Statistics, Uncertainties and Linear Fitting

Jorge A. Garcia

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1 Uncertainties

Diffraction equation:

$$m\lambda = dsin\theta$$

Derivation of expression for $\Delta\lambda$:

$$(\Delta\lambda)^2 = (\frac{1}{m}sin\theta)^2(\Delta d)^2 + (\frac{1}{m}dcos\theta)^2(\Delta\theta)^2$$
$$= (\frac{1}{m})^2(sin^2\theta(\Delta d)^2 + d^2cos^2\theta(\Delta\theta)^2)$$

$$\boxed{\Delta\lambda = \frac{1}{m}\sqrt{sin^2\theta(\Delta d)^2 + d^2cos^2\theta(\Delta\theta)^2}}$$

Given the values:

$$m = 1$$
$$d = (600)^{-1}mm$$
$$\Delta d/d = 1\%$$
$$\theta = 15deg$$
$$\Delta\theta = 0.5deg$$

Evaluation for λ:

$$\lambda = \frac{dsin\theta}{m} = \frac{[(600)^{-1}mm]sin(15deg)}{1}$$

$$\boxed{\lambda = 4.3 \times 10^{-4}m = 430\mu m}$$

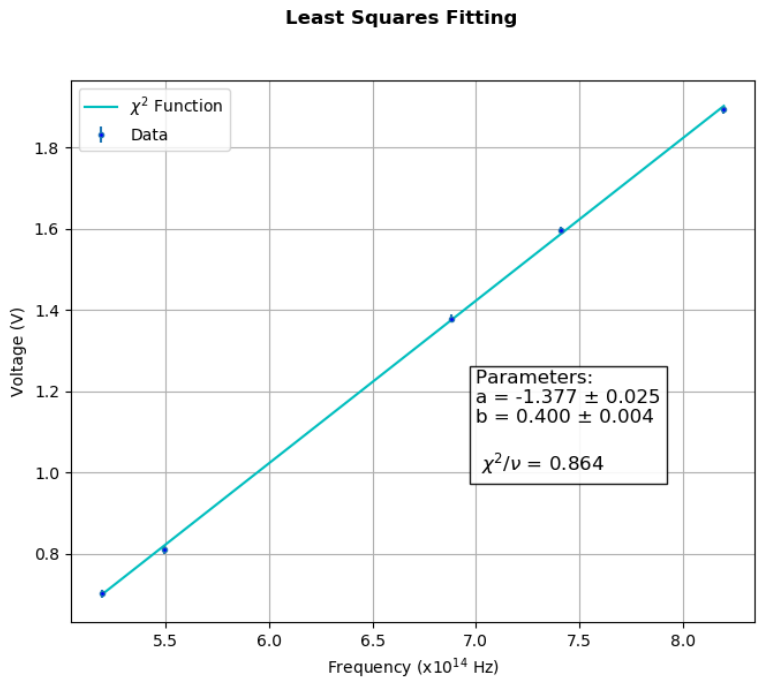
Evaluation for $\Delta\lambda$:

$$\Delta\lambda = \frac{1}{m}\sqrt{sin^2\theta(\Delta d)^2 + d^2cos^2\theta(\Delta\theta)^2}$$
$$= \frac{1}{1}\sqrt{[sin^2(15deg)](\frac{1}{6000}m)^2 + (\frac{1}{600}m)^2[cos^2(15deg)](0.5deg)^2}$$

$$\boxed{\Delta\lambda = 8.1 \times 10^{-4}m = 810\mu m}$$

2 Linear Least Squares Fit

Having programmed an analysis routine on Python and plotting the data and fit, these are the results:



The χ^2/ν is close to the ideal value of 1, so the linear fit is a good approximation to the system's function.

3 Geiger Counter - Radiation Decay

3.1 Low Count-Rate Data

The mean number of counts $\bar{r}$, and the standard deviation s without the radioactive sample is:

$$\bar{r} = \frac{1}{N}\sum_{n=1}^{N} r_n = 1.92$$

$$s = \sqrt{\frac{1}{N-1}\sum_{r=1}^{N}(r-\bar{r})^2} = 1.37$$

Since we are counting a random event, the data should follow a Poisson distribution. This means that the standard deviation squared should approximate the mean. Given our obtained data:

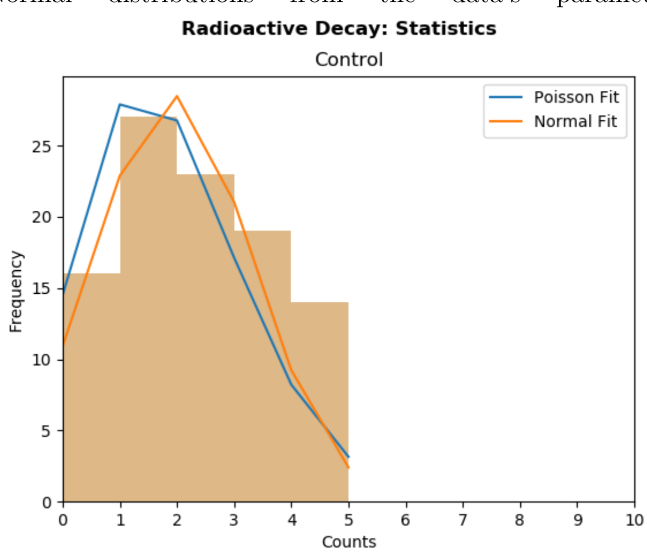
$$s^2 = 1.87 \approx \bar{r} = 1.92$$

The fraction of trials with an absolute deviation from the mean larger than the standard deviation s is approximately 30% of the sample. If one were to look at the probability of getting a result above that of the mean of the ideal Poisson distribution(≈ 2), then:

$$\sum_{r=3}^{5} P_\mu(r) = \sum_{r=3}^{5} \frac{\mu^r e^{-\mu}}{r!} = 29\%$$

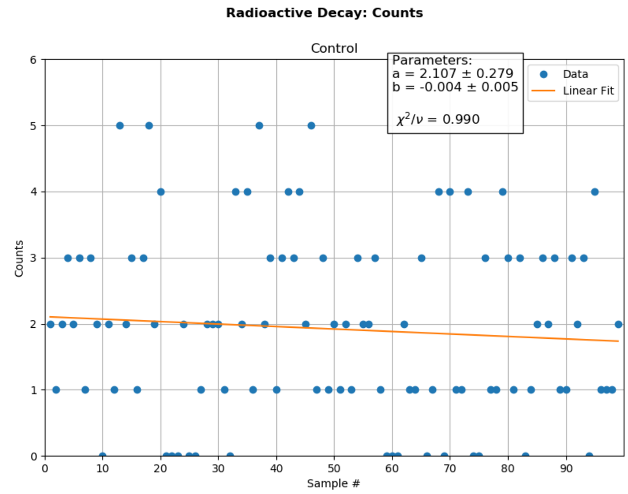
Giving indication that our results are very close to that of a Poisson distribution with the same mean.

The following figure shows a histogram of the data taken, as well as calculated Poisson and Normal distributions from the data's parameters:



As can be seen, the Poisson distribution gives a much better model for the data, showing that the number of counts from the background detected by the Geiger-Mller Counter is entirely random.

The following figure shows a plot of the data points of the experiment, displaying counts against sample number:



A linear Least Squares Fit was used in attempt to recreate the trend of the data. While the experiment assumed that the probability of detecting a radioactive decay product was constant in time ($a = \bar{r} = 1.92, b = 0$), the calculated parameters were very close to the ideal values, within their uncertainties. Therefore, it can be assumed that the detection is constant throughout time.

3.2 High Count-Rate Data

The same process is used to analyze the high count-rate data, which includes the sample.

The mean and the standard deviation of this set:

$$\bar{r} = 54.23$$
$$s = 9.09$$
$$s^2 = 82.54 \neq \bar{r} = 54.23$$

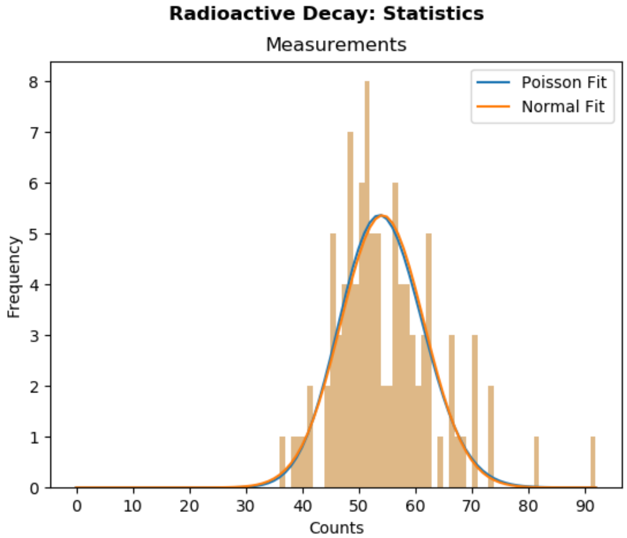
The comparison of the standard deviation and the mean means that this should not follow a Poisson distribution, since now there is a source of decay.

The fraction of trials with an absolute deviation from the mean larger than the standard deviation s is approximately 26% of the sample. A Poisson distribution would expect to see:

$$\sum_{r=55}^{92} P_\mu(r) = 48\%$$

There's a big discrepancy between both of these values as well.

Plotting the histogram of the data and fitting both a Poisson and Normal distribution:



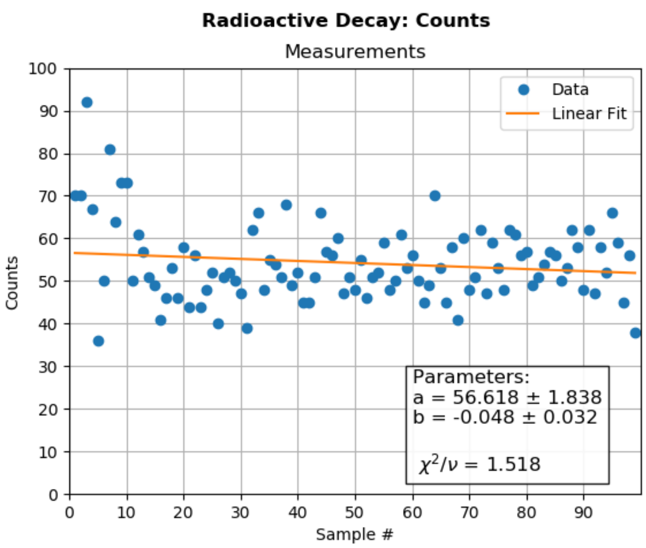
It can be seen that at this point the Poisson and Normal distributions are very similar, which makes sense since the Normal distribution is just a limiting case of the Poisson distribution ($\mu \gg 1$). Their numerical differences on the plot is due to how the Normal distribution is approximated, being:

$$P(r) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(r-\bar{r})^2/2\sigma^2}$$

While that of the Poisson distribution is:

$$P(r) = \frac{\bar{r}^r e^{-\bar{r}}}{r!}$$

The following figure shows the plot of the data points of the experiment and its appropriate Linear Least Squares Fit as well:



A good approximation is achieved, with the parameters being close to their expected values ($a = \bar{r} = 54.23, b = 0$).

3.3 Determining the Source Count Rate