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An Experimental Investigation of Resource Allocation in Multiactivity Projects

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This paper describes a computer-based experiment in the scheduling of multi-activity projects with limited resources. The main interest lay in a comparison of the traditional sorting criteria used to allot priority to activities competing for resources inadequate to satisfy all the demands and criteria newly defined for this work. One aim was to isolate, if possible, a criterion equally as good as the Critical Path parameters usually employed and yet easier to compute in practice. A similar experiment was carried out by Pascoe² but none of the criteria used satisfied the second condition. It was also possible to monitor the effects on the scheduling of network parameters such as shape and connectivity.

INTRODUCTION

CRITICAL PATH ANALYSIS is applicable to any multiactivity project which can be represented diagrammatically by a network of ordered arrows. Each arrow denotes one job or activity with a stated duration required for its execution and the project logic is ensured by the network configuration. Under the assumption of the availability of unlimited resources this method computes for each activity its earliest and latest start and finish times. For each activity the difference between the earliest start and latest finish times defines a time envelope during which it must be completed. The difference between this time and its duration measures the total float for the activity: this is the amount of time that it can be held up without causing delay to succeeding activities. Critical activities are those with zero total float defining the Critical Path(s) from the start to the end of the project. Their summed durations along any one path gives the overall duration which is the minimum time in which the project can be carried out.

But in practice there are limitations on the quantities of resources available and there arise in the scheduling process points of obstruction at which the resources then free (i.e. not in use for other activities currently in progress) are insufficient to meet the demands of all the activities logically free to start, all those preceding them having been completed. At such a point the conflicting activities must be sorted into an order of priority. The sorting criteria traditionally employed have been based on values of the Critical Path parameters. For instance, to give priority to that activity with the least total float would

obviously be valid as it would minimize delay to critical and near-critical activities. But the use of these criteria ideally involves constant updating of the Critical Path analysis as delay builds up, and they do not reflect the resource demand aspect of the project which is the cause of the delay. For these reasons new criteria were defined and their efficiency compared with the Critical Path ones. To be acceptable any new criterion would have to be equally as good and more easy to compute.

The sorting heuristic will not be described in detail: a parallel method was used in which all the activities free to start were resorted with reference to the ruling criterion and as many as possible satisfied within the resource limitation.

THE NETWORKS AND NETWORK PARAMETERS

In order to obtain data amenable to a statistical analysis all the networks used to simulate projects were generated by computer. Generation was random except that they were constrained to have single start and end events (to facilitate comparison) and to be not unconnected.

Each had 16 nodes, a limitation imposed by computer storage and running time, and fixed values of the following parameters:

DUM—the percentage of dummy activities (required by the project logic) with zero duration and, necessarily, zero resource demand was in the range 0–20.

CC—the complexity (or connectivity) of the network was defined by:

$$CC = \frac{2(AC - N + 1)}{(N - 1)(N - 2)}, \quad 0 < CC \leq 1,$$

where AC = number of activities and N = number of nodes. Values of CC in the range 0.1–0.5 gave rise to networks ($N = 16$) with 25–68 activities. Higher values of CC would seem unrealistic.

DENS—the density of a network is a measure of the free float under Critical Path conditions. Free float can absorb delay; only when the free float along a chain is exceeded by the delay is the overall duration increased. If activity duration and free float are denoted by d and ff then

$$DENS = \frac{\sum_{i=1}^{AC} d_i}{\left(\sum_{i=1}^{AC} d_i + \sum_{i=1}^{AC} ff_i \right)}$$

and lies in the range 0.5–0.9.

PRES—this was not a parameter related to the topography of the network but reflected the degree of resource limitation. It stated the proportion of the expected maximum demand for resources which was actually available, the expected maximum demand being the product of the average activity demand and the maximum number of activities (omitting the dummies) in parallel, i.e. which could theoretically be in progress simultaneously and in competition

for the same resources. PRES took values from 0.1 to 0.5 and gave rise to the actual number of units of resource available for each network. To avoid extraneous random effects the resource availability was held constant throughout a project though in practice idle resources tend to be deployed elsewhere.

