Trajectory Module for Launcher Multidisciplinary Design, Analysis and Optimization

S2 Research Project Presentation

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Annafederica Urbano [2] Mathieu Balesdent [3] Loïc Brevault [3]

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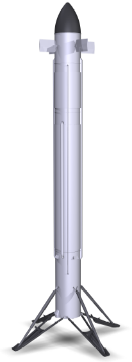
June 25, 2020

Contents of the presentation:

Context

- Launchers are highly optimized to reduce cost and improve performance



Callisto flight demonstrator. Credits[a]

[a] Sagliano et al. 2019.

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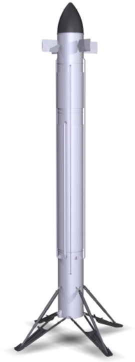
- Launchers are highly optimized to reduce cost and improve performance
- Reusability is changing the market, the architecture and the trajectories



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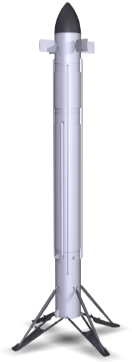
Context

- Launchers are highly optimized to reduce cost and improve performance
- Reusability is changing the market, the architecture and the trajectories
- Trajectory optimization is central for launch vehicle Multidisciplinary Design, Analysis and Optimization (MDAO)



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Context - Tools

Tools involved in the project

- LAST (Launcher Analysis and Sizing Tool)
- OpenMDAO
- DYMOS

Trajectory Module for Launcher MDAO

J. Valderrama

Context - Tools

Tools involved in the project

- LAST (Launcher Analysis and Sizing Tool)
- OpenMDAO
- DYMOS

Some reference tools

- FELIN (Framework for Evolutive Launcher optImizatioN)
- Dr. Brevault's Dymos code
- LATOM (LAuncher Trajectory Optimization Module)

Aims and objectives

Aim

To develop and integrate an efficient and accurate 3D trajectory optimization module for a launcher into the LAST tool.

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To develop and integrate an efficient and accurate 3D trajectory optimization module for a launcher into the LAST tool.

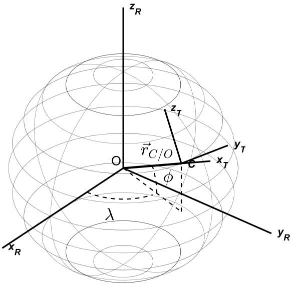
Objectives

- To implement trajectories of several level of complexity , including 2D and 3D
- To include different command laws for expendable and reusable launchers under different stage recovery scenarios
- To provide the analytic derivatives required for the MDAO for each case

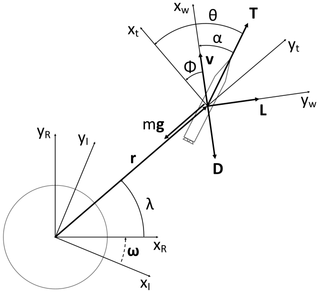
Contents of section

Equations of motion

- 3 DOF
- Cartesian and Spherical coordinates
- 3D and 2D



3D Spherical coordinates

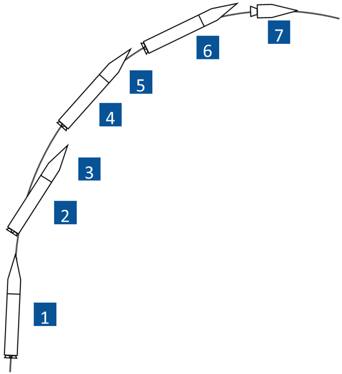


2D Polar coordinates

Guidance laws

Division of the trajectory into phases[1,2]

Control of pitch angle



Control of yaw angle

[1] Pagano and Mooij 2010.

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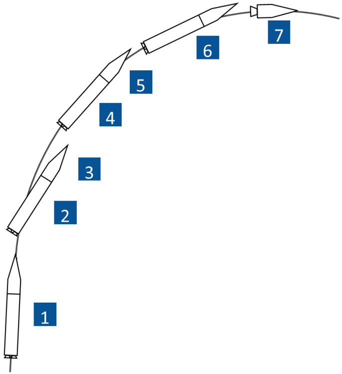

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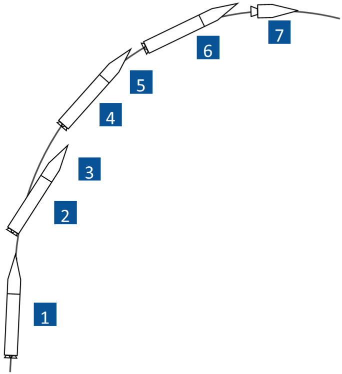

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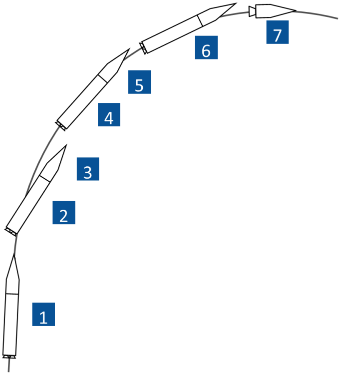

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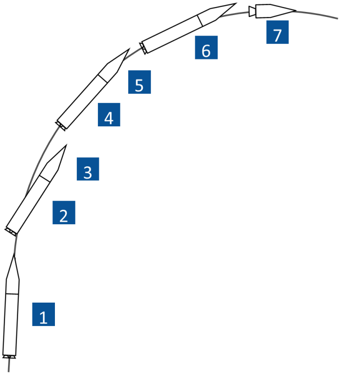

