Measurement of the Gravitational Constant via the Cavendish Method

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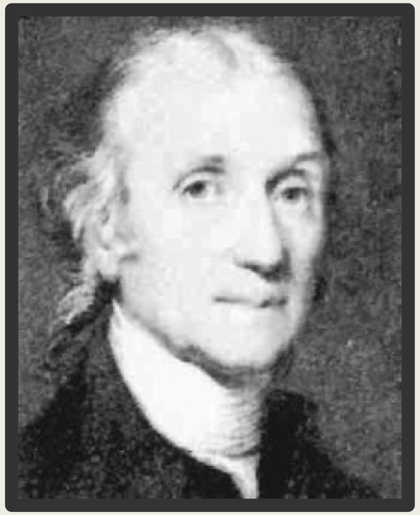
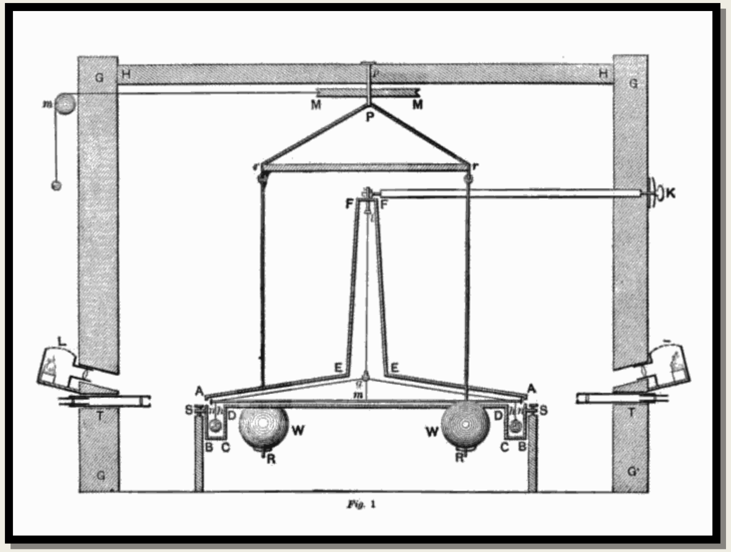
Purpose

- To measure the Universal Gravitational Constant (G) utilizing a torsional oscillator, two large and two small masses, and finding a change in torque by measuring the gravitational force between the large and small masses.

Physics Background



- The Cavendish experiment (1797-1798)
- Apparatus based on the torsion balance first created by geologist John Mitchell.
- Reconstructed by Henry Cavendish to measure the mass of the Earth.
- No notion of gravitational constant.



Physics Background

$$\tau_{net} = \sum \tau_n$$

$$\tau_{net} = F_1 \cdot d_1 + F_2 \cdot d_2$$

$$\boxed{d_1 = d_2 = L/2}$$

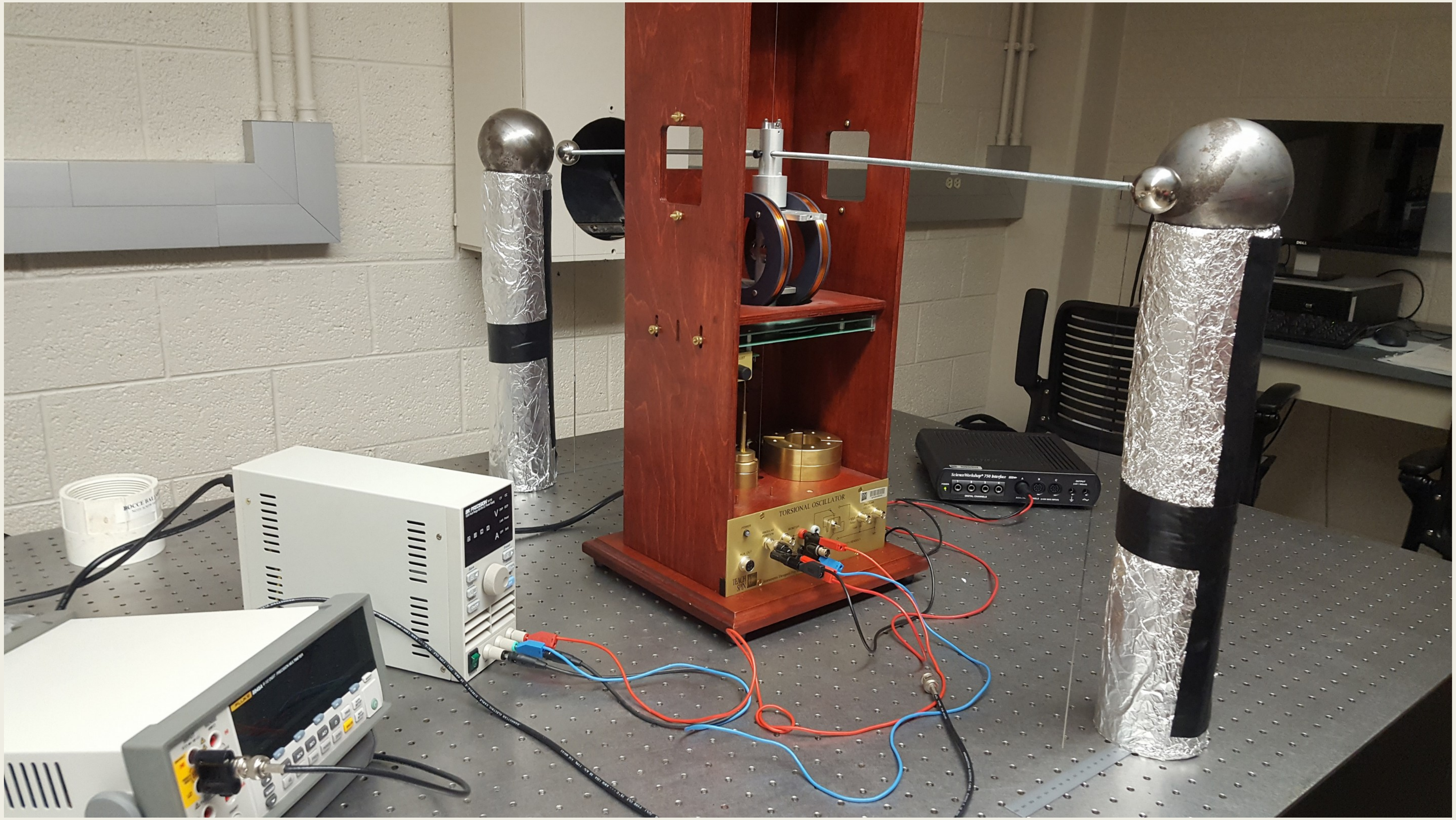
$$\tau_{net} = (F_1 + F_2)\frac{L}{2} = \kappa\theta$$

