T1 - Introduction To Circuit Analysis

Integrated Master in Physics Engineering

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Contents

1 Introduction

The objective of this laboratory assignment is to study a circuit containing various resistors, two voltage sources and two current sources. The circuit can be seen in Figure 1.

In Section 2, a theoretical analysis of the circuit is presented. In Section 3, the circuit is analysed by simulation, and the results are compared to the theoretical results obtained in Section 2. The conclusions of this study are outlined in Section 4.

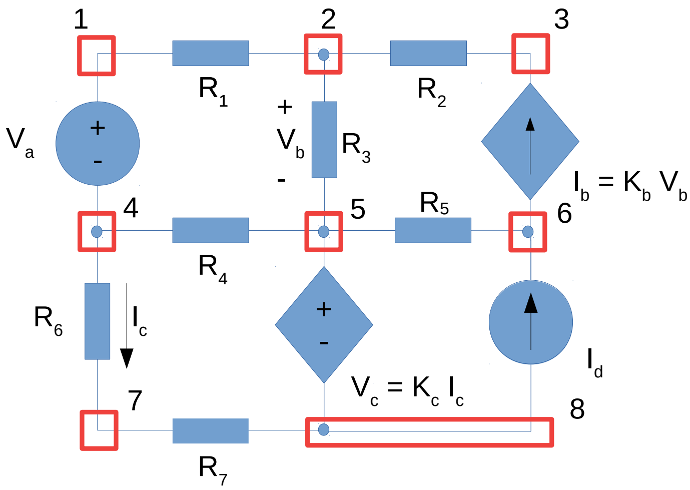

Figure 1: Circuit to be studied

2 Theoretical Analysis

In this section, the circuit shown in Figure 1 is analysed theoretically, first we aproach the circuit using the mesh analysis, and later we analyse the circuit using the nodal analysis.

2.1 Mesh Analysis

As seen during theoretical lessons, we can use a mesh analysis to analyse the circuit. This method is built upon Kirchhoff's Voltage Law that states:

In a mesh, the sum of all voltages equals 0.

$$\sum_{i=0}^{n} V_i = 0 \tag{1}$$

The method consists of identifying every mesh, labeling its current, and choosing the direction of the currents. Then the KVL equations are written for each mesh, and we can solve the system of equations, solving consequently the circuit.

As seen in in Figure 2, there are four meshes, each with currents Y_a, Y_b, Y_c, Y_d, we will label each mesh as A, B, C, D respectively.

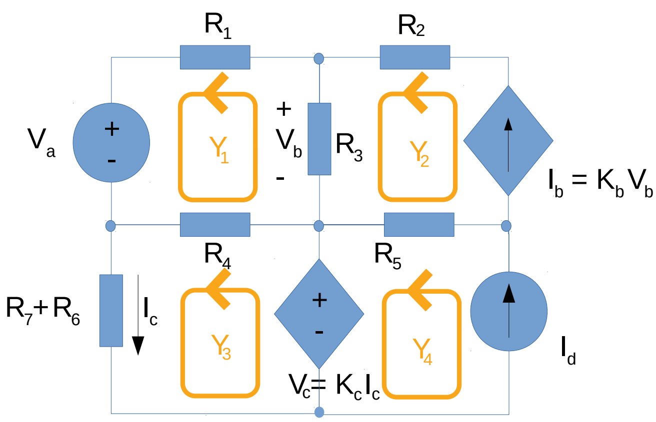

Figure 2: Current choosen for each mesh

The equations for each mesh are therefore:

$$\mathbf{E_1} : y_1 + R_4 \cdot (y_1 - y_3) + R_3 \cdot (y_1 - y_2) + y_1 \cdot R_1 = 0$$

$$\mathbf{E_2} : y_2 \cdot (1 - R_3 \cdot K_b) + K_b \cdot R_3 \cdot y_1 = 0$$

$$\mathbf{E_3} : y_3 \cdot R_6 + y_3 \cdot R_7 - V_c + (y_3 - y_1) \cdot R_4 = 0$$

