Homework 5

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Course: PHYS 520

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TEXTBOOK PROBLEMS

Problem 1.

Let us consider each gate throughout the circuit, and how it affects the input qubits. Let $|q_4, q_3, q_2, q_1, q_0\rangle$ be the state of the 5-qubit system, with the subindex corresponding a distinct qubit. Let U_{xy-z} denote a gate U with control qubits x and y acting on qubit z.

Here, consider the output of the previous gate as the input of each gate. The initial input is the state $|c, x, y, 0, 0\rangle$:

$$\begin{aligned}
T_{32-0} &: |c, x, y, 0, xy \oplus 0\rangle = |c, x, y, 0, xy\rangle \\
T_{43-0} &: |c, x, y, 0, cx \oplus xy\rangle \\
T_{42-0} &: |c, x, y, 0, cy \oplus (cx \oplus xy)\rangle \\
CNOT_{4-1} &: |c, x, y, c \oplus 0, cy \oplus (cx \oplus xy)\rangle = |c, x, y, c, cy \oplus (cx \oplus xy)\rangle \\
CNOT_{3-1} &: |c, x, y, x \oplus c, cy \oplus (cx \oplus xy)\rangle \\
CNOT_{2-1} &: |c, x, y, y \oplus (x \oplus c), cy \oplus (cx \oplus xy)\rangle
\end{aligned}$$

Therefore, the given circuit takes an input state $|c, x, y, s = 0, c' = 0\rangle$ and outputs the state $|c, x, y, s = y \oplus (x \oplus c), c' = cy \oplus (cx \oplus xy)\rangle$. Qubits $|c\rangle$, $|x\rangle$ and $|y\rangle$ are the ones being added.

The truth table for this circuit can be then seen in Table I, which is exactly the truth table for a full adder (as seen in [1]). Thus this circuit performs as a full adder.

$\|x\rangle$	$\|y\rangle$	$\|c\rangle$	$\|c'\rangle = \|cy \oplus (cx \oplus xy)\rangle$	$\|s\rangle = \|y \oplus (x \oplus c)\rangle$
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

TABLE I. Truth table for the given circuit.

COMPUTER PROBLEMS

Problem 2.

Functions that implement the XOR, AND, NAND and OR gates are in the source code provided. The first three gates are easy to implement, as only a CNOT or Toffoli gate with the proper qubit arrangement and input state is required.

XOR is implemented as $CNOT\,|x, y\rangle$, with the ouput being the second qubit. AND is implemented as $T\,|x, y, 0\rangle$, with the output being the third qubit. NAND is implemented as $T\,|x, y, 1\rangle$, with the output being the third qubit.

The OR gate was trickier, given the constraint of only using CNOT and Toffoli gates. It was implemented as $T(X \otimes X \otimes I)\,|x, y, 1\rangle$, with the output being the third qubit.

Problem 3.

In order to fine the optimal θ, I create an array from 0 to π, with a step size of $\pi/16$. The H-approximation routine is used at every θ, and the one with the minimum error is then the best to use. This optimal value was found to be $\theta = 11/16\pi$. Figure 1 shows the probability of measuring $|1\rangle$ as a function of n for this optimal value of θ. The error seems to decrease exponentially more so than quadratically.