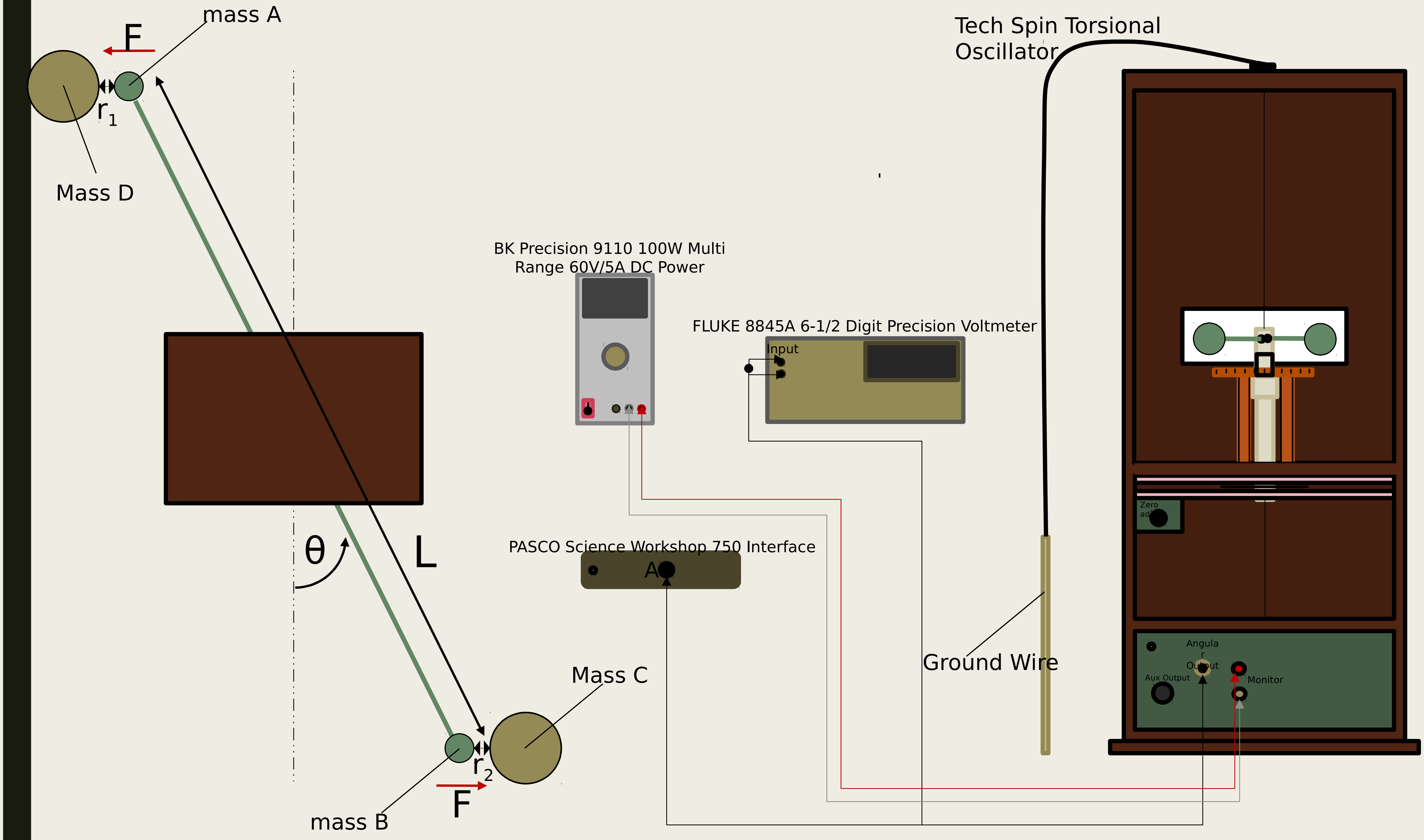
$$(G\frac{M_1 m_1}{r_1^2} + G\frac{M_2 m_2}{r_2^2})\frac{L}{2} = \kappa\theta$$

