

1. (20 pts) Given $F = \{ a \rightarrow b, b \rightarrow c, c \rightarrow \{d, e\} \}$. What is $\{b\}^+$ (i.e. the closure of b)? Show your steps to achieve the answer.

$$\begin{aligned}\{b\}^+ &= \{b\} \\ &= \{b, c\} = \{b, c, d, e\}\end{aligned}$$

2. (20 pts) Given $F = \{ a \rightarrow b, c \rightarrow d, b \rightarrow \{d, e\}, \{a, b\} \rightarrow c \}$. What is $\{a\}^+$ (i.e. the closure of a)? Show your steps to achieve the answer.

$$\begin{aligned}\{a\}^+ &= \{a\} \\ &= \{a, b\} \\ &= \{a, b, d, e\} \\ &= \{a, b, d, e, c\}\end{aligned}$$

3. (30 pts) Given $R(a, b, c, d, e)$ with two keys, (a, b) and c , and given the following set of functional dependencies $F = \{ \{a, b\} \rightarrow \{c, d, e\}, c \rightarrow \{a, b, d\} \}$.

1. Is R in 1NF? Justify your answer.

1. There is not enough information to determine it. There is no semantics given for the the relation. I have no idea if an attribute is a multi-value attribute or not. I assume it is in 1NF.

2. Is R in 2NF? Justify your answer.

1. Non-primary key attribute 'd' is fully functional dependent on $\{a, b\}$ or $\{c\}$. Non-primary key attribute 'e' is fully functional dependent on $\{a, b\}$ so R is in 2NF. Assuming R is already in 1NF.

3. Is R in 3NF? Justify your answer.

1. Every FD's left side is a superkey so this passes the test for 3NF, so R is in 3NF. Also, no non-prime attribute functionally assuming R is already in 1NF determines a non-prime attribute.

4. (30 pts) Given $R(a, b, c, d, e)$ with a key (a, b) and given the following set of functional dependencies $F = \{ a \rightarrow b, \{a, b\} \rightarrow c, b \rightarrow \{d, e\} \}$.
1. Is R in 1NF? Justify your answer.
 1. Given no information about the semantics of the columns. I will not be able to determine R is in 1NF or not. I will assume R is in 1NF.
 2. Is R in 2NF? Justify your answer.
 1. Non prime attribute d is have a partial functional dependency. It is being determined by part of a prime attribute. This violates the rule for 2NF. Attribute d also have this same problem. So R is not in 2NF.
 3. Is R in 3NF? Justify your answer.
 1. For the FD $a \rightarrow b$ a is not a superkey so violates the rule to be 3NF. Also in order to be in 3NF, R must be in 2NF first and R is not in 2NF so is not in 3NF as well.