

Trajectory
Module for
Launcher
MDAO

J. Valderrama

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Trajectory Module for Launcher Multidisciplinary Design, Analysis and Optimization

S2 Research Project Presentation

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Questions

 Launchers are highly optimized to reduce cost and improve performance



Callisto flight demonstrator. Credits^a

^aSagliano et al. 2019.



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Jugetion

- Launchers are highly optimized to reduce cost and improve performance
- Reusability is changing the market, the architecture and the trajectories



Callisto flight demonstrator. Credits^a

^aSagliano et al. 2019.



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Question

- Launchers are highly optimized to reduce cost and improve performance
 - Reusability is changing the market, the architecture and the trajectories
- Trajectory optimization is central for launch vehicle Multidisciplinary Design, Analysis and Optimization (MDAO)



Callisto flight demonstrator. Credits^a



Context - Tools

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Tools involved in the project

- LAST (Launcher Analysis and Sizing Tool)
- OpenMDAO
- DYMOS



Context - Tools

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Questions

Tools involved in the project

- LAST (Launcher Analysis and Sizing Tool)
- OpenMDAO
- DYMOS

Some reference tools

- FELIN (Framework for Evolutive Launcher optImizatioN)
- Dr. Brevault's Dymos code
- LATOM (LAuncher Trajectory Optimization Module)



Aims and objectives

Trajectory Module for Launcher MDAO

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Aim

To develop and integrate an efficient and accurate 3D trajectory optimization module for a launcher into the LAST tool.



Aims and objectives

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Aim

To develop and integrate an efficient and accurate 3D trajectory optimization module for a launcher into the LAST tool.

Objectives

- To implement trajectories of several level of complexity, including 2D and 3D
- To include different command laws for expendable and reusable launchers under different stage recovery scenarios
- To provide the analytic derivatives required for the MDAO for each case



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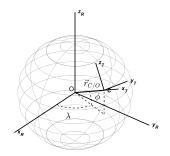
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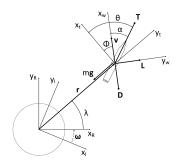
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- 3 DOF
- Cartesian and Spherical coordinates
- 3D and 2D



3D Spherical coordinates



2D Polar coordinates



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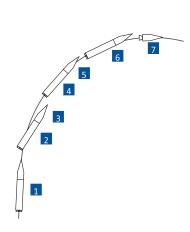
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Division of the trajectory into phases¹²

Control of pitch angle



¹Pagano and Mooij 2010.

²Castellini 2012.



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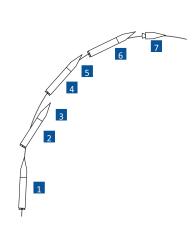
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Division of the trajectory into phases¹²

Control of pitch angle

1 Lift-off: (*h*_{lo})



¹Pagano and Mooij 2010.

²Pagano and Mooij 2010 ²Castellini 2012



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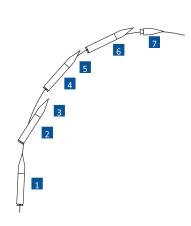
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Division of the trajectory into phases¹²

Control of pitch angle

- 1 Lift-off: (h_{lo})
- 2 Linear pitch-over: $(\Delta \theta_{lpo}, \Delta t_{lpo})$



¹Pagano and Mooij 2010.

²Castellini 2012.



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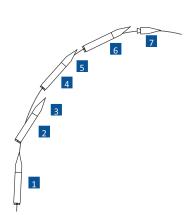
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Division of the trajectory into phases¹²

Control of pitch angle

- 1 Lift-off: (*h*_{lo})
- 2 Linear pitch-over: $(\Delta \theta_{lpo}, \Delta t_{lpo})$
- 3 Exponential decay of pitch: (Δt_{edp})



¹Pagano and Mooij 2010.

²Castellini 2012.



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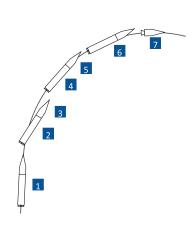
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- 1 Lift-off: (h_{lo})
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- 3 Exponential decay of pitch: (Δt_{edp})
- 4 Gravity turn:



¹Pagano and Mooij 2010.