$$G = \frac{2\kappa\theta}{(\frac{M_1 m_1}{r_1^2} + \frac{M_2 m_2}{r_2^2})L}$$

$$\boxed{G = \frac{2\kappa\theta}{\left(\frac{M_1(m_1+l_1\mu_{rod})}{r_1^2} + \frac{M_2(m_2+l_2\mu_{rod})}{r_2^2}\right)L}}$$

Apparatus





mass A
F
r1
Mass D
θ
L
r2
F
mass B
Mass C
BK Precision 9110 100W Multi Range 60V/5A DC Power
FLUKE 8845A 6-1/2 Digit Precision Voltmeter
Input
PASCO Science Workshop 750 Interface
A
Tech Spin Torsional Oscillator
Ground Wire
Zero adj
Angular Output
Aux Output
Monitor

Procedure

- Before the experiment can begin, an inventory of measurements describing the physical characteristics of the system are taken.
  - *Length and mass of the rod connecting two small spherical masses.*
  - *Diameter of the two small and two large spherical masses.*
  - *Mass of the two small and two large spherical masses (Metrology Lab).*
- To begin the experiment, we calibrated angle monitor voltage proportional to the angle of rotation.
  - *Record zero-angle and corresponding voltage monitored by Fluke voltmeter.*
  - *Apply voltage using BK precision power supply, inducing a rotation of a pre-decided angle.*
  - *Interpolate the data, applied voltage vs. angle of rotation.*

Procedure

- Determine the torsional constant of the steel torsion wire.
  - *Systematically increase the moment of inertia of the system and the measure corresponding periods.*
- Prepare the apparatus for measurement of the universal gravitational constant.
  - *Center rod, attach small masses, hang needle from thread, and situate grounded PVC stands.*
- Measurement of the universal gravitational constant
  - *Place large masses on stands, record the two center-to-center radii, and measure the voltage due to the force of mutual attraction between small and large masses.*
  - *Convert the voltage to a change in angle.*
  - *Plug-n-chug*

$$G = \frac{2\kappa\theta}{\left(\frac{M_1(m_1+l_1\mu_{rod})}{r_1^2} + \frac{M_2(m_2+l_2\mu_{rod})}{r_2^2}\right)L}$$

Data & Calculations

Small Mass A (m_A)	$164.5211 \pm 0.0011\ g$	Small Mass B (m_B)	$167.1469 \pm 0.0011\ g$
Depth of Hole Mass A (l_A)	$2.91 \pm 0.01\ cm$	Depth of Holes Mass B (l_B)	$2.90 \pm 0.01\ cm$
Large Mass D (M_D)	$4281.787 \pm 0.030\ g$	Large Mass C (M_C)	$4279.259 \pm 0.030\ g$
Length of Rod (L)	$91.7 \pm 0.1\ cm$	Mass of Rod (m_{rod})	$241.3 \pm 0.1\ g$
Mass/Length of Rod (μ_{rod})	$2.631 \pm 0.003\ g/cm$	Mass of Brass Quadrant (m_{brass})	$214.5 \pm 0.5\ g$
Inner Radius of Brass Quadrant (r_{inner})	$0.86\ in$	Outter Radius of Brass Quadrant (r_{outter})	$1.86\ in$
Distance Masses A - D (r_{AD})	$69.1 \pm 0.1\ mm$	Distance Masses B - C (r_{BC})	$71.5 \pm 0.1\ mm$

