Gradualism

ECN 490

April 12, 2018

Lobby

$$\max_{m^t, l^t} \sum_{t=1}^{\infty} \beta^{t-1} \left\{ A(m_t) F^{\alpha} \cdot l_t^{1-\alpha} \cdot P^W - l_t - \mu_t \right\} \quad \text{s.t.} \quad m_t = m_{t-1} + \mu_t$$

Bellman Equation

$$V_l(m_t) = \max_{m_t, l_t} \left\{ A(m_t) F^{\alpha} \cdot (l_t)^{1-\alpha} \cdot P^W - l_t - (m_t - m_{t-1}) + \beta V_l(m_{t+1}) \right\}$$

Value Function

$$V_l(e_t^*, m_t^*) = \left\{ A(m_t^*) F^{\alpha} \cdot (l_t^*)^{1-\alpha} \cdot P^W - l_t^* - (m_t^* - m_{t-1}) + \beta V_l(m_{t+1}^*) \right\}$$

FOCs

$$\frac{\delta V_L}{\delta m_t^*} = \frac{\delta A}{\delta m_t^*} \cdot F^{\alpha} \cdot l_t^{1-\alpha} \cdot P^W + \beta [\frac{\delta V_L(m_{t+1^*})}{\delta (m_{t+1^*})} \cdot 1] = 1$$

$$\frac{\delta V_L}{\delta \mu_t} = \frac{d^A}{dM_t} \cdot \frac{dM_t}{d\mu_t} \cdot F^{\alpha} \cdot l_t^{1-\alpha} \cdot P^W + \beta [\frac{\delta V_L(m_{t+1})}{\delta (m_{t+1})} \cdot 1] = 1$$

$$\frac{\delta V_L}{\delta l_t} = (1-\alpha) A(m_{t-1} + \mu_t) \cdot F^{\alpha} \cdot 1^{-\alpha} \cdot P^W = 1$$

$$\frac{\frac{\delta A}{\delta m_t} \cdot l^t}{(1-\alpha) A(m_{t+1} + \mu_t)} + \beta \frac{\delta V_L(m_{t+1})}{\delta m_{t+1}} = 1$$

$$t = \text{this period}$$

$$t+1 = \text{next period}$$

$$\beta = \text{discount factor}$$

$$l = \text{lobby}$$

note: 1 comes from $\frac{\delta m_{t+1}}{\delta \mu}$