

Gradualism

March 27, 2018

Lobby

$$\max_{m^t, l^t} \sum_{t=1}^{\infty} \beta^{t-1} \{A(m_t)F^\alpha \cdot l_t^{1-\alpha} \cdot P^W - l_t - \mu_t\} \quad \text{s.t.} \quad m_t = m_{t-1} + \mu_t$$

Bellman Equation

$$V_l(m_t) = \max_{m_t, l_t} \{A(m_t)F^\alpha \cdot (l_t)^{1-\alpha} \cdot P^W - l_t - (m_t - m_{t-1}) + \beta V_l(m_{t+1})\}$$

Value Function

$$V_l(e_t^*, m_t^*) = \{A(m_t^*)F^\alpha \cdot (l_t^*)^{1-\alpha} \cdot P^W - l_t^* - (m_t^* - m_{t-1}) + \beta V_l(m_{t+1}^*)\}$$

FOCs

$$\frac{\delta V_L}{\delta m_t^*} = \frac{\delta A}{\delta m_t^*} \cdot F^\alpha \cdot l_t^{1-\alpha} \cdot P^W + \beta \left[\frac{\delta V_L(m_{t+1}^*)}{\delta(m_{t+1}^*)} \cdot 1 \right] = 1$$

$$\frac{\delta V_L}{\delta \mu_t} = \frac{dA}{dM_t} \cdot \frac{dM_t}{d\mu_t} \cdot F^\alpha \cdot l_t^{1-\alpha} \cdot P^W + \beta \left[\frac{\delta V_L(m_{t+1})}{\delta(m_{t+1})} \cdot 1 \right] = 1$$

$$\frac{\delta V_L}{\delta l_t} = (1 - \alpha)A(m_{t-1} + \mu_t) \cdot F^\alpha \cdot 1^{-\alpha} \cdot P^W = 1$$

$$\frac{\frac{\delta A}{\delta m_t} \cdot l^t}{(1 - \alpha)A(m_{t+1} + \mu_t)} + \beta \frac{\delta V_L(m_{t+1})}{\delta m_{t+1}} = 1$$

$t = \text{this period}$

$t + 1 = \text{next period}$

$\beta = \text{discount factor}$

$l = \text{lobby}$

note: 1 comes from $\frac{\delta m_{t+1}}{\delta \mu}$