Hoja de trabajo #3

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1. Ejercicio #1

Sumar Tres $[s(s(s(0)))]$ y cuatro $[s(s(s(s(0))))]$

- $[s(s(s(0)))] \oplus [s(s(s(s(0))))]$
- $s(s(s(0 \oplus s(s(s(s(0)))))))$
- $s(s(s(s(0 \oplus s(s(s(0)))))))$
- $s(s(s(s(s(0 \oplus s(s(0)))))))$
- $s(s(s(s(s(s(0 \oplus 0)))))))$
- $s(s(s(s(s(s(0)))))))$

2. Ejercicio #2

Multiplicación:

$$a \otimes b := \begin{cases} a & \text{si } b = s(0) \\ b & \text{si } a = s(0) \\ 0 & \text{si } a = 0 \vee m = 0 \\ 0 & \text{si } a = 0 \wedge m = 0 \\ a \oplus (a \otimes j) & \text{si } b = s(j) \end{cases}$$

3. Ejercicio #3

Utilizar la definición de multiplicación.

1. $s(s(s(0))) \otimes 0$
   - Por definición es 0
2. $s(s(s(0))) \otimes s(0)$
   - Por definición es s(s(s(0)))
3. $s(s(s(0))) \otimes s(s(0))$
   - $s(s(s(0))) \oplus [s(s(s(0))) \otimes s(0)]$
   - $s(s(s(0))) \oplus s(s(s(0))) \oplus [s(s(s(0))) \otimes 0]$
   - $s(s(s(0))) \oplus s(s(s(0))) \oplus 0$
   - $s(s(s(0))) \oplus s(s(s(0)))$
   - $s(s(s(0 \oplus s(s(s(0))))))$
   - $s(s(s(s(0 \oplus s(s(0))))))$
   - $s(s(s(s(s(0 \oplus s(0))))))$
   - $s(s(s(s(s(s(0 \oplus 0))))))$
   - $s(s(s(s(s(s(0))))))$

4. Ejercicio #4

Definir por inducción.

1. $a \oplus s(s(0) = s(s(a)$
   Caso base:
   - $0 \oplus s(s(0) = s(s(0)$
   - $s(s(0 \oplus 0) = s(s(0)$
   - $s(s(0) = s(s(0)$

   Hipótesis inductiva:
   - $a \oplus s(s(0) = s(s(a)$
   - $s(s(0) = s(s(a \ominus a)$
   - $s(s(0) = s(s(0)$
2. $a \otimes b = b \otimes a$
   Caso base:
   - $0 \otimes 0 = 0 \otimes 0$
   - $0 = 0$

   Hipótesis inductiva:
   - $a \otimes b = b \otimes a$
   - $(a + 1) \otimes (b + 1) = (b + 1) \otimes (a + 1)$
   - $a \otimes b + a + b + 1 = b \otimes a + b + a + 1$
   - $a \otimes b = b \otimes a + b - b + a - a + 1 - 1$
   - $a \otimes b = b \otimes a + 0 + 0 + 0$
   - $a \otimes b = b \otimes a$
3. $a \otimes (b \otimes c) = (a \otimes b) \otimes c$
   Caso base:
   - $0 \otimes (0 \otimes 0) = (0 \otimes 0) \otimes 0$
   - $0 \otimes (0) = (0) \otimes 0$
   - $0 = 0$

   Hipótesis inductiva:
   - $a \otimes (b \otimes c) = (a \otimes b) \otimes c$
   - $s(i) \otimes (b \otimes c) = (s(i) \otimes b) \otimes c$
   - $s(i) \oplus (s(i) \otimes (b \otimes c)) = (s(i) \oplus (s(i) \otimes b)) \otimes c$
   - $s(i) \oplus (s(i) \otimes (b \otimes c)) = s(i) \oplus (s(i) \otimes (b \otimes c))$
4. $(a \oplus b) \otimes c = (a \otimes c) \oplus (b \otimes c)$
   Caso base:
   - $(0 \oplus 0) \otimes 0 = (0 \otimes 0) \oplus (0 \otimes 0)$
   - $0 \otimes 0 = 0 \oplus 0$
   - $0 = 0$

   Hipótesis inductiva:
   - $(a \oplus b) \otimes c = (a \otimes c) \oplus (b \otimes c)$
   - $(a \oplus b) \otimes (c + 1) = (a \otimes c + 1)) \oplus (b \otimes (c + 1))$

- $(a \oplus b) \otimes c = (a \otimes c) \oplus (b \otimes c)$
- $(a \oplus b) \otimes (c + 1) = (a \otimes (c + 1)) \oplus (b \otimes (c + 1))$
- $(a \oplus b) \otimes c + (a \oplus b) = ac + a + bc + b$
- $(a \oplus b) \otimes c + (a \oplus b) = (ac + bc) + a \oplus b$
- $(a \oplus b) \otimes c + (a \oplus b) = (a \oplus b) \otimes c + (a \oplus b)$