1, 5, 7)

$r = x\hat{\imath} + y\hat{\jmath} + z\hat{k} = x^i \hat{\imath}_i$

$a = a(r) = a(x,y,z) = a^i(x,y,z)\hat{\imath}_i$

$b = b(r) = b(x,y,z) = b^i(x,y,z)\hat{\imath}_i$

$\varphi = \varphi(r) = \varphi(x,y,z)$

$\psi = \psi(r) = \psi(x,y,z)$

2a) $\nabla(\varphi\psi) = \varphi\nabla\psi + \psi\nabla\varphi$

① $\nabla(\varphi\psi) = \underbrace{\frac{\partial(\varphi\psi)}{\partial x}}\hat{\imath} + \underbrace{\frac{\partial(\varphi\psi)}{\partial y}}\hat{\jmath} + \underbrace{\frac{\partial(\varphi\psi)}{\partial z}}\hat{k}$

Desarrollando:

② $\frac{\partial(\varphi\psi)}{\partial x}\hat{\imath} = \frac{\partial\varphi}{\partial x}\psi + \frac{\partial\psi}{\partial x}\varphi$

③ $\frac{\partial(\varphi\psi)}{\partial y}\hat{\jmath} = \frac{\partial\varphi}{\partial y}\psi + \frac{\partial\psi}{\partial y}\varphi$

④ $\frac{\partial(\varphi\psi)}{\partial z} = \frac{\partial\varphi}{\partial z}\psi + \frac{\partial\psi}{\partial z}\varphi$

②, ③ y ④ en ①

$\nabla(\varphi\psi) = \frac{\partial\varphi}{\partial x}\psi + \frac{\partial\psi}{\partial x}\varphi + \frac{\partial\varphi}{\partial y}\psi$

$+ \frac{\partial\psi}{\partial y}\varphi + \frac{\partial\varphi}{\partial z}\psi + \frac{\partial\psi}{\partial z}\varphi$

Agrupando:

$\nabla(\varphi\psi) = \psi\left(\frac{\partial\varphi}{\partial x} + \frac{\partial\varphi}{\partial y} + \frac{\partial\varphi}{z}\right) +$

$\varphi\left(\frac{\partial\psi}{\partial x} + \frac{\partial\psi}{\partial y} + \frac{\partial\psi}{\partial z}\right)$

$\boxed{* \frac{\partial\varphi}{\partial x} + \frac{\partial\varphi}{\partial y} + \frac{\partial\varphi}{z} = \nabla\varphi}$

$\boxed{* \frac{\partial\psi}{\partial x} + \frac{\partial\psi}{\partial y} + \frac{\partial\psi}{\partial z} = \nabla\psi}$

$\nabla(\varphi\psi) = \psi\nabla\varphi + \varphi\nabla\psi$ //

2d) $\nabla \cdot (\nabla \times \vec{a}) = 0$, $\nabla \times (\nabla \cdot \vec{a}) = ?$

$\nabla \times (\nabla \vec{a})$ no tiene sentido porque

$\nabla \cdot a$ es escalar, porque un producto escalar da como resultado un escalar, y no tiene sentido hacer un producto cruz entre un escalar y un vector.

2f) $\nabla \times (\nabla \times a) = \nabla (\nabla \cdot a) - \nabla^2 a$

① Rotacional de a

$(\nabla \times a)_i = \epsilon_{ijk} \partial_j a_k$

Rotacional del rotacional:

$(\nabla \times (\nabla \times a))_i = \epsilon_{ijk} \partial_j (\epsilon_{klm} \partial_l a_m)$

② Identidad de Levi-Civita.

$\epsilon_{ijk} \epsilon_{klm} = \delta_{il} \delta_{jm} - \delta_{im} \delta_{jl}$

$(\nabla \times (\nabla \times a))_i = (\delta_{il} \delta_{jm} - \delta_{im} \delta_{jl}) \partial_j \partial_l a_m$

③ Delta de Kronecker:

$(\nabla \times (\nabla \times a))_i = \partial_j \partial_i a_j - \partial_j \partial_j a_i$

$$\left.\begin{aligned} \partial_j \partial_i a_j &= \nabla (\nabla \cdot a)_i \\ \partial_j \partial_j a_i &= \nabla^2 a \end{aligned}\right\} \quad \nabla \times (\nabla \times a) = \nabla (\nabla a) - \nabla^2 a //$$