Universidad Nacional Autónoma de México

Facultad de Estudios Superiores Acatlán

Apuntes

Ecuaciones Diferenciales

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Índice

1. Teoremas de la tranformada de laplace

$$\mathcal{L}\{c\} = \frac{c}{s}$$

$$\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}$$

$$\mathcal{L}\{sen(\alpha t)\} = \frac{\alpha}{s^2+\alpha^2}$$

$$\mathcal{L}\{cos(\alpha t)\} = \frac{s}{s^2+\alpha^2}$$

$$\mathcal{L}\{e^{\alpha t}\} = \frac{1}{s-\alpha}$$

$$\mathcal{L}\{senh(\alpha t)\} = \frac{\alpha}{s^2-\alpha^2}$$

$$\mathcal{L}\{cosh(\alpha t)\} = \frac{s}{s^2-a^2}$$

2. Tarea 4

1. $y' + 4y = e^{-4t}, \quad y(0) = 2 \quad y'(0) = 0$
2. $y' - y = 1 - te^t, \quad y(0) = 0$
3. $y'' + 2y' + y = 0, \quad y(0) = y'(0) = 1$
4. $y'' - 4y' + 4y = t^3e^{2t}, \quad y(0) = y'(0) = 0$
5. $y'' - 6y' + 9y = t, \quad y(0) = 0, y'(0) = 1$
6. $y'' - 4y' + 4y = t^3, \quad y(0) = 1, y'(0) = 0$
7. $y'' - 6y' + 13y = 0, \quad y(0) = 0, y'(0) = -3$

8. $2y'' + 20y' + 51y = 0, \qquad y(0) = 2, y'(0) = 0$
9. $y'' - y' = e^t cos(t), \qquad y(0) = 0, y'(0) = 0$
10. $y'' - 2y' + 5y = 1 + t, \qquad y(0) = 0, y'(0) = 4$

2.1. Solución

2.1.1. Problema 1

$$y' + 4y = e^{-4t}, \qquad y(0) = 2$$

Aplicar la Transformada de Laplace en la ecuación es:

$$\mathcal{L}\{y'\} + 4\mathcal{L}\{y\} = \mathcal{L}\{e^{-4t}\}.$$

Resolviendo $\mathcal{L}\{e^{-4t}\}$:

$$\begin{aligned}
\mathcal{L}\{e^{-4t}\} &= \int_0^\infty e^{-st} e^{-4t}\, dt \\
&= \int_0^\infty e^{-(s+4)t}\, dt \\
&= \left[-\frac{1}{s+4} e^{-(s+4)t} \right]_0^\infty \\
&= \frac{1}{s+4}
\end{aligned}$$

Resolviendo $\mathcal{L}\{y'\}$ y $\mathcal{L}\{y\}$

$$\begin{aligned}
\mathcal{L}\{y'\} &= sY(s) - y(0), \\
\mathcal{L}\{y\} &= Y(s),
\end{aligned}$$

Sustituyendo con la condicion inicial $y(0) = 2$ en la ecuación:

$$sY(s) - 2 + 4Y(s) = \frac{1}{s+4}$$

Esto simplifica a:

$$Y(s)(s+4) = 2 + \frac{1}{s+4}$$

Despejamos $Y(s)$:

$$Y(s) = \frac{2}{s+4} + \frac{1}{(s+4)^2}$$

$$Y(s) = \frac{2s+9}{(s+4)^2}$$

$$\frac{2s+9}{(s+4)^2} = \frac{A}{s+4} + \frac{B}{(s+4)^2}$$

$$2s + 9 = A(s+4) + B$$

$$2s + 9 = As + 4A + B$$

$$2s + 9 = A(s+4) + B$$

Para $s = -4$

$$2(-4) + 9 = A(-4+4) + B$$

$$-8+9=A(0)+B$$

$$1=B$$

Como sabemos que $B=1$

$$2s+9=A(s+4)+1$$

Para $s=1$

$$2(1)+9=A(1+4)+1$$

$$11=A5+1$$

$$2=A$$

Por lo tanto los valores de A y B son

$$A=2 \qquad B=1$$

$$\frac{2s+9}{(s+4)^2}=\frac{2}{s+4}+\frac{1}{(s+4)^2}$$

Ahora aplicamos la tranfsormada inversa de laplace

$$\mathcal{L}^{-1}\{\frac{2}{s+4}\}+\mathcal{L}^{-1}\{\frac{1}{(s+4)^2}\}=2\mathcal{L}^{-1}\{\frac{1}{s+4}\}+\mathcal{L}^{-1}\{\frac{1}{(s+4)^2}\}$$

