Transformada de Laplace
Guía de estudio

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Ejercicios

Usando la definición de transformada de Laplace calcule $\mathscr{L}\{f(t)\}$

1. $f(t) = \begin{cases} t & \text{si} \quad 0 \le t < t_0 \\ 2t_0 - t & \text{si} \quad t_0 \le t \le 2t_0 \\ 0 & \text{si} \quad t > 2t_0 \end{cases}$

2. $f(t) = \begin{cases} t + t_0 & \text{si} \quad 0 \le t < t_0 \\ t_0 - t & \text{si} \quad t_0 \le t \le t_0 \\ 0 & \text{si} \quad t > t_0 \end{cases}$

3. $f(t) = \begin{cases} t & \text{si} \quad 0 \le t < t_0 \\ 2t_0 - 2t & \text{si} \quad t_0 \le t \le 2t_0 \\ 0 & \text{si} \quad t > 2t_0 \end{cases}$

4. $f(t) = \begin{cases} t & \text{si} \quad 0 \le t < t_0 \\ t_0 - t & \text{si} \quad t_0 \le t \le 2t_0 \\ t_0 & \text{si} \quad t > 2t_0 \end{cases}$

Calcule las siguientes transformadas de Laplace.(Usando tablas de transformadas)

1. $f(t) = (1+t)^2$
2. $f(t) = \sin(2t + \frac{\pi}{2})$
3. $f(t) = e^{3t}$
4. $f(t) = (e^{2t} + e^{-2t})^2$
5. $f(t) = \left(2 + \frac{t}{2}\right)^3$
6. $f(t) = t + e^{2t}$
7. $f(t) = \frac{(t^2+t)^2}{t}$
8. $f(t) = t^2 + e^t + 1$
9. $f(t) = \left(1 + \frac{1}{e^t}\right)^2$
10. $f(t) = \cos(2t)\sin t \cos t$
11. $f(t) = \cos(2t)\sin t$
12. $f(t) = 3t^3 + \cos(\sqrt{2}t)$
13. $f(t) = \frac{1}{2}t^2 - t + 1$
14. $f(t) = 2t + 1 - e^{2t}$
15. $f(t) = e^{4t+1}$
16. $f(t) = \frac{e^t + e^{-t}}{2}$
17. $f(t) = \frac{1}{e^{2t}} + \frac{1}{e^t}$
18. $f(t) = \sin(2t)\cos(3t)$
19. $f(t) = \sin^4 t$
20. $f(t) = \sin^3(4t)$
21. $f(t) = \cos^3(2t)$

Una definición de la función gamma esta dada por la integral impropia $\Gamma(\alpha) = \int_0^\infty t^{\alpha-1}e^{-t}\mathrm{d}t, \ \alpha > 0$, usando integración por partes muestre y la definición de la transformada de Laplace muestre que:

1. $\Gamma(\alpha+1) = \alpha\Gamma(\alpha)$
2. $\mathscr{L}\{t^\alpha\} = \frac{\Gamma(\alpha+1)}{s^{\alpha+1}}, \ \alpha > -1$

Usando el ejercicio anterior calcule la transformada de Laplace de las siguientes funciones,
Use el hecho que $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$:

1. $f(t) = \frac{1}{t^{1/2}}$
2. $f(t) = t^{1/2}$
3. $f(t) = t^2 + t^{1/2}$
4. $f(t) = t^{3/2}$
5. $f(t) = \frac{t + t^{1/2}}{t^{3/2}}$
6. $f(t) = \frac{t+2}{t^{1/2}}$

Transformadas Inversas: Calcule las siguientes transformadas inversas $\mathscr{L}^{-1}\{F(s)\}$de las siguientes funciones en muchos de los ejercicios necesitara utilizar fracciones parciales para resolverlos

