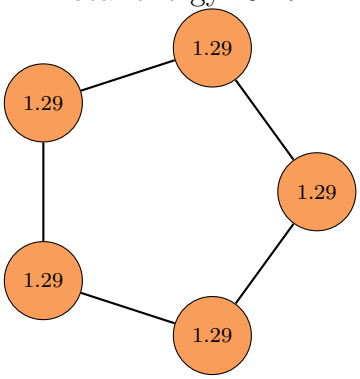
Graphs with large minimum vertex energy

Recall $\phi_i(|A|) \geq \sqrt{\frac{(\phi_i(A^2))^3}{\phi_i(A^4)}}$.

If G is a d-regular graph without 4-cycles we have $\phi_i(A^2) = d$ and $\phi_i(A^4)$ is the number of 4-walks on vertex v_i. Since no 4-cycles exist these walks are all of the form v_i, v_j, v_i, v_k, v_i or of the form v_i, v_j, v_k, v_j, v_i, the number of such paths is at most $d^2 + d^2$. We conclude $\phi_i(|A|) \geq \sqrt{\frac{d}{2}}$

For an example of such d regular graphs with $d = 2k$ consider the circulant graph of size $4^k + 1$ and distances $1, 4^1, \ldots, 4^{k-1}$

Total energy: 6.47



Total energy: 27.1