# All-At-Once MDO formulation for coupled optimization of launch vehicle design and its trajectory using a pseudo spectral method

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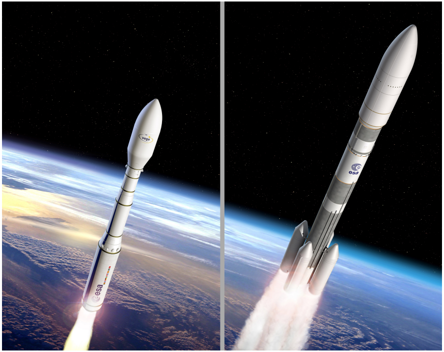
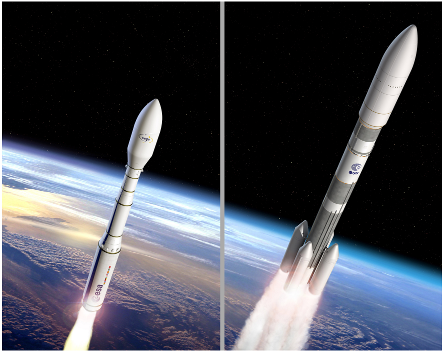
# Plan




## Launch vehicle system

- Complex system with multiple disciplines
- Coupled interactions between propulsion, structures, trajectory...

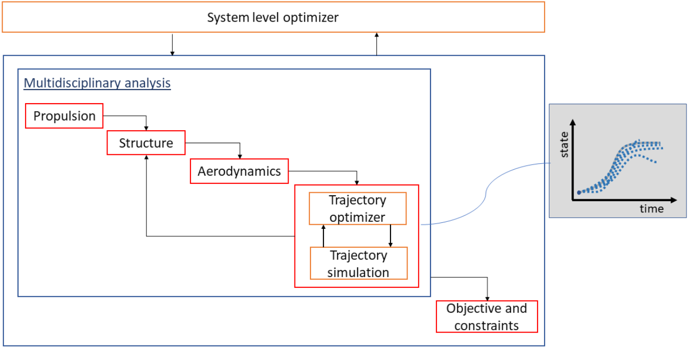
Credits: Huart and Ducros 2015




**Launch vehicle mission**

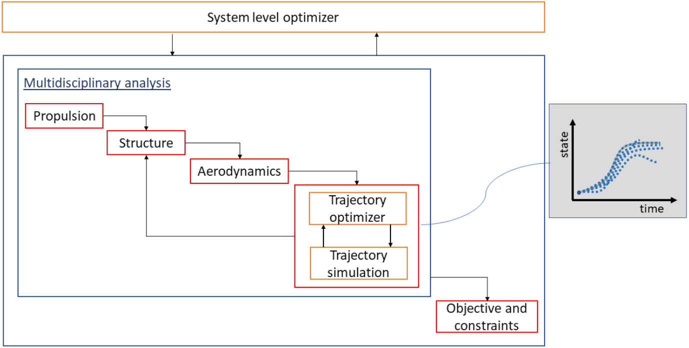
- Inject a payload into a determined orbit

Credits: Huart and Ducros 2015




## Multidisciplinary Design, Analysis and Optimization (MDAO)

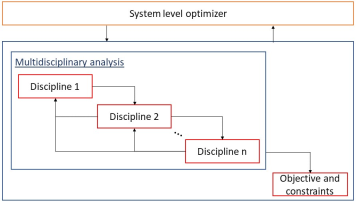
- Early design phase of launch vehicles
- Increase performance, minimize cost

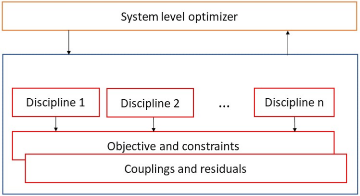



## A traditional approach

- Trajectory simulation nested into 3 loops
- High computational cost

# MDAO formulations




## MultiDiscipline Feasible (MDF)[1]

- MultiDisciplinary Analysis (MDA)
- Multidisciplinary feasibility at MDA convergence
- A few optimization variables

## All-At-Once (AAO)

- No MDA
- Multidisciplinary feasibility at optimizer convergence
- Higher dimensional optimization

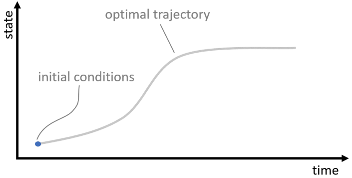
[1] Balesdent et al. 2012.