The above ranges of values were chosen after several preliminary runs of the program.

It did not prove possible to dictate the value for the SHAPE parameter; this was calculated after generation for each network.

Varying distributions, rectangular and several truncated binomial ones, for the activity durations and resource requirements per activity were introduced but the results were not meaningful. The resource demand by an activity was constant over its duration; variation could be simply achieved by subdivision of the activity.

Six hundred and forty-eight networks were analysed, each in both directions and each direction with either no or all activities liable to division into no more than two parts (i.e. an activity in progress could be interrupted in favour of one of higher priority and the remaining duration of the first returned to the queue). The choice of either no or all activities subject to division was made to avoid yet another random effect. No activities were “tied” in pairs though the program written in Algol will encompass this. The total number of runs exceeded 36,000.

SORTING CRITERIA

Priorities based on the Critical Path analysis which are functions of activity duration only were included. Ones new to this experiment were designed to reflect the resource demand and cost of the activity under consideration and also its relative position in the network. The principal ones were:

- (i) Total cost of the activity.
- (ii) Cost of the activity per unit time.
- (iii) Longest, and
- (iv) Shortest duration from the start of the activity along all unbroken paths to the end of the network.
- (v) Least, and
- (vi) Greatest cost measured similarly.

It is unlikely that, except near the end of a network, any two of the last four would be calculated along the same path. To vary the priority during any particular run would doubtless yield a better solution but requires an analysis such as the one described by Dr. Hastings.¹ A minor sort was only infrequently called upon and was held constant on ascending activity duration throughout. A random sort was included.

The objective function used to compare the relative virtues of the different priorities was OVDUR defined as the increase in overall project duration

over the Critical Path duration. This function rendered easy comparison of different networks.

THE DATA ANALYSIS

It must be stressed that an experimental method will not yield an optimum solution but only relatively “better” and “worse” ones measured by the objective function.

Application of analysis of variance (fixed effects model), multiple regression and some non-parametric tests yielded the following.

RESULTS

The investigation of the network SHAPE did not allow any stronger deduction than that with resource availability held constant, a network of “even” width is preferable. This has the effect of levelling out resource demands. Any longer term effects of “bottle-necks” would require separate analysis.

In the ranges considered the statistical significance of the other parameters was in all cases in the same order, viz: PRES, DENS, CC, DUM and PRIORITY CRITERION.

It is not surprising to find that PRES, the resource limitation, was the most significant; it is the sole cause of the scheduling delay compared with Critical Path conditions.

It is of interest to note that the amount of free float available (DENS) is more significant than the complexity (CC) though with the definitions adopted the latter is partly involved in PRES through the maximum number of parallel paths.

All the parameters analysed explained a significant amount of variation and there would be no justification for excluding any one of them.

Though the sorting criterion was the least significant it is the one over which a project manager might be said to have most control. Over all the 24 criteria used, the objective function (OVDUR) showed a variation of only 5–6 per cent between the “best” and “worst” cases. The best five criteria were:

- (i) Ascending total float,
- (ii) Ascending latest start,
- (iii) Descending activity cost per unit time,
- (iv) Descending longest duration to the end, and
- (v) Ascending latest finish,

between which the differences were not significant. (i), (ii) and (v) are Critical Path parameters and (iv) being difficult to compute is of academic interest only. (iii) seems to fulfil the aim of the work—it is equally as good as the traditional criteria and simple to calculate, requiring no previous Critical Path analysis and no up-dating—and did in fact have the most stable distribution.

An attempt to formulate a tentative model for networks in the ranges here specified under this criterion, giving priority to that activity with the greatest

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cost per unit time, yields an equation:

$$\text{OVDUR} = 1.52 + 0.02 \text{ DUM} + 2.05 \text{ CC} + 3.46 \text{ DENS} - 7.29 \text{ PRES}$$

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¹ N. A. J. HASTINGS (1972) Resource allocation in project networks. *Opl Res. Q.* **23**, 2.

² T. L. PASCOE (1965) An Experimental Comparison of Heuristic Methods for Allocating Resources. Thesis presented to the University of Cambridge, 1965.