Gradualism

ECN 490

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1 Original Model with Government / Lobbying Effort

Lobby

$$\max_{e^t, m^t, l^t} \sum_{t=1}^{\infty} \beta^{t-1} \left\{ A(m_t) F^{\alpha} \cdot l_t^{1-\alpha} \left[P^W + \tau(\gamma(e_{t-1})) \right] - l_t - \mu_t - e_t \right\} \tag{1}$$
$$\text{s.t. } m_t = m_{t-1} + \mu_t$$

Bellman Equation

$$V_l(e_t, m_t) = \max_{e_t, m_t, l_t} \left\{ A(m_t) F^{\alpha} \cdot (l_t)^{1-\alpha} \left[P^W + \tau(\gamma(e_{t-1})) \right] - l_t - \mu_t - e_t + \beta V_l(e_{t+1}, m_{m+t}) \right\} \tag{2}$$

Value Function

$$V_l(e_t^*, m_t^*) = \left\{ A(m_t^*) F^{\alpha} \cdot (l_t^*)^{1-\alpha} \left[P^W + \tau(\gamma(e_{t-1}^*)) \right] - l_t^* - \mu_t^* - e_t^* + \beta V_l(e_{t+1}^*, m_{t+1}^*) \right\} \tag{3}$$

Where e_t is the choice of lobbing effort at time t, m_t the level of technology at time t, μ_t the level of investment in technology in period t, β_t the discount factor, $\tau(\gamma(e_{t-1}))$ represent the tarif level as a function of the lobbying effort in the previous period. We use a Cobb-Douglas for the production function of the firm the lobbyst is representing in which $A(m_t)$ determines the level of technology in the production process, F^{α} is the amount of the fixed factor, l_t the labor at time t and P^W is the world price.

2 No Government / Lobbying Effort, Technology Depreciates

Lobby

$$\max_{m^t, l^t} \sum_{t=1}^{\infty} \beta^{t-1} \left\{ A(m_t) F^{\alpha} \cdot l_t^{1-\alpha} \cdot P^W - l_t - \mu_t \right\} \tag{4}$$
$$\text{s.t.} \quad m_{t+1} = (1-\delta) m_t + \mu_t$$

By rearranging the law of motion equation and then replacing it in the Bellman Equation

$$\mu_t = m_{t+1} - (1-\delta) m_t \tag{5}$$

Bellman Equation

$$V_l(m_t) = \max_{m_t, l_t} \left\{ A(m_t) F^{\alpha} \cdot (l_t)^{1-\alpha} \cdot P^W - l_t - (m_{t+1} - (1-\delta) m_t) + \beta V_l(m_{t+1}) \right\} \tag{6}$$

At the Lobby's optimum, we have that

$$m_t = m_t^* = m_{t+1}^* \tag{7}$$

Value Function

$$V_l(m_{t+1}^*) = \left\{ A(m_{t+1}^*) F^{\alpha} \cdot (l_t^*)^{1-\alpha} \cdot P^W - l_t^* - (m_{t+1}^* - (1-\delta) m_{t+1}^*) + \beta V_l(m_{t+1}^*) \right\} \tag{8}$$

Which implies that

$$V_l(m_{t+1}^*) = \frac{1}{1-\beta} \left[A(m_{t+1}^*) \cdot F^{\alpha} \cdot (l_t^*)^{1-\alpha} \cdot P^W - l_t^* - \delta(m_{t+1}^*) \right] \tag{9}$$

By taking the First Order Conditions and reajusting terms

$$\frac{\partial V_l(m_{t+1}^*)}{\partial m_{t+1}^*} = \frac{1}{1-\beta} \cdot \left[\frac{\partial A(m_{t+1}^*)}{\partial m_{t+1}^*} \cdot F^{\alpha} \cdot l_t^{*1-\alpha} \cdot P^W - \delta \right] \tag{10}$$

$$\frac{\partial V_l}{\partial m_t} = \frac{\partial A(m_t^*)}{\partial m_t^*} \cdot F^{\alpha} \cdot l_t^{*1-\alpha} \cdot P^W + \frac{\beta}{1-\beta} \cdot \left[\frac{\partial A(m_{t+1}^*)}{\partial m_{t+1}^*} \cdot F^{\alpha} \cdot l_t^{*1-\alpha} \cdot P^W - \delta \right] \cdot (1-\delta) = \delta \tag{11}$$

$$\frac{\partial V_L}{\partial l_t} = (1-\alpha) A(m_t^*) \cdot F^{\alpha} \cdot l^{-\alpha} \cdot P^W = 1 \tag{12}$$