## Examples of disciplines

Trajectory, Structures, Propulsion, Aerodynamics

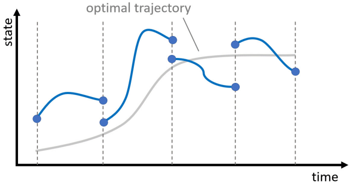
**The trajectory is a key discipline.**

Direct single shooting

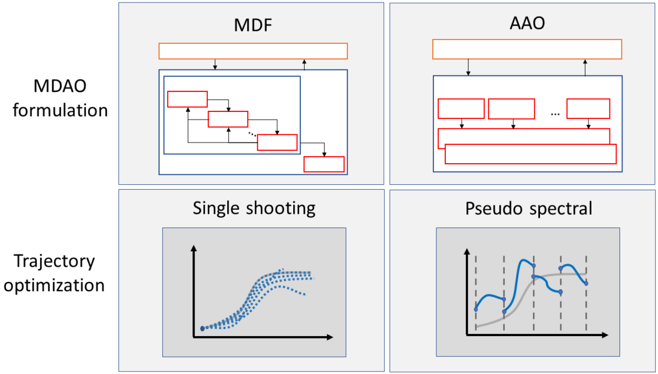



- Low-dimensional optimization
- Not suitable for gradient methods
- Feasibility ensured at each simulation

Pseudo spectral

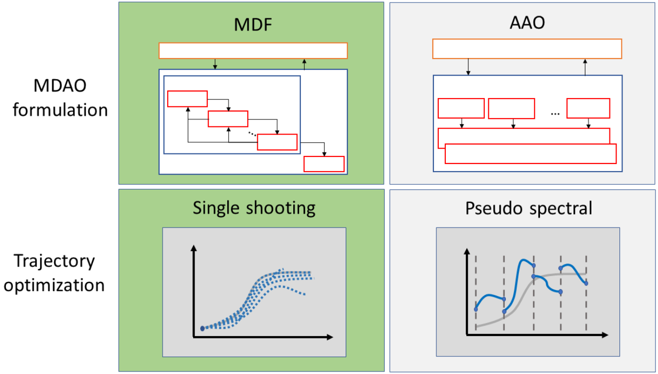



- High-dimensional optimization
- Fit for gradient methods
- Feasibility only ensured at convergence



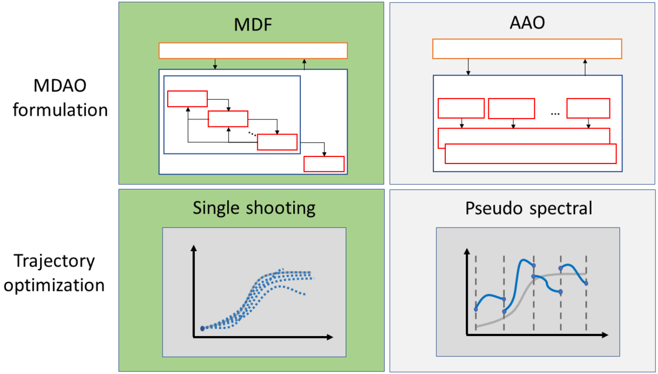

## Choice of methodologies

- Choice of MDAO formulation
- Choice of trajectory optimization method




## Characteristics

- Trajectory simulation is always feasible
- Multidisciplinary feasibility ensured at each MDA loop

