, the platform deadline is given and the objective is to minimise the resource consumption. Therefore, it can be classified as a Time Constrained Scheduling Problem (TCSP), which has been very little dealt with in the litterature. Although Möhring introduced it in 1984 [11], few recent references have been found: [12], [13] and [10]. Neither of them provide with an exact method for this problem and, moreover, the first two address some very specific mono-mode cases.

On this work, we have developed two MILP Event Based Formulations that cover the characteristics of an Aeronautical Platform Scheduling Problem. Actually, its contribution is threefold. To begin with, it provides with an exact MILP formulation for the TCSP. Moreover it includes the allowance of multiple modes per task, not only linked to the number of workers but also to their skill. Furthermore, it uses general temporal constraints, including some that have not yet been addresed for neither the TCSP nor the RCSP.

For the experimentation, we have created a new set of instances of up to 11 tasks each, due to the different structure of the problem with respect to the previous existing instance libraries. Throughout the computational results we have solved the instances up to optimallity with both models. As well as this, they have enabled as to make a comparative study between the two proposed event based formulations: SEE-M and OOE-M formulations. The results of this comparisons are coherent with the ones reported by Koné for the single mode RCSP with only precedence constraints model [9]. Finally, the major factors that have an impact on the instance hardness have been identified, providing with directions for a further study and algorithm optimization.

2. Problem Description - Aeronautical Platform Scheduling Problem

Aeronautical assembly lines consist on a series of platforms where different works are executed. Each product has to go through all the platforms. At the same time, the line is syncronised, which means that the time that each product remains on a platform is always the same and equal to the rate at which the assembly line produces its output.

Usually, both final and intermediate assembly lines are dedicated to only one product family. However, there may be some specific installations between the products of a same family. In that case, an effort is made in order to guarantee that the takt time remains standard throughout the line. Different strategies for that, refered to the aeronautical industry, can be found in [14].

The assignment of the works to a platform is in most of the cases related to industrial issues: assembly technologies and the need of specific jigs that cannot be easily moved. Therefore, the asssignment of works between platforms is made only once, during the line definition. Although there may be a small percentage of works that can be done in more than one platform, once the works are assigned they are rarely moved from one platform to another, so we can assume with no loss of generality that the task assignment is constant per platform.

Taking this into account, the scheduling decision consists on establishing the order in which the tasks will be done along with the resources allocated to each of them, given the line takt time and a set of works per platform.

It has therefore the structure of a Time Constraint Scheduling Problem (TCSP), that is, a project scheduling problem consisting on a time window and resource assignment for a set activities of known durations and resource requests, that must be executed guaranteeing some precedence relations, where given a time limit for the project duration the objective is to find the least resource consuming schedule.

Until 1999, there was not a common notation for scheduling problems. Herroelen provided a first classification on 1998 [2]. A year after, Brucker [3] proposed one based on the extension of the $\alpha|\beta|\gamma$ generalized scheme for the machine scheduling and resource constrained machine scheduling literature. In this notation, α refers to the resource environment, β to the activity characteristics and γ to the objective function.

In accordance to that classification, the Aeronautical Platform Scheduling Problem (APSP) is a $MPS_m, \sigma, \ \rho \mid prec, temp, LT \mid \sum C_o * num_o^{op}$ problem:

- $\alpha = MPS_m, \sigma, \rho$. This stands for a multimode resource constraint project where each activity can be processed in several alternative modes and there exists a set of renewable resources available for each time period during the project execution: m being the resources, σ the units of each resource available and ρ the maximum number of units of the resources demanded by an activity. For our particular problem, the activities are the work tasks assigned to each platform. The renewable resources are the number of operators (each of them belonging to a profile) and the space in each of the platform's working areas. As well as this, each mode for an activity defines a combination of operator profile, number of operators and durations. All the operators assigned to an activity must be from the same profile and the range of possible numbers of allocated operators per tasks is independent from the chosen profile.

- $\beta = temp$. There are precedence constraints (task w' can not start until task w has been completed), non-parallel constraints (tasks w and w' cannot be in progress at the same time, but there is no precedence relation between them), and maximal time lags between tasks (task w' must start within a maximal time after w has been completed). All the temporal constraints are independent from the mode in which a task is executed.

- $\gamma = \sum C_o * num_o^{op}$. The objective function is to minimise the resource investment. The total Lead Time is fixed by the assembly line Takt Time. In consequence, the objective function is to minimise the labor cost of the assembly. As the operators once assigned to a platform stay working on it for all the Takt Time, minimising the labor cost is equivalent to minimising the maximum number of operators needed throughout the Takt Time. We will consider that the cost of an operator does not depend on its profile.

3. Existing MILP Scheduling Formulations

Most of the research on scheduling problems has focused on the core single-mode Resource Constraint Scheduling Problem (RCSP): the scheduling problem where given constant limits for the available amount per resource and precedence relations between tasks, its objective is to find the shortest possible project duration. Its existing MILP formulations can be divided on three main groups:

- *Discrete Time Formulations:* In them, the time horizon is divided into time slots. The basic discrete time formulation was proposed by Pritsker [15]. Afterwards, Christofides [16] proposed the Disaggregated Discrete Time formulation (DDT) that implies a larger number of constraints but, on the other hand, is a tighter formulation and therefore its linear relaxation provides a better lower bound. The main drawback of discrete time formulations is the increase in the number of variables as the time horizon grows.

- *Continuous time formulation:* In this formulations, the time is represented by continuous variables. Alvarez-Valdés and Tamarit [17] studied Forbidden Sets Formulations which involve a high number of constraints that grows exponentially and cannot be used in practice. Flow-Based Continuous Time Formulations, described by Artigues [18], provide a poor relaxation, compared to discrete time formulations, although they can be preferable to them for instances involving large time scale.

- *Event Based Formulations:* Event Based Formulations for the RCSP where developed by Koné in 2011 [9] from a model introduced by Zapata [19]. These formulations define a series of events which correspond to the start or end of the different activities. They are based on the fact that for the RCSP it always exists an optimal semi-active schedule in which the start time of an activity is either 0 or coincides with the completion time of another activity [20]. Therefore, at most $n + 1$ events have to be considered. They have the advantage of not depending on the time horizon, making them specially relevant for long time projects, as is the case.

  Koné proposed two different Event Based Formulations: the Start/End Event Based Formulation involves two types of binary variables, x_{we} and y_{we}, that are equal to 1 if task w starts (x_{we}) or ends (y_{we}) at event e and 0 otherwise.

