Gradualism

ECN 490

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1 Original Model with Government / Lobbying Effort

Lobby

$$\max_{e^t, m^t, l^t} \sum_{t=1}^{\infty} \beta^{t-1} \left\{ A(m_t) F^{\alpha} \cdot l_t^{1-\alpha} \left[P^W + \tau(\gamma(e_{t-1})) \right] - l_t - \mu_t - e_t \right\} \quad \text{s.t.} \quad m_t = m_{t-1} + \mu_t$$

Bellman Equation

$$V_l(e_t, m_t) = \max_{e_t, m_t, l_t} \left\{ A(m_t) F^{\alpha} \cdot (l_t)^{1-\alpha} \left[P^W + \tau(\gamma(e_{t-1})) \right] - l_t - \mu_t - e_t + \beta V_l(e_{t+1}, m_{m+t} \right\}$$

Value Function

$$V_l(e_t^*, m_t^*) = \left\{ A(m_t^*) F^{\alpha} \cdot (l_t^*)^{1-\alpha} \left[P^W + \tau(\gamma(e_{t-1}^*)) \right] - l_t^* - \mu_t^* - e_t^* + \beta V_l(e_{t+1}^*, m_{t+1}^*) \right\}$$

$$t = \text{this period}$$

$$t + 1 = \text{next period}$$

$$\beta = \text{discount factor}$$

$$l = \text{lobby}$$

2 No Government / Lobbying Effort, Technology Depreciates

Lobby

$$\max_{m^t, l^t} \sum_{t=1}^{\infty} \beta^{t-1} \left\{ A(m_t) F^{\alpha} \cdot l_t^{1-\alpha} \cdot P^W - l_t - \mu_t \right\} \quad \text{s.t.} \quad m_{t+1} = (1-\delta) m_t + \mu_t$$

So $\mu_t = m_{t+1} - (1-\delta) m_t$

Bellman Equation

$$V_l(m_t) = \max_{m_t, l_t} \left\{ A(m_t) F^\alpha \cdot (l_t)^{1-\alpha} \cdot P^W - l_t - (m_{t+1} - (1-\delta) m_t) + \beta V_l(m_{t+1}) \right\}$$

In the optimal stage $m_t = m_t^* = m_{t+1}^*$

Value Function

$$V_l(m_{t+1}^*) = \left\{ A(m_{t+1}^*) F^\alpha \cdot (l_t^*)^{1-\alpha} \cdot P^W - l_t^* - (m_{t+1}^* - (1-\delta) m_{t+1}^*) + \beta V_l(m_{t+1}^*) \right\}$$

Which implies that

$$V_l(m_{t+1}^*) = \frac{1}{1-\beta} \left[A(m_{t+1}^*) \cdot F^\alpha \cdot (l_t^*)^{1-\alpha} \cdot P^W - l_t^* - \delta(m_{t+1}^*) \right]$$

FOCs

$$\frac{\partial V_l(m_{t+1}^*)}{\partial m_{t+1}^*} = \frac{1}{1-\beta} \cdot \left[\frac{\partial A(m_{t+1}^*)}{\partial m_{t+1}^*} \cdot F^\alpha \cdot l_t^{*1-\alpha} \cdot P^W - \delta \right]$$

$$\frac{\partial V_l}{\partial m_t} = \frac{\partial A(m_t^*)}{\partial m_t^*} \cdot F^\alpha \cdot l_t^{*1-\alpha} \cdot P^W + \frac{\beta}{1-\beta} \cdot \left[\frac{\partial A(m_{t+1}^*)}{\partial m_{t+1}^*} \cdot F^\alpha \cdot l_t^{*1-\alpha} \cdot P^W - \delta \right] \cdot (1-\delta) = \delta$$

$$\frac{\partial V_L}{\partial l_t} = (1-\alpha) A(m_t^*) \cdot F^\alpha \cdot l^{-\alpha} \cdot P^W = 1$$

$$\text{rearrange } l_t^{*1-\alpha} : (1-\delta) A(m_t^*) \cdot F^\alpha \cdot P^W = l_t^{*\alpha}$$

$$\text{square} \left(\frac{1-\alpha}{\alpha} \right) \text{on both sides} = \left[(1-\delta) A(m_t^*) \cdot F^\alpha \cdot P^W \right]^{\frac{1-\alpha}{\alpha}} = l_t^{*1-\alpha}$$