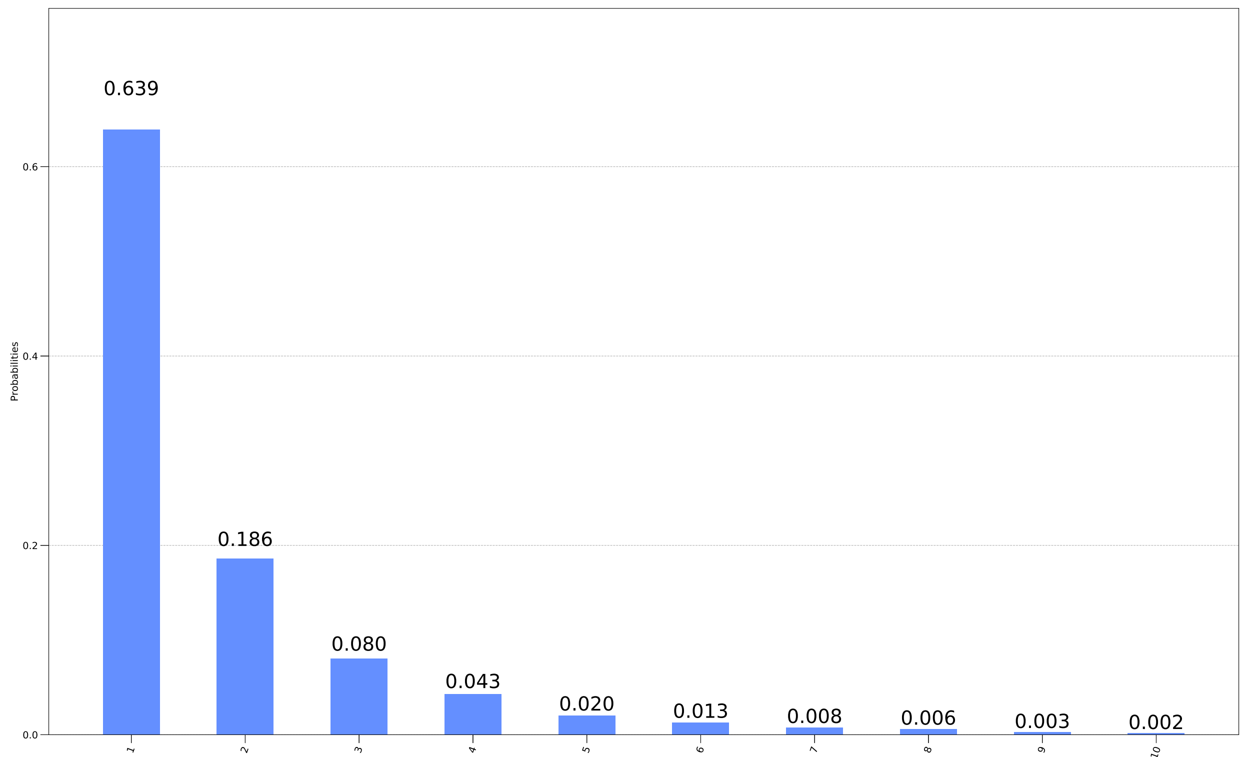

FIG. 1. Error vs n for the approximation $(R_x(\frac{\theta}{n})R_z(\frac{\theta}{n}))^n$, for $\theta = 11/16\pi$.

Figure 2 shows the error of both approximations as a function of theta, with $n = 10$. As θ increases, the error in the first approximation grows in volatility. In contrast, the second approximation results in a more stable fluctuation of the error.

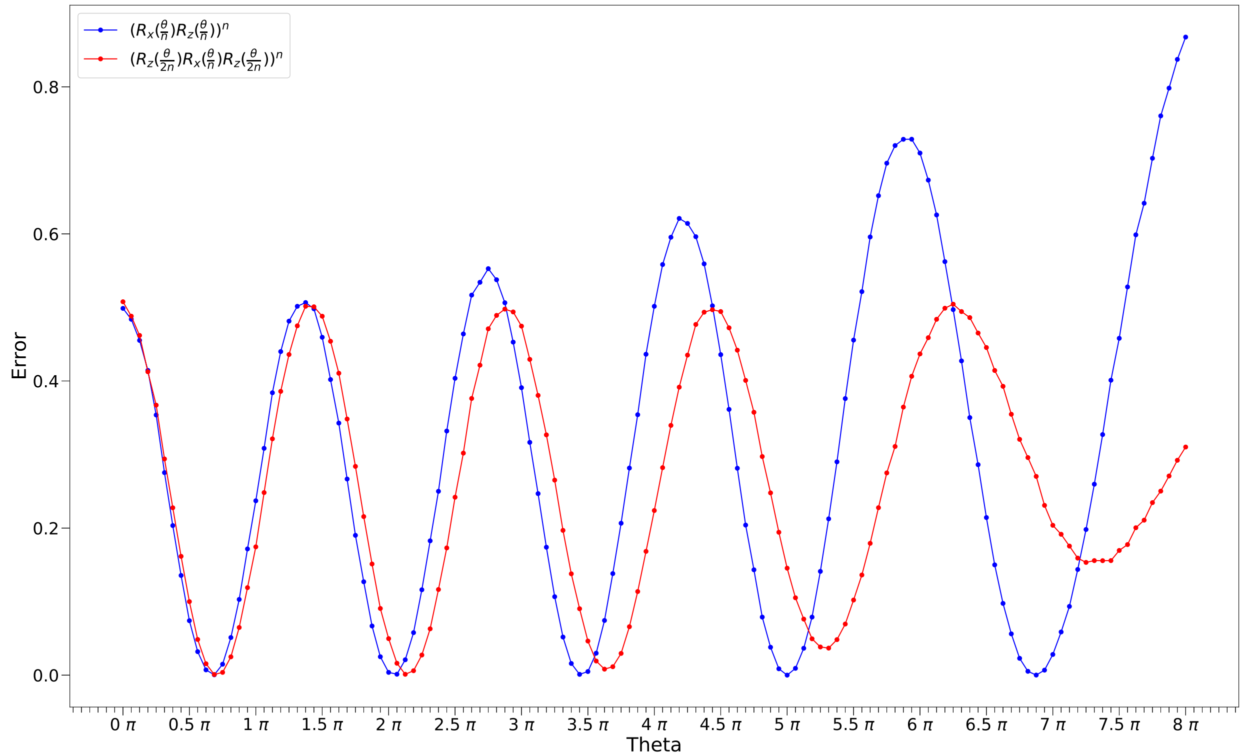

FIG. 2. Caption

Problem 4.

A more stable implementation of the AND gate was found using the circuit shown in Figure 3. This is the other implementation of AND discussed in Chapter 2 of the Qiskit book. The lowest probability was consistently above 0.9, surpassing that of a single Toffoli gate with ~ 0.89 as seen in the problem description.

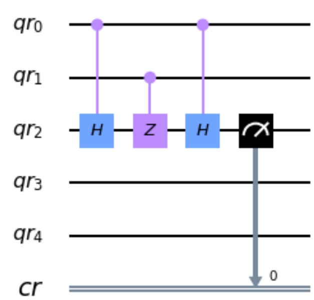

FIG. 3. Caption

[1] Wikipedia, "Adder (electronics)," `https://en.wikipedia.org/wiki/Adder_(electronics)`.