Sabemos:

$$\mathcal{L}^{-1}[\frac{1}{s-a}]=e^{at} \qquad \mathcal{L}^{-1}[\frac{1}{(s-a)^n}]=\frac{t^{n-1}e^{at}}{(n-1)!}$$

Por lo tanto:

$$\begin{aligned}\mathcal{L}^{-1}\left\{\frac{2}{s+4}\right\}+\mathcal{L}^{-1}\left\{\frac{1}{(s+4)^2}\right\}&=2e^{-4t}+\frac{t^{2-1}e^{-4t}}{(2-1)!}\\&=2e^{-4t}+\frac{te^{-4t}}{1!}\\&=2e^{-4t}+te^{-4t}\\&=e^{-4t}(2+t)\end{aligned}$$

Podemos confirmar este resultado con Mathematica

```
In[2]:= (*Problema 1*)
        ResolverEcuacionDiferencialConTransformadaDeLaplace[y'[t] + 4 y[t] == E^(-4 t),
                                                                               número e
         {2, 0}]
        9 + 2 s
        --------
        (4 + s)^2
Out[2]= e^(-4 t) (2 + t)
```

2.1.2. Problema 2

$$y' - y = 1 - te^t, \qquad y(0) = 0$$

Aplicamos la Transformada de Laplace a ambos lados de la ecuación:

$$\mathcal{L}\{y'\} - \mathcal{L}\{y\} = \mathcal{L}\{1 - te^t\}$$

Utilizamos las siguientes propiedades de la Transformada de Laplace:

$$\mathcal{L}\{1\} = \frac{1}{s}, \quad \mathcal{L}\{te^{\alpha t}\} = \frac{1}{(s-\alpha)^2}$$

Por lo tanto, para $\mathcal{L}\{1 - te^t\}$ tenemos:

$$\mathcal{L}\{1\} - \mathcal{L}\{te^t\} = \frac{1}{s} - \frac{1}{(s-1)^2}$$

Para $\mathcal{L}\{y'\}$ y $\mathcal{L}\{y\}$, considerando la condición inicial $y(0) = 0$, obtenemos:

$$\mathcal{L}\{y'\} = sY(s) - y(0) = sY(s)$$

$$\mathcal{L}\{y\} = Y(s)$$

Sustituyendo en la ecuación transformada:

$$sY(s) - Y(s) = \frac{1}{s} - \frac{1}{(s-1)^2}$$

Resolviendo para $Y(s)$:

$$Y(s)(s-1) = \frac{1}{s} - \frac{1}{(s-1)^2}$$

$$Y(s) = \frac{1}{s(s-1)} - \frac{1}{(s-1)^3}$$

Ahora debemos buscar las fracciones parciales de:

$$\frac{1}{s(s-1)} - \frac{1}{(s-1)^3}$$

Calculamos las fracciones parciales de $\frac{1}{s(s-1)}$

$$\frac{1}{s(s-1)} = \frac{A}{s} + \frac{B}{s-1}$$

$$1 = A(s-1) + Bs$$

Para $s = 0$

$$1 = A(0-1) + B(0)$$

$$1 = -A$$

$$A = -1$$

Para $s = 1$

$$1 = A(1-1) + B(1)$$

$$1 = A(0) + B$$

$$1 = B$$

$$\frac{1}{s(s-1)} = -\frac{1}{s} + \frac{1}{s-1}$$

Ahora aplicamos la tranfsormada inversa de laplace

$$-\mathcal{L}^{-1}\{\frac{1}{s}\} + \mathcal{L}^{-1}\{\frac{1}{s-1}\} - \mathcal{L}^{-1}\{\frac{1}{(s-1)^3}\}$$

Sabemos:

$$\mathcal{L}^{-1}[\frac{a}{s}] = a \qquad \mathcal{L}^{-1}[\frac{1}{(s-a)^n}] = \frac{t^{n-1}e^{at}}{(n-1)!} \qquad \mathcal{L}^{-1}[\frac{1}{s-a}] = e^{at}$$

$$-\mathcal{L}^{-1}\{\frac{1}{s}\} = -1$$
$$\mathcal{L}^{-1}\{\frac{1}{s-1}\} = e^t$$
$$-\mathcal{L}^{-1}\{\frac{1}{(s-1)^3}\} = -\frac{t^2e^t}{2}$$

$$-1 + e^t - \frac{t^2e^t}{2}$$

Podemos confirmar este resultado con Mathematica

```
In[18]:= (*Problema 2*)
         ResolverEcuacionDiferencialConTransformadaDeLaplace[y'[t] - y[t] == 1 - t*E^t,
                                                                                  [númer
          {0}]
         1 - 3 s + s^2
         -------------
         (-1 + s)^3 s
Out[18]= -1 - 1/2 e^t (-2 + t^2)
```