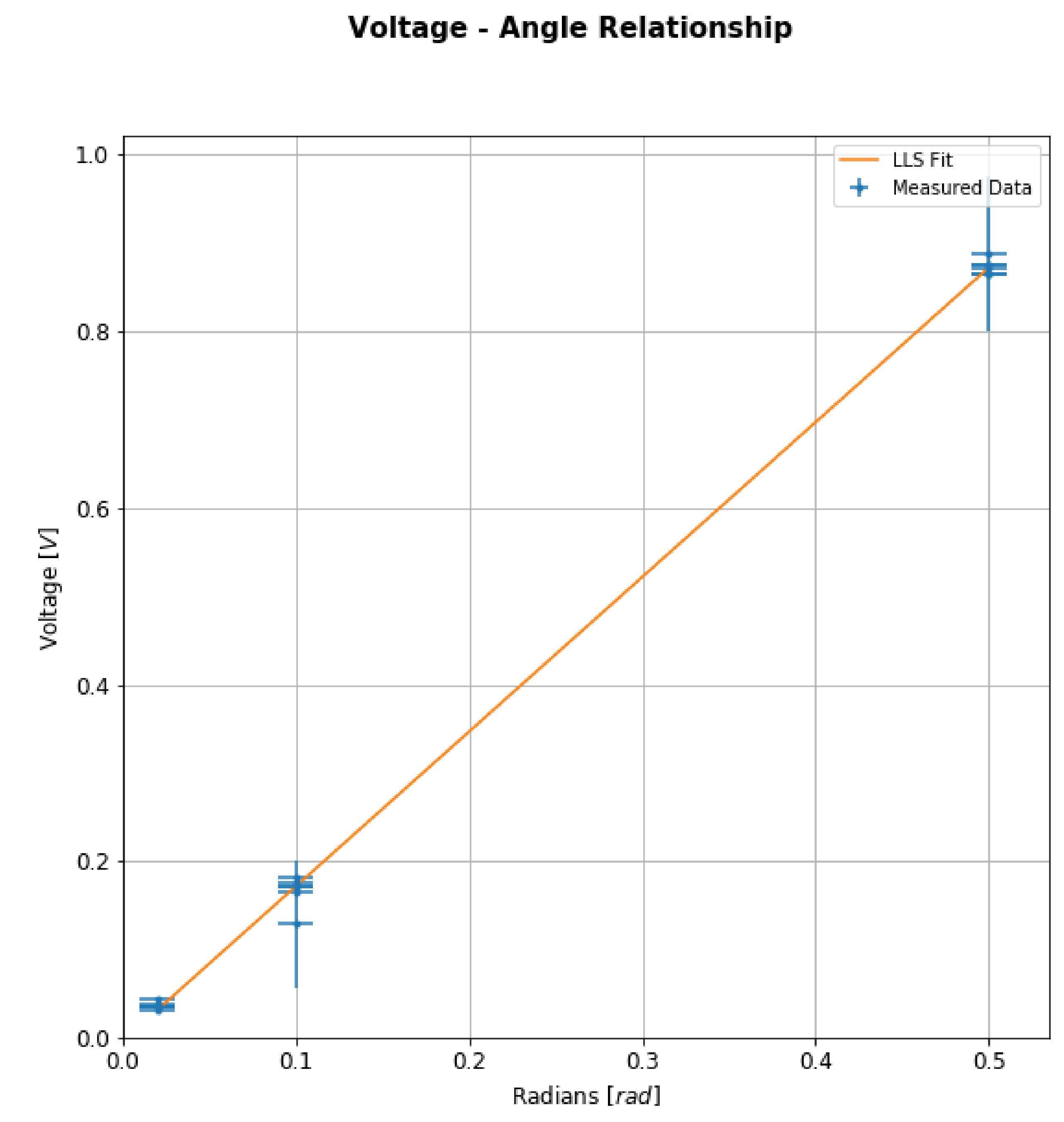
$$G = \frac{2\kappa\theta}{\left(\frac{M_1(m_1+l_1\mu_{rod})}{r_1^2} + \frac{M_2(m_2+l_2\mu_{rod})}{r_2^2}\right)L}$$

Data &

Angle ($\Delta\theta$)	Voltage ($V \pm \Delta V$) [μV]	
0.02	44.22759 ± 0.07881	Right
	35.46271 ± 0.16265	
	35.65744 ± 0.07063	
	36.87462 ± 0.05519	Left
	31.12234 ± 0.18475	
	35.40876 ± 0.05493	
0.1	129.0290 ± 72.2142	Right
	181.5122 ± 0.0755	
	170.9361 ± 0.1016	
	171.4923 ± 0.0751	Left
	166.1184 ± 0.0247	
	176.3709 ± 0.0617	
0.5	887.7279 ± 87.4721	Right
	875.4954 ± 0.0565	
	875.4954 ± 0.1897	
	865.5919 ± 0.0376	Left
	865.3777 ± 0.0375	
	870.7073 ± 0.0741	

