Recursion (8 points)

1. [2 points] Using the master method in Section 4.5, you can show that the solution to the recurrence $T(n) = 4T(n/3) + n$ is $T(n) = \Theta\left(n^{\log_3 4}\right)$. Show that a substitution proof with the assumption $T(n) \leq cn^{\log_3 4}$ fails. Then show how to subtract off a lower-order term to make a substitution proof work.

2. [2 points] How would you modify Strassen's algorithm to multiply $n \times n$ matrices in which n is not an exact power of 2? Show that the resulting algorithm runs in time $\Theta\left(n^{\lg 7}\right)$. (Read the details about the Strassen's algorithm in Section 4.2. Hint: Pad out the matrix to an exact power of 2.)

3. Give asymptotic upper and lower bounds for $T(n)$ in each of the following recurrences. Assume that $T(n)$ is constant for $n \leq 2$. Make your bounds as tight as possible, and justify your answers.
   (a) [1 point] $T(n) = 2T(n/2) + n^4$

   (b) [1 point] $T(n) = T(7n/10) + n$

   (c) [1 point] $T(n) = 16T(n/4) + n^2$

   (d) [1 point] $T(n) = T(\sqrt{n}) + \Theta(\lg \lg n)$