  On the other hand, the On / Off Event Based Formulation involves a single type of binary variables z_{we}, that is equal to 1 if task w is active inmediately after event e.

4. APSP Specific Features

Despite the differences between the general RCSP and our APSP, all of the three formulations could be taken as a starting point for a APSP MILP formulation. Taking into account the advantages and disadvantages of the previous formulations, we have chosen a Event Based Formulation. Its main advantage is the fact that the size of the formulation does not depend on the time horizon, which is specially suitable for our long scheduling period problem.

Also, we have taken into account that Event Based Formulations have outperformed previous formulations and even a specific heuristic [9]. These formulations rely on the fact that, according to Sprecher [20], when a regular measure of performance is concerned, active schedules are the minimal set for optimal solutions. Also, they take into account that active schedules can be characterized with a limited number of events ($W+1$ for the SEE formulation and W for the OOE, being W the number of work tasks to be scheduled).

However, up to now, Event Based formulations had been applied for the resource constrained scheduling problem (RCSP). We have stated on section 2 that the APSP is not a RCSP but a Time Constraint Scheduling Problem. Also, the previous Event Based formulations ([9] and [19]) included only precedence constraints whereas we have included the use of generalized temporal constraints: maximal time lags and non-parallel constraints.

Therefore, before developping an Event Based Formulation for the APSP, we must proof that the fact that we address a TCSP instead of a RCSP and the introduction of general temporal constraints do not have an impact on the two main Event Based Formulation hypothesis: the existance of a regular measure of performance and the minimum number of events needed to guarantee an optimal solution. We will firstly review the regularity of our objective function, then the structure of the active schedules for the APSP.

4.1. Objective Function Regularity

Möhring [11] studied in 1984 a TCPS, where the objective function is to minimize the project cost. He proved that for TCSP the active schedules are also the smallest subset containing optimal solutions. After characterising the objective funtion as a regular measure, he established duality relations with the RCSP.

Later, Guldemond [12] refered to a TCSP with regular and overtime time windows, and for that problem he concluded the objective function was not a regular measure.

In order to characterise the APSP objective function, we will refer to the definition proposed by Sprecher [20]: *"Given a standard RCSP monomode, for each numerically labelled task w we define a finish time FT_w. A performance measure is then a maping which assigns to each W−tuple $(FT_1, ..., FT_w)$ of finish times a performance value $\phi(FT_1, ..., FT_w)$. If ϕ is monotically increasing with respect to componentwise ordering, that is:*

$$\phi(FT_1, ..., FT_w) \quad > \quad \phi(FT'_1, ..., FT'_w) \text{ implies:}$$
$$\forall w : FT_w \geq FT'_w$$
$$\exists w : FT_w > FT'_w$$

and in addition, minimization is considered, then we call the performance measure regular."

For the APSP, a solution must assign a set of resources to each work task. Given a feasible solution, for each time instant there will be a number of in progress work tasks, each of them consuming a number of renewable resources. If we number $1...K$ the available resources from a specific profile, O, each of the active work tasks will use a set of resources $\{k_i..k_j\}$. We will define $R^o_w = 1..K$ as the ordinal of the last resource of type O assigned to task w. Then, our performance measure $\phi = num^{op}_o$ is a regular performance measure as:

$$\phi(R^o_1, ...R^o_W) > \phi(R'^o_1, ...R'^o_W) \Rightarrow num^{op}_o > num'^{op}_o$$
$$\text{implies:}$$
$$\forall w : R^o_w \geq R'^o_w$$
$$\exists w : R^o_w > R'^o_w$$

Which means that, for reducing the total number of resources needed on the overall project at least one of the tasks has to be done with a lower ranged resource.

In fact, there is a relationship between the RCSP and the TCSP. In the RCSP, given a resource availability we wil search the shortest possible Takt Time, whereas in the TCSP we look for the lower resource consumption given a Takt Time. In both cases, the set of optimal solutions is the same. Figure 1 represents the minimum number of resources needed for several Takt Times. The first area on the left is the infeasibility area, where no solution is posible for the given Takt Time. On the middle area, the resource needs decrease as the Takt Time increases. Finally, on the area on the right, the resource needs are stable no matter how long is the Takt Time.

4.2. APSP Active Schedule Structure

Once our performance measure has been proven to be regular, we must proof that the structure of active schedules (which we are sure now that is the minimum dominant set) is suitable for the Event Based Formulation.

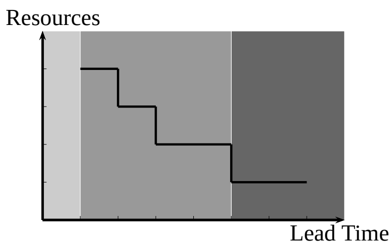

Figure 1: Relationship between the Minimum number of resources and the Project Lead Time

Schedules are called semi-active whenever no local left shift is posible for any ot its tasks and active if no local nor global left shift is posible. Moreover, if no local or global shift is posible even considering preemption the schedule is called a non-delay schedule [20].

Taking as an example a five tasks and one resource (R_1) scheduling problem (figure 2), we will see how the definitions of global / local left shifts, active and semi-active schedules apply to the APSP.

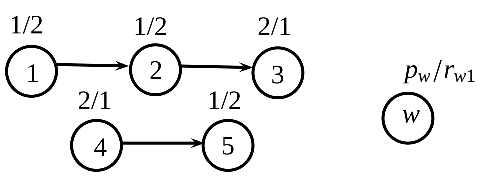

Figure 2: RCSP Instance

If we consider an instance with a five period platform Takt Time, $TT = 5$, the schedule on figure 3 is a feasible schedule. However, *task* 5 can be done by resources 2 and 3 instead of 3 and 4. Doing this left shift leads to the schedule of figure 4. On a next step, *task* 5 can be further left shifted, and done by ressources 1 and 2. Also, *task* 4 can have a global left shift, if it is done by resource 1, starting 2 periods later. Those last shifts result on the schedule on figure 5, which is an active schedule: tasks can no longer be left or global shifted in terms of ressource consumption. Note that when a task is left/globally shifted its starting time can be delayed, as far as the Takt Time restriction is not violated.

4.3. Minimum number of events needed

In this section, we will identify the minimal number of events needed to characterise the active schedules and, as a result, the minimal number of events that needs to be taken into account for solving the APSP.

