Graduate Program in Economics
IDEA

Final Project

Quantitative Macroeconomics

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1 Introduction

The main objective of this term paper in to introduce my self to the job search and matching model. In the first section I explain in detail the model proposed by **Lise & Robin (2017)**. They developed a model of job search and matching with ex ante heterogeneous workers (heterogeneous in ability) and firms (heterogeneous in technology), aggregate uncertainty, and vacancy creation. As can be seen in figure 1 of the paper the model replicates quite well the unemployment dynamics of the US economy, they use aggregate labor market data from 1951-2012.

This kind of models are useful to answer the following questions. How does the distribution of skills among the unemployed vary with the business cycle? How does the quality of matches for workers transiting from unemployment vary with the aggregate state? Similarly, how is the reallocation of currently employed workers to more appropriate matches related to the business cycle? They find that when the economy recovers from a recession, employment expands as the result of improving employment opportunities for the low ability workers and expanded hiring by low technology firms. Also, when the economy enters a recession, low ability workers are fired, in particular those matched with high technology firms. At the same time, low technology firms hire less and medium/high technology firms hire relatively more medium/high ability unemployed workers.

This results follow from the way that the value of a match is defined in the model. The value of the match depend on the ability of the worker, the technology of the firm and the aggregate state of the economy, for a given technology and a given aggregate state, the value of the match is higher as higher is the ability of the worker. Therefore when there is negative aggregate shock the low ability workers are the ones that are first fired, which is something that makes sense, however there something missing in the model, they assume that there is no severance payment, since they are comparing the model with the US economy where there legal protection of the worker is relatively low, this assumption might not be a problem. However, this assumption is quite unrealistic if we compare with a labor market like the Spanish one. I would like to introduce some rigidity in the model, and check if the previous results hold, in particular I would like to check if the result that low ability workers are the ones that are first fired when there is a negative aggregate shock. It could be the case that once we introduce some severance payment that is increasing with the years that individuals remain in the same firm, the firms might discharge some high skill worker before a low skill worker because it is cheaper.

In the second section I introduce my self to programming in python this kind of models. In particular I program some of the most standard model of job search and matching, the McCall Model and the Lake Model. For that section I have followed **Sargent & Stachurski (2020)**.

2 Model with Heterogeneous Workers and Firms

In this section I present the main features of a model of job search and matching, that I would like to use in a future extension of this project. The model is an equilibrium model with ex ante heterogeneous workers and firms, aggregate uncertainty and vacancy creation. The model is proposed by **Lise & Robin (2017)**.

2.1 Heterogeneous Agents and Aggregate Shocks

In this economy there is a continuum of infinitely lived workers indexed by ability (x) and a continuum of firms indexed by technology (y). Both the total measure of workers and firms is normalized to 1. The distribution of ability is denoted by l(x) is exogenous. The distribution of technology across firms is uniform, so that the technology (y) is just a ranking of firms. The aggregate state of the economy is indexed by z_t and follows a Markov process denoted by $\pi(z, z')$.

2.2 The Meeting Technology

Timing:
-First at the beginning of period t a measure $u_t(x)$ of unemployed workers type x and a measure $h_t(x, y)$ employed workers of type x at firm type y are inherit from t-1, therefore the distribution of ability inherit is given by:

$$l(x) = u_t(x) + \int h_t(x, y)dy \tag{1}$$

-Second there is a new realization of the aggregate state of the economy, the economy move from z_{t-1} to z_t.
-Third, the separation happen, then, the new measure of unemployed denoted by u_{t+} (that is the new stock of unemployed just after the realization of the shock) and the individuals that remain employed denoted by $h_{t+}(x, y)$ get a chance to draw a new offer. Both u_{t+} and h_{t+} together form the total job search effort denoted by:

$$L_t = \int u_{t+}(x)dx + s\int h_{t+}(x, y)dxdy \tag{2}$$

The job search effort of unemployed it has been normalized to 1, therefore s in the relative job search effort of the employed. Now let $v_t(y)$ be the total job opportunities chosen by a firm type y. Therefore the total amount of job opportunities is given by:

$$V_t = \int v_t(y)dy \tag{3}$$

The total number of meetings in period t is denoted by $M_t(V_t, L_t)$. Define $\lambda_t = \frac{M_t}{L_t}$ as the probability that an unemployed searcher contacts a vacancy and $s\lambda_t$ the probability that an employed searcher contacts a vacancy in period t. Finally let me denote $q_t = \frac{M_t}{V_t}$ as the probability per unit of recruiting effort $v_t(y)$ that a firm contacts any searcher.

