1. Answer this question using UF_QuickFind

a) Show the contents of id[] after connecting the following pairs (0, 1), (0,2) (0,3) (0,4), assume S = {0, 1, 2, 3, 4}.

b) What is the total running time for connecting N-1 pairs like these: (0,1), (0, 2), … (0, N-1)?

Answer: a)

	0	1	2	3	4
Initial	0	1	2	3	4
union (0,1)	1	1	2	3	4
union (0, 2)	2	2	2	3	4
union (0, 3)	3	3	3	3	4
union (0, 4)	4	4	4	4	4

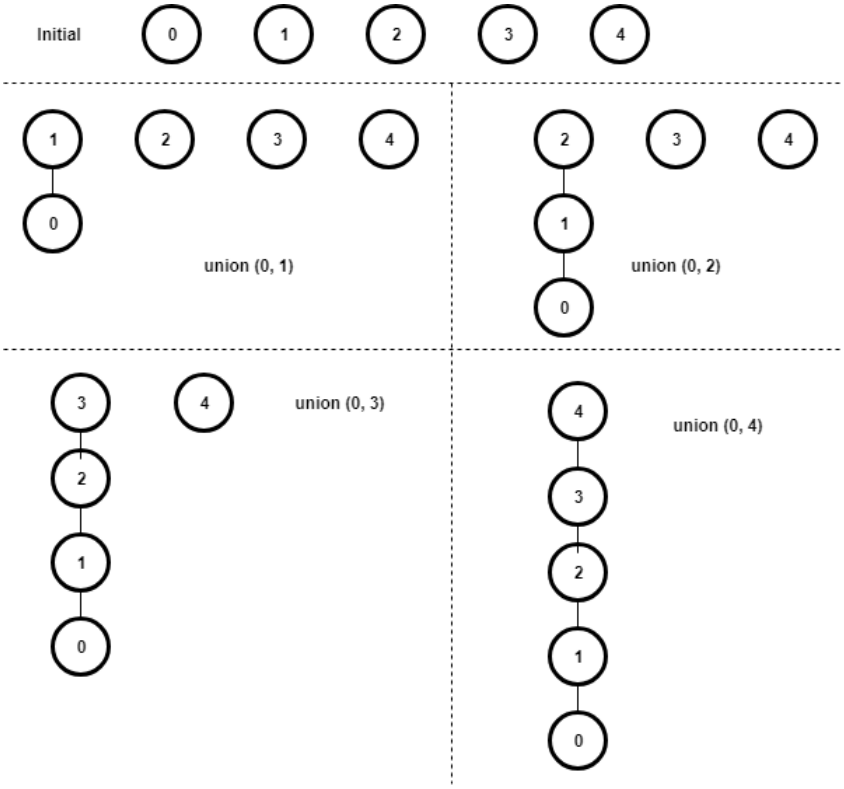
b) Each argument to the union like those comes from different component, thus union has O(n) running time. So (n-1) these kind of unions have (n-1)*O(n)=O(n^2) running time.

2. Answer this question using UF_QuickUnion

a) Show the contents of id[] after connecting the following pairs (0, 1), (0,2) (0,3) (0,4), assume S = {0, 1, 2, 3, 4}.

b) What is the total running time for connecting N-1 pairs like these: (0,1), (0, 2), … (0, N-1)?

Answer: a)



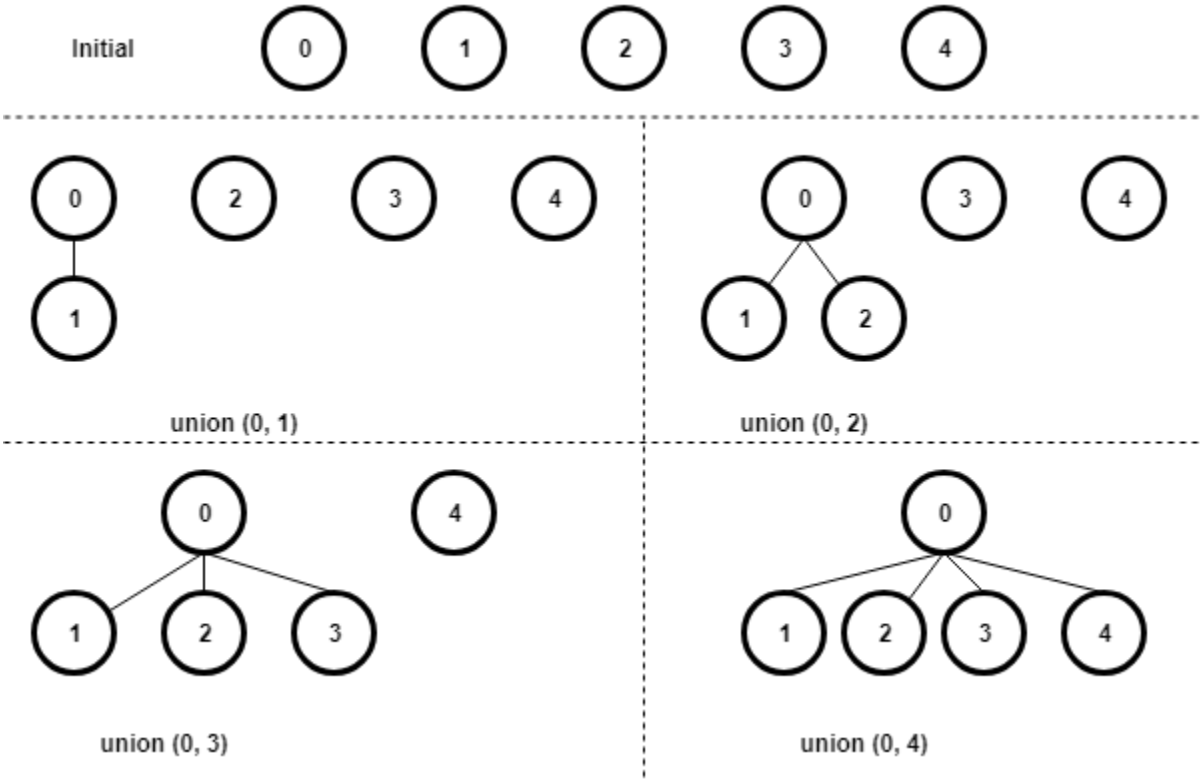
b) For each union (p, q) like those, the total number of comparisons for find(p) is 1+2+…+(n-2)=O(n^2), the total number for find(q) is n=O(n), thus the total running time for (n-1) unions is O(n^2).