Euler Equation:

$$\begin{aligned} \frac{\partial A(m_t^*)}{\partial m_t^*} \cdot F^\alpha \cdot \left[(1-\delta) A(m_t^*) \cdot F^\alpha \cdot P^W \right]^{\frac{1-\alpha}{\alpha}} \cdot P^W + \frac{\beta}{1-\beta} \cdot (1-\delta) \\ \cdot \left[\frac{\partial A(m_{t+1}^*)}{\partial m_{t+1}^*} \cdot F^\alpha \cdot \left[(1-\delta) A(m_t^*) \cdot F^\alpha \cdot P^W \right]^{\frac{1-\alpha}{\alpha}} \cdot P^W - \delta \right] = \delta \end{aligned} \tag{1}$$

Simplified Euler Equation:

$$\text{Set}\left[\tfrac{\partial A(m_t^*)}{\partial m_t^*} \cdot F^\alpha \cdot P^W \right] = \Pi \text{ and Set}\left[(1-\delta) A(m_t^*) \cdot F^\alpha \cdot P^W \right]^{\frac{1-\alpha}{\alpha}} = Z$$

$$\text{Then } \Pi + Z + \tfrac{\beta}{1-\beta} \cdot (1-\delta) \cdot \Pi \cdot Z - \tfrac{\beta}{1-\beta} \cdot (1-\delta) \cdot \delta = \delta$$

$$\Pi \cdot Z \left[1 + \tfrac{\beta}{1-\beta} \cdot (1-\delta) \right] = \delta + \left[\tfrac{\beta}{1-\beta} \cdot (1-\delta) \right]$$

$$\Pi \cdot Z \left(\tfrac{1-\beta+\beta-\beta\delta}{1-\beta} \right) = \tfrac{\delta(1-\beta)}{1-\beta} + \tfrac{\beta\delta - \beta\delta^2}{1-\beta}$$

$$\Pi \cdot Z \left(\tfrac{1-\beta\delta}{1-\beta} \right) = \tfrac{\delta - \beta\delta^2}{1-\beta}$$

$$\Pi \cdot Z = \tfrac{\delta(1-\beta\delta)}{1-\beta} \cdot \tfrac{1-\beta}{1-\beta\delta}$$

$$\Pi \cdot Z = \delta$$

Change $\left[(1-\delta)A(m_t^*)\cdot F^{\alpha}\cdot P^{W}\right]^{\frac{1-\alpha}{\alpha}}$ to $\left[(1-\delta)^{\frac{1-\alpha}{\alpha}}\cdot A(m_t^*)^{\frac{1-\alpha}{\alpha}}\cdot F^{1-\alpha}\cdot P^{\frac{W-W\alpha}{\alpha}}\right]$

$$\frac{\partial A(m_t^*)}{\partial m_t^*}\cdot F\cdot P^{\frac{W}{\alpha}}\cdot(1-\delta)^{\frac{1-\alpha}{\alpha}}\cdot A(m_t^*)^{\frac{1-\alpha}{\alpha}}=\delta$$
$$\frac{\partial A(m_t^*)}{\partial m_t^*}\cdot A(m_t^*)^{\frac{1-\alpha}{\alpha}}=\frac{\delta}{(1-\delta)^{\frac{1-\alpha}{\alpha}}\cdot F\cdot P^{\frac{W}{\alpha}}}$$

t = this period

$t+1$ = next period

β = discount factor

δ = depreciation rate

l = lobby