T1 - Introduction To Circuit Analysis

Integrated Master in Physics Engineering

João Lehodey (96538), Jorge Silva (96545), Pedro Monteiro (93156)

March 22, 2021

Contents

1 Introduction

The objective of this laboratory assignment is to study a circuit containing a various resistors, two voltage sources and two current sources. The circuit can be seen if Figure 1.

In Section 2, a theoretical analysis of the circuit is presented. In Section 3, the circuit is analysed by simulation, and the results are compared to the theoretical results obtained in Section 2. The conclusions of this study are outlined in Section 4.

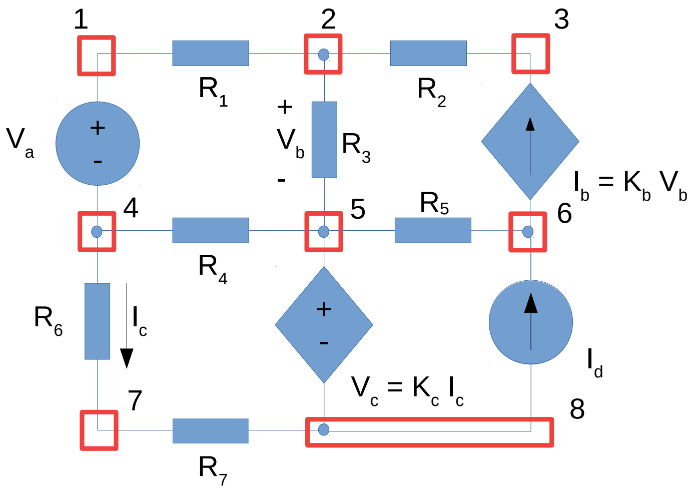

Figure 1: Circuit to be studied

2 Theoretical Analysis

In this section, the circuit shown in Figure 1 is analysed theoretically, first we aproach the circuit using the mesh analysis, and later we analyse the circuit using the nodal analysis.

2.1 Mesh Analysis

As seen during theoretical lessons, we can use a mesh analysis to analyse the circuit. This method is built upon Kirchhoff's Voltage Law that states:

In a mesh, the sum of all voltages equals 0.

$$\sum_{i=0}^{n} V_i = 0 \tag{1}$$

The method consists of identifying every mesh, labeling its current and choosing the currents direction. Then the KVL equations are written for each mesh, and we can solve the system of equations, solving consequently the circuit.

As seen in in Figure 3, there are four meshes, each with currents Y_a, Y_b, Y_c, Y_d, we will label each mesh as A, B, C, D respectively.

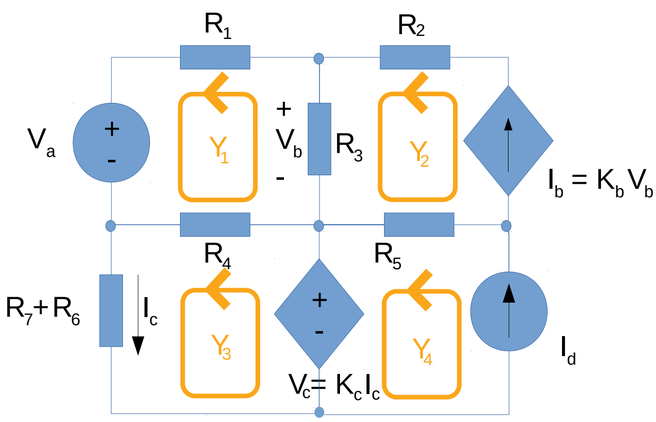

Figure 2: Current choosen for each mesh

The equations for each mesh are therefore:

$$\mathbf{E_1} : y_1 + R_4 \cdot (y_1 - y_3) + R_3 \cdot (y_1 - y_2) + y_1 \cdot R_1 = 0$$

$$\mathbf{E_2} : y_2 \cdot (1 - R_3 \cdot K_b) + K_b \cdot R_3 \cdot y_1 = 0$$

$$\mathbf{E_3} : y_3 \cdot R_6 + y_3 \cdot R_7 - V_c + (y_3 - y_1) \cdot R_4 = 0$$

$$\mathbf{E_4} : y_4 = I_d$$

In determining equation*s $\mathbf{E_1}$ to $\mathbf{E_4}$, we have used the following relations:

$$I_b = K_b \times V_b$$

$$V_b = R_3 \times (y_3 - y_1)$$

$$V_c = K_c \times I_c$$

$$I_c = y_3$$

In matrix form, the system looks like the following:

$$\begin{bmatrix} R_1 + R_2 + R_3 & -R_3 & -R_4 & 0 \\ K_b R_3 & 1 - R_3 K_b & 0 & 0 \\ -R_4 & 0 & R_6 + R_7 + R_4 - K_c & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} -V_a \\ 0 \\ 0 \\ I_d \end{bmatrix}$$

Solving this system of equations, using the following values generated with the t1_datagen.py script using the number 93156:

R1	1.03919759193
R2	2.06836523173
R3	3.03375774261
R4	4.12779067183
R5	3.11985677803
R6	2.04513887844
R7	1.04289965713
Va	5.00439410964
Id	1.04536428769
Kb	7.25705461539
Kc	8.23640363075

