Asymptotic Analysis (12 points)

1. Show that
   (a) [1 point] $5n^2 = O\left(n^2\right)$

   (b) [1 point] $n = O\left(n^2\right)$

   (c) [1 point] $5n^2 = \Omega\left(n^2\right)$

   (d) [1 point] $n \neq \Theta\left(5n^2\right)$

   (e) [1 point] $a^n = O\left(b^n\right), b > a > 1$

2. For each of the following statements, decide whether it is **always true**, **never true**, or **sometimes true** for asymptotically nonnegative functions f and g . If it is **always true** or **never true**, explain why. If it is **sometimes true**, give one example for which it is true, and one for which it is false.
   (a) [1 point] $f(n) = O\left(f(n)^2\right)$

   (b) [1 point] $f(n) + g(n) = \Theta(\max(f(n), g(n)))$

   (c) [1 point] $f(n) = \Omega(g(n))$ and $f(n) = o(g(n))$ (note the little-o notation: for any real constant $c > 0$, there exists an integer constant $n_0 \geq 1$ such that $0 \leq f(n) < c \cdot g(n)$)

3. [1 point] What is the smallest value of n such that an algorithm whose running time is $100n^2$ runs faster than an algorithm whose running time is 2^n on the same machine?

4. [1 point] Express the function $n^3/1000 - 100n^2 - 100n + 3$ in terms of Θ-notation.

5. [2 points] Is $2^{n+1} = O(2^n)$? Is $2^{2n} = O(2^n)$?