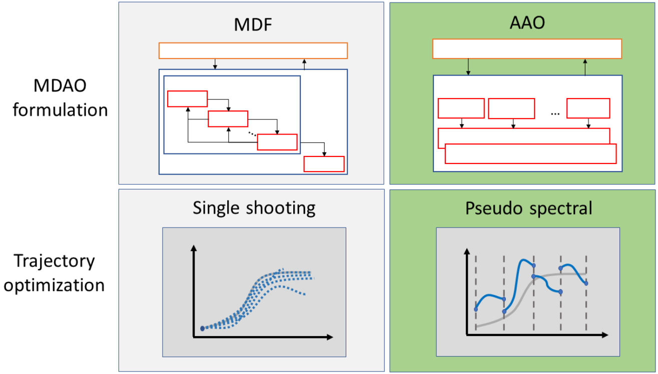



## Drawbacks

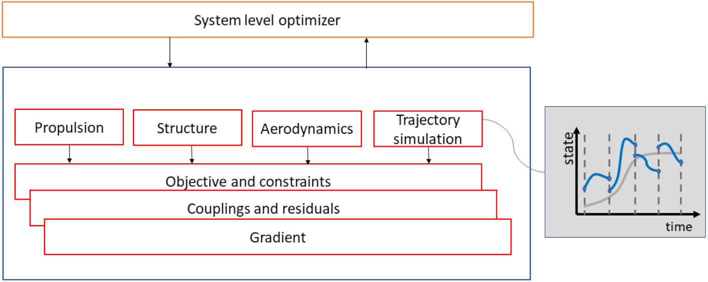
- Trajectory simulation nested into 3 loops
- High computational cost

# Plan

# Proposed method: AAO-PS




Coupling AAO formulation and Pseudo Spectral (AAO-PS) formulation for trajectory optimization to exploit their commonalities into an integrated MDO formulation

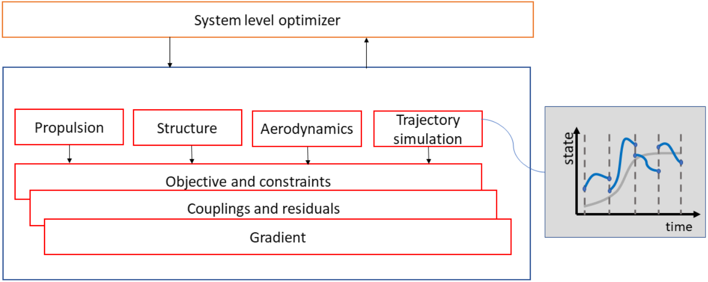



## Characteristics

- Legendre-Gauss-Lobatto pseudo spectral method [2]
- Analytic gradient calculation for all disciplines
- Chain rule for multidisciplinary gradient calculation

[2] Herman and Conway 1996.

# Proposed method: AAO-PS



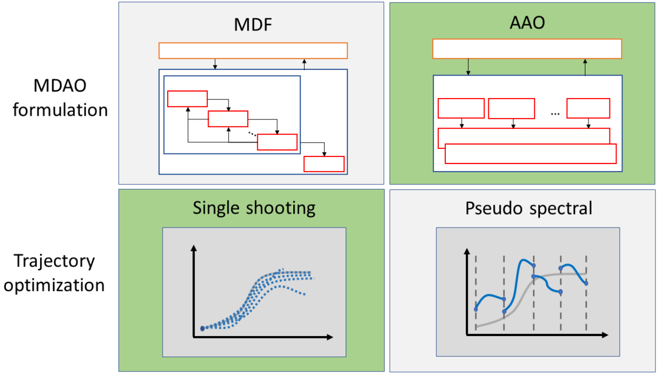

## Characteristics

- The whole MDAO of the launch vehicle is driven by a single gradient-based optimizer: SLSQP [3]
- Use of Dymos [4] and OpenMDAO [5]

[3] Kraft 1988, [4] Falck and Gray 2019, [5] Gray et al. 2019

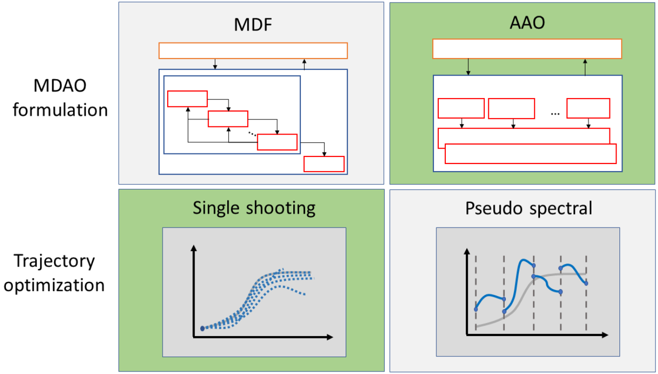
# Plan

# Reference method: AAO-SS




Reference method comprised of AAO formulation and single shooting (AAO-SS) to asses the influence of the trajectory optimization methods