Rearrange $l_t^{*1-\alpha}$ as:

$$(1-\delta) A(m_t^*) \cdot F^{\alpha} \cdot P^W = l_t^{*\alpha} \tag{13}$$

Square to the power of $\frac{1-\alpha}{\alpha}$

$$\left[(1-\delta)A(m_t^*)\cdot F^\alpha \cdot P^W\right]^{\frac{1-\alpha}{\alpha}} = l_t^{*1-\alpha} \tag{14}$$

Euler Equation:

$$\begin{aligned}&\frac{\partial A(m_t^*)}{\partial m_t^*}\cdot F^\alpha \cdot \left[(1-\delta)A(m_t^*)\cdot F^\alpha \cdot P^W\right]^{\frac{1-\alpha}{\alpha}} \cdot P^W + \frac{\beta}{1-\beta}\cdot(1-\delta)\\ &\cdot\left[\frac{\partial A(m_{t+1}^*)}{\partial m_{t+1}^*}\cdot F^\alpha \cdot \left[(1-\delta)A(m_t^*)\cdot F^\alpha \cdot P^W\right]^{\frac{1-\alpha}{\alpha}} \cdot P^W - \delta\right] = \delta\end{aligned} \tag{15}$$

In order to simplify the analysis, we define the next expessions as:

$$\frac{\partial A(m_t^*)}{\partial m_t^*}\cdot F^\alpha \cdot P^W = \Pi \text{ and } \left[(1-\delta)A(m_t^*)\cdot F^\alpha \cdot P^W\right]^{\frac{1-\alpha}{\alpha}} = Z \tag{16}$$

Then we replace in the Euler Equation and rearrange terms

$$\Pi\ + Z + \frac{\beta}{1-\beta}\cdot(1-\delta)\cdot\Pi\cdot Z - \frac{\beta}{1-\beta}\cdot(1-\delta)\cdot\delta = \delta \tag{17}$$

$$\Pi\cdot Z\left[1+\frac{\beta}{1-\beta}\cdot(1-\delta)\right] = \delta + \left[\frac{\beta}{1-\beta}\cdot(1-\delta)\right] \tag{18}$$

$$\Pi\cdot Z\left(\frac{1-\beta+\beta-\beta\delta}{1-\beta}\right) = \frac{\delta(1-\beta)}{1-\beta} + \frac{\beta\delta-\beta\delta^2}{1-\beta} \tag{19}$$

$$\Pi\cdot Z\left(\frac{1-\beta\delta}{1-\beta}\right) = \frac{\delta-\beta\delta^2}{1-\beta} \tag{20}$$

$$\Pi\cdot Z = \frac{\delta(1-\beta\delta)}{1-\beta}\cdot\frac{1-\beta}{1-\beta\delta} \tag{21}$$

$$\Pi\cdot Z = \delta \tag{22}$$

Expand Z to $\left[(1-\delta)^{\frac{1-\alpha}{\alpha}}\cdot A(m_t^*)^{\frac{1-\alpha}{\alpha}}\cdot F^{1-\alpha}\cdot P^{W\frac{1-\alpha}{\alpha}}\right]$ and replace Z and Π in (22)

$$\frac{\partial A(m_t^*)}{\partial m_t^*}\cdot F\cdot P^{W\frac{1}{\alpha}}\cdot(1-\delta)^{\frac{1-\alpha}{\alpha}}\cdot A(m_t^*)^{\frac{1-\alpha}{\alpha}} = \delta \tag{23}$$

$$\frac{\partial A(m_t^*)}{\partial m_t^*} \cdot A(m_t^*)^{\frac{1-\alpha}{\alpha}} = \frac{\delta}{(1-\delta)^{\frac{1-\alpha}{\alpha}} \cdot F \cdot P^{W\frac{1}{\alpha}}} \tag{24}$$

δ= depreciation rate