²Castellini 2012.



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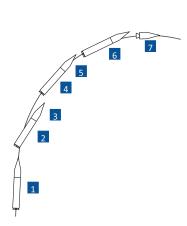
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- 1 Lift-off: (*h*_{lo})
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- 3 Exponential decay of pitch: (Δt_{edp})
- 4 Gravity turn:
- 5 Bilinear tangent law: $(\xi, \Delta\theta_{gt}, \theta_{btl_f})$



¹Pagano and Mooij 2010.

²Castellini 2012.



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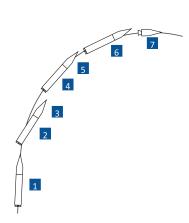
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- 2 Linear pitch-over: $(\Delta \theta_{lpo}, \Delta t_{lpo})$
- 3 Exponential decay of pitch: (Δt_{edp})
- 4 Gravity turn:
- Bilinear tangent law: $(\xi, \Delta \theta_{gt}, \theta_{btl_f})$
- 6 Coast phases



¹Pagano and Mooij 2010.

²Castellini 2012.



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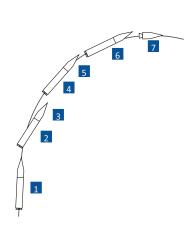
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Division of the trajectory into phases¹²

Control of pitch angle

- Lift-off: (h_{lo})
- 2 Linear pitch-over: $(\Delta \theta_{lpo}, \Delta t_{lpo})$
- Exponential decay of pitch: (Δt_{edp})
- Gravity turn:
- Bilinear tangent law: $(\xi, \Delta \theta_{at}, \theta_{btl})$
- Coast phases
- 7 Stage burns



¹Pagano and Mooij 2010.

²Castellini 2012.



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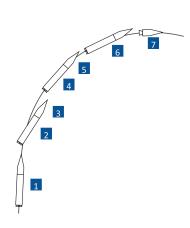
Division of the trajectory into phases¹²

Control of pitch angle

- 1 Lift-off: (h_{lo})
- 2 Linear pitch-over: $(\Delta \theta_{lpo}, \Delta t_{lpo})$
- 3 Exponential decay of pitch: (Δt_{edp})
- 4 Gravity turn:
- 5 Bilinear tangent law: $(\xi, \Delta \theta_{gt}, \theta_{btl_f})$
- 6 Coast phases
- 7 Stage burns

Control of yaw angle

Target inclination law



¹Pagano and Mooij 2010.

²Castellini 2012.



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- Trajectory optimization as a subset of optimal control
- Classification of main optimal control methods
 Direct
 Indirect



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Trajectory optimization as a subset of optimal control

Classification of main optimal control methods

Direct Indirect

Advantages

Low sensitivity to initial guess

Disadvantages

High dimensional problem



High dimensional problem

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Classification of main optimal control methods
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Advantages
High accuracy
Disadvantages

High sensitivity to initial guess



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Trajectory optimization as a subset of optimal control

Classification of main optimal control methods

Direct	Indirect

Advantages
High accuracy
Disadvantages
High sensitivity to initial guess

Justification for choice of Direct methods

In MDAO of launch vehicles trajectories vary significantly



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Pseudospectral methods

- Legendre-Gauss-Lobatto³
- Legendre Gauss-Radau⁴

Characteristics

- Direct methods validated with indirect methods. High accuracy.
- Low sensitivity to initial guess
- Good parallelization

³Herman and Conway 1996.

⁴Garg et al. 2012.



Multidisciplinary Design, Analysis and Optimization

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Questio

- Disciplines involved in launcher MDAO: Trajectory, Aerodynamics, structures...
- Provide partials derivatives of outputs w.r.t. Inputs.
 Compute total derivatives

Optimal control in the context of MDAO:

⁵Falck and Gray 2019.



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 Disciplines involved in launcher MDAO: Trajectory, Aerodynamics, structures...

