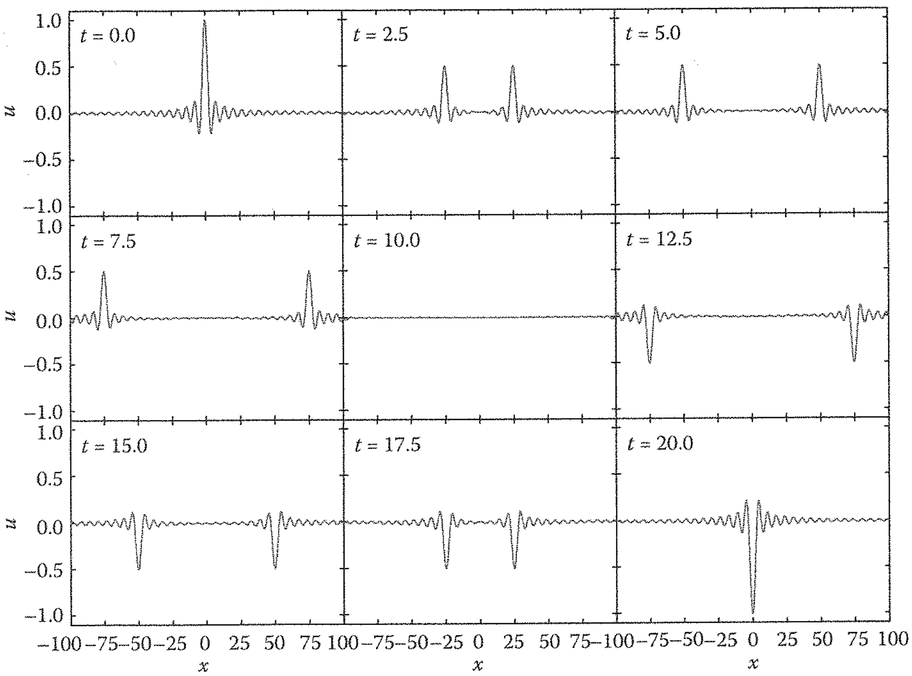

FIGURE 13.14 Numerical solution of problem (13.140–13.142) for the classical wave equation, yielded by Program 13.17.

with zero initial time derivative, $(\partial u/\partial t)(x, 0) = 0$, and subject to constant-value Dirichlet boundary conditions:

$$u(\pm x_{\max}, t) = \frac{\sin x_{\max}\Delta k}{x_{\max}\Delta k}, \quad t > 0. \tag{13.142}$$

Δk denotes the width of the interval of wave numbers contributing to the wave packet, and the larger its value, the smaller the packet's half-width.

Specifically, we employ $c = 10$, $\Delta k = 1$, and $x_{\max} = 100$, and propagate the solution with the step sizes $h_x = 5 \cdot 10^{-2}$ and $h_t = 5 \cdot 10^{-3}$ up to $t_{\max} = 40$. The chosen parameters correspond to $\lambda = 1$, which guarantees the stable propagation of the solution.

Output is produced every $n_{\text{out}} = 500$ time steps to files having the current propagation time marked in their names. Snapshots are shown in Figure 13.14 and illustrate the reflection of the two emerging independent solutions on the domain boundaries, as well as the inverted reconstruction of the wave packet at the origin at $t = 20$.

Continuing the propagation up to $t_{\max} = 40$, the original wave packet is reconstructed at the origin with an amplitude reduced to 0.99958 (instead of 1), which provides an estimate of $4 \cdot 10^{-3}$ for the relative error.

13.6 Implementations in C/C++

Listing 13.18 shows the content of the file `pde.h`, which contains equivalent C/C++ implementations of the Python functions developed in the main text and included in the module `pde.py`. The corresponding routines have identical names, parameters, and functionalities.