2.3 The Value of Unemployed

Let $B_t(x)$ denote the value of an unemployed worker of type x at time t.

$$B_t(x) = b_t(x, z_t) + \frac{1}{1+r}\mathbb{E}_t\left[(1-\lambda_{t+1})B_{t+1}(x) + \lambda_{t+1}\int W_{0,t+1}(x,y)\frac{v_{t+1}(y)}{V_{t+1}}dy\right] \quad (4)$$

Where $b_t(x, z_t)$ is the amount that an unemployed type x at state z_t earns. After the revelation of the new state z_t the worker anticipates that he will meet a vacancy at firm type y with probability $\lambda_{t+1}\frac{v_{t+1}(y)}{V_{t+1}}$. It is assume that unemployed workers has zero bargaining power, therefore they are offered their reservation wage $W_{0,t+1} = B_{t+1}(x)$. Therefore.

$$B_t(x) = b_t(x, z_t) + \frac{1}{1+r}\mathbb{E}_t B_{t+1}(x)$$

The function $B_t(x)$ is independent of any worker-specific history as long as home production is $b(x, z)$.

2.4 The Value and Surplus of a Match

Firms have access to a production technology, defined at the match level, that combines the skill of the worker and the technology of a firm with aggregate productivity to create value added $p(x, y, z)$. Here the model allow that the value added require a threshold level of inputs before being positive, for example a minimum level of skill of the worker. There are complementarities between the skill of the worker and the technology of the firm $p_{xy} \neq 0$. However there is no complementarity across workers within a firm.

$P_t(x, y)$ denote the present value of a match (x,y), including the continuation values to the worker and firm upon separation, given the aggregate state of the economy at time t. Assuming zero fixed investment in job creation, any vacancy generated by job destruction is lost and has zero continuation value. There is no severance payment or experience rating. Therefore, after the realization of the new shock z_{t+1} the worker and the firm are better off separated of $B_{t+1}(x) > P_{t+1}(x, y)$, also it is allow for a source of idiosyncratic job destruction δ, therefore the match is destroyed with probability:

$$\mathbb{1}\{B_{t+1}(x) > P_{t+1}(x,y)\} + \delta \times \mathbb{1}\{B_{t+1}(x) \leq P_{t+1}(x,y)\} \quad (5)$$

Therefore the match continues with probability $(1-\delta)\mathbb{1}\{B_{t+1}(x) \leq P_{t+1}(x,y)\}$. Then the worker draws an alternative offer from a firm of type $y' \neq y$ with probability $s\lambda_{t+1}\frac{v_{t+1}(y')}{V_{t+1}}$.

Let $W_{1,t}(x,y,y')$ be the value offered at time t by a firm of type y to a worker of type x that has an alternative offer by a firm type y'. The incumbent employer can make new wage offers to their existing workers in an attempt to retain those with outside offers. Incumbent and poaching firms engage in Bertrand competition which grants the worker a value equal to the second highest bid. Specifically, either $P_{t+1}(x,y') > P_{t+1}(x,y)$ and the worker moves to firm y' and receives the incumbent employer's reservation value $W_{1,t}(x,y,y') = P_{t+1}(x,y)$ as continuation value; or $P_{t+1}(x,y') \leq P_{t+1}(x,y)$ and the worker stays with his current employer with continuation value equal the reservation value of firm y' $W_{1,t+1}(x,y,y') = P_{t+1}(x,y')$.[1]. So Bertrand competition makes the continuation value of the match independent of whether the employee is poached:

$$
\begin{aligned}
P_t(x,y) = p(x,y,z_t) + \frac{1}{1+r}\mathbb{E}_t[&(1-(1-\delta)\mathbb{1}\{B_{t+1}(x) \leq P_{t+1}(x,y)\})B_{t+1}(x) \\
&+ (1-\delta)\mathbb{1}\{B_{t+1}(x) \leq P_{t+1}(x,y)\}((1-s\lambda_{t+1})P_{t+1}(x,y) \\
&+ s\lambda_{t+1}\int max\{P_{t+1}(x,y), W_{1,t+1}(x,y',y)\}\frac{v_{t+1}(y')}{V_{t+1}}dy')]
\end{aligned} \tag{6}
$$

$$
\begin{aligned}
P_t(x,y) = p(x,y,z_t) + \frac{1}{1+r}\mathbb{E}_t[&(1-(1-\delta)\mathbb{1}\{B_{t+1}(x) \leq P_{t+1}(x,y)\})B_{t+1}(x) \\
&+ (1-\delta)\mathbb{1}\{B_{t+1}(x) \leq P_{t+1}(x,y)\}P_{t+1}(x,y))]
\end{aligned}
$$

