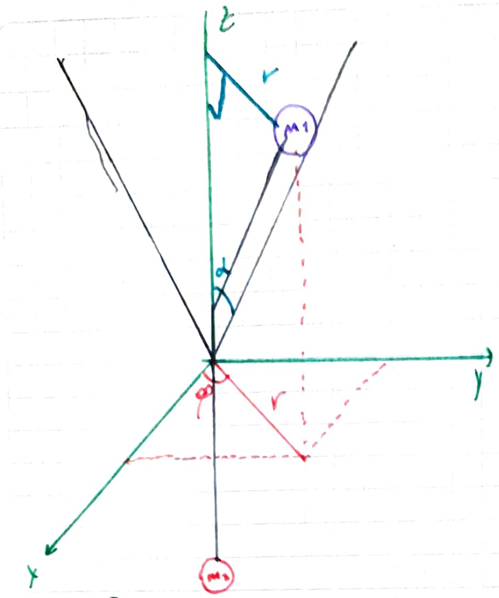
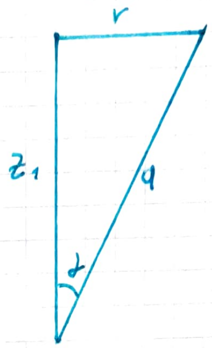
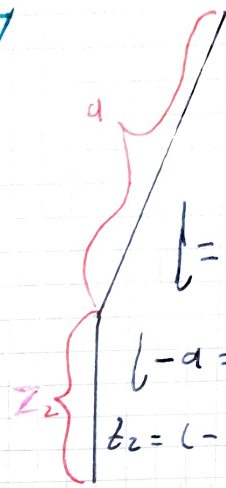
coordenadas cilíndricas.

$$(r, \varphi, z), \ (\dot r, \dot\varphi, \dot z)$$



Ligaduras:





$$l = a + z_2$$

$$l - a = z_2$$

$$z_2 = l - r\csc\alpha$$

$$\dot z_2 = -\dot r\csc\alpha$$

$$\cos\alpha = \frac{z_1}{a}, \quad a = z_1 \cdot \frac{1}{\cos\alpha} \qquad \tan\alpha = \frac{r}{z_1}$$

$$\operatorname{sen}\alpha = \frac{r}{a}, \quad a = r \cdot \frac{1}{\operatorname{sen}\alpha} \qquad z_1 = r\cot\alpha$$

Lagrangiano: $T - V$

$$T = T_{m_1} + T_{m_2}$$

$$T_{m_1} = \frac{1}{2} m_1 \left(\dot r^2 + (r\dot\varphi)^2 + \dot z_1^2\right) = \frac{1}{2} m_1 \left(\dot r^2 + r^2\dot\varphi^2 + \dot z_1^2\right)$$

$$T_{m_2} = \frac{1}{2} m_2 (-\dot z_2)^2 = \frac{1}{2} m_2 \dot z_2^2$$

$$T = \frac{1}{2}\left(m_1\left(\dot r^2 + r^2\dot\varphi^2 + \dot z_1^2\right) + m_2 \dot z_2^2\right) = \frac{1}{2}\left(m_1\left(\dot r^2 \csc^2\alpha + r\dot\varphi^2\right) + m_2 \dot z_2^2\right)$$

$$V = V_{m_1} + V_{m_2}$$

$$V_{m_1} = m_1 g z_1 = m_1 g r\cot\alpha$$

$$V_{m_2} = m_2 g z_2 = m_2 g\left[(-r\csc\alpha\right] = m_2 g (r\csc - l)$$

$$V = m_1 g r\cot\alpha + m_2 g (r\csc - l)$$

$$\mathcal{L} = \frac{1}{2}\left[m_1\left(\dot r^2 \csc^2\alpha + r^2\dot\varphi^2\right) + m_2 \dot z_2^2\right] - m_1 g r\cot\alpha - m_2 g (r\csc - l)$$

$$\mathcal{L} = \frac{1}{2}\left[m_1\left(\dot r^2 \csc^2\alpha + r^2\dot\varphi^2\right) + m_2 \dot r^2 \csc^2\alpha\right] - m_1 g r\cot\alpha - m_2 g (r\csc\alpha - l) \ //$$

Para r:

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot r}\right)-\frac{\partial \mathcal{L}}{\partial r}=0$$

$$\frac{\partial \mathcal{L}}{\partial \dot r}=(m_1+m_2)\dot r\csc^2\alpha$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot r}\right)=(m_1+m_2)\ddot r\csc^2\alpha$$

$$\frac{\partial \mathcal{L}}{\partial r}=m_1 r\dot\varphi^2-m_1 g\cot\alpha-m_2 g\csc\alpha$$

$$(m_1+m_2)\ddot r\csc^2\alpha-m_1 r\dot\varphi^2+m_1 g\cot\alpha+m_2 g\csc\alpha=0 \quad // \text{RTA}$$

Para φ:

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot\varphi}\right)-\frac{\partial \mathcal{L}}{\partial \varphi}=0$$

$$\left.\begin{aligned}\frac{\partial \mathcal{L}}{\partial \dot\varphi}&=m_1 r^2\dot\varphi\\ \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot\varphi}\right)&=2m_1 r\dot\varphi\end{aligned}\right\}\quad 2m_1 r\dot\varphi=0 \quad // \text{RTA.}$$

$$\frac{\partial \mathcal{L}}{\partial \varphi}=0$$

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b) r de equilibrio. $\ddot r=0$, $\dot\varphi=cte.$

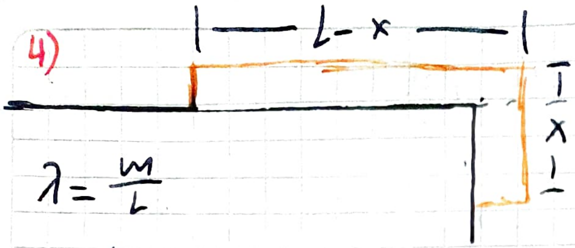
Ecuación de movimiento de r

$$(m_1+m_2)\cancelto{0}{\ddot r}\csc^2\alpha-m_1 r\dot\varphi^2+m_1 g\cot\alpha+m_2 g\csc\alpha=0$$

$$m_1 r\dot\varphi^2=m_1 g\cot\alpha+m_2 g\csc\alpha$$

$$r=\frac{m_1 g\cot\alpha+m_2 g\csc\alpha}{m_1\dot\varphi^2} \quad // \text{RTA.}$$

4)



$\lambda = \frac{m}{L}$

$m = L\lambda$

coordenadas:

La longitud que cae será $x(t)$

$y(t)$ será la porción de la mesa

$y(t) = L - x(t)$

Lagrangiano: $\mathcal{L} = T - V$

$T = T_{mesa} + T_{cuerda}$

$$T_{mesa} = \frac{1}{2} m v^2 = \frac{1}{2}\lambda(L - x(t))\,\dot{x}(t)^2$$

$$T_{cuerda} = \frac{1}{2} m v^2 = \frac{1}{2}\lambda x(t) \cdot \dot{x}(t)^2$$

$$\left.\begin{matrix} T_{mesa} \\ T_{cuerda} \end{matrix}\right\} T = \frac{1}{2}\lambda(L - x)\dot{x}^2 + \frac{1}{2}\lambda x \dot{x}^2$$

$$T = \frac{1}{2}\lambda L \dot{x}^2$$

$V = V_{cuerda}$

$$V_{cuerda} = -mgh = -\lambda x \cdot g \cdot \frac{x}{2} = \frac{1}{2}\lambda x^2 g$$

$$\mathcal{L} = \frac{1}{2}\lambda L \dot{x}^2 - \frac{1}{2}\lambda x^2 g = \frac{\lambda}{2}\left(L\dot{x}^2 - g x^2\right)$$

Ecuación de Euler-Lagrange:

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}}\right) - \frac{\partial \mathcal{L}}{\partial x} = 0$$

$$\frac{\partial \mathcal{L}}{\partial \dot{x}} = \lambda L \dot{x}$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}}\right) = \lambda L \ddot{x}$$

$$\frac{\partial \mathcal{L}}{\partial x} = -\lambda g x$$

$$\lambda L \ddot{x} - (\lambda g x) = 0$$

$$\ddot{x} + \frac{g}{L} x = 0 \longrightarrow \text{Se puede solucionar.}$$

$$\ddot{x} + \omega^2 x(t) = 0, \qquad \omega^2 = \frac{g}{L}, \quad \omega = \sqrt{\frac{g}{L}}$$

$$x(t) = A\cos(\omega t) + B\,\mathrm{sen}(\omega t)$$

$x(0) = L$

$\dot{x}(0) = 0$

$$x(t) = A\cos\left(\sqrt{\frac{g}{L}}\,t\right) + B\,\mathrm{sen}\left(\sqrt{\frac{g}{L}}\,t\right)$$

$$x(t) = L\cos\left(\sqrt{\frac{g}{L}}\,t\right) \text{ // RTA.}$$