

T1 - Introduction To Circuit Analysis

Integrated Master in Physics Engineering

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1 Introduction

The objective of this laboratory assignment is to study a circuit containing various resistors, two voltage sources and two current sources. The circuit can be seen in Figure 1.

In Section 2, a theoretical analysis of the circuit is presented. In Section 3, the circuit is analysed by simulation, and the results are compared to the theoretical results obtained in Section 2. The conclusions of this study are outlined in Section 4.

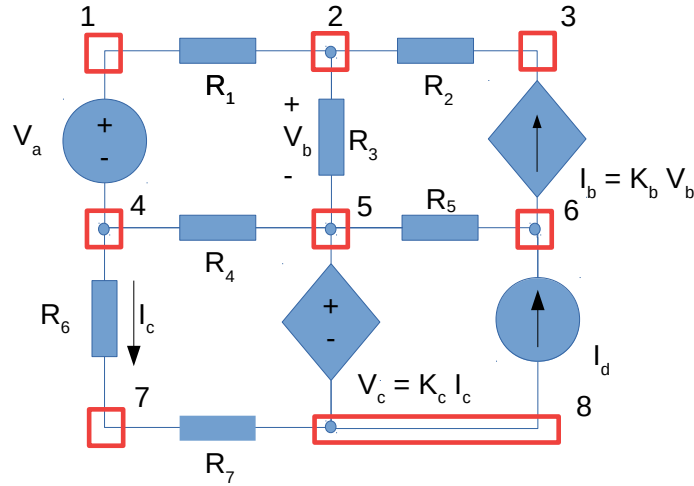


Figure 1: Circuit to be studied

2 Theoretical Analysis

In this section, the circuit shown in Figure 1 is analysed theoretically, first we approach the circuit using the mesh analysis, and later we analyse the circuit using the nodal analysis.

2.1 Mesh Analysis

As seen during theoretical lessons, we can use a mesh analysis to analyse the circuit. This method is built upon Kirchhoff's Voltage Law that states:

In a mesh, the sum of all voltages equals 0.

$$\sum_{i=0}^n V_i = 0 \quad (1)$$

The method consists of identifying every mesh, labeling its current, and choosing the direction of the currents. Then the KVL equations are written for each mesh, and we can solve the system of equations, solving consequently the circuit.

As seen in Figure 2, there are four meshes, each with currents Y_a, Y_b, Y_c, Y_d , we will label each mesh as A, B, C, D respectively.

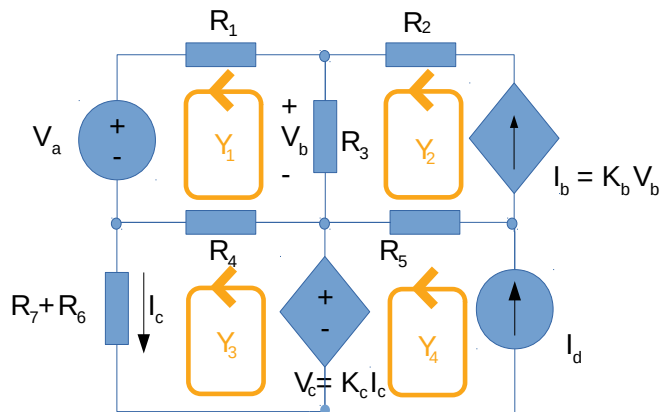


Figure 2: Current chosen for each mesh

The equations for each mesh are therefore:

$$\mathbf{E}_1 : y_1 + R_4 \cdot (y_1 - y_3) + R_3 \cdot (y_1 - y_2) + y_1 \cdot R_1 = 0$$

$$\mathbf{E}_2 : y_2 \cdot (1 - R_3 \cdot K_b) + K_b \cdot R_3 \cdot y_1 = 0$$

$$\mathbf{E}_3 : y_3 \cdot R_6 + y_3 \cdot R_7 - V_c + (y_3 - y_1) \cdot R_4 = 0$$

$$\mathbf{E}_4 : y_4 = I_d$$

In determining equations \mathbf{E}_1 to \mathbf{E}_4 , we have used the following relations:

$$I_b = K_b \times V_b$$

$$V_b = R_3 \times (y_2 - y_1)$$

$$V_c = K_c \times I_c$$

$$I_c = y_3$$

In matrix form, the system looks like the following:

$$\begin{bmatrix} R_1 + R_2 + R_3 & -R_3 & -R_4 & 0 \\ K_b R_3 & 1 - R_3 K_b & 0 & 0 \\ -R_4 & 0 & R_6 + R_7 + R_4 - K_c & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} -V_a \\ 0 \\ 0 \\ I_d \end{bmatrix}$$