Provide partials derivatives of outputs w.r.t. Inputs.
 Compute total derivatives

Optimal control in the context of MDAO:

■ The Dymos tool⁵

⁵Falck and Gray 2019.



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2D trajectory optimization with shooting method (Reference method)



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- 2 2D trajectory optimization with pseudospectral method (Partial implementation)

Next steps



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MDAO of launch vehicle with pseudospectral method



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Next steps

- MDAO of launch vehicle with pseudospectral method
- Extension to 3D trajectory optimization and recovery scenarios



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- 2D trajectory optimization with pseudospectral method (Partial implementation)

Next steps

- MDAO of launch vehicle with pseudospectral method
- 4 Extension to 3D trajectory optimization and recovery scenarios
- 5 Integration with LAST



Equations of motion in 2D

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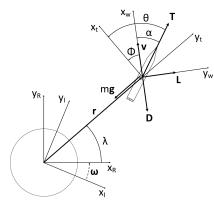
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Results expressed in $F_R := \{x_R, y_R, z_R\}$, Rotating Earth Centered reference frame

State vector

$$\bar{x} = \begin{bmatrix} r \\ \lambda \\ v \\ \phi \\ m \end{bmatrix}$$

 $\begin{array}{ll} \mathbf{r} & \text{Radius} \\ \lambda & \text{Longitude angle} \\ \mathbf{v} & \text{Speed} \\ \phi & \text{Flight path angle} \\ \mathbf{m} & \text{Mass} \end{array}$



Force models

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■ Gravity model: Newton's law of universal gravitation

⁶Tewari 2007.



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- Gravity model: Newton's law of universal gravitation
- Atmospheric models:

⁶Tewari 2007.



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Gravity model: Newton's law of universal gravitation

- Atmospheric models:
 - Atmospheric model based on the 1976 and 1962 U.S.
 Standard Atmospheres⁶ for shooting method

⁶Tewari 2007.



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Gravity model: Newton's law of universal gravitation

Atmospheric models:

Atmospheric model based on the 1976 and 1962 U.S. Standard Atmospheres⁶ for shooting method

Exponential atmosphere for pseudospectral method

⁶Tewari 2007.



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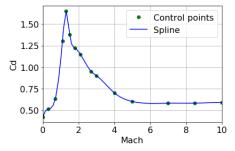
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■ Aerodynamic model: $C_l = 0$, $C_d = f(M)$ Cubic spline interpolation for Ariane 5^7



⁷Pagano 2010.

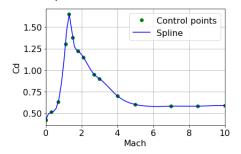


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Force models

Aerodynamic model: $C_l = 0$, $C_d = f(M)$ Cubic spline interpolation for Ariane 57



Propulsion model: Losses as a function of atmospheric pressure. No throttle control

⁷Pagano 2010.



Optimal control formulation - Specific case study

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Minimize Cost function: Mass of consumed propellants

$$J=m(t_0)-m(t_f)$$

With B.C.

 $\bar{x}(t_0)$ and $\bar{x}(t_f)$

and dynamics

$$\dot{\bar{x}} = f_{ode}(\bar{x}, t, \bar{u}, \bar{d})$$

Where the state vector (\bar{x}) and the design parameter vector (\bar{d}) are

$$ar{x} = egin{bmatrix} r \ \lambda \ v \ \phi \ m \end{bmatrix} \quad ar{d} = egin{bmatrix} h_{lo} \ \Delta heta_{lpo} \ \Delta t_{lpo} \ \Delta t_{edp} \ \xi \ \Delta heta_{btl} \ \theta_{btl_f} \ m_p \end{bmatrix}$$



Direct single-shooting method

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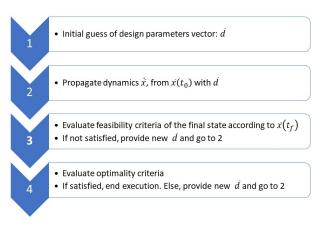
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Convert the two-point boundary value problem