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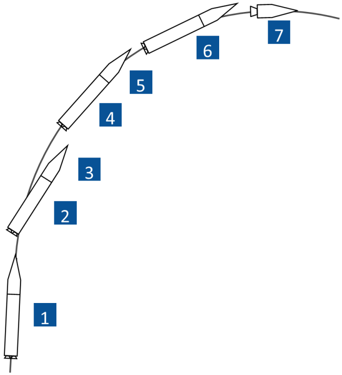

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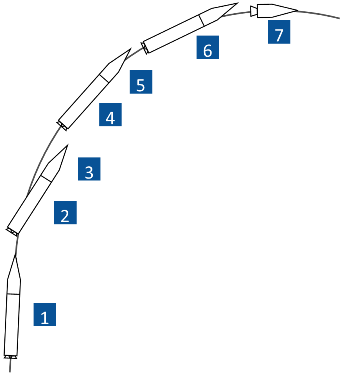
6. Coast phases



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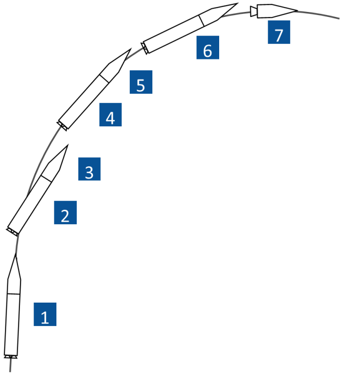
6. Coast phases
7. Stage burns

Control of yaw angle



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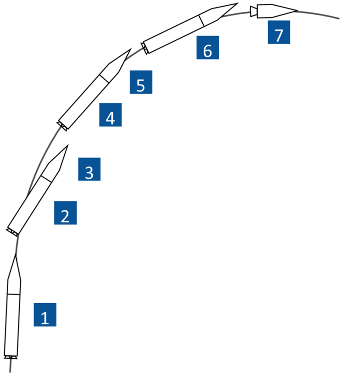
Control of yaw angle

- Target inclination law



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Optimal control

Trajectory Module for Launcher MDAO

J. Valderrama

- Trajectory optimization as a subset of optimal control
- Classification of main optimal control methods

Direct **Indirect**

Optimal control

- Trajectory optimization as a subset of optimal control
- Classification of main optimal control methods

Direct **Indirect**

Advantages
Low sensitivity to initial guess

Disadvantages
High dimensional problem

Optimal control

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Direct	**Indirect**
Advantages	Advantages
Low sensitivity to initial guess	High accuracy
Disadvantages	Disadvantages
High dimensional problem	High sensitivity to initial guess

Optimal control

- Trajectory optimization as a subset of optimal control
- Classification of main optimal control methods

	Direct	**Indirect**
Advantages	Low sensitivity to initial guess	High accuracy
Disadvantages	High dimensional problem	High sensitivity to initial guess

Justification for choice of Direct methods

In MDAO of launch vehicles trajectories vary significantly

Optimal Control

Pseudospectral methods

- Legendre-Gauss-Lobatto[3]
- Legendre Gauss-Radau[4]

Characteristics

- Direct methods validated with indirect methods. High accuracy.
- Low sensitivity to initial guess
- Good parallelization

[3]Herman and Conway 1996.
[4]Garg et al. 2012.

- Disciplines involved in launcher MDAO: Trajectory, Aerodynamics, structures...
- Provide partials derivatives of outputs w.r.t. Inputs. Compute total derivatives

Optimal control in the context of MDAO:

[5] Falck and Gray 2019.

- Disciplines involved in launcher MDAO: Trajectory, Aerodynamics, structures...
- Provide partials derivatives of outputs w.r.t. Inputs. Compute total derivatives

Optimal control in the context of MDAO:

- The Dymos tool[5]

[5] Falck and Gray 2019.

Contents of section

Research project Plan

What I have done

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1. 2D trajectory optimization with shooting method (Reference method)

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Next steps

Research project Plan

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Next steps

3. **MDAO of launch vehicle with pseudospectral method**

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3. **MDAO of launch vehicle with pseudospectral method**
4. Extension to 3D trajectory optimization and recovery scenarios

Research project Plan

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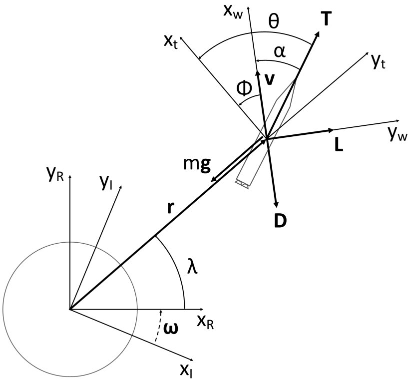
3. **MDAO of launch vehicle with pseudospectral method**
4. Extension to 3D trajectory optimization and recovery scenarios
5. Integration with LAST

Equations of motion in 2D



Results expressed in $F_R := \{x_R, y_R, z_R\}$, **Rotating Earth Centered reference frame**

State vector

$$\bar{x} = \begin{bmatrix} r \\ \lambda \\ v \\ \phi \\ m \end{bmatrix}$$

r	Radius
λ	Longitude angle
v	Speed
ϕ	Flight path angle
m	Mass

Force models

- Gravity model: Newton's law of universal gravitation

[6] Tewari 2007.