2.1.3. Problema 3

$$y'' + 2y' + y = 0, \qquad y(0) = y'(0) = 1$$

Aplicamos la Transformada de Laplace a ambos lados de la ecuación:

$$\mathcal{L}\{y''\} + 2\mathcal{L}\{y'\} + \mathcal{L}\{y\} = \mathcal{L}\{0\}$$

Para $\mathcal{L}\{y''\}$, $\mathcal{L}\{y'\}$ y $\mathcal{L}\{y\}$:

$$\begin{aligned}\mathcal{L}\{y''\} &= s^2Y(s) - sy(0) - y'(0)\\ \mathcal{L}\{y'\} &= sY(s) - y(0)\\ \mathcal{L}\{y\} &= Y(s)\end{aligned}$$

Considerando la condición inicial $y(0) = y'(0) = 1$, tenemos:

$$\begin{aligned}\mathcal{L}\{y''\} &= s^2Y(s) - s(1) - 1\\ \mathcal{L}\{y'\} &= sY(s) - 1\\ \mathcal{L}\{y''\} &= Y(s)\end{aligned}$$

Sustituyendo:

$$s^2Y(s) - s - 1 + 2(sY(s) - 1) + Y(s) = 0$$

$$s^2Y(s) - s - 1 + 2sY(s) - 2 + Y(s) = 0$$

$$Y(s)(s^2 + 2s + 1) - s - 3 = 0$$

$$(s^2 + 2s + 1)Y(s) = s + 3$$

Despejando $Y(s)$

$$Y(s) = \frac{s+3}{(s+1)^2}$$

Calculamos las fracciones parciales

$$\frac{s+3}{(s+1)^2} = \frac{A}{(s+1)} + \frac{B}{(s+1)^2}$$

$$s + 3 = A(s+1) + B$$

Para $s = -1$

$$\begin{aligned}-1 + 3 &= A(0) + B\\ 2 &= B\end{aligned}$$

Para $s - 3$

$$-3 + 3 = A(-3+1) + B$$

$$0 = A(-2) + B$$

Sabemos que $B = 2$

$$0 = A(-2) + 2$$

$$-2A = -2$$

$$A = 1$$

$$Y(s) = \frac{s+3}{(s+1)^2} = \frac{1}{s+1} + \frac{2}{(s+1)^2}$$

Aplicamos la tranformada inversa de laplace

$$\mathcal{L}^{-1}\{\frac{1}{s+1}\} + 2\mathcal{L}^{-1}\{\frac{1}{(s+1)^2}\}$$

Sabemos:

$$\mathcal{L}^{-1}[\frac{1}{(s-a)^n}] = \frac{t^{n-1}e^{at}}{(n-1)!} \qquad \mathcal{L}^{-1}[\frac{1}{s-a}] = e^{at}$$

$$\mathcal{L}^{-1}\{\frac{1}{s+1}\} = e^{-t}$$

$$2\mathcal{L}^{-1}\{\frac{1}{(s+1)^2}\} = 2te^{-t}$$

La solucion es:

$$e^{-t} + 2te^{-t} = e^{-t}(1+2t)$$

Podemos confirmar este resultado con Mathematica

```
In[6]:= (*Problema 3*)
        ResolverEcuacionDiferencialConTransformadaDeLaplace[y''[t] + 2 y'[t] + y[t] == 0,
         {1, 1}]
          3 + s
        ────────
        (1 + s)^2
Out[6]= e^-t (1 + 2 t)
```