Finally, defining match surplus as $S_t(x,y) = P_t(x,y) - B_t(x)$ and combining 4 and 6 you can get:

$$
\begin{aligned}
P_t(x,y) - B_t(x) &= p(x,y,z_t) - b_t(x,z_t) \\
&\quad + \frac{1}{1+r}\mathbb{E}_t[(1-(1-\delta)\mathbb{1}\{B_{t+1}(x) \leq P_{t+1}(x,y)\})B_{t+1}(x) \\
&\quad + (1-\delta)\mathbb{1}\{B_{t+1}(x) \leq P_{t+1}(x,y)\}P_{t+1}(x,y)) - B_{t+1}(x)] \\
&= p(x,y,z_t) - b_t(x,z_t) \\
&\quad + \frac{1-\delta}{1+r}\mathbb{E}_t[\{B_{t+1}(x) \leq P_{t+1}(x,y)\}[P_{t+1}(x,y) - B_{t+1}(x)]]
\end{aligned} \tag{7}
$$

In a simple expression:

$$
P_t(x,y) - B_t(x) = p(x,y,z_t) - b_t(x,z_t) + \frac{1-\delta}{1+r}\mathbb{E}_t[max\{S_{t+1}(x,y), 0\}] \tag{8}
$$

[1]In this case, the aggregate shock may still force the employer to renegotiate if $P_{t+1} \geq B_{t+1}$ but the existing contract would imply either $W_{t+1} < B_{t+1}$ or $W_{t+1} > P_{t+1}$.

With all that we have so far, I can state the main result of the model. This result means that the model is trackable and can be estimated.

Proposition 1: *The surplus from an (x, y) match at time t depends on time only through the current aggregate productivity shock z and does not depend on the distributions of vacancies, unemployed workers, or worker-firm matches. Specifically,* $S_t(x, y) \equiv S(x, y, z)$ *such that*

$$S(x, y, z) = s(x, y, z) + \frac{1-\delta}{1+r} \int S(x, y, z')^{+} \pi(z, z') dz' \tag{9}$$

Where $s(x, y, z) = p(x, y, z) - b(x, z)$ *and they denote* $x^{+} = max\{x, 0\}$

Outside offers do not change the size of the match surplus, only how it is shared between the worker and the firm. When a firm counters an outside offer to retain the worker this is done by transferring more of the match surplus to the worker, but has no impact on the total surplus in the match. Similarly, when a worker is poached by another firm, Bertrand competition ensures that the value to the worker of moving to the new match is exactly the total surplus at the previous match. The previous firm is then left with zero since vacancies do not have a continuation value.

The explicit form of $u_{t+}(x)$ and $h_{t+}(x, y)$ is given by:

$$u_{t+}(x) = u_t(x) + \int [\mathbb{1}\{S_t(x, y) < 0\} + \delta \mathbb{1}\{S_t(x, y) \geq 0\}] h_t(x, y) dy \tag{10}$$

$$h_{t+}(x, y) = (1-\delta)\mathbb{1}\{S_t(x, y) \geq 0\} h_t(x, y) \tag{11}$$

2.5 Vacancy Creation

Each period firms can buy the advertising of v job opportunities from job placement agencies at a price $c(v) \geq 0$ that is assumed to be independent of the firm's type. In the equilibrium is determined by equating the marginal cost to the expected value of a job opening.

$$c'[v_t(y)] = q_t J_t(y) \tag{12}$$

where $J_t(y)$ denotes the expected value of a contact by a vacancy of type y. The expected value of a contact is calculated as follows:

$$J_t(y) = \int \frac{u_{t+}(x)}{L_t} S(x, y, z)^{+} dx + \int \int \frac{s h_{t+}(x, y')}{L_t} [S(x, y, z) - S(x, y', z)]^{+} dx dy' \tag{13}$$

The contact is with an unemployed worker of type x with probability $\frac{u_{t+}(x)}{L_t}$ and a match is formed if the match surplus is positive. The contact is with a worker of type x that is currently employed at a firm of type y' with complementary probability $\frac{sh_{t+}(x,y')}{L_t}$. Poaching is successful if $S(x, y, z) > S(x, y', z)$ and Bertrand competition grants the poacher a value $S(x, y, z) - S(x, y', z) = P(x, y, z) - P(x, y', z)$.