Force models

- Gravity model: Newton's law of universal gravitation
- Atmospheric models:

[6] Tewari 2007.

Force models

- Gravity model: Newton's law of universal gravitation
- Atmospheric models:
  - Atmospheric model based on the 1976 and 1962 U.S. Standard Atmospheres[6] for shooting method

[6] Tewari 2007.

Force models

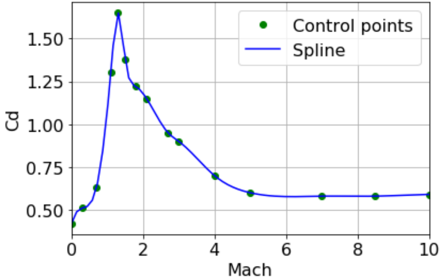
- Gravity model: Newton's law of universal gravitation
- Atmospheric models:
  - Atmospheric model based on the 1976 and 1962 U.S. Standard Atmospheres[6] for shooting method
  - Exponential atmosphere for pseudospectral method

[6] Tewari 2007.

Force models

- Aerodynamic model: $C_l = 0$, $C_d = f(M)$

  Cubic spline interpolation for Ariane 5[7]



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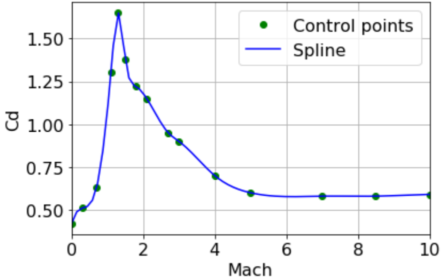


- Propulsion model: Losses as a function of atmospheric pressure. No throttle control

[7] Pagano 2010.

Optimal control formulation - Specific case study

Minimize Cost function: Mass of consumed propellants

$$J = m(t_0) - m(t_f)$$

With B.C.

$$\bar{x}(t_0) \text{ and } \bar{x}(t_f)$$

and dynamics

$$\dot{\bar{x}} = f_{ode}(\bar{x}, t, \bar{u}, \bar{d})$$

Where the state vector ($\bar{x}$) and the design parameter vector ($\bar{d}$) are

$$\bar{x} = \begin{bmatrix} r \\ \lambda \\ v \\ \phi \\ m \end{bmatrix} \quad \bar{d} = \begin{bmatrix} h_{lo} \\ \Delta\theta_{lpo} \\ \Delta t_{lpo} \\ \Delta t_{edp} \\ \xi \\ \Delta\theta_{btl} \\ \theta_{btl_f} \\ m_p \end{bmatrix}$$

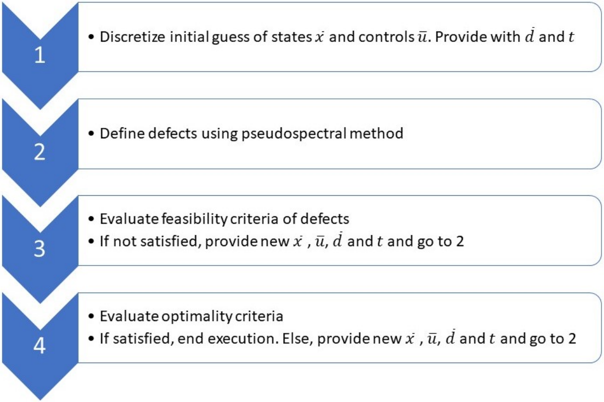
Direct single-shooting method

Convert the two-point boundary value problem

1.
   - Initial guess of design parameters vector: $\bar{d}$
2.
   - Propagate dynamics $\dot{\bar{x}}$, from $\bar{x}(t_0)$ with $\bar{d}$
3.
   - Evaluate feasibility criteria of the final state according to $\bar{x}(t_f)$
   - If not satisfied, provide new $\bar{d}$ and go to 2
4.
   - Evaluate optimality criteria
   - If satisfied, end execution. Else, provide new $\bar{d}$ and go to 2

- Step 2: Runge Kutta 5(4) numerical integration method
- Steps 3 and 4: COBYLA gradient-free optimizer

Pseudospectral method



- Step 2: Legendre-Gauss-Lobatto (Order 3 and 48 segments)
- Step 3 and 4: SLSQP (Analytic Jacobian information)

Pseudospectral method

Legendre-Gauss-Lobatto transcription (LGL)

- Discretize time into segments, discretize segments with LGL nodes.
  State discretization nodes (even LGL nodes)
  Collocation nodes (odd LGL nodes)

Pseudospectral method

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- Evaluate ODE at state discretization nodes to fit an Hermite polynomial

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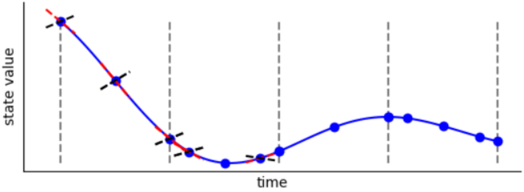
- Evaluate ODE at state discretization nodes to fit an Hermite polynomial
- Calculate state rate at collocation nodes using the Hermite polynomial

Credits[8]



[8] *Dymos: Open-Source Optimal Control for Multidisciplinary Systems — Dymos 0.15.0 Documentation.*

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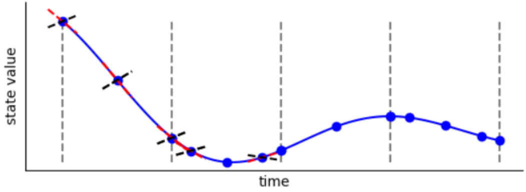

- Calculate state rate at collocation nodes by evaluating the ODE

[8] *Dymos: Open-Source Optimal Control for Multidisciplinary Systems — Dymos 0.15.0 Documentation*.