$$\mathbf{E_4} : y_4 = I_d$$

In determining equations $\mathbf{E_1}$ to $\mathbf{E_4}$, we have used the following relations:

$$I_b = K_b \times V_b$$

$$V_b = R_3 \times (y_2 - y_1)$$

$$V_c = K_c \times I_c$$

$$I_c = y_3$$

In matrix form, the system looks like the following:

$$\begin{bmatrix} R_1 + R_2 + R_3 & -R_3 & -R_4 & 0 \\ K_b R_3 & 1 - R_3 K_b & 0 & 0 \\ -R_4 & 0 & R_6 + R_7 + R_4 - K_c & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} -V_a \\ 0 \\ 0 \\ I_d \end{bmatrix}$$

Solving this system of equations, using the following values generated with the t1_datagen.py script using the number 93156:

R1	1.03919759193
R2	2.06836523173
R3	3.03375774261
R4	4.12779067183
R5	3.11985677803
R6	2.04513887844
R7	1.04289965713
Va	5.00439410964
Id	1.04536428769
Kb	7.25705461539
Kc	8.23640363075

Table 1: Data generated using number 93156

We reach the following results, using octave:

2.2 Nodal Analysis

The general point of the node analysis method is to figure out the node voltages of all the nodes in our circuit (in relation to a reference node, which we call ground, and whose voltage we set to 0). Having figured this out, it's straightforward to determine the branch currents and voltages, thus completely solving the circuit.

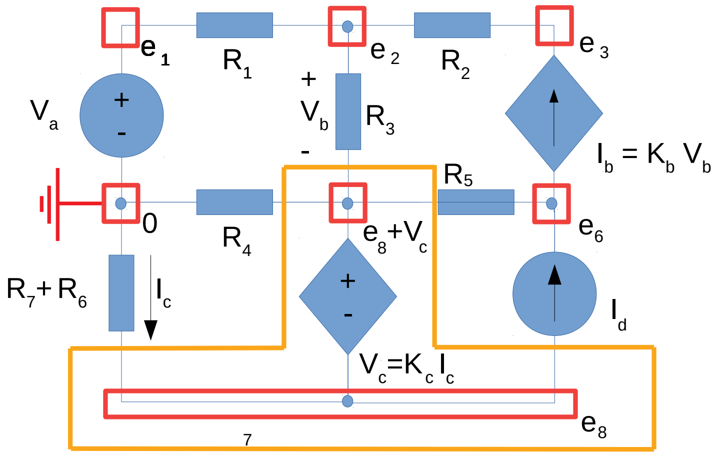

Figure 3: Nodes used in Nodal Analysis

To do this, we first need to identify and label all the nodes in our system as seen in Figure 3, as well as choose the ground node. In our case, we defined the node connected to resistance R_4 and to voltage source V_a as ground. The second step consists in determining the voltages of the easy nodes. In our circuit, it is clear to see, for example, that $e_1 = V_a$. To solve the other non-trivial nodes, we proceed to write Kirchhoff's current law for each one of them, which states that the sum of currents going into a node must be 0; or, in other words, that charges may not accumulate in one singular node:

$$\sum_i y_i = 0$$

where y_i is a current defined as going **into** the node.
Using Ohm's Law, which states that $I = U/R$, and assuming we know the values of the resistances of the elements in each branch (also noticing that we can write the branch voltages as differences between node voltages), we get a system of equations that allows us to determine each node voltage.
Since this method seems to rely upon Ohm's law, it seems to be a fatal problem that we have a dependent voltage source in our circuit, V_c, for which we **can't** write ohm's law. To deal with this, we create a super-node by lumping together the nodes to which V_c is connected and we write KCL for the super-node. To find the missing equation, we simply note that V(negative terminal of dependent voltage source) + V_c = V(positive terminal of dependent voltage source), thus getting two equations for our two "problematic" nodes.
We can therefore write the following equations:

$$\mathbf{Node_1} : e_1 = V_a$$

$$\mathbf{Node_2} : -C_1 \cdot (e_1 - e_2) - C_3 \cdot (e_8 - K_c \cdot C_{6,7} \cdot e_8 - e_2) - C_2 \cdot (e_3 - e_2) = 0$$

$$\textbf{Node}_{\mathbf{3}} : C_2 \cdot (e_3 - e_2) - K_b \cdot (e_2 - e_8 + K_c \cdot C_{6,7} \cdot e_8) = 0$$

$$\textbf{Node}_{\mathbf{6}} : C_5 \cdot (e_6 - e_8 + K_c \cdot C_{6,7} \cdot e_8) + K_b \cdot (e_2 - e_8 + K_c \cdot C_{6,7} \cdot e_8) - I_d = 0$$

$$\textbf{SuperNode} : -C_4 \cdot (-e_8 + K_c \cdot C_{6,7} \cdot e_8) + C_3 \cdot (e_8 - k_c \cdot C_{6,7} \cdot e_8 - e_2) - C_5(e_6 - e_8 + k_c \cdot C_{6,7} \cdot e_8) + e_8 \cdot C_{6,7} + I_d = 0$$

In determining these equations, we have used, as for the the mesh analysis, the relations:

$$I_b = K_b \times V_b$$

$$V_b = (e_2 - e_8 + V_c)$$

$$V_c = K_c \times I_c$$

$$I_c = -e_8 \times C_{6,7}$$

In matrix form, the system of looks like this:

$$\begin{bmatrix} C_1 + C_2 + C_3 & -C_2 & 0 & -C_3(1 - K_c \cdot C_{6,7}) \\ -C_2 - K_b & C_2 & 0 & K_b \cdot (1 - K_c \cdot C_{6,7}) \\ K_b & 0 & C_5 & -(C_5 + K_b)(1 - K_c C_{6,7}) \\ -C_3 & 0 & -C_5 & (1 - K_c \cdot C_{6,7})(C_4 + C_3 + C_5) + C_{6,7} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} = \begin{bmatrix} V_a \cdot C_1 \\ 0 \\ I_d \\ -I_d \end{bmatrix}$$

Solving the system with octave and the previous data we reach the following results:

3 Simulation Analysis

3.1 Operating Point Analysis

Table 2 shows the simulated operating point results for the circuit under analysis.

Name	Value [A or V]
c[i]	0.000000e+00
gb[i]	-1.78554e-03
r1[i]	1.785457e-03
r2[i]	1.785538e-03
r3[i]	-8.11013e-08
r4[i]	-7.62925e-04
r5[i]	-1.78554e-03
r6[i]	-1.02253e-03
r7[i]	-1.02253e-03
v(1)	5.004394e+00
v(2)	3.148950e+00
v(3)	-5.44195e-01
v(5)	3.149196e+00
v(6)	8.719821e+00
v(7)	2.091220e+00
v(8)	3.157618e+00
na	0.000000e+00

Table 2: Operating point. Variables v(i) are of type *voltage* and expressed in Volt; other variables are of type *current* and expressed in Ampere

The results are the same as the ones obtained using Octave. The subject will be further developed in Section 4.

4 Conclusion

In this laboratory assignment the objective of analysing a simple circuit has been achieved. The analysis of the circuit was done both theoretically, using the Ocatve maths tool to solve the systems of equations obtained by using the mesh and node analysis, and by circuit simulation, using Ngspice. The simulation results perfectly matched the theoretical results. For example, if we look at the results tables for ngspice and octave we may note that the following relations hold:

$$y_1 = -r1[i]$$

$$y_2 = r2[i]$$

$$y_3 = r6[i] = r7[i]$$

$$C_i = 1/r[i]$$

which is to be expected. The reason for this perfect match is that the analysed circuit is linear, so that any method for analysing consists in solving a linear system of equations.