1. $\mathscr{L}^{-1}\left\{\dfrac{4}{s^2}\right\}$
2. $\mathscr{L}^{-1}\left\{\dfrac{(s^2+3)^2}{s^5}\right\}$
3. $\mathscr{L}^{-1}\left\{\dfrac{2s+3}{s^2+4}\right\}$
4. $\mathscr{L}^{-1}\left\{\dfrac{2s^2+2s+3}{s(s^2+4)}\right\}$
5. $\mathscr{L}^{-1}\left\{\dfrac{1}{s(s^2+1)}\right\}$
6. $\mathscr{L}^{-1}\left\{\dfrac{s+1}{s(s+2)}\right\}$
7. $\mathscr{L}^{-1}\left\{\dfrac{s^2+1}{(s^2+4)(s^2+16)}\right\}$
8. $\mathscr{L}^{-1}\left\{\dfrac{s+1}{3s^2+1}\right\}$
9. $\mathscr{L}^{-1}\left\{\dfrac{s^2+3s+1}{s^3+s}\right\}$
10. $\mathscr{L}^{-1}\left\{\dfrac{1}{s(s-1)}\right\}$
11. $\mathscr{L}^{-1}\left\{\dfrac{s+2}{s^2(s-1)}\right\}$
12. $\mathscr{L}^{-1}\left\{\dfrac{s^2+2s+3}{s(s^2+1)}\right\}$
13. $\mathscr{L}^{-1}\left\{\dfrac{6s+3}{s^4+5s^2+4}\right\}$
14. $\mathscr{L}^{-1}\left\{\dfrac{2s-4}{(s^2+s)(s^2+1)}\right\}$
15. $\mathscr{L}^{-1}\left\{\dfrac{s}{(s+2)(s^2+4)}\right\}$
16. $\mathscr{L}^{-1}\left\{\dfrac{s^2+1}{s(s-1)(s+1)(s-2)}\right\}$
17. $\mathscr{L}^{-1}\left\{\dfrac{s}{(s-2)(s-3)(s-6)}\right\}$
18. $\mathscr{L}^{-1}\left\{\dfrac{2s+1}{s^2+s-20}\right\}$
19. $\mathscr{L}^{-1}\left\{2s^{-1}e^{-3s}\right\}$
20. $\mathscr{L}^{-1}\left\{\dfrac{10s-3}{25-s^2}\right\}$
21. $\mathscr{L}^{-1}\left\{\dfrac{9+s}{4-s^2}\right\}$
22. $\mathscr{L}^{-1}\left\{\dfrac{5-3s}{s^2+9}\right\}$

Propiedades Operacionales I: Calcule las siguientes transformadas usando las propiedades operacionales(traslaciones en el eje s)

1. $\mathscr{L}\left\{\sin^2(3t)e^{4t}\right\}$
2. $\mathscr{L}\left\{\sin^3(3t)e^{5t}\right\}$
3. $\mathscr{L}\left\{(t+1)^2e^{4t}\right\}$
4. $\mathscr{L}\left\{(1+e^t+3e^{3t})\cos(4t)\right\}$
5. $\mathscr{L}\left\{\sin(3t)\cos(2t)e^{2t}\right\}$
6. $\mathscr{L}\left\{\cos(2t)\sin(2t)e^{5t}\right\}$
7. $\mathscr{L}\left\{t^4e^{4t}\right\}$
8. $\mathscr{L}\left\{\left(t+\dfrac{1}{t^{1/2}}\right)e^{5t}\right\}$
9. $\mathscr{L}\left\{\sqrt{t}e^{2t+3}\right\}$
10. $\mathscr{L}\left\{\left(e^t+e^{3t}\right)t^2\right\}$
11. $\mathscr{L}\left\{(2-t+e^t)e^{4t-1}\right\}$
12. $\mathscr{L}\left\{t^{3/2}e^{2t}\right\}$
13. $\mathscr{L}\left\{(t^2+t)e^{-2t}\right\}$
14. $\mathscr{L}\left\{\left(\dfrac{t^2+t}{\sqrt{t}}\right)e^t\right\}$
15. $\mathscr{L}\left\{\left(\dfrac{t^3+2t^2}{t^2}\right)e^t\right\}$
16. $\mathscr{L}\left\{(t+e^t)^2e^{-2t}\right\}$
17. $\mathscr{L}\left\{(1-t)e^{2t}\right\}$
18. $\mathscr{L}\left\{t^2e^{3t}\right\}$