2.1.4. Problema 4

$$y'' - 4y' + 4y = t^3 e^{2t}, \qquad y(0) = y'(0) = 0$$

Aplicamos la Transformada de Laplace a ambos lados de la ecuación:

$$\mathcal{L}\{y''\} - 4\mathcal{L}\{y'\} + 4\mathcal{L}\{y\} = \mathcal{L}\{t^3 e^{2t}\}.$$

para $\mathcal{L}\{t^3 e^{2t}\}$ tenemos:

$$\mathcal{L}\{t^3 e^{2t}\} = \frac{3!}{(s-2)^4} = \frac{6}{(s-2)^4}.$$

Para $\mathcal{L}\{y''\}$, $\mathcal{L}\{y'\}$ y $\mathcal{L}\{y\}$:

$$\begin{aligned}
\mathcal{L}\{y''\} &= s^2 Y(s) - sy(0) - y'(0) \\
\mathcal{L}\{y'\} &= sY(s) - y(0) \\
\mathcal{L}\{y\} &= Y(s)
\end{aligned}$$

Considerando la condición inicial $y(0) = y'(0) = 0$, tenemos:

$$\begin{aligned}
\mathcal{L}\{y''\} &= s^2 Y(s) - s(0) - 0 \\
\mathcal{L}\{y'\} &= sY(s) - 0 \\
\mathcal{L}\{y\} &= Y(s)
\end{aligned}$$

Sustituyendo:

$$s^2 Y(s) - 4sY(s) + 4Y(s) = \frac{6}{(s-2)^4}.$$

Factorizamos el término en $Y(s)$ y simplificamos:

$$Y(s)(s^2 - 4s + 4) = \frac{6}{(s-2)^4},$$

$$Y(s)(s-2)^2 = \frac{6}{(s-2)^4}.$$

Despejamos $Y(s)$:

$$Y(s) = \frac{6}{(s-2)^6}.$$

Aplicamos la transformada inversa de Laplace:

$$\begin{aligned}
\mathcal{L}^{-1}\left[\frac{1}{(s-a)^n}\right] &= \frac{t^{n-1}e^{at}}{(n-1)!} \\
6\mathcal{L}^{-1}\left[\frac{1}{(s-2)^6}\right] &= 6\frac{t^5 e^{2t}}{5!} \\
&= 6\frac{t^5 e^{2t}}{120} \\
&= \frac{t^5 e^{2t}}{20}
\end{aligned}$$

Podemos confirmar este resultado con Mathematica

In[7]:=
```
(*Problema 4*)
ResolverEcuacionDiferencialConTransformadaDeLaplace[
 y''[t] - 4 y'[t] + 4 y[t] == t^3 E^(2 t), {0, 0}]
```
número e

$$\frac{6}{(-2+s)^6}$$

Out[7]= $\frac{1}{20} e^{2t} t^5$

2.1.5. Problema 5

$$y'' - 6y' + 9y = t, \qquad y(0) = 0, y'(0) = 1$$

Aplicamos la Transformada de Laplace a ambos lados de la ecuación:

$$\mathcal{L}\{y''\} - 6\mathcal{L}\{y'\} + 9\mathcal{L}\{y\} = \mathcal{L}\{t\}.$$

para $\mathcal{L}\{t\}$ tenemos:

$$\mathcal{L}\{t\} = \frac{1}{s^2}$$

Para $\mathcal{L}\{y''\}$, $\mathcal{L}\{y'\}$ y $\mathcal{L}\{y\}$:

$$\begin{aligned}
\mathcal{L}\{y''\} &= s^2Y(s) - sy(0) - y'(0) \\
\mathcal{L}\{y'\} &= sY(s) - y(0) \\
\mathcal{L}\{y\} &= Y(s)
\end{aligned}$$

Considerando la condición inicial $y(0) = 0$ y $y'(0) = 1$, tenemos:

$$\begin{aligned}
\mathcal{L}\{y''\} &= s^2Y(s) - s(0) - 1 \\
\mathcal{L}\{y'\} &= sY(s) - 0 \\
\mathcal{L}\{y\} &= Y(s)
\end{aligned}$$

Sustituyendo:

$$s^2Y(s) - 1 - 6sY(s) + 9Y(s) = \frac{1}{s^2}$$

$$Y(s)(s^2 - 6s + 9) - 1 = \frac{1}{s^2}$$

$$Y(s)(s^2 - 6s + 9) = \frac{1}{s^2} + 1$$

$$Y(s)(s - 3)^2 = \frac{1}{s^2} + 1$$

$$Y(s)(s - 3)^2 = \frac{1 + s^2}{s^2}$$

$$Y(s) = \frac{1 + s^2}{s^2(s - 3)^2}$$

Calculamos las fracciones parciales

$$\frac{1 + s^2}{s^2(s - 3)^2} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{(s - 3)} + \frac{D}{(s - 3)^2}$$