For RCSP in any left-shifted schedule with finish-to-start precedence relations, with zero time lag, the start

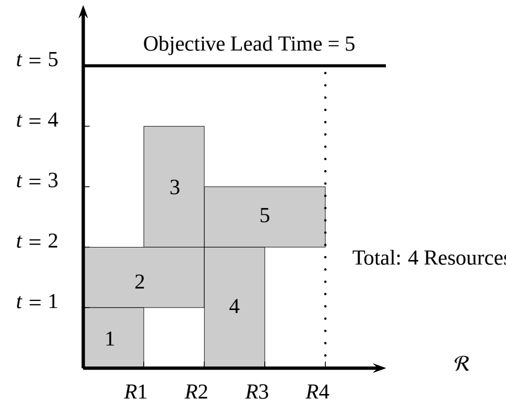

Figure 3: Feasible Schedule

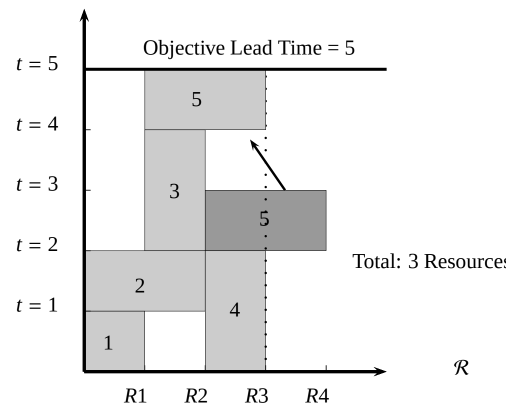

Figure 4: Local Left Shift

time of an activity is either 0 or coincides with the end time of some other activity [9].

As we have seen on the previous subsection and on figure 1, for a set of tasks and precedences, if the optimal solutions given an amount of resources R will lead to a platform Takt Time TT then the TCSP with Takt Time TT will lead to the same optimal solution, with R resources.

Therefore, the number of events needed for a TCSP Event Based formulation is the same that for a RCSP, given a number of tasks.

However, the original Event Based Model does not deal with non-parallel or maximal/minimal time lag constraints either. Therefore, we will now check the impact on this constraints on the number of events expected for an optimal solution.

Whenever we introduce non-parallel constraints, the minimum number of needed events does not go further than the preliminar $E = W + 1$ ($E' = W$), as the solution with all the tasks in series does not violate any possible

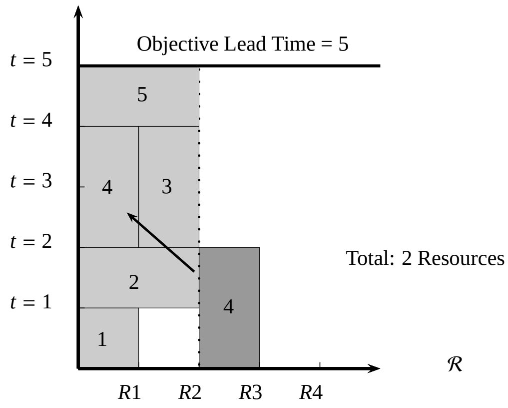

Figure 5: Active schedule

non parallel constraints. Anyway, the non-parallel constraints have to be taken into account as a stopper when expecting optimal solutions with less than $E = W + 1$ ($E' = W$) events. In this case, the number of tasks involved in non-parallel constraints will be a lower bound for the number of events.

As for maximal/minimal time lag constraints, if the time lag is equal to zero, there will be no need of more events. In the case of minimal time lags greater than zero, there is one task per constraint whose start time may be different form the ending time of all the other activities. Therefore, one extra event will be needed per minimal / maximal time lag contraints with nonzero time lag.

5. APSP Event Based Formulations

We will now propose two new Event Based Formulations for the APSP. They are inspired on the Start/End and On-Off Event Based Formulations presented by Koné [9].

Extension of previously existing event-based models to the APSP is not inmediate. The main variables x_{we}, y_{we} and z_{we} have been modified with two new subindex in order to deal with the multiple modes per task. As well as this, the original formulations included only general precedence constraints. New constraints and variables have been added in order to take into account the maximal time lag and non-parallel constraints. Finally, the objective function has been modified to takle with the time constraint approach. As in the other Event Based formulations, the number of events will be, at most, the number of scheduled tasks +1 + the number of tasks with nonzero time lag precedences.

Both formulations include four different sets. Set $\mathcal{W}$ includes the work tasks, set O the operator profiles and set $\mathcal{A}$ the platform working areas. Finally, due to the formulation chosen we will need a set $\mathcal{E}$ of events.

As well as this, each task can be performed by operators of different profiles, defined by P_{ow} (= 1 if task $w \in \mathcal{W}$ can be done by operators with profile o and 0 otherwise) and a range of posible number of operators, between MAX_w^{op} and MIN_w^{op}. The duration of a task w is defined by both its workload D_w and a reduction coefficient to obtain the task's w's makespan when it is done by p operators Γ_{pw}. Finally, the platform area it occupies is defined by $AREA_{aw}$ ($AREA_{aw} = 1$ if task w belongs to area a).

The different relations between two tasks will be expressed by parameters $PRE_{ww'}$, $NONP_{ww'}$, $MTL_{ww'}$, which will be equal to 1 if there is a precedence / non-parallel / maximal time lag constraint between tasks w and w'.

To end with, we must take into account two platform defining characteristics: the capacity per area, CAP_a, and the expected Takt Time, TT.

5.1. APSP Start End Event Based Multimode Formulation: SEE-M

For our APSP Start End Event Based Multimode Formulation (SEE-M), the sets of variables x_{weop} and y_{weop} are used to define the start and end events of each tasks, as $x_{weop} = 1$ ($y_{weop} = 1$) if task w starts (ends) at event e using p operators of profile o.

On top of this, additional constraints have been added to guarantee the same asignation of operator profiles and number throughout the task fullfilment. Also, for tasks involved on some non-parallel constraint ($w \in \mathcal{W}$ and $\sum_{w'} MTL_{ww'} + MTL_{w'w} > 0$) we need to define a continuous variable, t_w^i which represents the starting time of task w. Another new set of variables, num_o^{op}, represents the total number of operators of profile o needed. All the model variables are listed on table 1.