2.6 Labor Market Flows

The law of motion for unemployment resulting from meetings between unemployed workers and vacant jobs is therefore

$$u_{t+1}(x) = u_{t+}(x)\left[1 - \int \lambda_t \frac{v_t(y)}{V_t} \mathbb{1}\{S_t(x, y) \geq 0\}dy\right] \tag{14}$$

and for employment

$$h_{t+1} = h_{t+}(x, y)\left[1 - \int \lambda_t \frac{v_t(y)}{V_t} \mathbb{1}\{S_t(x, y') > S_t(x, y)\}dy'\right] \tag{15}$$

$$+ \int h_{t+}(x, y')s\lambda_t \frac{v_t(y)}{V_t} \mathbb{1}\{S_t(x, y) > S_t(x, y')\}dy' \tag{16}$$

$$+ u_{t+}(x)\lambda_t \frac{v_t(y)}{V_t} \mathbb{1}\{S_t(x, y) \geq 0\} \tag{17}$$

2.7 Extension

In this section I propose an extension to the model in order to introduce some rigidity in the labor market allowing for severance payment. In section 2.4 I have defined a match destruction as:

$$\mathbb{1}\{B_{t+1}(x) > P_{t+1}(x, y)\} + \delta \times \mathbb{1}\{B_{t+1}(x) \leq P_{t+1}(x, y)\}$$

Where δ was an idiosyncratic job destruction. We can include in this expression the severance payment. When $\{B_{t+1}(x) > P_{t+1}(x, y)\}$ we can consider this as a voluntary separation, that is the worker is better off unemployed than working and therefore he decided to quit, in this case there is no severance payment. However when there is a destruction when $\{B_{t+1}(x) \leq P_{t+1}(x, y)\}$ the firm will need to compensate the worker. Let n be the number of periods that a worker has worked for the same firm, and $\phi(n)$ the factor that determines the severance payment and increases with the number of years in the same firm, we can denote the severance payment as $\phi(n)W_{1,t}(x, y)$ where $W_{1,t}(x, y)$ is the actual wage of the worker with ability x at firm y in time t. Therefore, assuming again Bertrand competition we can define the value of the match as follows:

$$P_t(x,y) = p(x,y,z_t) + \frac{1}{1+r}\mathbb{E}_t[(1-(1-\delta)\mathbb{1}\{B_{t+1}(x) \leq P_{t+1}(x,y)\})B_{t+1}(x) + (1-\delta)\mathbb{1}\{B_{t+1}(x) \leq P_{t+1}(x,y)\}P_{t+1}(x,y))] + \delta\phi(n)W_{1,t}(x,y)$$

And the surplus of the match:

$$P_t(x,y) - B_t(x) = p(x,y,z_t) - b_t(x,z_t) + \frac{1-\delta}{1+r}\mathbb{E}_t[max\{S_{t+1}(x,y),0\}] + \delta\phi(n)W_{1,t}(x,y)$$

An important remark is that one of the main results of the original model is that this surplus from an (x, y) match at time t depended on time only through the current aggregate productivity shock z and did not depend on the distributions of vacancies, unemployed workers, or worker-firm matches, this allowed the model to be track-able. The way that I propose to introduce the severance payment keep most of this results, however now it also depend on the number of year that the match has lasted. I would need to get deeper into the model to be sure that the model is trackable with this extension.

3 Solving McCall Model

In this section a solve with python one of the most standard model of job search, I solve a discrete version of the model proposed in **McCall (1970)**.

3.1 Model

I'm going to introduce the theoretical framework of the McCall's model. In this model we focus in an unemployed worker who is searching for job. Each period that he search the is a new job offer, that offers a wage ω. The worker has two alternatives, he can reject the job and wait until the next period to receive a new job offer, in that case he will receive a compensation c in this period, that is going to be exogenous determine in this model, or he can accept the job, in which case he or she receives the wage ω forever, in this simple model there is no option to quit or to get fired.

Let y_t be the income in period t. Therefore $y_t = c$ if he is unemployed and $y_t = \omega$ if he is employed. The unemployed worker want to maximize $\mathbb{E}\sum_{t=0}^{\infty}\beta^t y_t$ where $\beta \in (0,1)$ is the discount factor.

The wage offer is going to be a non-negative function of some underlying state:

$$\omega_t = \omega(s_t) \quad \text{where} \quad s_t \in S$$

Where s_t is going to be a realization of a shock that affects the wages, in this case is going to be assume that s_t are iid, with q(s) as the probability of observing state s. The worker observes s_t at the begging of period t, therefore he knows ω_t.

The worker face a trade off between accepting too soon or waiting too much, waiting too long is costly, since the future is discounted. Accepting too soon is also costly, since better offers can arrive in the future.

Therefore the value function of an unemployed worker is given by:

$$v(s) = max\left\{\frac{\omega(s)}{1-\beta}, c + \beta\sum_{s'\in S} v(s')q(s')\right\} \tag{18}$$

Where $\frac{\omega(s)}{1-\beta}$ is the lifetime payoff of accepting the job for a wage $\omega(s)$ at a state s, and $c+\beta\sum_{s'\in S} v(s')q(s')$ is the lifetime payoff of rejecting the job and then behaving optimally in all subsequent periods.