$$1 + s^2 = As(s - 3)^2 + B(s - 3)^2 + Cs^2(s - 3) + Ds^2$$

Expandiendo:

$$\begin{aligned}
1 + s^2 &= As^3 - 6As^2 + 9As + Bs^2 - 6Bs + 9B + Cs^3 - 3Cs^2 + Ds^2 \\
1 + s^2 &= (A + C)s^3 + (-6A + B - 3C + D)s^2 + (9A - 6B)s + 9B
\end{aligned}$$

$$A + C = 0$$
$$-6A + B - 3C + D = 1$$
$$9A - 6B = 0$$
$$9B = 1$$

DE este sistema observamos:

$$B = \frac{1}{9}$$

Sustituyendo arriba

$$9A - 6(\frac{1}{9}) = 0$$
$$9A = \frac{6}{9}$$
$$A = \frac{6}{81}$$
$$A = \frac{2}{27}$$

Sustituyendo en la primera expresion

$$A + C = 0$$
$$\frac{2}{27} + C = 0$$
$$C = -\frac{2}{27}$$

Resolviendo la ultima expresion

$$-6A + B - 3C + D = 1$$
$$-6(\frac{2}{27}) + \frac{1}{9} - 3(-\frac{2}{27}) + D = 1$$
$$-\frac{12}{27} + \frac{1}{9} + \frac{6}{27} + D = 1$$
$$-\frac{12}{27} + \frac{1}{9} + \frac{6}{27} + D = 1$$
$$-\frac{1}{9} + D = 1$$
$$D = 1 + \frac{1}{9} = \frac{10}{9}$$

$$\frac{1 + s^2}{s^2(s-3)^2} = \frac{2/27}{s} + \frac{1/9}{s^2} + \frac{-2/27}{(s-3)} + \frac{10/9}{(s-3)^2}$$

Aplicar la tranformada inversa de laplace
Sabemos:

$$\mathcal{L}^{-1}[\frac{a}{s}] = a \qquad \mathcal{L}^{-1}[\frac{1}{(s-a)^n}] = \frac{t^{n-1}e^{at}}{(n-1)!} \qquad \mathcal{L}^{-1}[\frac{1}{s-a}] = e^{at} \qquad \mathcal{L}^{-1}\left[\frac{1}{s^n}\right] = \frac{t^{n-1}}{(n-1)!}$$

$$\mathcal{L}^{-1}[\frac{2/27}{s}] + \mathcal{L}^{-1}[\frac{1/9}{s^2}] + \mathcal{L}^{-1}[\frac{-2/27}{(s-3)}] + \mathcal{L}^{-1}[\frac{10/9}{(s-3)^2}]$$

$$\begin{aligned}
\mathcal{L}^{-1}[\frac{2/27}{s}] &= \frac{2}{27} \\
\mathcal{L}^{-1}[\frac{1/9}{s^2}] &= \frac{1}{9} \cdot \frac{t^{2-1}}{(2-1)!} = \frac{1}{9}t \\
-2/27\mathcal{L}^{-1}[\frac{1}{(s-3)}] &= \frac{-2}{27}e^{3t} \\
10/9\mathcal{L}^{-1}[\frac{1}{(s-3)^2}] &= \frac{10}{9}\frac{te^{3t}}{1}
\end{aligned}$$

La tranformada inversa de laplace completa es:

$$\frac{2}{27} + \frac{1}{9}t - \frac{2}{27}e^{3t} + \frac{10}{9}te^{3t}$$

2.1.6. Problema 6

$$y'' - 4y' + 4y = t^3, \qquad y(0) = 1, y'(0) = 0$$

Aplicamos la Transformada de Laplace a ambos lados de la ecuación:

$$\mathcal{L}\{y''\} - 4\mathcal{L}\{y'\} + 4\mathcal{L}\{y\} = \mathcal{L}\{t^3\}.$$

para $\mathcal{L}\{t^3\}$ tenemos:

$$\mathcal{L}\{t^3\} = \frac{6}{s^4}$$

Para $\mathcal{L}\{y''\}$, $\mathcal{L}\{y'\}$ y $\mathcal{L}\{y\}$:

$$\begin{aligned}
\mathcal{L}\{y''\} &= s^2Y(s) - sy(0) - y'(0) \\
\mathcal{L}\{y'\} &= sY(s) - y(0) \\
\mathcal{L}\{y\} &= Y(s)
\end{aligned}$$