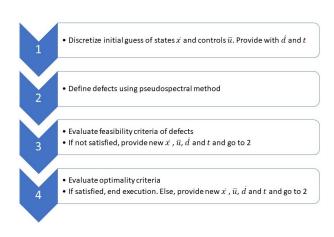
- Step 2: Runge Kutta 5(4) numerical integration method
- Steps 3 and 4: COBYLA gradient-free optimizer



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- Step 2: Legendre-Gauss-Lobatto (Order 3 and 48 segments)
- Step 3 and 4: SLSQP (Analytic Jacobian information)



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Legendre-Gauss-Lobatto transcription (LGL)

 Discretize time into segments, discretize segments with LGL nodes.

State discretization nodes (even LGL nodes)
Collocation nodes (odd LGL nodes)



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Legendre-Gauss-Lobatto transcription (LGL)

- Discretize time into segments, discretize segments with LGL nodes.
 - State discretization nodes (even LGL nodes)
 Collocation nodes (odd LGL nodes)
- Evaluate ODE at state discretization nodes to fit an Hermite polynomial



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Legendre-Gauss-Lobatto transcription (LGL)

Discretize time into segments, discretize segments with IGI nodes

State discretization nodes (even LGL nodes) Collocation nodes (odd LGL nodes)

- Evaluate ODE at state discretization nodes to fit an Hermite polynomial
- Calculate state rate at collocation nodes using the Hermite polynomial



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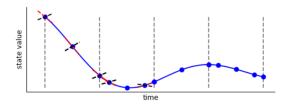
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Credits⁸



— Dymos 0.15.0 Documentation.

⁸Dymos: Open-Source Optimal Control for Multidisciplinary Systems



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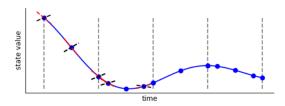
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 Calculate state rate at collocation nodes by evaluating the ODE

— Dymos 0.15.0 Documentation.

⁸Dymos: Open-Source Optimal Control for Multidisciplinary Systems



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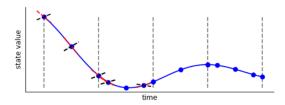
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- Calculate state rate at collocation nodes by evaluating the ODE
- The defect is the error

— Dymos 0.15.0 Documentation.

⁸ Dymos: Open-Source Optimal Control for Multidisciplinary Systems

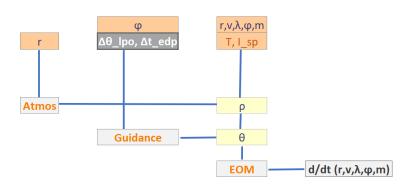


Pseudospectral method - N2 diagram

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Pseudospectral method



N2 diagram - Exponential decay of pitch phase

Simplified models of gravity, drag coefficient, exponential atmosphere



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Case study of single-stage-to-orbit vehicle

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Input parameter	Value	Units	Description
m_e	35	ton	Empty mass
μ	1990	kg/s	Mass flow rate
I_{sp}	400	S	Specific impulse
I _{sp} S	37.5	m^2	Reference area
A_e	10	m ²	Nozzle expansion area

Vehicle parameters



Case study of single-stage-to-orbit vehicle

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Final B.C.	Value	Tolerance	Units
$r(t_f)$	6378.135 + 200	2	km
$V(t_f)$	7319.16	500	m/s
$\phi(t_f)$	0	15	deg

Final boundary conditions for shooting method



Direct single-shooting method - Initial guess

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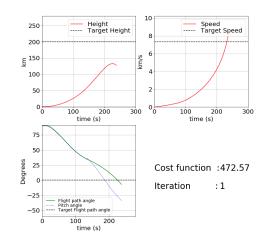
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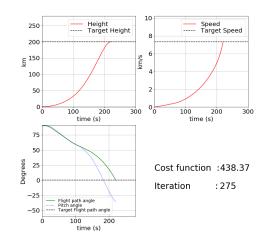
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Pseudospectral method - Initial guess



