T1 - Introduction To Circuit Analysis

Integrated Master in Physics Engineering

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March 22, 2021

Contents

1 Introduction

The objective of this laboratory assignment is to study a circuit containing various resistors, two voltage sources and two current sources. The circuit can be seen in Figure 1.

In Section 2, a theoretical analysis of the circuit is presented. In Section 3, the circuit is analysed by simulation, and the results are compared to the theoretical results obtained in Section 2. The conclusions of this study are outlined in Section 4.

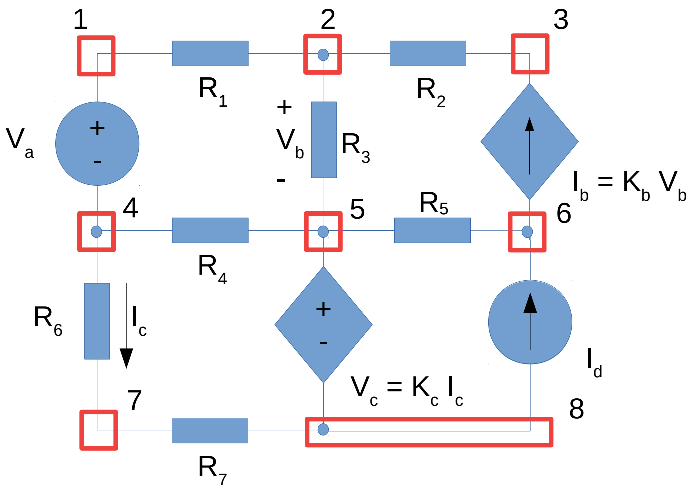

Figure 1: Circuit to be studied

2 Theoretical Analysis

In this section, the circuit shown in Figure 1 is analysed theoretically, first we aproach the circuit using the mesh analysis, and later we analyse the circuit using the nodal analysis.

2.1 Nodal Analysis

The general point of the node analysis method is to figure out the node voltages of all the nodes in our circuit (in relation to a reference node, which we call ground, and whose voltage we set to 0). Having figured this out, it's straightforward to determine the branch currents and voltages, thus completely solving the circuit.

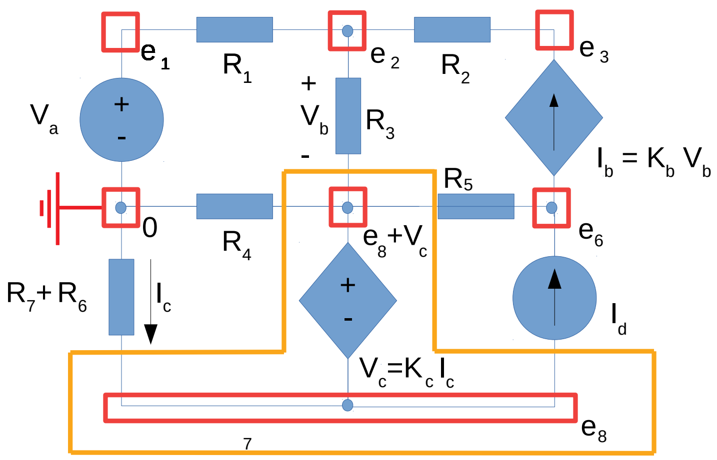

Figure 2: Nodes used in Nodal Analysis

To do this, we first need to identify and label all the nodes in our system as seen in Figure 2, as well as choose the ground node. In our case, we defined the node connected to resistance R_4 and to voltage source V_a as ground. The second step consists in determining the voltages of the easy nodes. In our circuit, it is clear to see, for example, that $e_1 = V_a$. To solve the other non-trivial nodes, we proceed to write Kirchhoff's current law for each one of them, which states that the sum of currents going into a node must be 0; or, in other words, that charges may not accumulate in one singular node:

$$\sum_i y_i = 0$$

where y_i is a current defined as going **into** the node.

Using Ohm's Law, which states that $I = U/R$, and assuming we know the values of the resistances of the elements in each branch (also noticing that we can write the branch voltages as differences between node voltages), we get a system of equations that allows us to determine each node voltage.

Since this method seems to rely upon Ohm's law, it seems to be a fatal problem that we have a dependent voltage source in our circuit, V_c, for which we **can't** write ohm's law. To deal with this, we create a super-node by lumping together the nodes to which V_c is connected and we write KCL for the super-node. To find the missing equation, we simply note that V(negative terminal of dependent voltage source) + V_c = V(positive terminal of dependent voltage source), thus getting two equations for our two "problematic" nodes.