Considerando la condición inicial $y(0) = 1$ y $y'(0) = 0$, tenemos:

$$\begin{aligned}
\mathcal{L}\{y''\} &= s^2Y(s) - s(1) - 0 \\
\mathcal{L}\{y'\} &= sY(s) - 1 \\
\mathcal{L}\{y\} &= Y(s)
\end{aligned}$$

Sustituyendo:

$$\begin{aligned}
s^2Y(s) - s - 4(sY(s) - 1) + 4Y(s) &= \frac{6}{s^4} \\
s^2Y(s) - s - 4sY(s) + 4 + 4Y(s) &= \frac{6}{s^4} \\
s^2Y(s) - 4sY(s) + 4 + 4Y(s) &= \frac{6}{s^4} + s \\
Y(s)(s^2 - 4s + 4) + 4 &= \frac{6}{s^4} + s \\
Y(s)(s-2)^2 &= \frac{6}{s^4} + s - 4 \\
Y(s)(s-2)^2 &= \frac{s^5 - 4s^4 + 6}{s^4} \\
Y(s) &= \frac{s^5 - 4s^4 + 6}{s^4(s-2)^2}
\end{aligned}$$

Buscamos las fracciones parciales de esta expresion

$$\frac{s^5 - 4s^4 + 6}{s^4(s-2)^2} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{D}{s^4} + \frac{E}{s-2} + \frac{F}{(s-2)^2}$$

podemos calcular las fracciones parciales con `Mathematica`

```
In[5]:= Apart[(s^5 - 4*s^4 + 6) / (s^4*(s - 2)^2)]
        separa fracciones simples
```

$$\text{Out[5]=} -\frac{13}{8(-2+s)^2} + \frac{1}{4(-2+s)} + \frac{3}{2s^4} + \frac{3}{2s^3} + \frac{9}{8s^2} + \frac{3}{4s}$$

$$\frac{13}{8(s-2)^2}+\frac{1}{4(s-2)}+\frac{3}{2s^4}+\frac{3}{2s^3}+\frac{9}{8s^2}+\frac{3}{4s}$$

Calculamos la inversa

$$\mathcal{L}^{-1}[\frac{13}{8(s-2)^2}]+\mathcal{L}^{-1}[\frac{1}{4(s-2)}]+\mathcal{L}^{-1}[\frac{3}{2s^4}]+\mathcal{L}^{-1}[\frac{3}{2s^3}]++\mathcal{L}^{-1}[\frac{9}{8s^2}]+\mathcal{L}^{-1}[\frac{3}{4s}]$$

$$\mathcal{L}^{-1}\left[\frac{13}{8(s-2)^2}\right]=\mathcal{L}^{-1}\left[\frac{13}{8}\cdot\frac{1}{(s-2)^2}\right]=\frac{13}{8}e^{2t}t$$

$$\mathcal{L}^{-1}\left[\frac{1}{4(s-2)}\right]=\mathcal{L}^{-1}\left[\frac{1}{4}\cdot\frac{1}{s-2}\right]=\frac{1}{4}e^{2t}$$

$$\mathcal{L}^{-1}\left[\frac{3}{2s^4}\right]=\mathcal{L}^{-1}\left[\frac{3}{2}\cdot\frac{1}{s^4}\right]=\frac{3}{2}\cdot\frac{t^3}{3!}=\frac{t^3}{4}$$

$$\mathcal{L}^{-1}\left[\frac{3}{2s^3}\right]=\mathcal{L}^{-1}\left[\frac{3}{2}\cdot\frac{1}{s^3}\right]=\frac{3}{2}\cdot\frac{t^2}{2!}=\frac{3t^2}{4}$$

$$\mathcal{L}^{-1}\left[\frac{9}{8s^2}\right]=\mathcal{L}^{-1}\left[\frac{9}{8}\cdot\frac{1}{s^2}\right]=\frac{9}{8}\cdot\frac{t}{1!}=\frac{9t}{8}$$

$$\mathcal{L}^{-1}\left[\frac{3}{4s}\right]=\mathcal{L}^{-1}\left[\frac{3}{4}\cdot\frac{1}{s}\right]=\frac{3}{4}$$