Pseudospectral method

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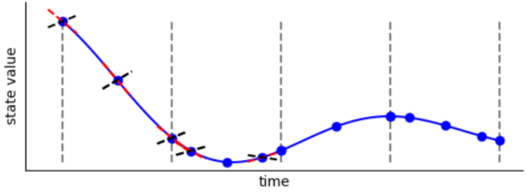
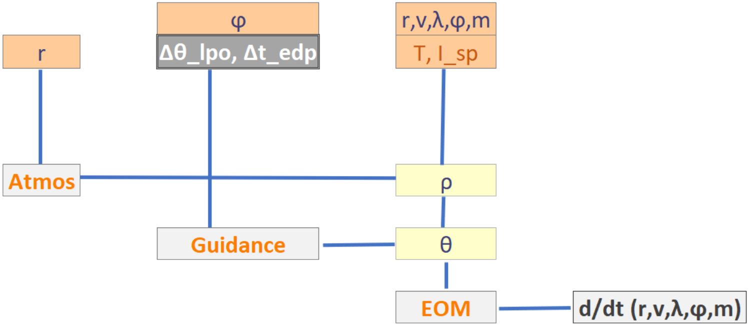


- Calculate state rate at collocation nodes by evaluating the ODE
- The defect is the error

[8] *Dymos: Open-Source Optimal Control for Multidisciplinary Systems — Dymos 0.15.0 Documentation*.

Pseudospectral method - N2 diagram



N2 diagram - Exponential decay of pitch phase

Simplified models of gravity, drag coefficient , exponential atmosphere

Contents of section

Case study of single-stage-to-orbit vehicle

Input parameter	Value	Units	Description
m_e	35	ton	Empty mass
μ	1990	kg / s	Mass flow rate
I_{sp}	400	s	Specific impulse
S	37.5	m^2	Reference area
A_e	10	m^2	Nozzle expansion area

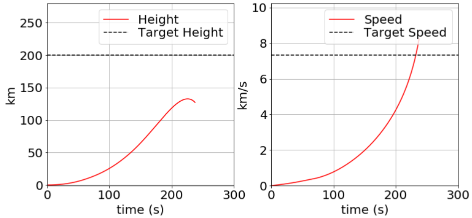
Vehicle parameters

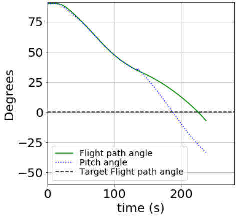
Case study of single-stage-to-orbit vehicle

Final B.C.	**Value**	**Tolerance**	**Units**
$r(t_f)$	$6378.135 + 200$	2	km
$v(t_f)$	7319.16	500	m/s
$\phi(t_f)$	0	15	deg

Final boundary conditions for shooting method

Direct single-shooting method - Initial guess

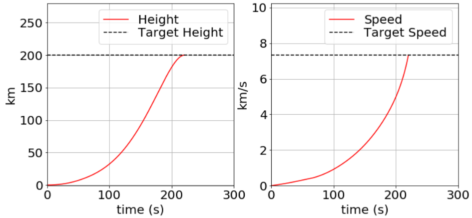


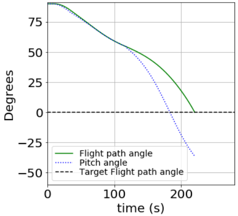


Cost function :472.57

Iteration :1

Direct single-shooting method - Opt. results

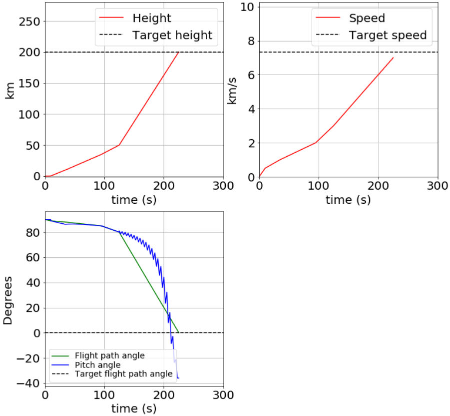




Cost function :438.37

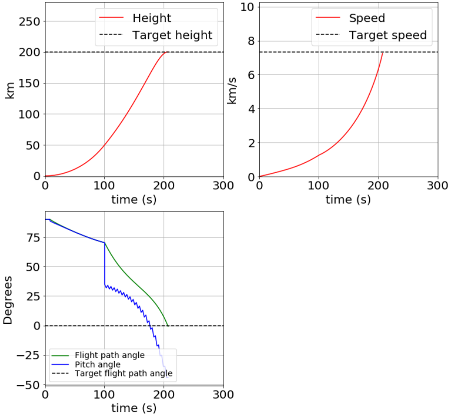
Iteration : 275

Pseudospectral method - Initial guess



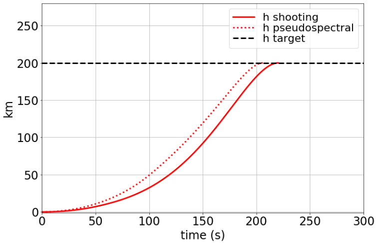
- Initial guess of $\bar{d}$ is the output of the shooting method
- Guess of states for different phases with linear interpolation