Propiedades Operacionales I: Calcule las siguientes transformadas inversas

1. $\mathscr{L}^{-1}\left\{\dfrac{2s+1}{s^2-6s+1}\right\}$
2. $\mathscr{L}^{-1}\left\{\dfrac{s^3+s}{(s-1)^4}\right\}$
3. $\mathscr{L}^{-1}\left\{\dfrac{s-5}{s^2+2s+10}\right\}$
4. $\mathscr{L}^{-1}\left\{\dfrac{3s+4}{s^2+6s+34}\right\}$
5. $\mathscr{L}^{-1}\left\{\dfrac{s^2+3s+5}{(s+1)^3}\right\}$
6. $\mathscr{L}^{-1}\left\{\dfrac{4s+3}{s^2(s-3)^2}\right\}$
7. $\mathscr{L}^{-1}\left\{\dfrac{2s+1}{s^2(s+1)^2}\right\}$
8. $\mathscr{L}^{-1}\left\{\dfrac{s+3}{s^2+4s+7}\right\}$
9. $\mathscr{L}^{-1}\left\{\dfrac{s-3}{s(s^2+4s+7)}\right\}$
10. $\mathscr{L}^{-1}\left\{\dfrac{s+4}{(s-1)(s+2)^2}\right\}$
11. $\mathscr{L}^{-1}\left\{\dfrac{s^2+3s+1}{s^2(s+2)}\right\}$
12. $\mathscr{L}^{-1}\left\{\dfrac{3s^2+2s+4}{(s-1)^2(s^2+2s+2)}\right\}$
13. $\mathscr{L}^{-1}\left\{\dfrac{1}{\sqrt{s-1}}\right\}$
14. $\mathscr{L}^{-1}\left\{\dfrac{2}{(s-\pi)^{3/2}}\right\}$
15. $\mathscr{L}^{-1}\left\{\dfrac{1}{\sqrt[3]{(s-3)}(s-3)}\right\}$

Propiedades Operacionales II: Calcule las siguientes transformadas usando las propiedades operacionales(traslaciones en el eje t)

1. $\mathscr{L}\left\{\cos(3t)\sin(2t)e^{4t}\mathscr{U}(t-\pi)\right\}$
2. $\mathscr{L}\left\{e^t \sin t\ \mathscr{U}\left(t-\frac{\pi}{2}\right)\right\}$
3. $\mathscr{L}\left\{\cos^3(2t)\mathscr{U}\left(t-\frac{\pi}{2}\right)\right\}$
4. $\mathscr{L}\left\{t+t^2\mathscr{U}(t-2)\right\}$
5. $\mathscr{L}\left\{\sin(2t)\mathscr{U}\left(t-\frac{\pi}{2}\right)\right\}$
6. $\mathscr{L}\left\{e^{2t}+t\sin t\mathscr{U}(t-\pi)\right\}$
7. $\mathscr{L}\left\{t^{1/2}\mathscr{U}(t-\pi)\right\}$
8. $\mathscr{L}\left\{t^2e^{5t}\mathscr{U}(t-1)\right\}$
9. $\mathscr{L}\left\{t\sin(at)e^{kt}\mathscr{U}(t-k)\right\}$

Propiedades Operacionales II: Calcule las siguientes transformadas inversas

1. $\mathscr{L}^{-1}\left\{\dfrac{e^{-2s}}{s^4}\right\}$
2. $\mathscr{L}^{-1}\left\{\dfrac{e^{-\pi s}}{s^{3/2}}\right\}$
3. $\mathscr{L}^{-1}\left\{\dfrac{(e^{-\pi s}+e^{-2\pi s})^2}{s^2+2s+5}\right\}$
4. $\mathscr{L}^{-1}\left\{\dfrac{s\sqrt{e^{-\pi s}}}{s^2+1}\right\}$
5. $\mathscr{L}^{-1}\left\{\dfrac{\left(1+e^{-2s}\right)^2}{s^2-1}\right\}$
6. $\mathscr{L}^{-1}\left\{\dfrac{s^2+s+2}{e^{3s}\ s\left(s^2+4s+5\right)}\right\}$
7. $\mathscr{L}^{-1}\left\{\dfrac{e^{-2s}}{s^2-4}\right\}$
8. $\mathscr{L}^{-1}\left\{\dfrac{e^{-\pi s}}{s^2+2s+10}\right\}$
9. $\mathscr{L}^{-1}\left\{\dfrac{s^2+4s+1}{e^s s^3(s-1)}\right\}$
10. $\mathscr{L}^{-1}\left\{\dfrac{e^{-\pi s}}{\sqrt{s-\pi}}\right\}$
11. $\mathscr{L}^{-1}\left\{\dfrac{e^{-2\pi s}}{(s^2+1)^2}\right\}$
12. $\mathscr{L}^{-1}\left\{\dfrac{2ks\ e^{-\pi k}}{(s^2+k^2)^2}\right\}$