$$y(t)=\mathcal{L}^{-1}\left[\frac{13}{8(s-2)^2}\right]+\mathcal{L}^{-1}\left[\frac{1}{4(s-2)}\right]+\mathcal{L}^{-1}\left[\frac{3}{2s^4}\right]+\mathcal{L}^{-1}\left[\frac{3}{2s^3}\right]+\mathcal{L}^{-1}\left[\frac{9}{8s^2}\right]+\mathcal{L}^{-1}\left[\frac{3}{4s}\right]$$
$$=\frac{13}{8}e^{2t}t+\frac{1}{4}e^{2t}+\frac{t^3}{4}+\frac{3t^2}{4}+\frac{9t}{8}+\frac{3}{4}$$

```
(*Problema 6*)
ResolverEcuacionDiferencialConTransformadaDeLaplace[
 y''[t] - 4 y'[t] + 4 y[t] == t^3, {1, 0}]
```

$$\frac{6-4s^4+s^5}{(-2+s)^2 s^4}$$

$$\frac{1}{8}\left(6+e^{2t}(2-13t)+9t+6t^2+2t^3\right)$$

2.1.7. Problema 7

$$y'' - 6y' + 13y = 0, \qquad y(0) = 0, y'(0) = -3$$

Aplicamos la Transformada de Laplace a ambos lados de la ecuación:

$$\mathcal{L}\{y''\} - 6\mathcal{L}\{y'\} + 13\mathcal{L}\{y\} = \mathcal{L}\{0\}$$

Para $\mathcal{L}\{y''\}$, $\mathcal{L}\{y'\}$ y $\mathcal{L}\{y\}$:

$$\begin{aligned} \mathcal{L}\{y''\} &= s^2Y(s) - sy(0) - y'(0) \\ \mathcal{L}\{y'\} &= sY(s) - y(0) \\ \mathcal{L}\{y\} &= Y(s) \end{aligned}$$

Considerando la condición inicial $y(0) = 0$ y $y'(0) = 3$, tenemos:

$$\begin{aligned} \mathcal{L}\{y''\} &= s^2Y(s) - s(0) + 3 \\ \mathcal{L}\{y'\} &= sY(s) - 0 \\ \mathcal{L}\{y\} &= Y(s) \end{aligned}$$

Sustituyendo:

$$s^2Y(s) + 3 - 6sY(s) + 13Y(s) = 0$$

$$Y(s)(s^2 - 6s + 13) + 3 = 0$$

$$Y(s) = -\frac{3}{(s^2 - 6s + 13)} = -\frac{3}{(s-3)^2 + 4}$$

$$L\left\{e^{at}\sin(bt)\right\} = \frac{b}{(s-a)^2 + b^2}$$

$$L^{-1}\left\{-\frac{3}{(s-3)^2 + 2^2}\right\} = -3e^{3t}\sin(2t)$$

```
In[4]:= (*Problema 7*)
        ResolverEcuacionDiferencialConTransformadaDeLap
         y''[t] - 6 y'[t] + 13 y[t] == 0, {0, -3}]
```

$$-\frac{3}{13 - 6\,s + s^2}$$

```
Out[4]= -3 e^(3 t) Cos[t] Sin[t]
```

2.1.8. Problema 8

$$2y'' + 20y' + 51y = 0, \qquad y(0) = 2, y'(0) = 0$$

Aplicamos la Transformada de Laplace a ambos lados de la ecuación:

$$2\mathcal{L}\{y''\} + 20\mathcal{L}\{y'\} + 51\mathcal{L}\{y\} = \mathcal{L}\{0\}.$$

Para $\mathcal{L}\{y''\}$, $\mathcal{L}\{y'\}$ y $\mathcal{L}\{y\}$:

$$\begin{aligned}
\mathcal{L}\{y''\} &= s^2Y(s) - sy(0) - y'(0) \\
\mathcal{L}\{y'\} &= sY(s) - y(0) \\
\mathcal{L}\{y\} &= Y(s)
\end{aligned}$$

Considerando la condición inicial $y(0) = 2$ y $y'(0) = 0$, tenemos:

$$\begin{aligned}
\mathcal{L}\{y''\} &= s^2Y(s) - s(2) - 0 \\
\mathcal{L}\{y'\} &= sY(s) - 2 \\
\mathcal{L}\{y\} &= Y(s)
\end{aligned}$$