The complete APSP SEE-M formulation is as follows:

$$Minimize \sum_o num_o^{op} \tag{1}$$

Subject to:

Table 1: APSP SEE-M Variables

x_{weop} :	1 if work task w starts at event e with p operators of profile o and 0 otherwise; $\forall w \in \mathcal{W}, e <> last(\mathcal{E}), MIN_w^{op} \leq p \leq MAX_w^{op}, \forall o/P_{ow} = 1$
y_{weop} :	1 if work task w ends at event e with p operators of profile o and 0 otherwise; $\forall w \in \mathcal{W}, e <> first(\mathcal{E}), MIN_w^{op} \leq p \leq MAX_w^{op}, \forall o/P_{ow} = 1$
r^*_{oe} :	Number of operators of type o required by the tasks in progress immediately after event e, $\forall o \in \mathcal{O}, e \in \mathcal{E}$
s^*_{ae} :	Number of operators on zone a required by the tasks in progress immediately after event e, $\forall a \in \mathcal{A}, e \in \mathcal{E}$
$\alpha_{ww'}$:	1 if w ends before w' starts and 0 viceversa. Defined $\forall w, w'/NONP_{ww'} = 1$
t_e :	Time of event e
num_o^{op}:	Total number of operators of profile o needed, $o \in \mathcal{O}$
t_w^i :	Defines the starting time of task w. This will be used for maximal time lag constraints, and therefore defined $\forall w \in \mathcal{W}$ and $\sum_{w'} MTL_{ww'} + MTL_{w'w} > 0$

$$t_0 = 0 \tag{2}$$

$$t_e \leq TT \ \forall e \neq \{0\} \tag{3}$$

$$t_{e+1} - t_e \geq 0 \ \forall e \neq last(e) \tag{4}$$

$$\sum_{eop} e y_{weop} - \sum_{eop} e x_{weop} \geq 1 \quad \forall w \in \mathcal{W} \tag{5}$$

$$\sum_{eop} x_{weop} = 1 \ \forall w \in \mathcal{W} \tag{6}$$

$$\sum_{eop} y_{weop} = 1 \ \forall w \in \mathcal{W} \tag{7}$$

$$t_f - t_e \geq \sum_o D_w \Gamma_{pw} x_{weop} + (D_w \Gamma_{pw})(1 - \sum_o y_{wfop})$$
$$\forall (f, e) \in \mathcal{E}, lf > e, w \in \mathcal{W}, p \in (MIN_w^{op}, MAX_w^{op}) \tag{8}$$

$$\sum_{eo} x_{weop} = \sum_{eo} y_{weop}$$
$$\forall w \in \mathcal{W}, MIN_w^{op} \leq p \leq MAX_w^{op} \tag{9}$$

$$\sum_{ep} x_{weop} = \sum_{ep} y_{weop} \ \forall w \in \mathcal{W}, o \in \mathcal{O} \tag{10}$$

$$\sum_{\substack{e''=0\\o,p}}^{e-1} x_{w'eop} + \sum_{\substack{e'=e\\o,p}}^{E} y_{we'op} \leq 1$$
$$\forall e \in \mathcal{E}, (w, w') \in \mathcal{W}/PRE_{ww'} = 1 \tag{11}$$

$$t_w^i \geq t_e - M(1 - \sum_{op} x_{weop})$$
$$\forall w \in \mathcal{W} \ / \ \sum_{w'} (MTL_{ww'} + MTL_{w'w}) > 0 \tag{12}$$

$$t_w^i \leq t_e + M(1 - \sum_{op} x_{weop})$$
$$\forall w \in \mathcal{W} \ / \ \sum_{w'} (MTL_{ww'} + MTL_{w'w}) > 0 \tag{13}$$

$$t_{w'}^i - t_w^i - \sum_{eop} \Gamma_{wp} D_w x_{weop} \leq \Delta$$
$$\forall (w, w') \in \mathcal{W} \ / \ MTL_{ww'} = 1 \tag{14}$$

$$\sum_{eop} e y_{weop} - \sum_{eop} e x_{w'eop} \leq M(1 - \alpha_{ww'})$$
$$\forall (w, w') \in \mathcal{W}/NONP_{ww'} = 1 \tag{15}$$

$$\sum_{eop} e y_{w'eop} - \sum_{eop} e x_{weop} \leq M(\alpha_{ww'})$$
$$\forall (w, w') \in \mathcal{W}/NONP_{ww'} = 1 \tag{16}$$

$$r^*_{o0} - \sum_{wp} p x_{w0op} = 0 \ \forall o \in \mathcal{O} \tag{17}$$

$$r^*_{oe} - r^*_{oe-1} + \sum_{\substack{w\\P_{ow}=1}} (\sum_p p y_{weop} - \sum_p p x_{weop}) = 0$$
$$\forall o \in \mathcal{O}, e \in \mathcal{E} - \{0\} \tag{18}$$

$$r^*_{oe} \leq num_o^{op} \ \forall o \in \mathcal{O}, e \in \mathcal{E} \tag{19}$$

$$s^*_{a0} - \sum_{wop} p x_{w0op} AREA_{aw} = 0 \ \forall a \in \mathcal{A} \tag{20}$$

$$\sum_w (\sum_{op} p y_{weop} AREA_{aw} - \sum_{op} p x_{weop} AREA_{aw}) =$$
$$s^*_{ae-1} - s^*_{ae}$$
$$\forall a \in \mathcal{A}, e \in \mathcal{E} - \{0\} \tag{21}$$

$$x_{ewop} \in \{0, 1\} \ \ \forall e \in \mathcal{E}/\{0\}, \ w \in \mathcal{W},$$
$$o \in \mathcal{O}/P_{ow} = 1, MIN_w^{op} \leq p \leq MAX_w^{op} \tag{22}$$

$$y_{ewop} \in \{0, 1\} \ \forall e \in \mathcal{E}/\{last(\mathcal{E})\}, \ w \in \mathcal{W},$$
$$o \in \mathcal{O}/P_{ow} = 1, MIN_w^{op} \leq p \leq MAX_w^{op} \tag{23}$$

$$r^*_{oe} \geq 0 \ \forall e \in \mathcal{E}, \ o \in \mathcal{O} \tag{24}$$

$$0 \leq s^*_{ae} \leq CAP_a \ \forall e \in \mathcal{E}, a \in \mathcal{A} \tag{25}$$

$$t_e \geq 0 \ \forall e \in \mathcal{E}, (w, w') \in \mathcal{W} \ /\alpha_{w,w'} \in \{0, 1\} \tag{26}$$

The objective function, (1) is to minimise the total project cost. This cost depends only on the labor costs, which are proportional to the maximum number of operators needed during the planning horizon, as resources allocated to a work station will remain in it throughout the complete Takt Time. Constraint (2) forces the first event to begin at t=0 and constraint (3) assures that there is no delay in the platform completion. The order of the events on time is imposed by constraint (4). Constraint (5) states that the start event of a task must precede its end event. Constraints (6) and (7) limit to one the start and end event per work task. Constraint (8) fixes the minimum time difference between the start and the end events to the duration of the task.