Derivadas de Transformadas: Usando el teorema de derivadas de transformadas calcule:

1. $\mathscr{L}\left\{t^2e^{3t}\right\}$
2. $\mathscr{L}\left\{t^3\sin^2(3t)\right\}$
3. $\mathscr{L}\left\{t\cos(2t)\sin(4t)\right\}$
4. $\mathscr{L}\left\{t^3e^{kt}\right\}$
5. $\mathscr{L}\left\{t^{5/2}\right\}$
6. $\mathscr{L}\left\{t^2e^{kt}f(t)\mathscr{U}(t-a)\right\}$

Convolución: Usando la definición de convolucion,

$$f*g=\int_0^t f(\tau)g(t-\tau)\mathrm{d}\tau$$

calcule las siguientes siguientes convoluciones

1. $t*e^t$
2. $\sin t*\sin t$
3. t^2*e^{3t}
4. $e^t*\sin t$
5. $\cos t*\sin t$
6. e^t*t^2

Transformadas de integrales: Calcule las transformadas de Laplace de las siguientes convoluciones, de dos formas:

a) Calculando las convoluciones, luego las transformadas

b) Usando el teorema de transformadas de integrales

1. $\mathscr{L}\left\{t*e^t\right\}$
2. $\mathscr{L}\left\{\sin t*\sin t\right\}$
3. $\mathscr{L}\left\{t^2*e^{3t}\right\}$
4. $\mathscr{L}\left\{e^t*\sin t\right\}$
5. $\mathscr{L}\left\{\cos t*\sin t\right\}$
6. $\mathscr{L}\left\{e^t*t^2\right\}$

Transformadas de Integrales: Calcule la transformada de Laplace(NO resuelva la integral):

1. $\mathscr{L}\left\{\int_0^t e^\tau\mathrm{d}\tau\right\}$
2. $\mathscr{L}\left\{\int_0^t \tau e^\tau\mathrm{d}\tau\right\}$
3. $\mathscr{L}\left\{\int_0^t (t-\tau)^2e^{2\tau}\mathrm{d}\tau\right\}$
4. $\mathscr{L}\left\{\int_0^t \tau(t-\tau)\mathrm{d}\tau\right\}$
5. $\mathscr{L}\left\{\int_0^t \sin(t-\tau)e^\tau\mathrm{d}\tau\right\}$
6. $\mathscr{L}\left\{\int_0^t \sin(\tau)\cos(t-\tau)\mathrm{d}\tau\right\}$
7. $\mathscr{L}\left\{t^2\int_0^t \sin\tau\mathrm{d}\tau\right\}$
8. $\mathscr{L}\left\{t^2e^{3t}\int_0^t e^{t-\tau}\mathrm{d}\tau\right\}$
9. $\mathscr{L}\left\{te^{kt}\int_0^t \sin(k\tau)\mathrm{d}\tau\right\}$

Transformadas Inversas Usando el teorema de convolución calcule las transformadas inversas

1. $\mathscr{L}^{-1}\left\{\dfrac{1}{(s^2+k^2)^2}\right\}$
2. $\mathscr{L}^{-1}\left\{\dfrac{1}{s^2(s-k)}\right\}, k \in \mathbb{R}$
3. $\mathscr{L}^{-1}\left\{\dfrac{1}{s^2(s-k)^2}\right\}, k \in \mathbb{R}$
4. $\mathscr{L}^{-1}\left\{\dfrac{k}{s^2(s^2+1)}\right\}$
5. $\mathscr{L}^{-1}\left\{\dfrac{1}{s^3(s^2+1)}\right\}$
6. $\mathscr{L}^{-1}\left\{\dfrac{s}{(s^2+1)^2}\right\}$
7. $\mathscr{L}^{-1}\left\{\dfrac{8k^3 s}{(s^2+k^2)^3}\right\}, k \in \mathbb{R}$
8. $\mathscr{L}^{-1}\left\{\dfrac{1}{s(s-k)^2}\right\}, k \in \mathbb{R}$
9. $\mathscr{L}^{-1}\left\{\dfrac{1}{(s-2)(s-4)}\right\}$

Ecuaciones Integrales e integro-diferenciales: Use la transformada de Laplace para resolver las ecuaciones integrales o integro-diferenciales

