

# Scientific Computing for Differential Equations

## Lecture 02A - The Explicit Euler Method

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# Ordinary Differential Equations

# The Explicit Euler Method

Ordinary Differential Equation (ODE)

$$\frac{d}{dt}x(t) = f(t, x(t)) \quad \text{or} \quad \dot{x}(t) = f(t, x(t)) \quad (1)$$

**Solution method by integration**

$$x(t_{k+1}) - x(t_k) = \int_{x(t_k)}^{x(t_{k+1})} dx(t) = \int_{t_k}^{t_{k+1}} f(t, x(t)) dt \quad (2)$$

Let  $x_k = x(t_k)$ . Then (2) can be approximated as

$$x_{k+1} = x_k + \Delta t_k f(t_k, x_k) \quad (3)$$

# The Explicit Euler Method

Ordinary Differential Equation (ODE)

$$\frac{d}{dt}x(t) = f(t, x(t)) \quad \text{or} \quad \dot{x}(t) = f(t, x(t)) \quad (4)$$

**Solution method by differentiation**

$$\frac{x(t_{k+1}) - x(t_k)}{\Delta t_k} \approx \frac{d}{dt}x(t_k) = f(t_k, x(t_k)) \quad (5)$$

Let  $x_k = x(t_k)$ . Then (2) can be approximated as

$$x_{k+1} = x_k + \Delta t_k f(t_k, x_k) \quad (6)$$

# Initial Value Problem and the Explicit Euler Method

Initial value problem

$$x(t_0) = \bar{x}_0 \quad (7a)$$

$$\frac{d}{dt}x(t) = f(t, x(t)) \quad t_0 \leq t \leq t_N \quad (7b)$$

Fixed time step

$$\Delta t = \frac{t_N - t_0}{N} = \frac{t_b - t_a}{N} \quad (8)$$

Euler's explicit method

$$x_0 = \bar{x}_0 \quad (9a)$$

$$x_{k+1} = x_k + \Delta t f(t_k, x_k) \quad k = 0, 1, \dots, N-1 \quad (9b)$$

# Matlab Implementation of Euler's Explicit Method

## Euler's Explicit Method

$$x_0 = \bar{x}_0 \quad (10a)$$

$$x_{k+1} = x_k + \Delta t f(t_k, x_k) \quad k = 0, 1, \dots, N-1 \quad (10b)$$

```
1 function [T,X] = ExplicitEulerFixedStepSize(fun,t0,tN,N,x0,varargin)
2
3 % Compute step size and allocate memory
4 dt = (tN-t0)/N;
5 nx = size(x0,1);
6 X = zeros(nx,N+1);
7 T = zeros(1,N+1);
8
9 % Eulers Explicit Method
10 T(:,1) = t0;
11 X(:,1) = x0;
12 for k=1:N
13     f = feval(fun,T(k),X(:,k),varargin{:});
14     T(:,k+1) = T(:,k) + dt;
15     X(:,k+1) = X(:,k) + f*dt;
16 end
17
18 % Form a nice table for the result
19 T = T';
20 X = X';
```

# Exercises

- ▶ Implement Euler's method with fixed step-size
- ▶ Test it for various step size using the following system
  - ▶  $\dot{x}(t) = \lambda x(t), x(t_0) = 1$
  - ▶  $\dot{x}(t) = \cos(t)x(t), x(t_0) = 1$
  - ▶ The van der Pol problem
  - ▶ The prey-predator problem