A single mode for performing the task is imposed by constraints (9) and (10). Note that the possible modes are limited by the definition of x_{weop} and y_{weop} as they are only defined for $MIN_w^{op} \leq p \leq MAX_w^{op}$ and $\forall o/P_{ow} = 1$, see (22) and (23).

As for the relations between tasks: (11) is the multimode expression for the precedence constraints. It states that if w must precede w' then, if w finishes at event e, w' cannot start until event $e+1$. Maximal time lags are imposed on constraints (12) to (14). Constraints (12) and (13) define the start time of a task. This constraints will only be calculated for the task involved on maximal time lag constraints Number (14) limits the time between the end of a task and the start of its succesor. This time lag is defined as a constant parameter Δ but it can be easily replaced by a task dependent parameter: $\Delta_{ww'}$. In these constraints, M (big enough number) could be replaced by the platform's Takt time, TT. Constraints (15) and (16) define the non-parallel constraints. In this constraints, M (big enough number) could be replaced by the number of events.

As for the resource consumption, (17) is the number of operators needed per profile inmedately after the first event. Constraint (18) deals with the number of operators per profile, o, for all the other events. Constraint (19) calculates the maximum need of operators per profile allover the events. Similarly, constraints (20) and (21) assure that the maximum occupation in the platform working areas is not exceeded.

Formulation SEE-M involves $2(W+1)W\sum_w((1+MAX_w^{op}-MIN_w^{op})\sum_o P_{ow}) + |NONP|$ binary variables and $(W+1)(O+A+1)+MTL_w+O$ continuous variables. Compared to the single-mode with only general precedence constraints, that had $2W(W+1)$ binary variables, we need $2W(W+1)$ extra variables per extra mode of a variable, as well as one more binary variable per non-parallel relation. However, the number of binary variable remains polynomial. Continuous variables grow by $W+1$ per each area and operator profile and by one for each MTL constraint and profile.

Table 2: APSP OOE-M Variables

z_{weop}	:	1 if work task w is active from event e to event $e+1$ with p operators of profile o , 0 otherwise; $\forall e \in \mathcal{E}, w \in \mathcal{W}, o \in \mathcal{O}/P_{ow} = 1, MIN_w^{op} \leq p \leq MAX_w^{op}$
β_{wop}	:	1 if work task w is performed by p operators of profile o, 0 otherwise; $\forall w \in \mathcal{W}, o \in \mathcal{O}/P_{ow} = 1, MIN_w^{op} \leq p \leq MAX_w^{op}$
t_e	:	Time of event e
num_o^{op}	:	Total number of operators of profile o needed, $o \in \mathcal{O}$
t_w^i	:	Defines the starting time of task w. This will be used for maximal time lag constraints, and therefore defined $\forall w \in \mathcal{W}$ and $\sum_{w'} MTL_{ww'} + MTL_{w'w} > 0$

5.2. APSP On-Off Event Based Multimode Formulation: OOE-M

The other Event Based Formulation we have implemented for the APSP deals with a single set of variables, z_{weop}, whose value is 1 if work task w is active from event e to event $e+1$ with p operators of profile o and 0 otherwise. Another new set of variables, β_{wop} will be used to choose the mode in which a task is performed. As resources are modeled in a simpler way, r_{oe}^* and s_{ae}^* are no longer needed. There is no need either for variables $\alpha_{ww'}$ for the non-parallel constraints. Variables, t_e, num_o^{op} and t_w^i are common to the SEE-M model. All the OOE-M variables are listed on table 2.

Given this variables, the OOE-M formulation is:

$$Minimize \sum_o num_o^{op} \tag{27}$$

Subject to:

$$t_0 = 0 \quad (28)$$

$$\sum_{wop} z_{w0op} \geq 1 \quad (29)$$

$$t_{e+1} - t_e \geq 0 \ \forall e \neq \text{first}(\mathcal{E}) \quad (30)$$

$$\sum_{eop} z_{weop} \geq 1 \ \forall w \in \mathcal{W} \quad (31)$$

$$\sum_{\substack{o \\ MIN_w^{op} \leq p \leq MAX_w^{op} o}} \beta_{wop} = 1 \ \forall w \in \mathcal{W} \quad (32)$$

$$z_{weop} \leq \beta_{wop} \forall w \in \mathcal{W}, o \in \mathcal{O}, e \in \mathcal{E}, \\ MIN_w^{op} \leq p \leq MAX_w^{op} \quad (33)$$

$$\sum_{e' \leq e-1} \sum_{op} z_{we'op} - e(1 - \sum_{op} (z_{weop} - z_{we-1op})) \leq 0 \\ \forall w \in \mathcal{W}, e \in \mathcal{E} \quad (34)$$

$$\sum_{e'=e}^{E-1} \sum_{op} z_{we'op} \leq (E - e)(1 + \sum_{op} (z_{weop} - z_{we-1op})) \\ \forall w \in \mathcal{W}, e \in \mathcal{E} \quad (35)$$

$$t_f - t_e + \sum_{op} DUR_w \Gamma_{pw} \beta_{wop} \geq \\ \sum_{op} DUR_w \Gamma_{pw} \beta_{wop} (z_{weop} - z_{we-1op} - (z_{wfop} - z_{wf-1op})) \\ \forall (f, e) \in \mathcal{E}, f > e, w \in \mathcal{W} \quad (36)$$

$$TT - t_e - \sum_{op} DUR_w \Gamma_{pw} (z_{weop} - z_{we-1op}) \geq 0 \\ \forall w \in \mathcal{W}, e \in \mathcal{E} \quad (37)$$

$$\sum_{o,p} z_{weop} + \sum_{e'=0}^{e} \sum_{op} z_{w'e'op} - (e - 1)(1 - \sum_{op} z_{weop}) \leq 1 \\ \forall e \in \mathcal{E}, (w, w') \in \mathcal{W}/PRE_{ww'} = 1 \quad (38)$$