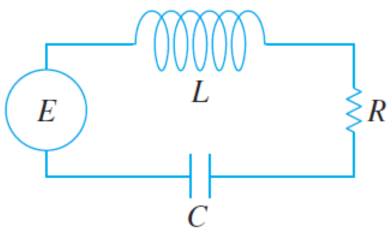
1. $f(t) + \displaystyle\int_0^t (t-\tau)f(\tau)\mathrm{d}\tau = t$
2. $f(t) = 2t - 4\displaystyle\int_0^t \sin\tau f(t-\tau)\mathrm{d}\tau$
3. $f(t) = te^t + \displaystyle\int_0^t \tau f(t-\tau)\mathrm{d}\tau$
4. $f(t) + 2\displaystyle\int_0^t f(\tau)\cos(t-\tau)\mathrm{d}\tau = 4e^{-t} + \sin t$
5. $f(t) + \displaystyle\int_0^t f(\tau)\mathrm{d}\tau = 1$
6. $f(t) = \cos t + \displaystyle\int_0^t e^{-\tau} f(t-\tau)\mathrm{d}\tau$
7. $f(t) = 1 + t - \dfrac{8}{3}\displaystyle\int_0^t (t-\tau)^3 f(\tau)\mathrm{d}\tau$
8. $t - 2f(t) = \displaystyle\int_0^t (e^\tau - e^{-\tau})f(t-\tau)$
9. $y'(t) = 1 - \sin t - \displaystyle\int_0^t y(\tau)\mathrm{d}\tau \ , y(0) = 0$
10. $0.1\dfrac{\mathrm{d}i}{\mathrm{d}t} + 2i + 10\displaystyle\int_0^t i(\tau)\mathrm{d}\tau = 120t - 120t\mathscr{U}(t-1),$
    $i(0) = 0$

Aplicaciones de ecuaciones integro-diferenciales: En una sola malla o circuito en serie, la segunda ley de *Kirchhoff* establece que las sumas de las caídas de voltaje en un inductor, resistor y capacitor es igual al voltaje aplicado $E(t)$. Ahora se sabe que las caídas de voltaje en un inductor, resistor y un capacitor son, respectivamente

$$L\frac{\mathrm{d}i}{\mathrm{d}t}, \quad Ri(t), \quad \frac{1}{C}\int_0^t i(\tau)\mathrm{d}\tau$$

Donde $i(t)$ es la corriente y L, R y C son constantes. Se deduce que la corriente en el circuito, esta gobernada por la ecuación integro-diferencial

$$L\frac{\mathrm{d}i}{\mathrm{d}t} + Ri(t) + \frac{1}{C}\int_0^t i(\tau)\mathrm{d}\tau = E(t)$$

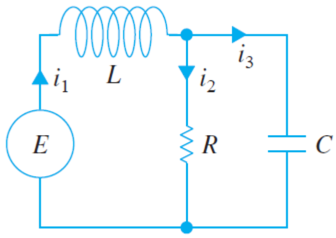


Determine la corriente $i(t)$ y ademas utilice un programa para dibujar la gráfica de la solución para el circuito de una sola malla RLC cuando:

1. $L = 0.1h,\ R = 2\Omega,\ C = 0.1f,$ $i(0) = 0$ y $E(t) = 120t - 120t\mathscr{U}(t-1)$
2. $L = 0.1h,\ R = 3\Omega,\ C = 0.05f,$ $i(0) = 0$ y $E(t) = 100\left[\mathscr{U}(t-1) - \mathscr{U}(t-2)\right]$
3. $L = 0.005h,\ R = 1\Omega,$ $C = 0.02f,\ i(0) = 0$ y $E(t) = 100\left[t - (t-1)\mathscr{U}(t-1)\right]$

Aplicaciones de sistemas de ecuaciones diferenciales(Redes):

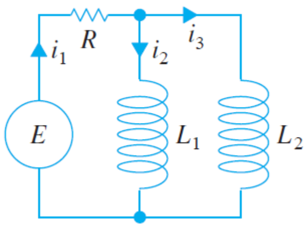
La ecuación diferencial para la red se modela por el sistema de ecuaciones diferenciales
$$\begin{cases} L\dfrac{\mathrm{d}i_1}{\mathrm{d}t} + Ri_2 = E(t) \\ RC\dfrac{\mathrm{d}i_2}{\mathrm{d}t} + i_2 - i_1 = 0 \\ i_1(0) = 0,\ i_2(0) = 0 \end{cases}$$