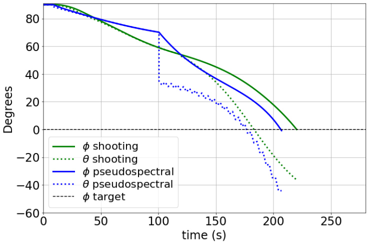
Pseudospectral method - Opt. results



- Duration of phase taking a null value
- Time management may be causing a bad definition of Jacobian matrix

Comparison of methods





- h = height
- ϕ = flight path angle
- θ = pitch angle

Contents of section

Conclusions and perspective

Conclusions

1. Implementation of 2D trajectory optimization with shooting method (Reference)

Conclusions and perspective

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2. Partial implementation of 2D trajectory optimization with pseudospectral method

Conclusions and perspective

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Perspective

1. Finish 3
2. MDAO of launch vehicle with pseudospectral method
3. Extension to 3D trajectory optimization and recovery scenarios
4. Integration with LAST

Betts, John T. (Jan. 2010). *Practical Methods for Optimal Control and Estimation Using Nonlinear Programming*. Advances in Design and Control. Society for Industrial and Applied Mathematics. ISBN: 978-0-89871-688-7. DOI: 10.1137/1.9780898718577.

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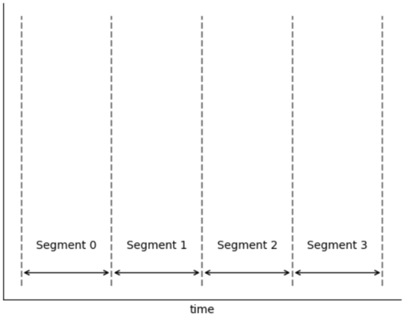
Questions

Legendre-Gauss-Lobatto transcription I

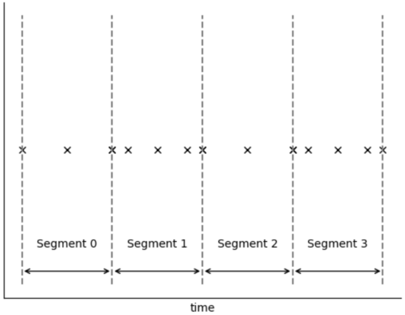
The LGL transcription as described in Dymos documentation[9]

1. Phase segmentation: Discretize time into segments



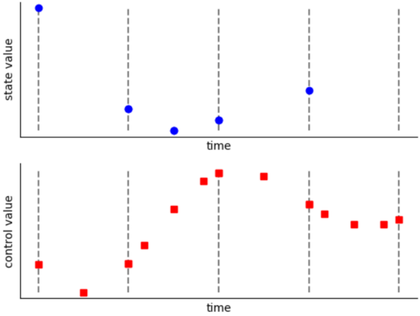
Legendre-Gauss-Lobatto transcription II

2. Discretization: Each segment is discretized by using the LGL nodes that depend on the order chosen for the method.



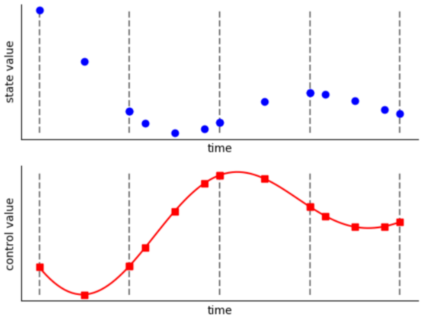
Legendre-Gauss-Lobatto transcription III

3. Input: Initial guess for states at discretization nodes (Even LGL nodes). Initial guess for control at all nodes.

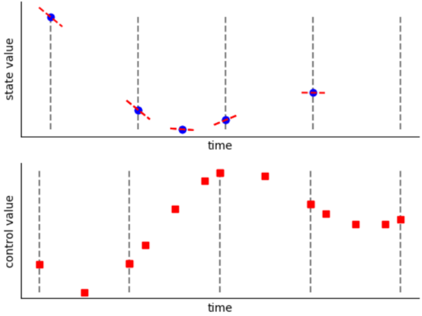


Legendre-Gauss-Lobatto transcription IV

4. Control rate interpolation: Lagrange interpolating polynomial. This allows to provide derivative values of the dynamic controls as inputs for the ODE