$$t_w^i \geq t_e - M(1 + z_{we-1op} - z_{weop}) \quad (39)$$

$$t_w^i \leq t_e + M(1 + z_{we-1op} - z_{weop}) \\ \forall e \in \mathcal{E}, w \in \mathcal{W}/ \sum_{w'} (MTL_{w,w'} + MTL_{w',w}) \geq 1 \quad (40)$$

$$t_{w'}^i - (t_w^i + \sum_{op} \beta_{op} DUR_w \Gamma_{wp}) \leq \Delta \\ \forall (w, w') \in \mathcal{W}/MTL_{w,w'} = 1 \quad (41)$$

$$\sum_{op} z_{weop} + \sum_{op} z_{w'eop} \leq 1 + M(1 - NONP_{ww'}) \\ \forall (w, w') \in \mathcal{W}/NONP_{w,w'} = 1 \quad (42)$$

$$\sum_{wp} pz_{weop} \leq num_o^{op}, \ \forall o \in \mathcal{O}, e \in \mathcal{E} \quad (43)$$

$$\sum_{wop} pz_{weop} AREA_{aw} \leq CAP_a, \ \forall a \in \mathcal{A}, e \in \mathcal{E} \quad (44)$$

$$z_{ewop} \in \{0, 1\} \\ \forall e \in \mathcal{E}, w \in \mathcal{W}, o \in \mathcal{O}, MIN_w^{op} \leq p \leq MAX_w^{op} \quad (45)$$

$$z_{w-1op} = 0 \\ \forall w \in \mathcal{W}, o \in \mathcal{O}, MIN_w^{op} \leq p \leq MAX_w^{op} \quad (46)$$

$$\beta_{wop} \in \{0, 1\} \\ \forall w \in \mathcal{W}, o \in \mathcal{O}, MIN_w^{op} \leq p \leq MAX_w^{op} \quad (47)$$

$$t_e \geq 0 \ \forall e \in \mathcal{E} \quad (48)$$

$$t_w^i \geq 0 \ \forall w \quad (49)$$

The objective function is to minimise the total project cost (27). The first event on the project starts at $t = 0$ per constraint (28) and at least one task must be active after this event as per (29). Constraint (30) refers to the order of the events, allowing two of them to occur at the same time.

Each task must be active at least after one of the events (31) in order to assure the scheduling of all the tasks. Variables β_{wop} select the mode in which each task will be performed. Only one of the variables β_{wop} can be set to 1 per task, constraint (32) and the tasks can only be performed on the selected mode, as per (33).

Constraints (34) to (37) are based on the three values than can take the difference $z_{weop} - z_{we+1on}$:

- $z_{weop} - z_{we+1op}$= -1. When $e+1$ is the first event after which w is active, so $z_{weop} = 0$ and $z_{we+1op} = 1$.
- $z_{weop} - z_{we+1op}$=1. When e is the last event after which w is active, so $z_{weop} = 1$ and $z_{we+1op} = 0$.
- $z_{weop} - z_{we+1op}$=0. Otherwise

Constraints (34) and (35) refer to the continuous processing of each task: by (34) if task w begins after event e, then it can not be processed before $e - 1$. Similarly, by (35) if task w ends at event e then w is no longer active $\forall e' \geq e + 1$. The time of a task is measured by the difference between the start event (the first event e after which w is active) and the end event (the first the last event after which w is active).

The time difference between w's start event and its end event must be at least the work task's processing time (36) and none of the tasks can end after the platform Takt Time, see (37).

Regular precedence constraints are (38), as if w must precede w', then it must start at an event after which w is no more active. Maximal time lags are expressed by constraints (39) to (41). Constraints (39) and (40) define the start time of a task. This constraints will only be calculated for the tasks involved on maximal time lag constraints. Together with them, (41) limits the time between the end of a task and its succesor's start time. This time lag is defined as a unique parameter Δ but if needed it can be easily transformed on a parameter dependent on the pair of tasks, $\Delta_{ww'}$. Non-parallel constraints are (42), as two non-parallel tasks cannot be active at the same time.

Ressource constraints are simpler than for the SEE-M formulation. In this case, only one set of constraints is defined per scarce resource: (43) for the quantity of operators per type and (44) for the amount of operators per area, and no specific variables are defined for these constraints.

This formulation involves: $W^2 \sum_w ((1 + MAX_w^{op} - MIN_w^{op}) \sum_o P_{ow})$ event binary variables (z_{weop}) and $W \sum_w ((1 + MAX_w^{op} - MIN_w^{op}) \sum_o P_{ow})$ extra binary variables for the mode selection (β_{wop}). On all, there are $(W^2 + W) \sum_w ((1 + MAX_w^{op} - MIN_w^{op}) \sum_o P_{ow})$ binary variables, less than half the number of the ones needed for the SEE-M.

Also, there are $W + MTL_w + O$ continuous variables, corresponding to t_e, t_w^i and num_o^{op}. In this case, there are $(O + A)(W + 1)$ variables less than in the SEE-M formulatiom as no extra variables are needed for the resource constraints.

As for the number of constraints, for precedence and maximal time lag constraints, both models are similar. The SEE-M has twice as constraints as the OOE-M for defining the non-parallel constraints. On all, it is not possible to establish a general dominance rule.

6. Computational Results

As the standard PSPLIB instances are not valid for the structure of the problem, four new sets of 8 task instances were used. Moreover, Sets 3 and 4 were extended in order to create instances of up to 11 tasks. Their characteristics are listed on table 3.

On all, 75 different combinations of data sets, number of tasks, Takt Time and number of events were tested for each formulation. The detailed instances, together with the whole computational results are available on [21].

All instances were solved up to optimality for both formulations. For SEE-M formulation, the instances took times from seconds to fifteen minutes and from 0,1 seconds to eight minutes for the OOE-M formulation. The harder instance to solve was Set3-11, that took up to 4133 seconds for a $TT = 17$ days and 11 events on the SEE-M formulation and 452 on the OOE-M formulation.

Table 3: Instance Characteristics

Set	Tasks	Prec	MTL	NONP	O	$\mathcal{A}$	Modes
Set1-8	8	6	1	1	2	2	12
Set2-8	8	8	1	1	2	2	16
Set3-8	8	7	1	1	2	2	17
Set3-9	9	8	1	1	2	2	18
Set3-10	10	9	1	1	2	2	18
Set3-11	11	10	1	1	2	2	18
Set4-8	8	7	1	1	2	2	16
Set4-9	9	8	1	1	2	2	16
Set4-10	10	9	1	1	2	2	16
Set4-11	11	10	1	1	2	2	16

Both formulations have similar behaviour as far as the impact of variations on the number of events, number of tasks and Takt Time on the solution time. As for the evolution of the solution time throughout different Takt Times, on average the solution time also grew as the objective Takt Time got closer to the Critical Path Lead Time. Table 4 shows an example of this evolution for each of the formulations.