Linear Least Squares Fit

$$\theta(V) = bV + a$$

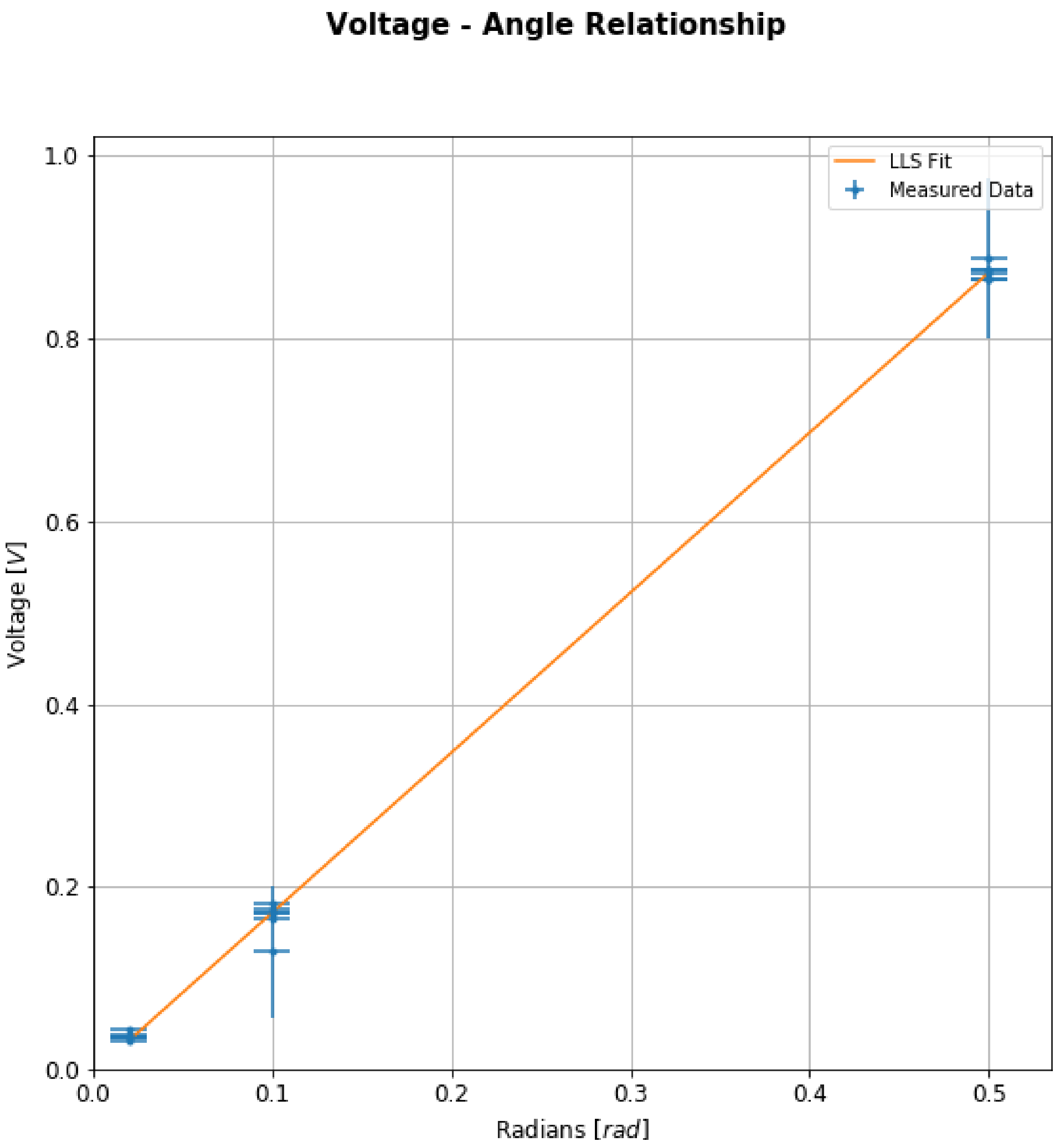


Data & Calculations

b	0.57086 ± 0.00463
a	0.00196 ± 0.00239
χ^2/ν	11875.46

$$\theta = 0.57086V + 0.00196$$

$$\Delta\theta = \sqrt{(0.00463)^2 V^2 + (0.00239)^2}$$