# Reference method: AAO-SS




## Characteristics

- Runge-Kutta integrator and gradient-free optimizer: CMA-ES [6]
- Using ONERA FELIN [7]

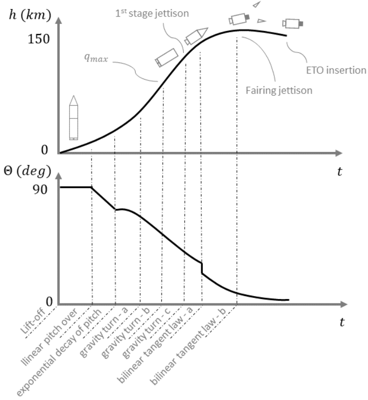
[6] Hansen 2016, [7] *Framework for Evolutive Launcher optImizatioN*

## Mission

- 11 tons of payload
- Equatorial circular orbit of 400 km

**Trajectory:**

- 2D polar coordinates
- 8 phases such as lift-off and gravity turn
- 4 design variables for parameterized pitch angle ($\theta$) guidance
- Constraints such as max. heat flux at fairing jettison

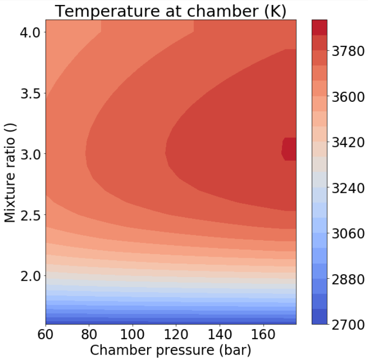


## Mission

- 11 tons of payload
- Equatorial circular orbit of 400 km

**Propulsion:**

- Metamodel of Rocket CEA [8]
- 8 design variables such as chamber pressure and thrust




[8] Sanford and McBride 1994.

## Mission

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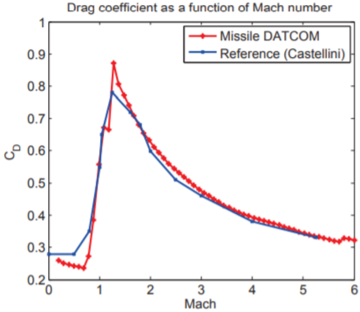
**Aerodynamics:**

- U.S Standard atmosphere
- Variation of drag coefficient and vehicle diameter
- Design variables such as stage diameter



## Mission

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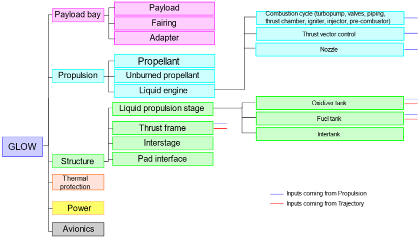
- Equatorial circular orbit of 400 km

**Structure:**

- Regressions on existing data [9]
- 3 design variables such as mass of propellants




[9] Castellini 2012.

# Definition and size of MDAO problem

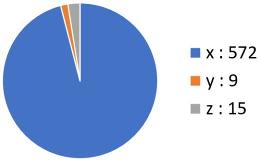
$$
\begin{aligned}
\text{minimize} &: \text{Gross lift-off weight (GLOW)} \\
\text{with respect to} &: [\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}] \in \mathbb{R}^{596} \\
\text{subject to} & \\
\text{equality constraints} &: \boldsymbol{h}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) = \boldsymbol{0} \quad , \quad \boldsymbol{h} \in \mathbb{R}^{567} \\
\text{inequality constraints} &: \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) \leq \boldsymbol{0} \quad , \quad \boldsymbol{g} \in \mathbb{R}^{18}
\end{aligned}
$$

where

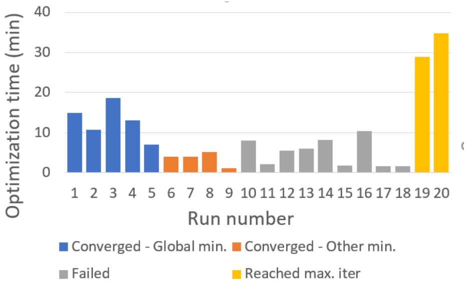
$\boldsymbol{x}$ : state variables

$\boldsymbol{y}$ : coupling variables

$\boldsymbol{z}$ : design variables



- Local optimizer in MDAO context: multistart
- 20 runs with partially random initialization
- 45 % of runs converged to a minimum
- 25 % of runs converged to the global minimum