Table 4: Sample Solving Time for Different Takt Times

	Instance	LT 31.5	LT 33	LT 34.75	LT 41
SEE-M	Set2-8	10.2s	6.65s	1.79s	0.83s
OOE-M	Set2-8	4.66s	0.83s	0.47s	0.2s

Finally, focusing on the influence of the number of tasks, most of the instances required more solution time with the same number of events when new tasks are added. Withall, some were solved faster with more tasks. This shows that in some cases the structure of the problem is more important than the number of tasks itself. We also stated that the hardening of the instances as we add new tasks is wider on the cases where we are using more events.

However, the major impact on the models' performance is related to the number of events used to solve an instance. For each set and Takt Time, different number of events were tested. Starting from the theoretical minimum number of events, they were reduced until solutions where no longer optimal. The solution time decreased exponentially with the number of events, even when solving the same set of instances.

Figure 6 shows the evolution of the SEE-M solution times for some of the instances whenever the number of events changed. On this figure the series data include information on the data set, the number of tasks and the input Takt Time: Set1-8-11.5 stands for the solution of data set 1, with 8 tasks and a TT=11.5 days. For example, for the SEE-M formulation the first Set (Set1-8), when solved for a $TT = 11,5$ days took from 2 to 281 seconds, depending on the number of events. Plotting the results for the OOE-M leads to a similar figure.

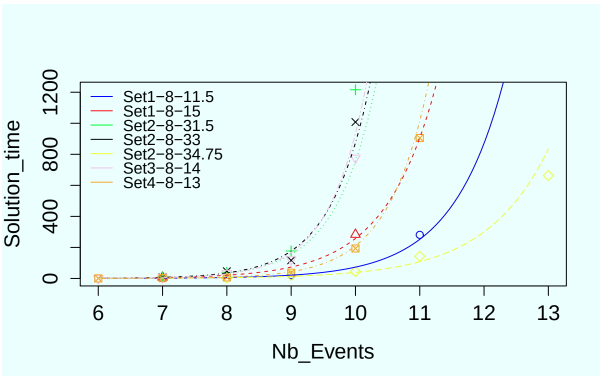

Figure 6: SEE-M Solution Time Vs number of Events

The event reduction not only lowers the solution time but also improves the quality of he linnear relaxation. The First Lower Bound (FLB), that is, the value of the firts solution for the Branch and Bound, is higher as the number of events goes down. In some cases, the integrality gap is reduced also by an improvement of the First Integer Solutions (FIS). However, the improvement on the First Integer Solution does not always happen. For example, with the SEE-M formulation, Set2-8 with $TT = 33$, if solved with 7 events has a First Lower Bound $FLB = 1,84$ and First Integer Solutions $FIS = 7$. If for the same set we use 10 events, the first solutions are $FLB = 0,29$ and $FIS = 10$. In fact, with 10 events, the number of explored nodes is 136 times bigger than with 7 events (405935 vs 2981).

Tables 5 and 6 include the main information about this different performance depending on the number of events for some of the tested instances. On the one hand, Table 5 refers to the values of the First Lower Bound (FLB) and the First Integer Solutions (FIS). On the other hand, Table 6 includes the information on the number of nodes explored before achieving the optimal solution together with the solution time (in seconds).On both of them, S stands for the SEE-M formulation and O for the OOE-M.

Taking into account that in most of the instances the precedence graph had at least two parallel paths, the possibility of event reduction could have been foreseen.

Table 5: FLB and FIS for different Number of Events

Inst	TT	E	Sol	FLB S	FIS S	FLB O	FIS O
Set1-8	11,5	7	8	2,3	10	2,4	10
Set1-8	11,5	9	8	1,92	9	2,29	10
Set1-8	11,5	11	8	1,63	11	2,22	10
Set2-8	34,75	7	5	1,84	7	3,6	7
Set2-8	34,75	8	5	1,19	10	3,5	7
Set2-8	34,75	9	5	0,64	11	3,43	6
Set2-8	34,75	10	5	0,296	9	3,34	6
Set2-8	34,75	11	5	0,07	8	3,33	7
Set2-8	34,75	13	5	0	10	3,3	5
Set3-8	14	6	6	2,8	8	2,58	7
Set3-8	14	7	6	1,7	8	2,14	7
Set3-8	14	8	6	1,13	7	1,85	7
Set3-8	14	9	6	0,69	7	1,64	7
Set3-8	14	10	6	0,36	8	1,47	8
Set3-9	17	8	5	1,18	7	2	6
Set3-9	17	9	5	0,73	7	1,78	5
Set3-9	17	10	5	0,38	7	1,6	6
Set3-9	17	11	5	0,08	9	1,45	11
Set3-10	17	8	5	1,64	11	2,25	7
Set3-10	17	9	5	1,07	11	2	8
Set3-10	17	10	5	0,67	8	1,8	9
Set3-10	17	11	5	0,36	7	1,63	7
Set3-11	17	8	6	1,93	9	2,5	9
Set3-11	17	9	6	1,31	9	2,22	8
Set3-11	17	10	6	0,86	9	2	8
Set3-11	17	11	6	0,54	12	1,82	7
Set4-8	13	6	7	3,05	7	2,5	9
Set4-8	13	7	7	2,14	8	2,14	8
Set4-8	13	8	7	1,43	8	1,88	7
Set4-8	13	9	7	0,92	9	1,67	9
Set4-8	13	10	7	0,55	9	1,5	8
Set4-8	13	11	7	0,26	9	1,36	8
Set4-9	13	8	7	1,57	8	2	8
Set4-9	13	9	7	1,04	8	1,78	8
Set4-9	13	10	7	0,65	8	1,6	8
Set4-10	13	8	8	1,87	9	2,25	9
Set4-10	13	9	8	1,29	10	2	8
Set4-10	13	10	8	0,86	10	1,8	8
Set4-11	13	8	8	2,02	8	2,38	9
Set4-11	13	9	8	1,4	9	2,11	9
Set4-11	13	10	8	0,94	10	1,9	8

