Universidad Nacional Autonoma de Honduras

Escuela de Matematicas
Tarea MM-411 Ecuaciones Diferenciales
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Encontrar dos soluciones en series de potencia de la ecuacion diferencial alrededor del punto ordinario $x = 0$

1. $y'' + y = 0$, tanto por series como por metodos elementales

2. $y'' + 3xy' + 3y = 0$ **Solucion:** $y = a_0\left[1 + \sum_{k=1}^{\infty} \frac{(-3)^k x^{2k}}{2^k k!}\right] + a_1\left[x + \sum_{k=1}^{\infty} \frac{(-3)^k x^{2k+1}}{3 \cdot 5 \cdot 7 \cdots (2k+1)}\right]$

3. $(1 + 4x^2)y'' - 8y = 0$ **Solucion:** $y = a_0\left[1 + 4x^2\right] + a_1\left[\sum_{k=0}^{\infty} \frac{(-1)^{k+1} 2^{2k} x^{2k+1}}{4k^2 - 1}\right]$

4. $(1 + x^2)y'' - 4xy' + 6y = 0$ **Solucion:** $y = a_0\left[1 - 3x^2\right] + a_1\left[x - \frac{1}{3}x^3\right]$

5. $(1 + x^2)y'' + 10xy' + 20y = 0$,

   Solucion: $y = \frac{a_0}{3}\left[\sum_{k=0}^{\infty}(-1)^k(k+1)(2k+1)(2k+3)x^{2k}\right] + \frac{a_1}{6}\left[\sum_{k=0}^{\infty}(-1)^k(k+1)(k+2)(2k+3)x^{2k+1}\right]$

6. $(x^2 + 4)y'' + 2xy' - 12y = 0$, **Solucion:** $a_0\left[1 + \sum_{k=1}^{\infty} \frac{3(-1)^k(k+1)x^{2k}}{2^{2k}(2k-1)(2k-3)}\right] + a_1\left[x + \frac{5}{12}x^3\right]$

7. $(x^2 - 9)y'' + 3xy' - 3y = 0$ **Solucion:** $a_0\left[1 - \sum_{k=1}^{\infty} \frac{3 \cdot 5 \cdot 7 \cdots (2k+1)x^{2k}}{(18)^k(2k+1)k!}\right] + a_1 x$

8. $y'' + 2xy' + 5y = 0$,

   Solucion: $y = a_0\left[1 + \sum_{k=1}^{\infty} \frac{(-1)^k[5 \cdot 9 \cdot 13 \cdots (4k+1)x^{2k}]}{(2k)!}\right] + a_1\left[x + \sum_{k=1}^{\infty} \frac{(-1)^k[7 \cdot 11 \cdot 15 \cdots (4k+3)]x^{2k+1}}{(2k+1)!}\right]$

9. $(x^2 + 4)y'' + 6xy' + 4y = 0$, **Solucion:** $a_0\left[1 + \sum_{k=1}^{\infty} \frac{(-1)^k(k+1)x^{2k}}{2^{2k}}\right] + a_1\left[x + \sum_{k=1}^{\infty} \frac{(-1)^k(2k+3)x^{2k+1}}{3 \cdot 2^{2k}}\right]$

10. $2y'' + xy' - 4y = 0$, **Solucion:** $y = a_0\left[1 + x^2 + \frac{1}{12}x^4\right] + a_1\left[\sum_{k=0}^{\infty} \frac{3(-1)^k x^{2k+1}}{2^{2k}k!(2k-3)(2k-1)(2k+1)}\right]$

11. $(1 + 2x^2)y'' - 5xy' + 3y = 0$ **Solucion:** $y = a_0\left[1 + \sum_{k=1}^{\infty} \frac{3(-1)^k[(-1) \cdot 3 \cdot 7 \cdots (4k+5)]x^{2k}}{2^{2k}k!(2k-3)(2k-1)}\right] + a_1\left[x + \frac{1}{3}x^3\right]$

12. $y'' + x^2 y = 0$ **Solucion:** $y = a_0\left[1 + \sum_{k=1}^{\infty} \frac{(-1)^k x^{4k}}{2^{2k}k! \cdot 3 \cdot 5 \cdot 9 \cdot 13 \cdots (4k+1)}\right] + a_1\left[x + \sum_{k=1}^{\infty} \frac{(-1)^k x^{4k+1}}{2^{2k}k! \cdot 5 \cdot 9 \cdot 13 \cdots (4k+1)}\right]$

13. $y'' - 2(x+3)y' - 3y = 0$ resolver alrededor del punto $x = -3$ **Solucion:**

$$y = a_0\left[1 + \sum_{k=1}^{\infty} \frac{3\cdot 7\cdot 11\cdots(4k-1)(x+3)^{2k}}{(2k)!}\right] + a_1\left[(x+3) + \sum_{k=1}^{\infty} \frac{5\cdot 9\cdot 13\cdots(4k+1)(x+3)^{2k+1}}{(2k+1)!}\right]$$

14. $y'' + (x-2)y = 0$. Resolver alrededor de $x = 2$, **Solucion:**

$$y = a_0\left[1 + \sum_{k=1}^{\infty} \frac{(-1)^k (x-2)^{3k}}{3^k k![2\cdot 5\cdot 8\cdots(3k-1)]}\right] + a_1\left[(x-2) + \sum_{k=1}^{\infty} \frac{(-1)^k (x-2)^{3k+1}}{3^k k![4\cdot 7\cdot 10\cdots(3k+1)]}\right]$$

15. $(1-4x^2)y'' + 6xy' - 4y = 0$, **Solucion:** $y = a_0\left[1 + 2x^2\right] + a_1\left[x - \sum_{k=1}^{\infty} \frac{1\cdot 5\cdot 9\cdots(4k-3)x^{2k+1}}{k!(4k^2-1)}\right]$

16. $(1+2x^2)y'' + 3xy' - 3y = 0$, **Solucion:** $y = a_1 x + a_0\left[1 + \sum_{k=1}^{\infty} \frac{(-1)^{k+1} 3\cdot 7\cdot 11\cdots(4k-1)x^{2k}}{2^k(2k-1)k!}\right]$