Resuelva el sistema con los siguientes datos:

1. $R = 50\Omega$, $L = 1\ h$, $C = 10^{-4}\ f$, $E = 60V$, $i_2(0) = 0,\ i_3(0) = 0$
2. $R = 50\Omega$, $L = 0.5\ h$, $C = 10^{-4}\ f$, $E = 60V$, $i_2(0) = 0,\ i_3(0) = 0$
3. $R = 50\Omega$, $L = 2\ h$, $C = 10^{-4}\ f$, $E = 60V$, $i_2(0) = 0,\ i_3(0) = 0$

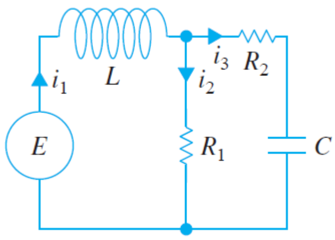
La ecuación diferencial para la red se modela por el sistema de ecuaciones diferenciales
$$\begin{cases} L_1\dfrac{\mathrm{d}i_2}{\mathrm{d}t} + Ri_2 + Ri_3 = E(t) \\ L_2\dfrac{\mathrm{d}i_3}{\mathrm{d}t} + Ri_2 + Ri_3 = E(t) \\ i_2(0) = 0,\ i_3(0) = 0 \end{cases}$$



Resuelva el sistema con los siguientes datos:

1. $R = 5\Omega$, $L_1 = 0.01\ h$, $L_2 = 0.0125\ h$, $E = 100V$, $i_2(0) = 0,\ i_3(0) = 0$

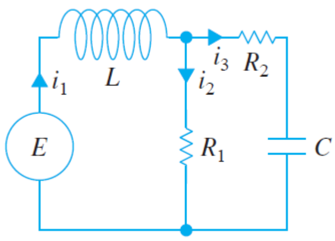
La ecuación diferencial para la red se modela por el sistema de ecuaciones diferenciales
$$\begin{cases} L\dfrac{\mathrm{d}i_2}{\mathrm{d}t} + L\dfrac{\mathrm{d}i_3}{\mathrm{d}t} + R_1 i_2 = E(t) \\ -R_1\dfrac{\mathrm{d}i_2}{\mathrm{d}t} + R_2\dfrac{\mathrm{d}i_3}{\mathrm{d}t} + \dfrac{1}{C}i_3 = 0 \\ i_2(0) = 0,\ i_3(0) = 0 \end{cases}$$



Resuelva el sistema con los siguientes datos:

1. $R_1 = 10\Omega$, $R_2 = 5\ \Omega$, $L1\ h$, $C = 0.2\ f$, $E(t) = 120 - 120\mathscr{U}(t-2)$, $i_2(0) = 0,\ i_3(0) = 0$

La ecuación diferencial para la red se modela por el sistema de ecuaciones diferenciales
$$\begin{cases} R_1\dfrac{\mathrm{d}q}{\mathrm{d}t} + \dfrac{1}{C}q + R_1 i_3 = E(t) \\ L\dfrac{\mathrm{d}i_3}{\mathrm{d}t} + R_2 i_3 - \dfrac{1}{C}q = 0 \\ i_3(0) = 0,\ q(0) = 0 \end{cases}$$



Determine la carga en el capacitor cuando :

1. $L = 1\ h$, $R_1 = 1\ \Omega$, $R_2 = 1\Omega$, $C = 1\ f$, $E(t) = 50e^{-t}\mathscr{U}(t-1)$, $i_3(0) = 0,\ q(0) = 0$

Resolución de ecuaciones diferenciales

1. $\begin{cases} y'' - 7y' + 6y = e^t + \delta(t-2) + \delta(t-4) \\ y(0) = 1,\ y'(0) = 0 \end{cases}$

2. $\begin{cases} y'' + 4y' + 13y = \delta(t-\pi) + \delta(t-3\pi) \\ y(0) = 1,\ y'(0) = 0 \end{cases}$

3. $\begin{cases} y'' + y = \sin t \\ y(0) = 1,\ y'(0) = -1 \end{cases}$

4. $\begin{cases} y'' + 9y = \cos(3t) \\ y(0) = 2,\ y'(0) = 5 \end{cases}$

5. $\begin{cases} y'' + 4y' + 3y = t - \mathscr{U}(t-2) - \mathscr{U}(t-4) \\ y(0) = 1,\ y'(0) = 0 \end{cases}$