We can therefore write the following equations:

$$\mathbf{Node_1} : e_1 = V_a$$

$$\mathbf{Node_2} : -C_1 \cdot (e_1 - e_2) - C_3 \cdot (e_8 - K_c \cdot C_{6,7} \cdot e_8 - e_2) - C_2 \cdot (e_3 - e_2) = 0$$

$$\mathbf{Node_3} : C_2 \cdot (e_3 - e_2) - K_b \cdot (e_2 - e_8 + K_c \cdot C_{6,7} \cdot e_8) = 0$$

$$\mathbf{Node_6} : C_5 \cdot (e_6 - e_8 + K_c \cdot C_{6,7} \cdot e_8) + K_b \cdot (e_2 - e_8 + K_c \cdot C_{6,7} \cdot e_8) - I_d = 0$$

$$\mathbf{SuperNode} : -C_4 \cdot (-e_8 + K_c \cdot C_{6,7} \cdot e_8) + C_3 \cdot (e_8 - k_c \cdot C_{6,7} \cdot e_8 - e_2) - C_5(e_6 - e_8 + k_c \cdot C_{6,7} \cdot e_8) + e_8 \cdot C_{6,7} + I_d = 0$$

In determining these equations, we have used, as for the the mesh analysis, the relations:

$$I_b = K_b \times V_b$$

$$V_b = (e_2 - e_8 + V_c)$$

$$V_c = K_c \times I_c$$

$$I_c = -e_8 \times C_{6,7}$$

In matrix form, the system of looks like this:

$$\begin{bmatrix} C_1 + C_2 + C_3 & -C_2 & 0 & -C_3(1 - K_c \cdot C_{6,7}) \\ -C_2 - K_b & C_2 & 0 & K_b \cdot (1 - K_c \cdot C_{6,7}) \\ K_b & 0 & C_5 & -(C_5 + K_b)(1 - K_c C_{6,7}) \\ -C_3 & 0 & -C_5 & (1 - K_c \cdot C_{6,7})(C_4 + C_3 + C_5) + C_{6,7} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} = \begin{bmatrix} V_a \cdot C_1 \\ 0 \\ I_d \\ -I_d \end{bmatrix}$$

Solving the system with octave and the previous data we reach the following results: Table with results created with octave

V(1)	5.00439410964
V(2)	3.14895076846
V(3)	-0.54419560583
V(5)	3.14919681026
V(6)	8.71982213858
V(7)	2.09122033060
V(8)	3.15761879808

Table 1: Results of Nodal Analysis using Octave

3 Simulation Analysis

3.1 Operating Point Analysis

3.1.1 First task

Table 2 shows ——————————————

Name	Value [A or V]
@c[i]	0.000000e+00
@gb[i]	-1.78554e-03
@r1[i]	1.785458e-03
@r2[i]	1.785539e-03
@r3[i]	-8.11013e-08
@r4[i]	-7.62926e-04
@r5[i]	-1.78554e-03
@r6[i]	-1.02253e-03
@r7[i]	-1.02253e-03
v(1)	5.004394e+00
v(2)	3.148951e+00
v(3)	-5.44196e-01
v(5)	3.149197e+00
v(6)	8.719822e+00
v(7)	2.091220e+00
v(8)	3.157619e+00
na	0.000000e+00

Table 2: Operating point. Variables v(i) are of type *voltage* and expressed in Volt; other variables are of type *current* and expressed in Ampere

The results are the same as the ones obtained using Octave. The subject will be further developed in Section 4.

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3.1.2 Second task

Table 3 ——————————————

Name	Value [A or V]
@gb[i]	8.548293e-19
@r1[i]	-8.54790e-19
@r2[i]	-8.54829e-19
@r3[i]	3.882738e-23
@r4[i]	-2.15170e-19
@r5[i]	-1.78284e-03
@r6[i]	-2.87619e-19
@r7[i]	-2.87619e-19
v(1)	0.000000e+00
v(2)	8.882962e-16
v(3)	2.656395e-15
v(5)	8.881784e-16
v(6)	5.562203e+00
v(7)	5.882207e-16
v(8)	8.881784e-16
na	0.000000e+00

Table 3: Operating point. Variables v(i) are of type *voltage* and expressed in Volt; other variables are of type *current* and expressed in Ampere

4 Conclusion

In this laboratory assignment the objective of analysing a simple circuit has been achieved. The analysis of the circuit was done both theoretically, using the Ocatve maths tool to solve the systems of equations obtained by using the mesh and node analysis, and by circuit simulation, using Ngspice. The simulation results perfectly matched the theoretical results. For example, if we look at the results tables for ngspice and octave we may note that the following relations hold:

$$y_1 = -r1[i]$$

$$y_2 = r2[i]$$

$$y_3 = r6[i] = r7[i]$$

$$C_i = 1/r[i]$$

which is to be expected. The reason for this perfect match is that the analysed circuit is linear, so that any method for analysing consists in solving a linear system of equations.