Sustituyendo:

$$2(s^2Y(s) - 2s) + 20(sY(s) - 2) + 51Y(s) = 0$$

$$2s^2Y(s) - 4s + 20sY(s) - 40 + 51Y(s) = 0$$

$$Y(s)(2s^2 + 20s + 51) - 4s - 40 = 0$$

$$Y(s)(2s^2 + 20s + 51) = 4s + 40$$

$$Y(s) = \frac{4s + 40}{2s^2 + 20s + 51}$$

$$Y(s) = \frac{4s + 40}{(s - 5)^2 + 50}$$

$$L^{-1}\left\{\frac{s - a}{(s - a)^2 + \omega^2}\right\} = e^{at}\cos(\omega t)$$

$$L^{-1}\left\{\frac{\omega}{(s - a)^2 + \omega^2}\right\} = e^{at}\sin(\omega t)$$

$$2\left(3\sqrt{2}\sin(5\sqrt{2}t) + 2\cos(5\sqrt{2}t)\right)e^{5t}u(t)$$

2.1.9. Problema 9

$$y'' - y' = e^t cos(t), \qquad y(0) = 0, y'(0) = 0$$

Aplicamos la Transformada de Laplace a ambos lados de la ecuación:

$$\mathcal{L}\{y''\} - \mathcal{L}\{y'\} = \mathcal{L}\{e^t \cos(t)\}.$$

Resolviendo $\mathcal{L}\{e^t \cos(t)\}$:
Sabemos:

$$\mathcal{L}\{e^{at} \cos(bt)\} = \frac{s-a}{(s-a)^2 + b^2}$$

$$\mathcal{L}\{e^t \cos(t)\} = \frac{s-1}{(s-1)^2 + 1^2}$$

Para $\mathcal{L}\{y''\}$, $\mathcal{L}\{y'\}$ y $\mathcal{L}\{y\}$:

$$\mathcal{L}\{y''\} = s^2Y(s) - sy(0) - y'(0)$$
$$\mathcal{L}\{y'\} = sY(s) - y(0)$$

Considerando la condición inicial $y(0) = 0$ y $y'(0) = 0$, tenemos:

$$\mathcal{L}\{y''\} = s^2Y(s)$$
$$\mathcal{L}\{y'\} = sY(s)$$

Sustituyendo:

$$s^2Y(s) - sY(s) = \frac{s-1}{(s-1)^2 + 1^2}$$

$$Y(s)(s^2 - s) = \frac{s-1}{(s-1)^2 + 1}$$

$$Y(s) = \frac{s-1}{(s^2 - s)((s-1)^2 + 1)}$$

$$Y(s) = \frac{s-1}{s(s-1)((s-1)^2 + 1)}$$

$$Y(s) = \frac{1}{s((s-1)^2 + 1)}$$

$$Y(s) = \frac{1}{s^3 - 2s^2 + 2s}$$

$$Y(s) = \frac{1}{s(s^2 - 2s + 2)}$$

2.1.10. Problema 10

$$y'' - 2y' + 5y = 1 + t, \qquad y(0) = 0, y'(0) = 4$$

Aplicamos la Transformada de Laplace a ambos lados de la ecuación:

$$\mathcal{L}\{y''\} - 2\mathcal{L}\{y'\} + 5\mathcal{L}\{y\} = \mathcal{L}\{1 + t\}$$

$$\mathcal{L}\{y''\} - 2\mathcal{L}\{y'\} + 5\mathcal{L}\{y\} = \frac{1}{s} + \frac{1}{s^2}$$

Para $\mathcal{L}\{y''\}$, $\mathcal{L}\{y'\}$ y $\mathcal{L}\{y\}$:

$$\begin{aligned}
\mathcal{L}\{y''\} &= s^2Y(s) - sy(0) - y'(0) \\
\mathcal{L}\{y'\} &= sY(s) - y(0) \\
\mathcal{L}\{y\} &= Y(s)
\end{aligned}$$

Considerando la condición inicial $y(0) = 0$ y $y'(0) = 4$, tenemos:

$$\begin{aligned}
\mathcal{L}\{y''\} &= s^2Y(s) - s(0) - 4 \\
\mathcal{L}\{y'\} &= sY(s) - (0) \\
\mathcal{L}\{y\} &= Y(s)
\end{aligned}$$

Sustituyendo:

$$s^2Y(s) - 4 - 2sY(s) + 5Y(s) = \frac{1}{s} + \frac{1}{s^2}$$

$$Y(s)(s^2 - 2s + 5) - 4 = \frac{1}{s} + \frac{1}{s^2}$$

$$Y(s)(s^2 - 2s + 5) = \frac{1}{s} + \frac{1}{s^2} + 4$$

$$Y(s)(s^2 - 2s + 5) = \frac{4s^2 + s + 1}{s^2}$$

$$Y(s) = \frac{4s^2 + s + 1}{s^2(s^2 - 2s + 5)}$$