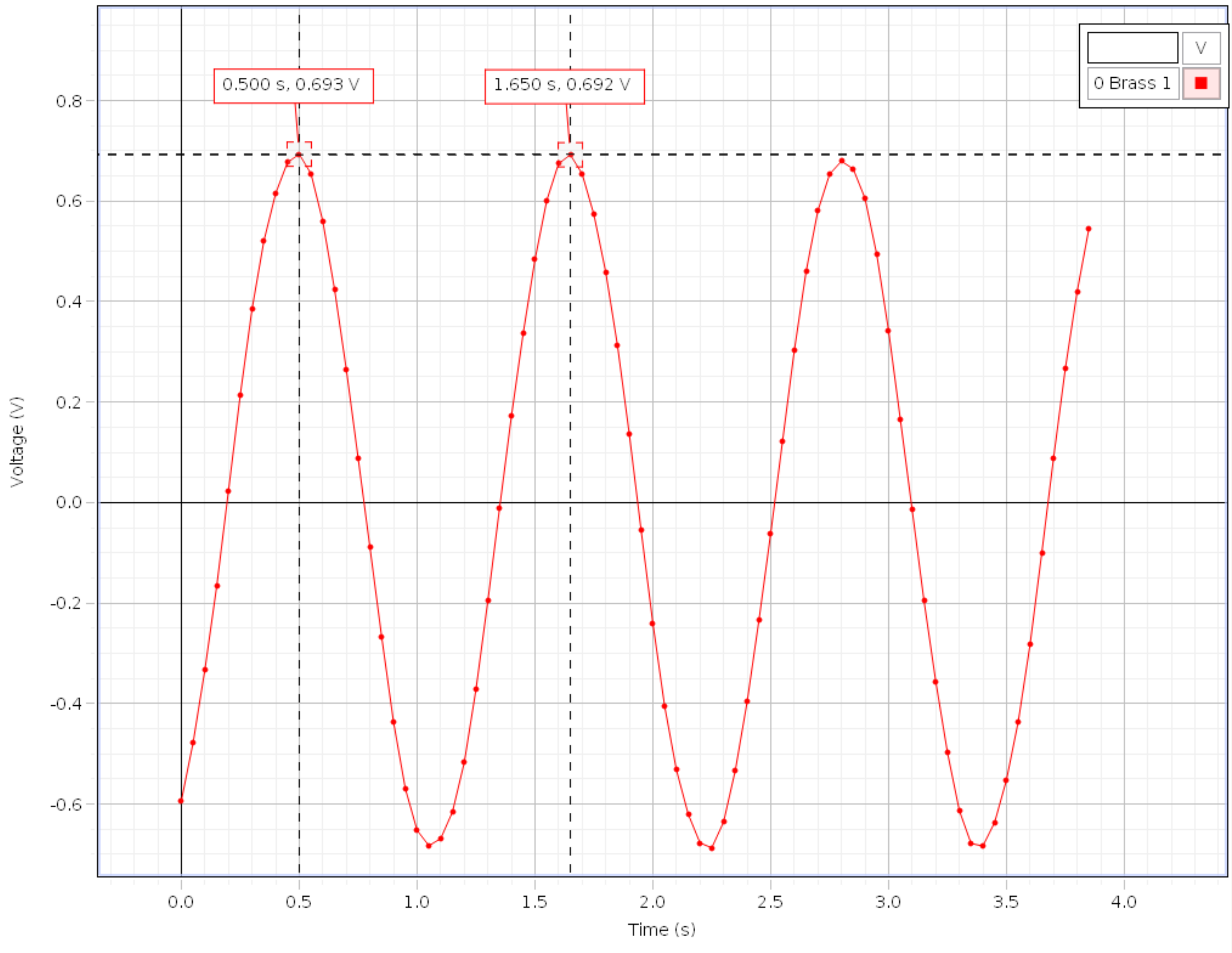
Data & Calculations

# Brass Quadrants	Period $(T \pm \Delta T)$ $[s]$
0	1.15 ± 0.05
2	1.3 ± 0.05
4	1.5 ± 0.05
6	1.55 ± 0.05
8	1.7 ± 0.05

Linear Least Squares Fit:

$$T = bn + a$$

$$T = \frac{4\pi^2 \Delta I}{\kappa} n + \frac{4\pi^2 I_0}{\kappa}$$



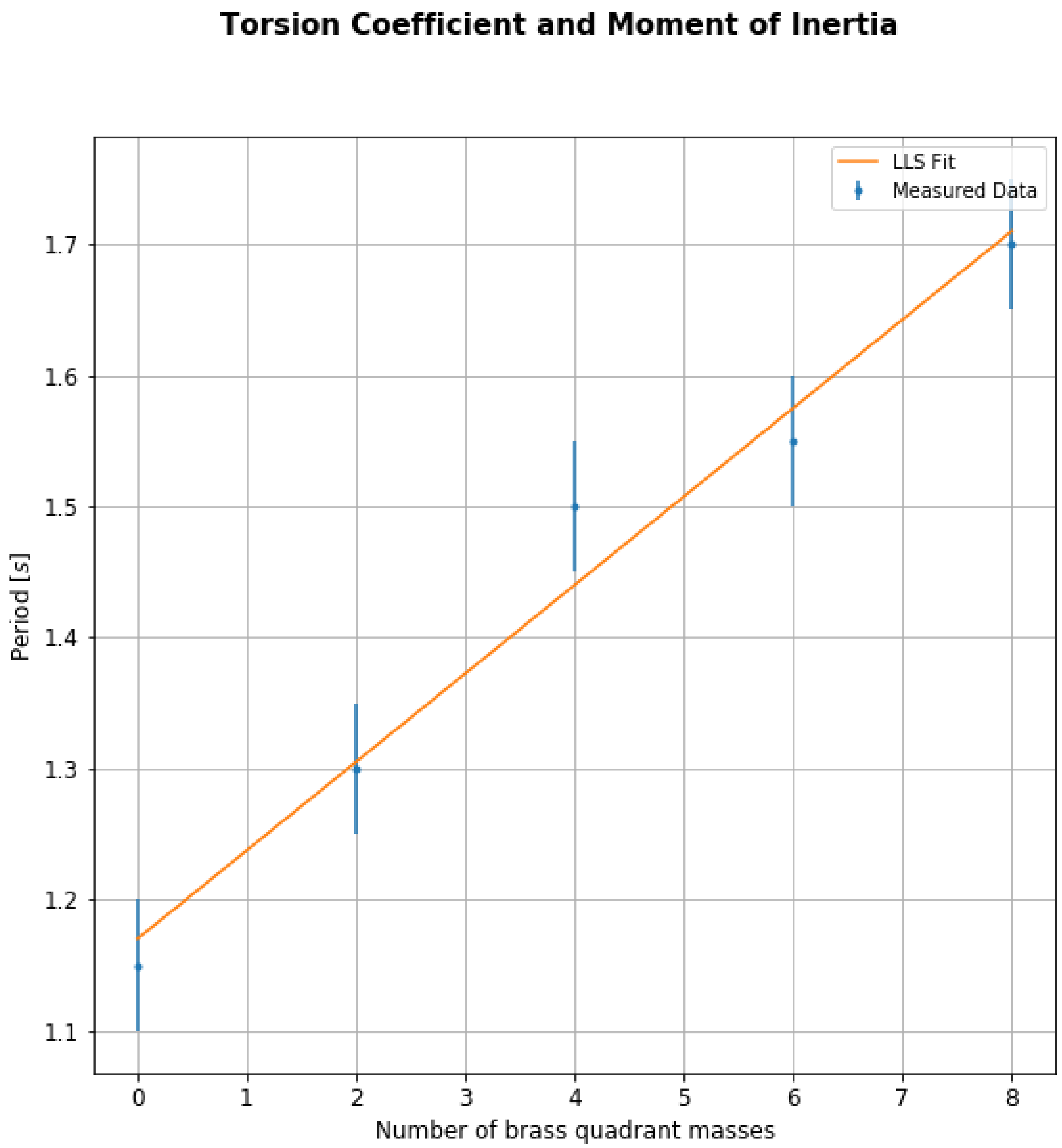
Data & Calculations

b	0.192 ± 0.016
a	1.341 ± 0.077
χ^2/ν	0.44

$$\Delta I = \frac{1}{2} m_{brass}(r_{outer}^2 + r_{inner}^2)$$

$$b = \frac{4\pi^2 \Delta I}{\kappa} \quad \Rightarrow \quad \kappa = \frac{4\pi^2 \Delta I}{b}$$

ΔI	$2905.6 \pm 6.8\ g \cdot cm^2$
κ	$0.16994 \pm 0.00040\ \frac{N \cdot m}{rad}$



Data & Calculations

	No Balls	Balls
1	$4.648660 \pm 0.0182331\ mV$	$6.436821 \pm 0.0176908\ mV$
2	$4.349725 \pm 0.0167990\ mV$	$5.253742 \pm 0.0188325\ mV$
3	$4.535125 \pm 0.0158124\ mV$	$5.737554 \pm 0.0196152\ mV$
4	$4.884205 \pm 0.0135203\ mV$	$5.868733 \pm 0.0159878\ mV$
5	$4.820535 \pm 0.0195627\ mV$	$5.466052 \pm 0.0168731\ mV$

$$\theta = 0.57086(V_{Balls} - V_{NoBalls})$$

$$G = \frac{2\kappa\theta}{\left(\frac{M_1(m_1+l_1\mu_{rod})}{r_1^2} + \frac{M_2(m_2+l_2\mu_{rod})}{r_2^2}\right)L}$$

Really long uncertainty…

Let Python handle it!

Evaluation of Uncertainties

- Systematic Errors:
  - *Many of the length measurements were not measured more than once*
    - Best guess of uncertainty is due to instrumental limitations
  - *Periods of oscillation were not measured across various peaks*

- Random Errors:
  - *Possible ferromagnetic force dominating measurement*
  - *Noise in Voltmeter*
    - Actually prevented us from being able to even measure G in the first place!

$$G = \frac{2\kappa\theta}{\left(\frac{M_1(m_1+l_1\mu_{rod})}{r_1^2} + \frac{M_2(m_2+l_2\mu_{rod})}{r_2^2}\right)L}$$