# Results for proposed method: AAO-PS

- Local optimizer in MDAO context: multistart
- 20 runs with partially random initialization
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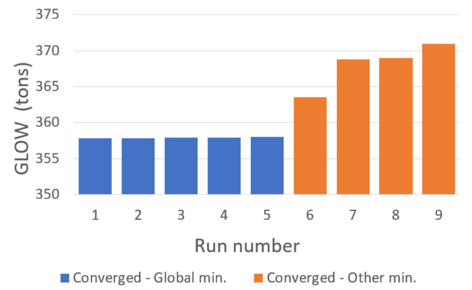
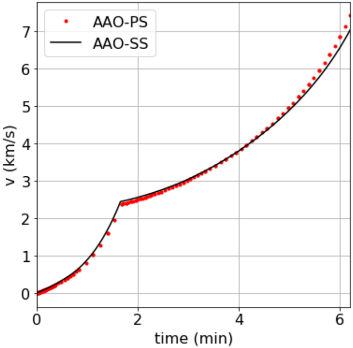


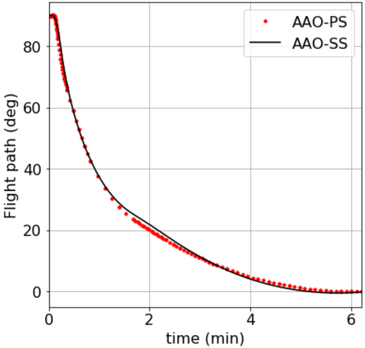
# Comparison with reference method

| | AAO-PS<br>Global minimum<br>Best run | AAO-PS<br>20 runs | AAO-SS<br>Unique run |
|---|---|---|---|
| GLOW (tons) | 357.99 | 357.99 | 360.13 |
| Function evaluations | 706 | 22 163 | 48 000 |
| Gradient evaluations | 371 | 7 331 | —- |

Minimum number of runs to get global minimum: 7
Average function evaluations for 7 runs : 7758
Average gradient evaluations for 7 runs : 2566

# Comparison with reference method



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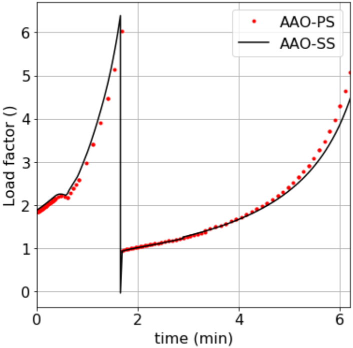


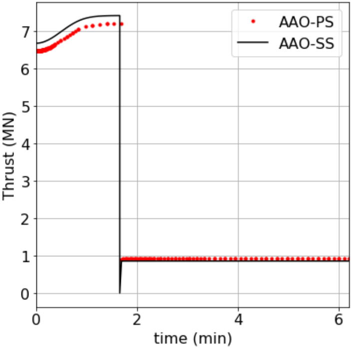

# Comparison with reference method

| | AAO-PS | AAO-SS |
|---|---|---|
| Type of search | Local | Global |
| Number of trajectory simulations | Low | High |
| Sensitivity to initial guess | High | Low |

# Plan

## Conclusions

- A methodology for the MDAO of launch vehicles is proposed relying on an AAO MDO formulation and a pseudo spectral method for the trajectory optimization
- The whole MDAO of the launch vehicle is driven by a single gradient-based optimizer
- A comparison with a reference method is done to analyze the convergence accuracy and to provide a comprehensive discussion of the advantages/drawbacks

## Perspectives

- Improve the approach to reduce the sensitivity to the initialization
- Apply the approach on more complex cases such as 3D trajectory or by involving more disciplines

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