3. Answer this question using UF_WeightedQuickUnion

a) Show the contents of id[] after connecting the following pairs (0, 1), (0,2) (0,3) (0,4), assume S = {0, 1, 2, 3, 4}.

b) What is the total running time for connecting N-1 pairs like these: (0,1), (0, 2), … (0, N-1)?

Answer: a)



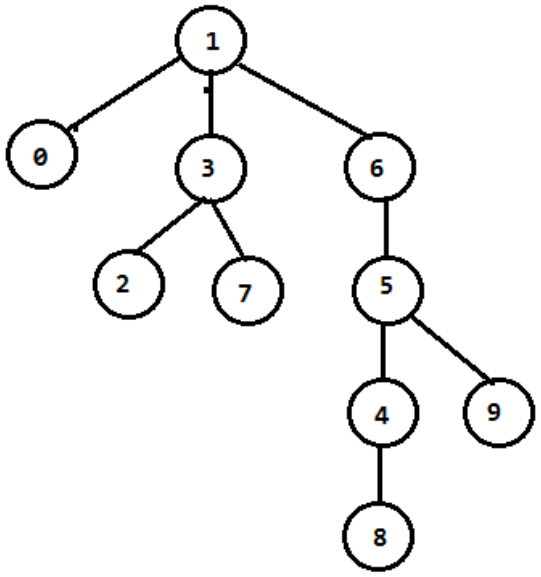
b) For each union (p, q) like those, find(p) and find(q) takes constant time. Thus (n-1) unions like those has O(n) total running time.

4. Draw the tree corresponding to the following id[] array. Can this be the result of running weighted quick-union? Explain why this is impossible or give a sequence of operations that result in this array.

i	0	1	2	3	4	5	6	7	8	9
id[i]	1	1	3	1	5	6	1	3	4	5

Answer:

The tree is shown below. It cannot be the result of running weighted quick-union because that algorithm guarantees the height of any tree built by it will be at most lgN. Here we have height=4, N=10, and height>lgN.



Note that graph in the following exercises refer to undirected graph.

5. Draw the adjacency matrix built by constructing the graph from input file depicted at Figure 1.

Answer: The following is the adjacency matrix representation. Note it cannot represent parallel edges between 3 and 10.

	0	1	2	3	4	5	6	7	8	9	10	11
0			T			T						
1					T			T			T	
2	T			T		T	T					
3			T			T				T		
4		T						T				
5			T							T		
6	T		T	T								
7								T			T	
8		T			T		T				T	
9												
10				T		T						
11		T					T	T				

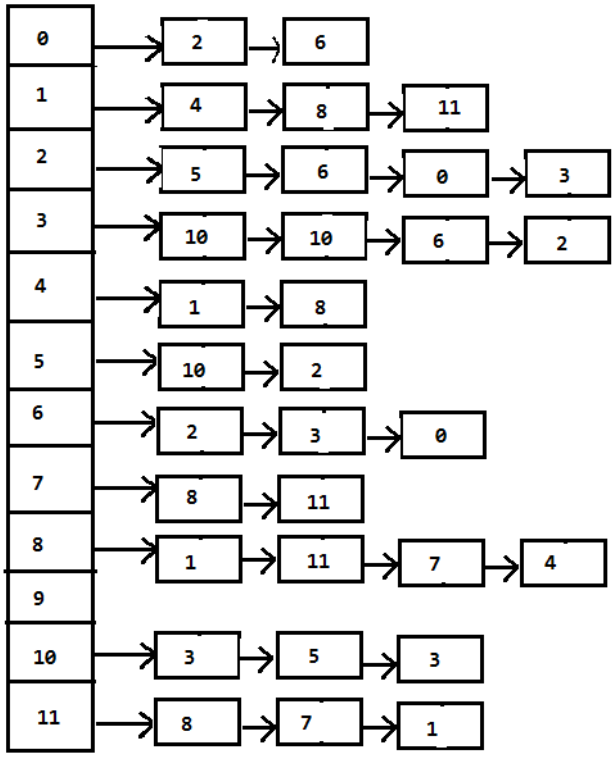
tinyGex2.txt

V → 12
16 ← E
8 4
2 3
1 11
0 6
3 6
10 3
7 11
7 8
11 8
2 0
6 2
5 2
5 10
3 10
8 1
4 1

Figure 1 Graph Input File

6. Draw the adjacency lists built by constructing the graph from input file depicted at Figure 1.

Answer: The following is the adjacency lists representation. Note it can represent parallel edges between 3 and 10.



7. What is the maximum number of edges in a graph with V vertices and no parallel or self-loop edges?
Answer: V*(V-1)/2

8. What is the minimum number of edges in a graph with V vertices, none of which are isolated?
Answer: V-1