Table 1: Data generated using number 93156

We reach the following results, using octave:

y_1	-0.001880
y_2	-0.001970
y_3	-0.001076
y_4	0.001045

Table 2: Results of Mesh Analysis using Octave

2.2 Nodal Analysis

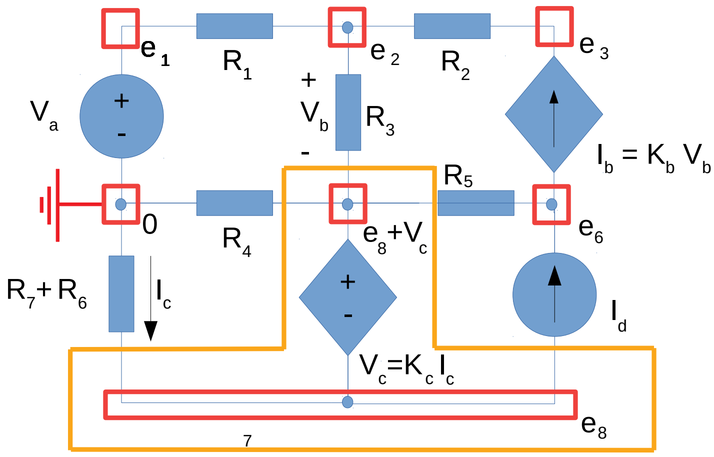

Figure 3: Nodes used in Nodal Analysis

SMALL INTRO

$$\mathbf{Nó_0} : e_0 = V_a$$

$$\mathbf{Nó_1} : -C_1 \cdot (e_0 - e_1) - C_3 \cdot (e_4 - K_c \cdot C_{6,7} \cdot e_4 - e_1) - C_2 \cdot (e_2 - e_1) = 0$$
$$\mathbf{Nó_2} : C_2 \cdot (e_2 - e_1) - K_b \cdot (e_1 - e_4 + K_c \cdot C_{6,7} \cdot e_4) = 0$$
$$\mathbf{Nó_3} : C_5 \cdot (e_3 - e_4 + K_c \cdot C_{6,7} \cdot e_4) + K_b \cdot (e_1 - e_4 + K_c \cdot C_{6,7} \cdot e_4) - I_d = 0$$
$$\mathbf{SuperNó} : -C_4 \cdot (-e_4 + K_c \cdot C_{6,7} \cdot e_4) + C_3 \cdot (e_4 - k_c \cdot C_{6,7} \cdot e_4 - e_1) - C_5(e_3 - e_4 + k_c \cdot C_{6,7} \cdot e_6) + e_4 C_{6,7} + I_d = 0$$

In determining these equations, we have used, as for the the mesh analysis, the relations:

$$I_b = K_b \times V_b$$

$$V_b = (e_1 - e_4 + V_c)$$
$$V_c = K_c \times I_c$$
$$I_c = -e_4 \times C_{6,7}$$

In matrix form, the system of looks like this:

$$\begin{bmatrix} C_1 + C_2 + C_3 & -C_2 & 0 & -C_3(1 - k_c \cdot C_{6,7}) \\ -C_2 - K_b & C_2 & 0 & K_b \cdot (1 - K_c \cdot C_{6,7}) \\ K_b & 0 & C_5 & -(C_5 + K_b)(1 - K_c C_{6,7}) \\ -C_3 & 0 & -C_5 & (1 - K_c \cdot C_{6,7})(C_4 + C_3 + C_5) + C_{6,7} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} = \begin{bmatrix} V_a \cdot C_1 \\ 0 \\ I_d \\ -I_d \end{bmatrix}$$

Reaching the following results with octave:

V2	3.050285
V3	-1.024137
V5	12.728845
V7	3.321737

Table 3: Results of Nodal Analysis using Octave

3 Simulation Analysis

3.1 Operating Point Analysis

Table 4 shows the simulated operating point results for the circuit under analysis. Compared to the theoretical analysis results, one notices the following differences: describe and explain the differences.

Name	Value [A or V]
gib[i]	-1.96988e-03
id[current]	1.045364e-03
r1[i]	1.880402e-03
r2[i]	-1.96988e-03
r3[i]	8.947414e-05
r4[i]	-8.04723e-04
r5[i]	3.015240e-03
r6[i]	-1.07568e-03
r7[i]	-1.07568e-03
v(1)	5.004394e+00
v(2)	3.050285e+00
v(3)	-1.02414e+00
v(4)	3.321728e+00
v(5)	1.272885e+01
v(6)	2.199912e+00
v(7)	3.321737e+00
v(8)	2.199912e+00
v(10)	0.000000e+00

Table 4: Operating point. Variables v(i) are of type *voltage* and expressed in Volt; other variables are of type *current* and expressed in Ampere

4 Conclusion

In this laboratory assignment the objective of analysing an RC circuit has been achieved. Static, time and frequency analyses have been performed both theoretically using the Octave maths tool and by circuit simulation using the Ngspice tool. The simulation results matched the theoretical results precisely. The reason for this perfect match is the fact that this is a straightforward circuit containing only linear components, so the theoretical and simulation models cannot differ. For more complex components, the theoretical and simulation models could differ but this is not the case in this work.

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.