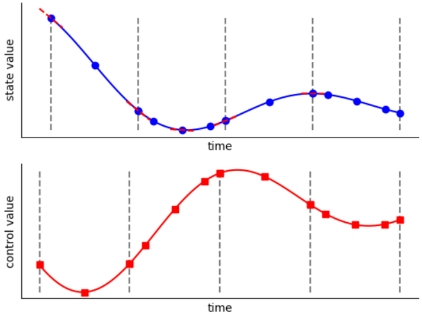


5. Evaluation of the ODE at the state discretization nodes:



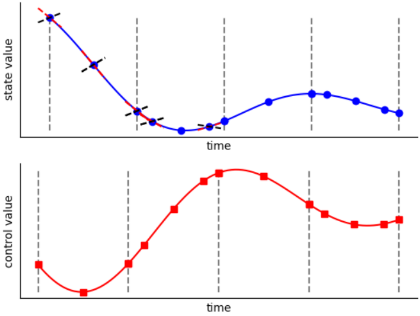
Legendre-Gauss-Lobatto transcription VI

6. State interpolation: Hermite polynomial. Calculate rate at collocation nodes (odd LGL nodes)



Legendre-Gauss-Lobatto transcription VII

7. Evaluation of the ODE at the collocation nodes:



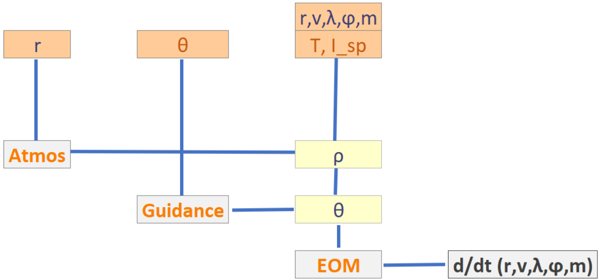
Legendre-Gauss-Lobatto transcription VIII

8. Evaluation of the collocation defects: Compare 6 and 7.
9. Iterate: Steps 3 to 8 are repeated until the solution reaches feasibility and optimality criteria.

[9] *Dymos: Open-Source Optimal Control for Multidisciplinary Systems — Dymos 0.15.0 Documentation*.

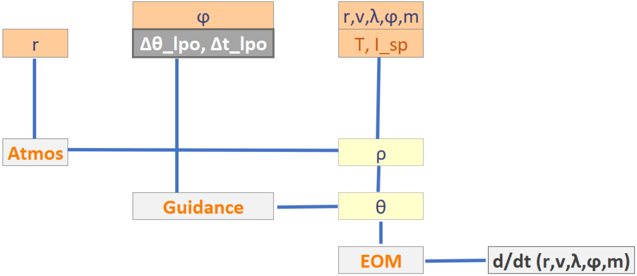
N2 diagrams I

- Lift-off:



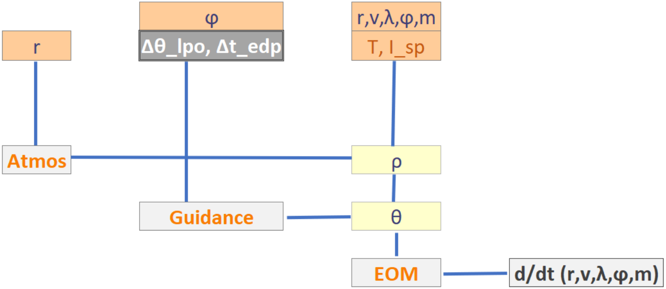
N2 diagrams II

- Linear pitch over:



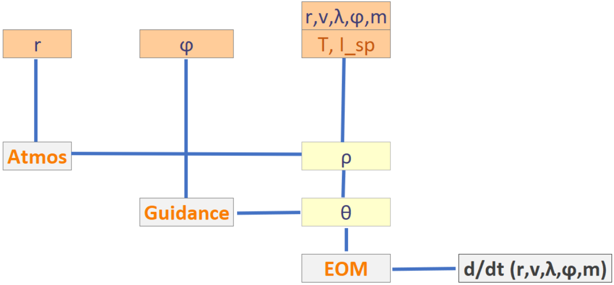
N2 diagrams III

- Exponential decay of pitch:



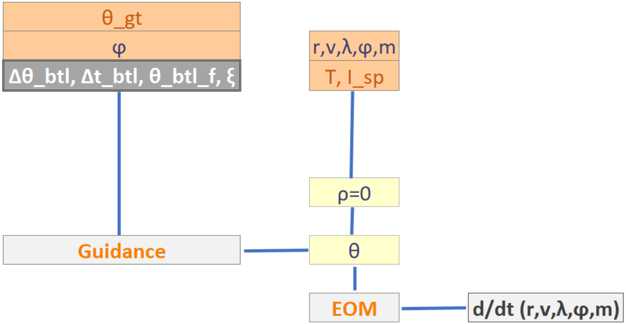
N2 diagrams IV

- Gravity turn:



N2 diagrams V

- Bilinear tangent law:



NLP solvers

- SNOPT
- iPOPT
- NPSOL

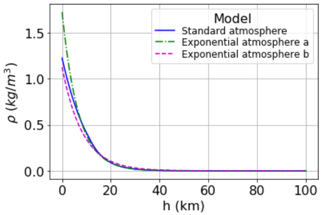
Force models

- Gravity: Axisymmetric planet model including up to the fourth Jeffery constant (J_4)[10][11]
- Atmosphere: *U.S Standard atmosphere 1976*, exponential atmosphere
- Aerodynamics: Drag coefficient data (C_d) data must be C_2 continuous for non-linear programming (NLP)[12]

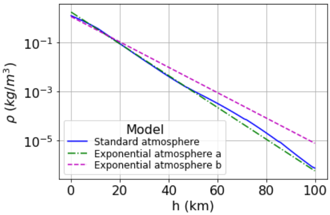
[10] Castellini 2012.
[11] Tewari 2007.
[12] Betts 2010.