A solution to this problem is a policy function, that maps states into actions, that is for every possible state s gives the optimal behavior. We can write the policy function as follows:

$$\sigma(s) := \mathbb{1}\left\{\frac{\omega(s)}{1-\beta} \geq c + \beta\sum_{s'\in S} v(s')q(s')\right\} \tag{19}$$

Where $\mathbb{1}\{X\} = 1$ if statement X is true and 0 otherwise.

From the previous equation we can isolate $\omega(s) = \bar{\omega}$ to get the reservation wage, that is the threshold for accepting or rejecting an offer. We can define the reservation wage as follows

$$\bar{\omega} := (1-\beta)\left\{c + \beta\sum_{s'} v(s')q(s')\right\} \tag{20}$$

3.2 Algorithm

I need to compute the value function at each possible state $s \in S$. For $S = \{1, \dots, n\}$.

The value function is then represented by the vector $v = (v(i)_{i=1}^{n}$.

Step 1: pick an arbitrary initial guess $v \in \mathbb{R}^n$.

Step 2: compute a new vector $v' \in \mathbb{R}^n$

$$v'(i) = max\left\{\frac{\omega(i)}{1-\beta}, c + \beta\sum_{1\leq j\leq n} v(j)q(j)\right\} \quad \text{for} \quad i = 1, \dots, n \tag{21}$$

Step 3: calculate a measure of the deviation between v and v', such as $max_i|v(i)v(i)|$.

Step 4: if the deviation is larger than some fixed tolerance, set $v = v$ and go to step 2, else continue.

Step 5: return v.

3.3 Results

In this section I present some basic results of the McCall Model, I followed the parametrization of **Sargent & Stachurski (2020)**. The distribution for the state process is assume to be a Beta-Binomial.

I create the set of all possible wage offers using linspace in python, setting the minimum wage equal to 10 the maximum wage equal to 60 and 51 spaces. The probabilities for each possible wage offer is determine by the Beta-Binomial distribution. In 1 I plot each possible wage offer with his respective probability.

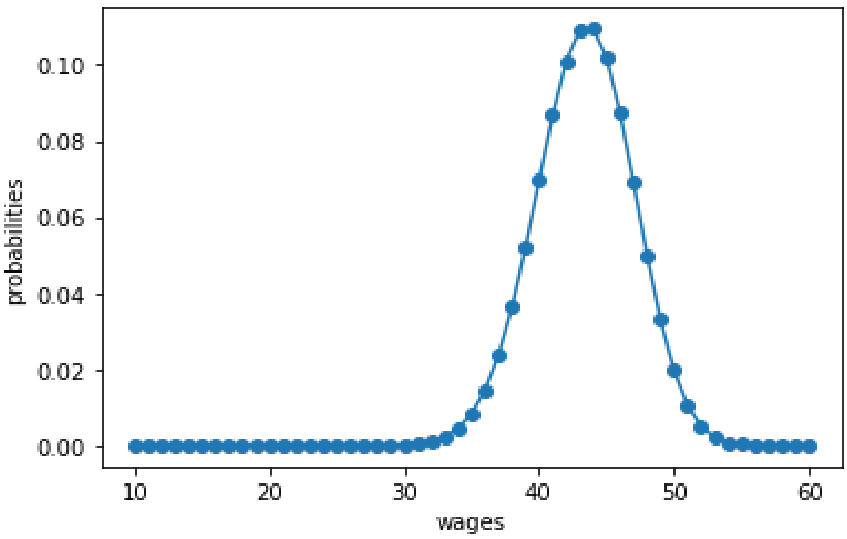

Figure 1: Wage probability density function

In figure 2, I plot some iteration of the value function of the unemployed worker, in iteration 0 I'm plotting my guess which in this case is that they accept the offer for every wage. Then you can see that in every step of the iteration the value functions converge increasing, the value functions are flat until some wage where they accept the offer, the threshold the value function stop being flat, converges to the reservation wage. The reservation wage for the given parametrization ($\beta = 0.99$ and $c = 25$) is equal to 47.32.

To finish with this model a final interesting exercise is to look how does the reservation wage changes when we change the discount factor and the unemployment compensation. In figure 3, you can see a heat map, for the reservation wage, for different possible values of the discount factor and the unemployment compensation, as expected the reservation wage increases with the compensation and with the patience.

