

Introduction

Multidisciplinary optimization is used in launch vehicle design to maximize performance and minimize cost. With these aims in mind, the LAST tool is being developed at SUPAERO's SacLab and the goal of this research project is to provide it with a 3D trajectory optimization module with 3DOF. State of the art trajectory optimization of launchers is performed with pseudospectral methods. To smoothen the learning curve for the implementation of these methods, it was decided to start by a 2D simplification and a shooting method. In this summary, the results of the implementation of a direct single shooting method and partial implementation of a pseudospectral method for the optimization of 2D launcher trajectories are discussed.

Literature review

Trajectory optimization can be considered as a sub-domain of Optimal control as proposed by Rao [1]. Often, optimal control methods are classified as either direct or indirect. Indirect methods are founded on the calculus of variations and the formulation of first-order optimality conditions. Although precise, indirect methods are difficult to implement because of their sensitivity to the initial guess. Direct methods work by discretizing the problem and converting it into a non-linear programming problem. They are less sensitive to the initial guess; a desired characteristic for multidisciplinary optimization where trajectories vary significantly from iteration to iteration. Direct pseudospectral methods use collocation of orthogonal polynomials to approximate the states and controls of the optimal control problem. For some pseudospectral methods the mapping between the direct and in-

direct formulation has been done, as in the case of the Direct high-order Gauss Lobatto Collocation scheme as presented by Herman and Conway [2] and the Radau Pseudospectral Method presented by Garg et al [3].

The LAST tool is built in OpenMDAO [4], a multidisciplinary design, analysis and optimization (MDAO) environment. OpenMDAO allows to divide a complex system into components that must provide the derivatives of its outputs with respect to its inputs, so that the software can compute the total derivative of the system using the chain rule to perform the optimization of the desired variables. Falck and Gray presented the Dymos package for OpenMDAO in [5]. Dymos implements the two aforementioned pseudospectral optimal control methods by using the OpenMDAO environment.

Methods

A set of 5 ODE describes the dynamics of the launcher in 2D. The forces it experiences are modeled by Newton's law of universal gravitation, exponential and standard atmospheres, an interpolated curve for the variation of the drag coefficient with the angle of attack, and a constant mass flow rate thrust model. Guidance laws based on static controls are used to control the attitude during 5 different phases.

A direct single-shooting method was used to minimize the consumed mass of propellants of a single-stage to orbit (SSTO) launcher aiming an orbit constrained in height (200 km), speed and flight path angle with an empty mass of 35 ton. A Runge-Kutta 5(4) integration scheme was used to propagate the dynamics of the initial boundary condition problem and the gradient-free constrained optimization algorithm COBYLA was used to find a feasible solution while minimizing the objective function.

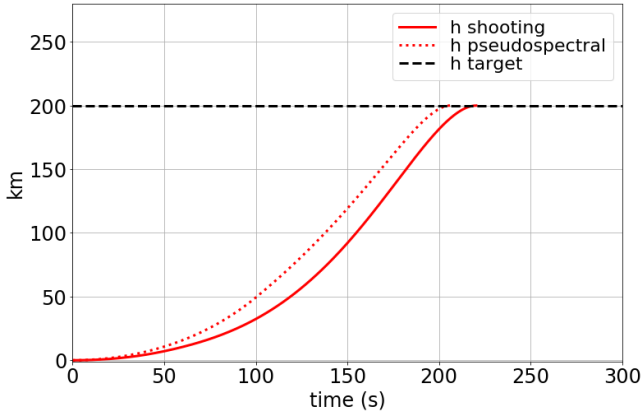


Figure 1: Final height (h)

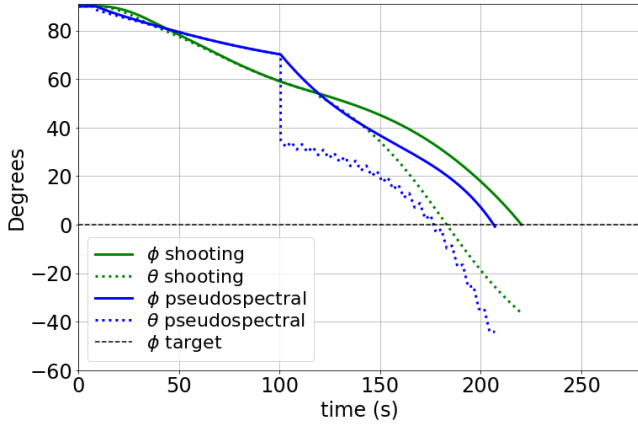


Figure 2: Final flight path (ϕ) and pitch (θ) ang.

8 optimization variables were used for the control of the trajectory and the initial mass of propellants of the launcher.

Simplified models of constant gravity and drag coefficient were used for the implementation of the pseudospectral method in Dymos. With the aim of comparing the results with those of the shooting method these models will be upgraded in the future. The Jacobian matrices of the different subsystems were defined analytically. The Legendre-Gauss-Lobatto transcription of order 3 was used with 48 segments in tandem with the non-linear programming solver SLSQP.

Results and discussion

The optimization result for the single-shooting method complies with the constraints of height (figure 1), speed and flight path angle (figure 2). The minimum cost function was found after 275 iterations and it is equivalent to a consumption of 438 ton of propellants.

For the pseudospectral method, the results show a history of height (figure 1) and speed that is similar to those of the shooting method. However, the shape of the flight path angle differs and the results of the pitch angle are noisy (figure 2). This may be linked to a violation on the bounds of a time variable taking a null value and the methodology used to define it, that could be affecting the definition of the Jacobian matrix for the guidance subsystem.

Conclusions

A direct single-shooting method for the optimization of 2D launcher trajectories was successfully implemented to obtain feasible solutions that minimize the mass of consumed propellants of a SSTO launcher. The partial development of a pseudospectral method for the same case studio is presented and the issues presented during the implementation were addressed. Future actions include a better management of the time variables and refinement of force models for the pseudospectral method in order to compare the results with those of the shooting method. Followed by the optimization of variables from multiple disciplines to study the suitability of pseudospectral methods for the MDAO of launch vehicles and the extension to 3D trajectories.

Selected References

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