Atmospheric models

- Atmospheric model based on the 1976 and 1962 U.S. Standard Atmospheres[13]
- Exponential atmosphere based reference height and reference pressure. Model ”a” is from Tewari and model ”b” from Falck.



Atmospheric models - linear



Atmospheric models - log

[13] Tewari 2007.

Thrust model

$$T = T_{vac} - A_e p_a$$

$$T_{vac} = g_0 I_{sp} \mu$$

Optimal control formulation

Minimize

$$\mathbf{J} = f_{obj}(\bar{x}, t, \bar{u}, \bar{d})$$

Subjected to the dynamics

$$\dot{\bar{x}} = f_{ode}(\bar{x}, t, \bar{u}, \bar{d})$$

Bounded as

$$\begin{aligned} \text{Time} &: t_{lb} \leq t \leq t_{ub} \\ \text{State Variables} &: \bar{x}_{lb} \leq \bar{x} \leq \bar{x}_{ub} \\ \text{Dynamic Controls} &: \bar{u}_{lb} \leq \bar{u} \leq \bar{u}_{ub} \\ \text{Design Parameters} &: \bar{d}_{lb} \leq \bar{d} \leq \bar{d}_{ub} \end{aligned}$$

Optimal control formulation

Also bounded by the non-linear constraints

Initial Boundary Constraints : $\bar{g}_{0,lb} \leq g_0\left(\bar{x}_0, t_0, \bar{u}_0, \bar{d}\right) \leq \bar{g}_{0,ub}$

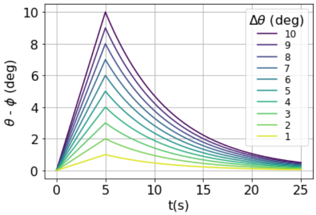
Final Boundary Constraints : $\bar{g}_{f,lb} \leq g_f\left(\bar{x}_f, t_f, \bar{u}_f, \bar{d}\right) \leq \bar{g}_{f,ub}$

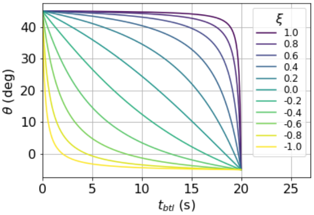
Path Constraints : $\bar{p}_{f,lb} \leq p_f(\bar{x}, t, \bar{u}, \bar{d}) \leq \bar{p}_{f,ub}$

Guidance laws

Control of pitch angle θ for a Single-Stage to Orbit launcher.
Definition of phases

1. Lift-off: $\theta = 90°$ (h_{lo})
2. Linear pitch over ($\Delta\theta_{lpo}$, Δt_{lpo})
3. Exponential decay of pitch (Δt_{edp})
4. Gravity turn: $\theta = \phi$
5. Bilinear tangent law (ξ, $\Delta\theta_{gt}$, θ_{btl_f})

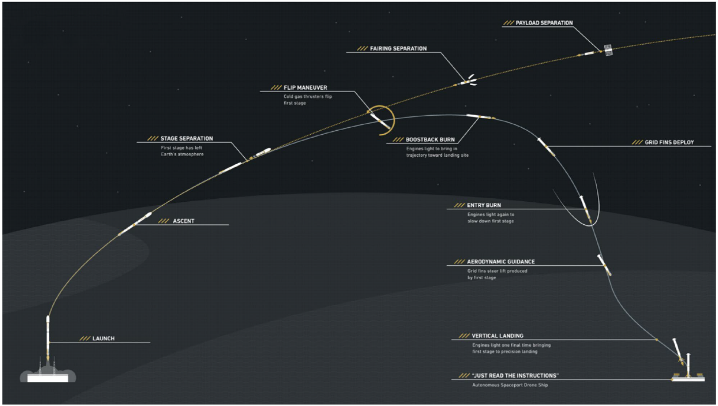




Guidance laws

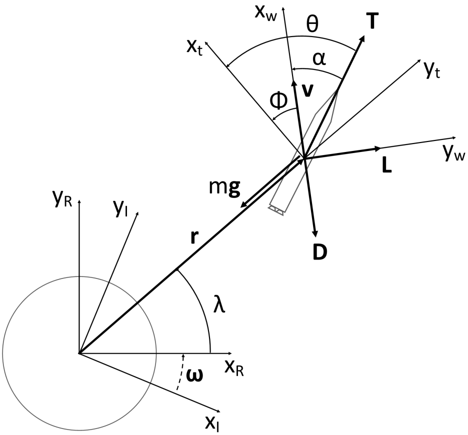
Recovery scenarios[14]

- Return to launch site
- Downrange landing (Image below. Credits[15])



[14] Sagliano et al. 2019.
[15] SpaceX 2020.