6. $\begin{cases} y'' - 5y' + 6y = 1 + \mathscr{U}(t-1) \\ y(0) = 0,\ y'(0) = 1 \end{cases}$

7. $\begin{cases} y'' + 4y = \sin t\,\mathscr{U}(t-2\pi) \\ y(0) = 1,\ y'(0) = 0 \end{cases}$

8. $\begin{cases} y'' + 4y' + 3y = \mathscr{U}(t-1) + \delta(t-2) \\ y(0) = 1,\ y'(0) = 0 \end{cases}$

9. $\begin{cases} y'' - 7y' + 6y = e^t + \mathscr{U}(t-1) + \delta(t-2) \\ y(0) = 1,\ y'(0) = 0 \end{cases}$

10. $\begin{cases} y'' - 4y' = 6e^{3t} - 3e^{-t} \\ y(0) = 1,\ y'(0) = -1 \end{cases}$

SELECCION MULTIPLE: Elegir la opcion que corresponde la transformada de Laplace de las siguientes funciones

1. $f(t) = \cos^2(2t)$
   - (a) $F(s) = \dfrac{1}{s} + \dfrac{s}{s^2+16}$
   - (b) $F(s) = \dfrac{1}{s} + \dfrac{s}{s^2+4}$
   - (c) $F(s) = \dfrac{s^2+2}{s(s^2+4)}$
   - (d) $F(s) = \dfrac{s^2+8}{s(s^2+16)}$

2. $f(t) = (\sin t + \cos t)^2$
   - (a) $F(s) = \left(\dfrac{1}{s^2+1} + \dfrac{s}{s^2+1}\right)^2$
   - (b) $F(s) = \dfrac{2}{s^2+4}$
   - (c) $F(s) = \dfrac{s^2+2s+4}{s(s^2+4)}$
   - (d) $F(s) = \dfrac{s^2+2}{s(s^2+4)}$

3. $f(t) = \left(1 + e^{-t}\right)e^t$
   - (a) $F(s) = \dfrac{2s+1}{s(s+1)}$
   - (b) $F(s) = \dfrac{2s-1}{s(s-1)}$
   - (c) $F(s) = \dfrac{2s+1}{s+1}$
   - (d) $F(s) = \dfrac{2s-1}{s-1}$

4. $f(t) = e^{-2t}\left(3\cos(6t) - 5\sin(6t)\right)$
   - (a) $F(s) = \dfrac{3s-24}{s^2+4s+40}$
   - (b) $F(s) = \dfrac{-30}{s^2+4s+40}$
   - (c) $F(s) = \dfrac{8-5s}{s^2+4s+40}$
   - (d) $F(s) = \dfrac{3s+2}{s^2+4s+40}$

5. $f(t) = \sin t \cos t$
   - (a) $F(s) = \dfrac{1}{s^2+2}$
   - (b) $F(s) = \dfrac{1}{2(s^2+4)}$
   - (c) $F(s) = \dfrac{2}{s^2+4}$
   - (d) $F(s) = \dfrac{1}{s^2+4}$