Results

	$G_{calc}\ [\frac{m^3}{k \cdot s^2}]$
1	$1.2581 \pm 0.0182 \times 10^{-6}$
2	$6.3605 \pm 0.1783 \times 10^{-7}$
3	$8.4601 \pm 0.1786 \times 10^{-7}$
4	$6.9270 \pm 0.1484 \times 10^{-7}$
5	$4.5418 \pm 0.1821 \times 10^{-7}$
Nominal	6.676×10^{-11}

$$Precision \equiv 1 - \frac{\Delta G_{exp}}{G_{exp}}$$

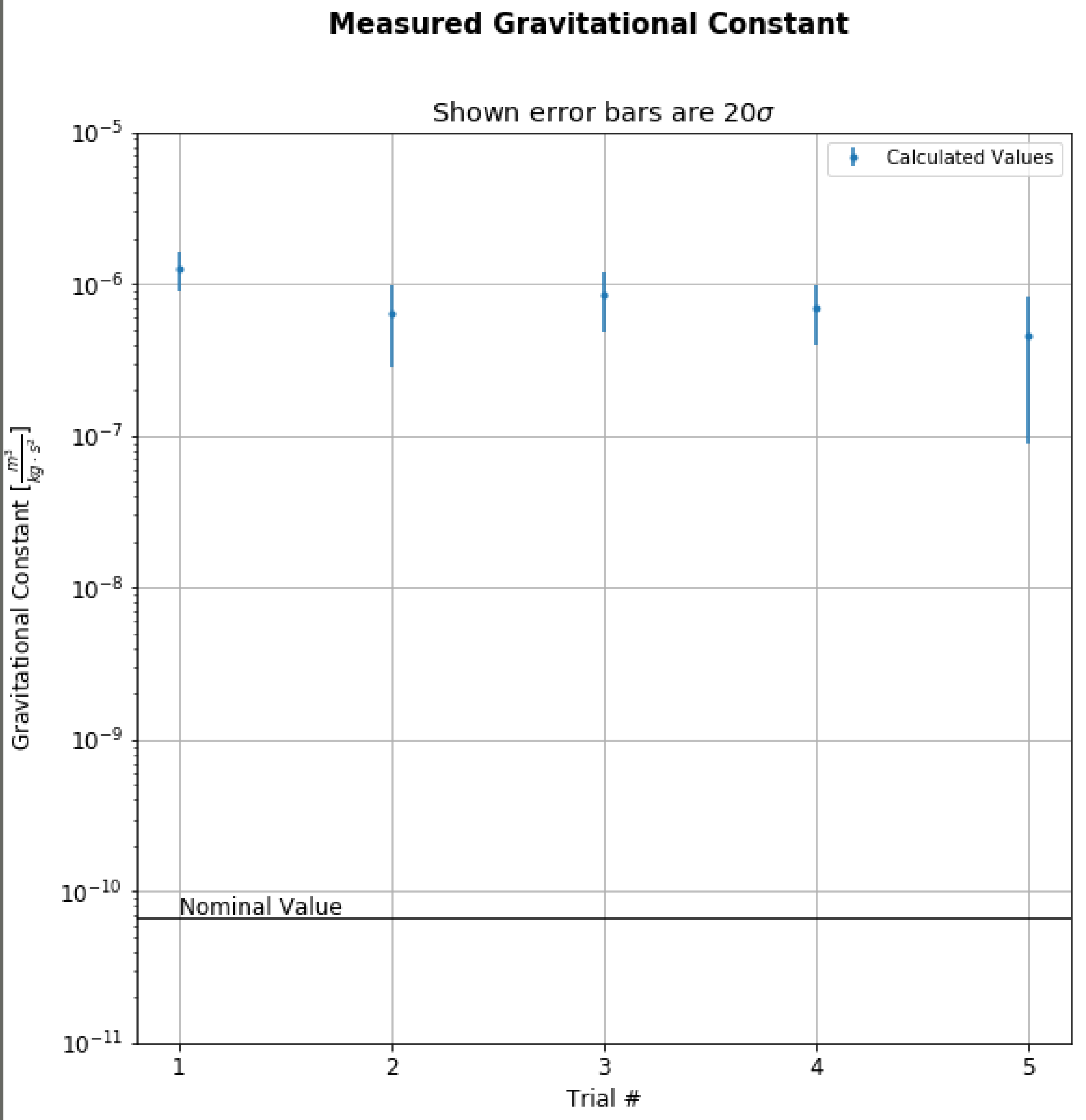
Trial 5: 95.90 %

Trial 1: 98.55 %

$$Accuracy \equiv 1 - \frac{G_{exp}}{G}$$

Trial 5: 680,213 %

Trial 1: 1,884,451 %



Results

- Change from steel to brass materials
  - *Paramagnetic*
- Thinner wire => Smaller torsion coefficient

$$\kappa \propto D^4$$

Conclusion

- Precise, but not accurate
- Really got to know the experiment and methodology behind measuring G
- Even attempted to improve experiment with modifications

GRACIAS