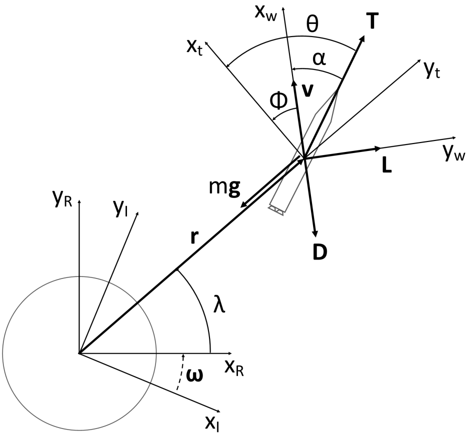
Equations of motion in 2D



Reference frames

- $\mathcal{F}_R := \{x_R, y_R, z_R\}$, **Rotating Earth Centered reference frame**
- $\mathcal{F}_I := \{x_I, y_I, z_I\}$, Inertial Earth Centered reference frame
- $\mathcal{F}_t := \{x_t, y_t, z_t\}$, Local Vertical - Local Horizontal reference frame
- $\mathcal{F}_w := \{x_w, y_w, z_w\}$, Wind reference frame

Equations of motion in 2D



Variables

θ	**Pitch angle**
ϕ	**Flight path angle**
λ	**Longitude angle**
r	**Radius**
v	**Velocity**
m	**Mass**
g	Gravity force
D	Drag force
L	Lift force
T	Thrust
ω	Earth's angular speed
α	Angle of attack

Equations of motion in 2D

$$\dot{r} = v \sin(\phi)$$

$$\dot{\lambda} = \frac{v \cos(\phi)}{r}$$

$$\dot{v} = \frac{-D + T \cos(\theta - \phi)}{m} + \left(-g + \omega^2 r\right) \sin(\phi)$$

$$\dot{\phi} = \frac{L}{mv} + \frac{T \sin(\theta - \phi)}{mv} + \frac{(\omega^2 r - g)\cos(\phi)}{v} + 2\omega + \frac{v \cos(\phi)}{r}$$

θ	**Pitch angle**	g	Gravity force
ϕ	**Flight path angle**	D	Drag force
λ	**Longitude angle**	L	Lift force
r	**Radius**	T	Thrust
v	**speed**	ω	Earth's angular speed
m	**Mass**	α	Angle of attack

Direct single-shooting - Reference method

Initial B.C.	Value	Units
$r(t_0)$	6378.135	km
$\lambda(t_0)$	0	deg
$v(t_0)$	1	m/s
$\phi(t_0)$	90	deg
$m(t_0)$	$m_e + m_p = 507.6$	ton

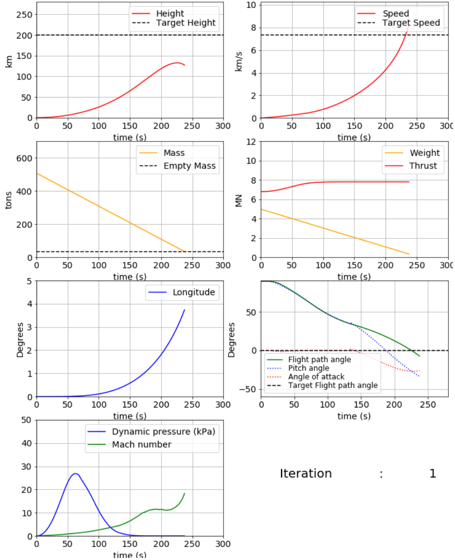
Initial boundary conditions

Direct single-shooting - Reference method

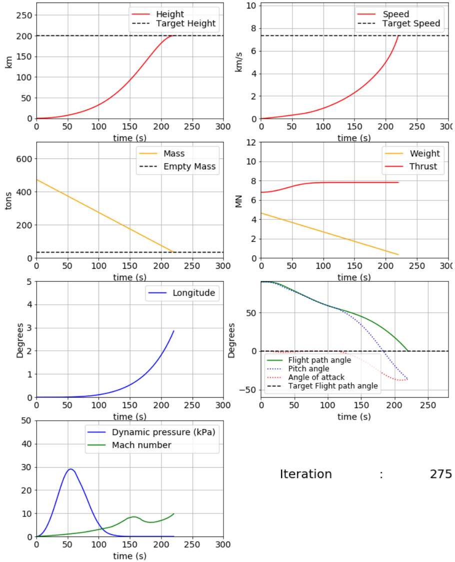
Design parameter	Value	lb	up	Units	Description
h_{lo}	222.45	150	300	m	Height end of lift-off
$\Delta\theta_{lpo}$	1.47	0.1	8	deg	Change in pitch angle
Δt_{lpo}	20.32	1	40	s	Phase duration
Δt_{edp}	18.4	1	25	s	Phase duration
ξ	-0.01	-1	1		Shape coefficient btl
$\Delta\theta_{btl}$	-1.66	-60	60	deg	Change in pitch angle
θ_{btl_f}	-33.56	-50	60	deg	Final pitch angle
m_p	472.6	370	800	ton	Mass of propellants

Initial guess - Design parameters for shooting method

Direct single-shooting method - Initial guess



Direct single-shooting method - Opt. results



Initial guess - Design parameters for pseudospectral method

Design parameter	**Value**	**lb**	**up**	**Units**	**Description**
h_{lo}	212.8	150	300	m	Height end of lift-off
$\Delta\theta_{lpo}$	1.77	0.1	8	deg	Change in pitch angle
Δt_{lpo}	25.33	1	40	s	Phase duration
Δt_{edp}	20.0	1	80	s	Phase duration
ξ	-0.01	-1	1		Shape coefficient btl
$\Delta\theta_{btl}$	0.9	-60	60	deg	Change in pitch angle
θ_{btl_f}	-36.4	-50	60	deg	Final pitch angle
$\mathbf{m_p}$	**438.4**	$-\infty$	∞	**ton**	**Mass of propellants**

Initial guess - Design parameters for pseudospectral method

These were taken from the optimization results of the shooting method