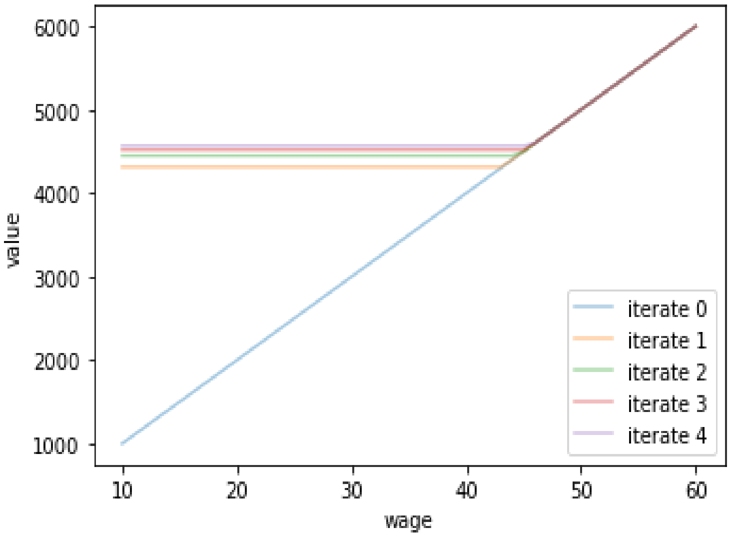

Figure 2: Value Function Iteration

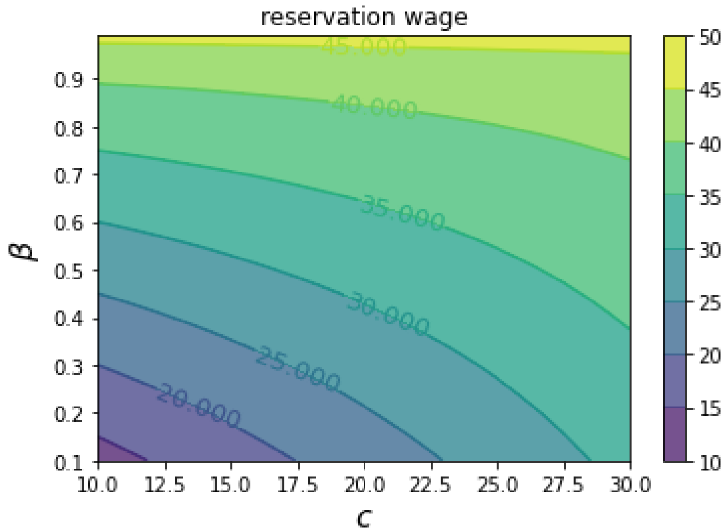

Figure 3: Value Function Iteration

4 Solving the Lake Model

The Lake model is a simple tool for modelling unemployment, it is called lake model, because we can interpret being unemployed and being employed as being in two different lake, in this simple model I'm going to assume that the parameters that determine the flows between employment and unemployment are exogenous.

4.1 Model

First, let's define the paremeters and the variables of the model:

- λ: Job finding rate for currently unemployed workers.

- α: Dismissal rate for currently employed workers.
- b: Entry rate into the labor force.
- d: Exit rate from the labor force.
- $g = b - d$: Growth rate of the labor force.
- E_t: Total employed in time t.
- U_t: Total unemployed in time t.
- N_t: Total workers in the labor force.
- $e_t := \frac{E_t}{N_t}$: Employment rate.
- $u_t := \frac{U_t}{N_t}$: Unemployment rate.

Using these definitions, I can define the law of motion of the aggregates E_t, U_t and N_t, from the employed and unemployed in time t we know the following:

- $(1-d)E_t$ remain in the labor force.
- $(1-\alpha)(1-d)E_t$ remain employed.
- $(1-d)U_t$ remain in the labor force.
- $\lambda(1-d)U_t$ will become employed.

Therefore, the number of workers who will be employed and unemployed at date t + 1 are given by:

$$E_{t+1} = (1-d)(1-\alpha)E_t + (1-d)\lambda U_t \tag{22}$$

$$U_{t+1} = (1-d)\alpha E_t + (1-d)(1-\lambda)U_t + b(E_t + U_t) \tag{23}$$

Where $b(E_t + U_t)$ is mass of new workers entering in the labor force.
The total stock of Workers in t+1:

$$N_{t+1} = (1 - d + b)N_t = (1+g)N_t \tag{24}$$

Writing this low of motions in matrix form $X_{t+1} = AX_t$, where X_t and A are defined as:

$$X_t := \begin{pmatrix} U_t \\ E_t \end{pmatrix} \tag{25}$$

$$A := \begin{pmatrix} (1-d)(1-\lambda)+b & (1-d)\alpha+b \\ (1-d)\lambda & (1-d)(1-\alpha) \end{pmatrix} \tag{26}$$

