Gradualism

ECN 490

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1 Original Model with Government / Lobbying Effort

Lobby

$$\max_{e^t,m^t,l^t} \sum_{t=1}^{\infty} \beta^{t-1} \left\{ A(m_t) F^{\alpha} \cdot l_t^{1-\alpha} \left[P^W + \tau(\gamma(e_{t-1})) \right] - l_t - \mu_t - e_t \right\} \quad \text{s.t.} \quad m_t = m_{t-1} + \mu_t$$

Bellman Equation

$$V_l(e_t, m_t) = \max_{e_t,m_t,l_t} \left\{ A(m_t) F^{\alpha} \cdot (l_t)^{1-\alpha} \left[P^W + \tau(\gamma(e_{t-1})) \right] - l_t - \mu_t - e_t + \beta V_l(e_{t+1}, m_{m+t} \right\}$$

Value Function

$$V_l(e_t^*, m_t^*) = \left\{ A(m_t^*) F^{\alpha} \cdot (l_t^*)^{1-\alpha} \left[P^W + \tau(\gamma(e_{t-1}^*)) \right] - l_t^* - \mu_t^* - e_t^* + \beta V_l(e_{t+1}^*, m_{t+1}^*) \right\}$$

$t =$ this period

$t + 1 =$ next period

$\beta =$ discount factor

$l =$ lobby

2 No Government / Lobbying Effort

Lobby

$$\max_{m^t,l^t} \sum_{t=1}^{\infty} \beta^{t-1} \left\{ A(m_t) F^{\alpha} \cdot l_t^{1-\alpha} \cdot P^W - l_t - \mu_t \right\} \quad \text{s.t.} \quad m_t = m_{t-1} + \mu_t$$

Bellman Equation

$$V_l(m_t) = \max_{m_t,l_t} \left\{ A(m_t) F^{\alpha} \cdot (l_t)^{1-\alpha} \cdot P^W - l_t - (m_t - m_{t-1}) + \beta V_l(m_{t+1}) \right\}$$

Value Function

$$V_l(e_t^*, m_t^*) = \left\{A(m_t^*)F^\alpha \cdot (l_t^*)^{1-\alpha} \cdot P^W - l_t^* - (m_t^* - m_{t-1}) + \beta V_l(m_{t+1}^*)\right\}$$

FOCs

$$\frac{\delta V_L}{\delta m_t^*} = \frac{\delta A}{\delta m_t^*} \cdot F^\alpha \cdot l_t^{1-\alpha} \cdot P^W + \beta[\frac{\delta V_L(m_{t+1^*})}{\delta(m_{t+1^*})} \cdot 1] = 1$$

$$\frac{\delta V_L}{\delta \mu_t} = \frac{d^A}{dM_t} \cdot \frac{dM_t}{d\mu_t} \cdot F^\alpha \cdot l_t^{1-\alpha} \cdot P^W + \beta[\frac{\delta V_L(m_{t+1})}{\delta(m_{t+1})} \cdot 1] = 1$$

$$\frac{\delta V_L}{\delta l_t} = (1-\alpha)A(m_{t-1} + \mu_t) \cdot F^\alpha \cdot 1^{-\alpha} \cdot P^W = 1$$

$$\frac{\frac{\delta A}{\delta m_t} \cdot l^t}{(1-\alpha)A(m_{t+1} + \mu_t)} + \beta \frac{\delta V_L(m_{t+1})}{\delta m_{t+1}} = 1$$

$t =$ this period

$t+1 =$ next period

$\beta =$ discount factor

$l =$ lobby

note: 1 comes from $\frac{\delta m_{t+1}}{\delta \mu}$