6. $f(t) = \cos t \cos(2t)$

(a) $F(s) = \dfrac{1}{3}\left(\dfrac{3}{s^2+9} - \dfrac{1}{s^2+1}\right)$

(b) $F(s) = \dfrac{1}{2}\left(\dfrac{3}{s^2+9} - \dfrac{1}{s^2+1}\right)$

(c) $F(s) = \dfrac{1}{3}\left(\dfrac{1}{s^2+9} - \dfrac{1}{s^2+1}\right)$

(d) $F(s) = \dfrac{1}{2}\left(\dfrac{2}{s^2+9} - \dfrac{1}{s^2+1}\right)$

7. $f(t) = e^{-t}\sin^2 t$

(a) $F(s) = \dfrac{1}{(s+1)(s^2+2s+5)}$

(b) $F(s) = \dfrac{3}{(s+1)(s^2+2s+5)}$

(c) $F(s) = \dfrac{2}{(s-1)(s^2+2s+5)}$

(d) $F(s) = \dfrac{2}{(s+1)(s^2+2s+5)}$

8. $f(t) = t^2\cos(\omega t)$, $\omega \in \mathbb{R}$

(a) $F(s) = \dfrac{2s^2+6s\omega^2}{(s^2+\omega^2)^3}$

(b) $F(s) = \dfrac{2s^2-6s\omega^2}{(s^2+\omega^2)^2}$

(c) $F(s) = \dfrac{s^2+6s\omega^2}{(s^2+\omega^2)^3}$

(d) $F(s) = \dfrac{2s^2-6s\omega^2}{(s^2+\omega^2)^3}$

9. $f(t) = e^{-t} * e^t \cos t$

(a) $F(s) = \dfrac{2s-1}{5\left[(s-1)^2+1\right]}$

(b) $F(s) = \dfrac{2s-1}{5(s-1)^2+1}$

(c) $F(s) = \dfrac{2s+1}{5(s-1)^2+1}$

(d) $F(s) = \dfrac{2s-1}{(s-1)^2+5}$

10. La transformada de la función periodica $\begin{cases} t & \text{si} \quad 0 < t < 3 \\ 3 & \text{si} \quad 3 < t < 6 \end{cases}$ es:

(a) $F(s) = \dfrac{s - 3se^{6s} - e^{-3s}}{s(1-e^{-6s})}$

(b) $F(s) = \dfrac{s + 3se^{-6s} - e^{-3s}}{s(1-e^{-6s})}$

(c) $F(s) = \dfrac{s - 3se^{-6s} - e^{-3s}}{s(1-e^{6s})}$

(d) $F(s) = \dfrac{s - 3se^{-6s} - e^{-3s}}{s(1-e^{-6s})}$

Transformadas de funciones periodicas: Calcule las transformadas de las siguientes funciones periodicas

1. $f(t) = \begin{cases} 1 & \text{si} \quad 0 < t < 1 \\ -1 & \text{si} \quad 1 < t < 2 \end{cases}$

2. $f(t) = \begin{cases} 1 & \text{si} \quad 0 < t < 1 \\ 0 & \text{si} \quad 1 < t < 2 \end{cases}$

3. $f(t) = \begin{cases} 0 & \text{si} \quad 0 < t < 1 \\ t & \text{si} \quad 1 < t < 2 \end{cases}$

4. $f(t) = \begin{cases} 2t & \text{si} \quad 0 < t < 2 \\ 4 & \text{si} \quad 2 < t < 4 \end{cases}$

5. $f(t) = \begin{cases} \sin t & \text{si} \quad 0 < t < \pi \\ 0 & \text{si} \quad \pi < t < 2\pi \end{cases}$

6. $f(t) = \begin{cases} 0 & \text{si} \quad 0 < t < 2\pi \\ \cos t & \text{si} \quad 2\pi < t < 4\pi \end{cases}$

Ejercicios Teoricos: Calcule la transformada de las siguientes funciones

1. $f(t) = [\![t]\!]$ para $t \geq 0$
2. Muestre que $\mathscr{L}\left\{\dfrac{\sin(3t)}{t}\right\} = \dfrac{\pi}{2} - \tan^{-1}\left(\dfrac{s}{3}\right)$
3. Muestre que $\mathscr{L}^{-1}\left\{\ln\left(\dfrac{s+a}{s+b}\right)\right\} = \dfrac{e^{-bt} - e^{-at}}{t}$
4. Muestre que $\mathscr{L}\left\{\dfrac{\sinh t}{t}\right\} = \ln\left(\sqrt{\dfrac{s+1}{s-1}}\right)$
5. Muestre que $\displaystyle\int_0^\infty \frac{e^{-3t} - e^{-6t}}{t}\mathrm{d}t$ usando transformada de Laplace
6. Muestre que $\mathscr{L}\left\{\dfrac{\cos(at) - \cos(bt)}{t}\right\} = \ln\left(\sqrt{\dfrac{s^2+b^2}{s^2+a^2}}\right)$
7. Muestre que $\displaystyle\int_0^\infty \frac{\cos(6t) - \cos(4t)}{t}\mathrm{d}t = \ln\left(\frac{2}{3}\right)$
8. Muestre que $\displaystyle\int_0^\infty \frac{\sin t}{t}\mathrm{d}t = \frac{\pi}{2}$
9. Muestre que $\mathscr{L}\left\{\dfrac{\sin(at)}{t}\right\} = \dfrac{\pi}{2} - \tan^{-1}\left(\dfrac{s}{a}\right)$