We can write this low of motions in terms of rates dividing both sides by N_{t+1}:

$$\begin{pmatrix} \frac{U_{t+1}}{N_{t+1}} \\ \frac{E_{t+1}}{N_{t+1}} \end{pmatrix} = \frac{1}{1+g} A \begin{pmatrix} \frac{U_t}{N_t} \\ \frac{E_t}{N_t} \end{pmatrix} \tag{27}$$

To simplify the notation let me write it as follows:

$$x_{t+1} = \hat{A} x_t \quad \text{where} \quad \hat{A} := \frac{1}{1+g} A \tag{28}$$

4.2 Implementation

I'm going to create a class for the Lake Model, that will do the following:

1. Store the know parameters α, λ, b, d.
2. Compute and store the objects that depend on the know parameters such as g, A, $\hat{A}$.
3. Compute the steady state of the rates.
4. Simulate the dynamics for the stocks and rates.

4.3 Results

In figure 4 I've plot the converge of the employment and unemployment rates to the steady state, to do so I followed the parametrization of **Sargent & Stachurski (2020)** ($\lambda = 0.283$, $\alpha = 0.013$, b=0.0124, d=0.00822). And used as initial conditions $u_t = 0.08$, $e_t = 0.92$ and $N_t = 150$, for this parameters the steady state values of employment and unemployment rates, are the red dashed lines.

Now I focus the analysis on the individual employment dynamics, a worker can be in two states, employed ($s_t = 1$) or unemployed ($s_t = 0$). Let me assume that $b = d = 0$, therefore there are no individuals leaving the labor force or new individuals entering in the labor force. In this case the transition matrix from one state to another for an individual is given by:

$$P = \begin{pmatrix} (1-\lambda) & \lambda \\ \alpha & (1-\alpha) \end{pmatrix} \tag{29}$$

Since $\alpha \in (0,1)$ and $\lambda \in (0,1)$ we can be sure that this markov process has a stationary distribution and it is unique. Let me call ψ_t the marginal distribution of the states employment and unemployment for a given worker in time t, then:

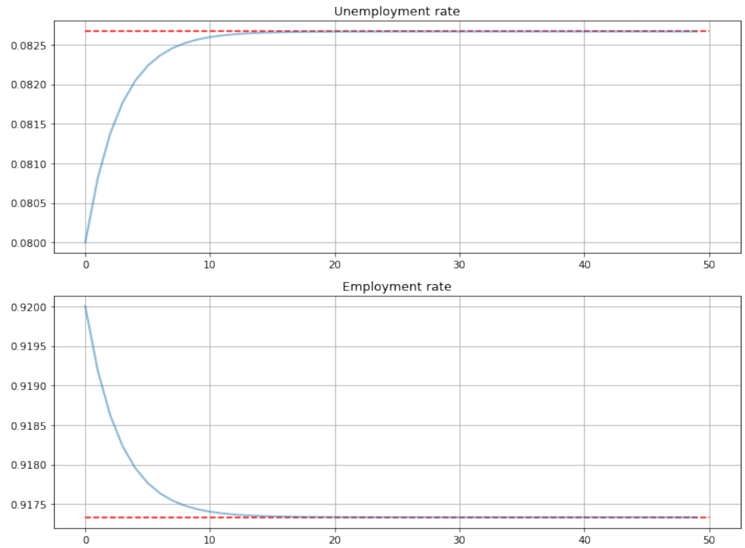

Figure 4: Employment and Unemployment rates

$$\psi_{t+1} = \psi_t P \quad \text{there is a unique } \psi^* \text{ such that} \quad \psi^* = \psi^* P$$

We can compute the average time that an infinitely lived worker spend employed and unemployed as follows:

$$\bar{s}_{u,T} := \frac{1}{T} \sum_{t=1}^{T} \mathbb{1}\{s_t = 0\}$$
$$\bar{s}_{e,T} := \frac{1}{T} \sum_{t=1}^{T} \mathbb{1}\{s_t = 1\}$$

These are the fraction of time that a worker spends unemployed and employed up to period T, again since $\alpha \in (0,1)$ and $\lambda \in (0,1)$ we can be sure that P is ergodic and therefore:

$$\lim_{T\to\infty} \bar{s}_{u,T} = \psi[0]$$
$$\lim_{T\to\infty} \bar{s}_{e,T} = \psi[1]$$

Therefore, an infinitely lived worker stays unemployed exactly the same proportion of time as the stationary unemployment rate. In figure 5 we can see that it takes a lot of periods

4.4 Joining McCall and Lake Model

Finally I will join the two presented models. To do so I endogenize the transition from unemployment to employment, this transition will be determine by the McCall Model, that I presented in the previous section.