Table 6: Number of Nodes and Solution time for different Number of Events

Inst	TT	E	Nodes S	t(s) S	Nodes O	t(s) O
Set1-8	11,5	7	1546	1,97	478	0,67
Set1-8	11,5	9	11203	21,09	1918	1,93
Set1-8	11,5	11	105195	281	10688	12,68
Set2-8	34,75	7	1180	1,79	141	0,47
Set2-8	34,75	8	1912	3,39	93	0,58
Set2-8	34,75	9	6926	19,66	630	1,45
Set2-8	34,75	10	10619	44,69	276	1,11
Set2-8	34,75	11	28112	142,52	177	1,14
Set2-8	34,75	13	75825	663,27	150	1,45
Set3-8	14	6	224	0,48	837	0,87
Set3-8	14	7	4631	6,35	2207	2,14
Set3-8	14	8	23407	34,4	8593	8,24
Set3-8	14	9	94129	121,98	21865	19,31
Set3-8	14	10	506132	776,76	261475	181,07
Set3-9	17	8	1799	4,87	110	0,83
Set3-9	17	9	5214	27,5	0	1,2
Set3-9	17	10	20731	100,5	154	1,67
Set3-9	17	11	24707	138,72	1128	3,95
Set3-10	17	8	1781	4,56	3087	3,93
Set3-10	17	9	14574	46,11	3852	7,86
Set3-10	17	10	17275	69,78	520	2,96
Set3-10	17	11	68836	270,86	977	3,4
Set3-11	17	8	5662	16,21	4081	5,16
Set3-11	17	9	29692	66,94	20542	22,99
Set3-11	17	10	183981	586,13	82736	82,21
Set3-11	17	11	1233159	4133	467471	452,53
Set4-8	13	6	112	0,3	408	0,65
Set4-8	13	7	1957	2,2	1819	2,06
Set4-8	13	8	3341	6,85	3460	3,82
Set4-8	13	9	29076	36,58	16935	17,18
Set4-8	13	10	127109	193,25	53020	45,66
Set4-8	13	11	422579	905,13	25773	29,28
Set4-9	13	8	5250	9,83	7490	7,83
Set4-9	13	9	20828	38,02	9479	11,18
Set4-9	13	10	162585	285,26	46245	47,58
Set4-10	13	8	2940	7,38	20033	18,55
Set4-10	13	9	75062	107,41	39242	46,15
Set4-10	13	10	151881	269,79	101903	112,63
Set4-11	13	8	16314	28,2	13620	13,07
Set4-11	13	9	108599	149,5	14976	16,29
Set4-11	13	10	202190	473,54	71714	76,32

Another important factor is the relationship between the shortest possible Takt Time and the objective Takt Time, as a tighter Takt Time would lead to more tasks to be performed in parallel and therefore less events to be needed.

As for the comparison between the two formulations, the results in terms of solution time, number of nodes and first lower bound have been better for all the instances with the OOE-M formulation than with the SEE-M formulation. Figure 7 shows the histogram for the division of the time spent for a solution with the OOE-M formulation between the time spent by the SEE-M formulation. The only two cases where the solution time is longer for the OOE-M formulation are instances with solution times within the range of 0,5 seconds, where the absolute difference is not relevant.

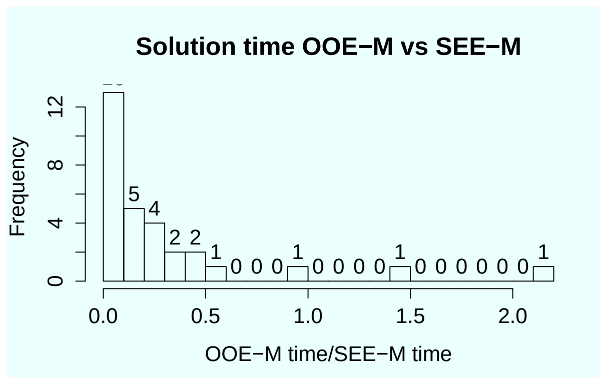

Figure 7: OOE-M Solution Time / SEE-M Solution Time

In fact, the difference between both formulations grows with the number of events. Therefore, as the complexity of the instances grows the use of the OOE-M formulation becomes more suitable. It is also important to rememember that, although the comparison has been made between instances with the same number of events, the OOE-M formulation is always capable of calculating a solution using one event less than the SEE-M formulation. As a result, the performance difference between both formulations is even greater.

These comparative results are coherent with the results of Koné [9] for the mono-mode problem with only precedence constraints, who concluded that the OOE outperformed the SEE formulation for all the instance sets. As stated by [22], a good formulation must have two characteristics: its tightness and its compactness. The tightness refers to the quality of its approximation to the convex hull of integer feasible solutions and the compactness is a measure of the number of variables and constraints that are used to formulate a given problem. The OOE-M is better than the SEE-M formulation in both aspects.

7. Conclusions

In this work, we have identified a knowledge gap related to the RCSP applicable for the scheduling of aeronautical assembly plaforms (or, the Aeronautical Platform Scheduling Problem, APSP). After a deep litterature review, where we have gone through the main existing models for the more general RCSP, we have chosen to develop an Event Based Formulation for our problem. To do so, we have dealt with general temporal constraints, maximal and minimal time lags and multimode scheduling. This problems had been rarely addressed on their own in the existing litterature and no exact method references have been found dealing with them together. Due to this features, the two new formulations we have developped are a contribution not only for the aeronautical industry but also for general scheduling.

The computational results from section 6 have proven that the model is able to solve up to optimality small instances. As well as this, they have enabled as to make a comparative study between the two event based formulations: SEE-M and OOE-M formulations. The results of this comparations are coherent with the ones reported by Koné for the single mode with only precedence constraints model, [9].

However, in order to extend it to bigger instances, it is necessary to improve the solver performances. Both SEE-M and OOE-M formulations have a number of constraints that grows as ($O(E^2W)$ where E is the number of events and W the number of work tasks to be scheduled. From the computational results we know that most of the instances can be solved with less events than the established upper bound of $W + 1$ for SEE-M and W for OOE-M. Therefore, the use of pre-processing to calculate the real needed number of events will lead to major performance improvements.

In that sense, a future research direction is the definition of a set of indicators that will help us for the characterisation of APSP instances and as a result, to the developpment of more suitable and efficient preprocessing techniques. Those indicators should be useful for a pre-processing including a more accurate definition of the number of events. Other future research opportunities include the improvent of the solution procedures by the introduction of decomposition or column generation techniques, as well as hybrid procedures including scheduling heuristics or metaheuristics.

8. References

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