Solving this system of equations, using the following values generated with the t1_datagen.py script using the number 93156:

R1	1.03919759193
R2	2.06836523173
R3	3.03375774261
R4	4.12779067183
R5	3.11985677803
R6	2.04513887844
R7	1.04289965713
Va	5.00439410964
Id	1.04536428769
Kb	7.25705461539
Kc	8.23640363075

Table 1: Data generated using number 93156

We reach the following results, using octave:

We can therefore write the following equations:

$$\text{Node}_1 : e_1 = V_a$$

$$\text{Node}_2 : -C_1 \cdot (e_1 - e_2) - C_3 \cdot (e_8 - K_c \cdot C_{6,7} \cdot e_8 - e_2) - C_2 \cdot (e_3 - e_2) = 0$$

$$\text{Node}_3 : C_2 \cdot (e_3 - e_2) - K_b \cdot (e_2 - e_8 + K_c \cdot C_{6,7} \cdot e_8) = 0$$

$$\text{Node}_6 : C_5 \cdot (e_6 - e_8 + K_c \cdot C_{6,7} \cdot e_8) + K_b \cdot (e_2 - e_8 + K_c \cdot C_{6,7} \cdot e_8) - I_d = 0$$

$$\text{SuperNode} : -C_4 \cdot (-e_8 + K_c \cdot C_{6,7} \cdot e_8) + C_3 \cdot (e_8 - K_c \cdot C_{6,7} \cdot e_8 - e_2) - C_5 \cdot (e_6 - e_8 + K_c \cdot C_{6,7} \cdot e_8) + e_8 \cdot C_{6,7} + I_d = 0$$

In determining these equations, we have used, as for the the mesh analysis, the relations:

$$I_b = K_b \times V_b$$

$$V_b = (e_2 - e_8 + V_c)$$

$$V_c = K_c \times I_c$$

$$I_c = -e_8 \times C_{6,7}$$

In matrix form, the system of looks like this:

$$\begin{bmatrix} C_1 + C_2 + C_3 & -C_2 & 0 & -C_3(1 - K_c \cdot C_{6,7}) \\ -C_2 - K_b & C_2 & 0 & K_b \cdot (1 - K_c \cdot C_{6,7}) \\ K_b & 0 & C_5 & -(C_5 + K_b)(1 - K_c C_{6,7}) \\ -C_3 & 0 & -C_5 & (1 - K_c \cdot C_{6,7})(C_4 + C_3 + C_5) + C_{6,7} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} = \begin{bmatrix} V_a \cdot C_1 \\ 0 \\ I_d \\ -I_d \end{bmatrix}$$

Solving the system with octave and the previous data we reach the following results:

V2	3.050285
V3	-1.024137
V5	12.728845
V7	3.321737

Table 3: Results of Nodal Analysis using Octave

3 Simulation Analysis

3.1 Operating Point Analysis

Table 4 shows the simulated operating point results for the circuit under analysis.

Name	Value [A or V]
gib[i]	-1.96988e-03
id[current]	1.045364e-03
r1[i]	1.880402e-03
r2[i]	-1.96988e-03
r3[i]	8.947414e-05
r4[i]	-8.04723e-04
r5[i]	3.015240e-03
r6[i]	-1.07568e-03
r7[i]	-1.07568e-03
v(1)	5.004394e+00
v(2)	3.050285e+00
v(3)	-1.02414e+00
v(4)	3.321728e+00
v(5)	1.272885e+01
v(6)	2.199912e+00
v(7)	3.321737e+00
v(8)	2.199912e+00
v(10)	0.000000e+00

Table 4: Operating point. Variables v(i) are of type *voltage* and expressed in Volt; other variables are of type *current* and expressed in Ampere

The results are the same as the ones obtained using Octave. The subject will be further developed in Section 4.

4 Conclusion

In this laboratory assignment the objective of analysing a simple circuit has been achieved. The analysis of the circuit was done both theoretically, using the Octave maths tool to solve the systems of equations obtained by using the mesh and node analysis, and by circuit simulation, using Ngspice. The simulation results perfectly matched the theoretical results. For example, if we look at the results tables for ngspice and octave we may note that the following relations hold:

$$y_1 = -r1[i]$$

$$y_2 = r2[i]$$

$$y_3 = r6[i] = r7[i]$$

$$C_i = 1/r[i]$$

which is to be expected. The reason for this perfect match is that the analysed circuit is linear, so that any method for analysing consists in solving a linear system of equations.