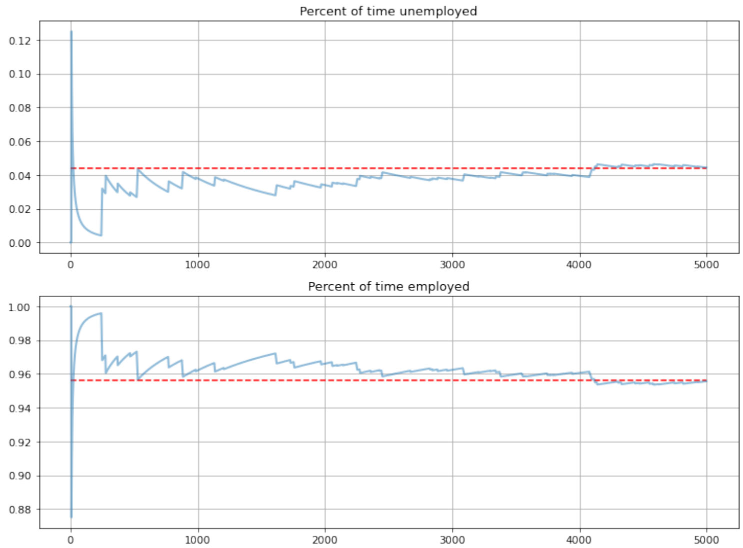

Figure 5: Percent of time employed and unemployed

Let me recover some parameters from McCall Model:

- β: Discount factor.
- γ: Offer arrival rate.
- c: Unemployment compensation.
- α: The same separation rate as in Lake Model.

Now assume that all workers inside a lake model take their decision of working or not following the McCall Model. Then their optimal decision rule determine the probability λ of leaving unemployment, λ is now given by the following expression:

$$\lambda = \gamma \mathbb{P}\{\omega_t \geq \bar{\omega}\} = \gamma \sum_{\omega' \geq \bar{\omega}} p(\omega') \tag{30}$$

Under this environment, I can add another element to the model, since the reservation wage and therefore the employment decisions depend on the compensation c, I can introduce a government that chooses the compensation c and finance this compensation through a lump-sum tax τ. To have balanced budget, I have to impose $\tau = uc$. The lump-sum tax will apply to everyone, so an employed worker will have a post-tax income equal to $\omega - \tau$, and an unemployed worker $c - \tau$.

So for each couple (c, τ), there is a different reservation, by changing the fiscal policy, I can change the unemployment rate of the steady state $u(c, \tau)$.

To evaluate the different fiscal policies, I need a welfare function, I will set a steady state criterion, that is I will focus on maximizing the welfare at the steady state, the welfare function will be given by:

$$W := e\mathbb{E}[V|employed] + uU \tag{31}$$

Where $\mathbb{E}[V|employed]$ is the expected utility of being employed and U the utility of unemployed.

I will assume a constant risk aversion utility function for the workers.

In figure 6, I provide the plots of the employment rate, unemployment rate, tax rate and welfare for different values of the unemployment compensation c, the grid for c goes from 5 to 140. We can see that the welfare first increases until it reach the maximum at $c = 62$ and then it decreases.

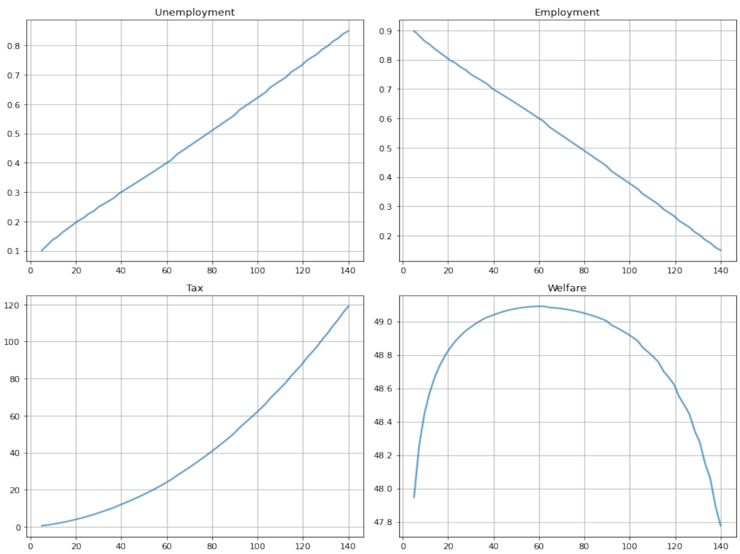

Figure 6: Welfare Analysis

In the future I would like to keep working in this kind of models to get to program more complex models like the one I presented in the first section of the term paper.

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