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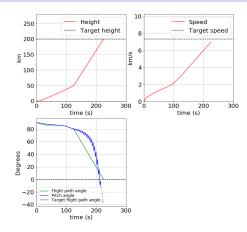
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- Initial guess of \bar{d} is the output of the shooting method
- Guess of states for different phases with linear interpolation



Pseudospectral method - Opt. results

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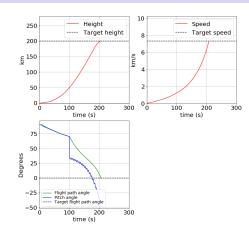
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- Duration of phase taking a null value
- Time management may be causing a bad definition of Jacobian matrix

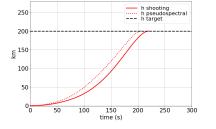


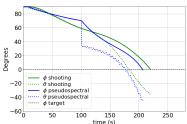
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Pseudospectral method





- h = height ϕ = flight path angle θ = pitch angle



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Implementation of 2D trajectory optimization with shooting method (Reference)



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- Partial implementation of 2D trajectory optimization with pseudospectral method



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Conclusions

- Implementation of 2D trajectory optimization with shooting method (Reference)
- Partial implementation of 2D trajectory optimization with pseudospectral method

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- 1 Finish 3
- 2 MDAO of launch vehicle with pseudospectral method
- 3 Extension to 3D trajectory optimization and recovery scenarios
- Integration with LAST





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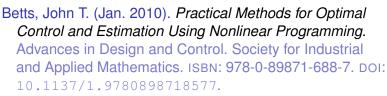
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Castellini, Francesco (2012). "MULTIDISCIPLINARY DESIGN OPTIMIZATION FOR EXPENDABLE LAUNCH VEHICLES" en In:

Dymos: Open-Source Optimal Control for Multidisciplinary Systems — Dymos 0.15.0 Documentation. https://openmdao.github.io/dymos/index.html.





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Falck, Robert D. and Justin S. Gray (2019). "Optimal Control within the Context of Multidisciplinary Design, Analysis, and Optimization". In: *AIAA Scitech 2019 Forum*.

American Institute of Aeronautics and Astronautics.

Garg, Divya et al. (June 2012). "Direct Trajectory Optimization and Costate Estimation of General Optimal Control Problems Using a Radau Pseudospectral Method". In:

Herman, Albert L. and Bruce A. Conway (1996). "Direct Optimization Using Collocation Based on High-Order Gauss-Lobatto Quadrature Rules". In: *Journal of Guidance, Control, and Dynamics*. DOI: 10.2514/3.21662.





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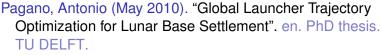
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Pagano, Antonio and Erwin Mooij (Aug. 2010). "Global Launcher Trajectory Optimization for Lunar Base Settlement". en. In: AIAA/AAS Astrodynamics Specialist Conference. Toronto, Ontario, Canada.

Sagliano, Marco et al. (July 2019). "Guidance and Control Strategy for the CALLISTO Flight Experiment". en. In: SpaceX (Apr. 2020). Falcon User's Guide. Tech. rep. Space

Exploration Technologies Corp.



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Tewari, Ashish (2007). Atmospheric and Space Flight Dynamics: Modeling and Simulation with MATLAB and Simulink. en. OCLC: 180887853. Boston, Mass.: Birkhäuser.



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Questions

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Legendre-Gauss-Lobatto transcription I

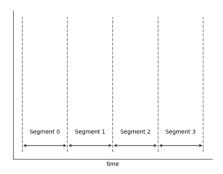
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LGL Transcription

The LGL transcription as described in Dymos documentation9

1 Phase segmentation: Discretize time into segments





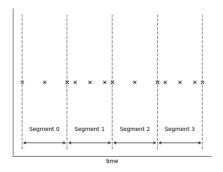
Legendre-Gauss-Lobatto transcription II

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LGL Transcription

Discretization: Each segment is discretized by using the LGL nodes that depend on the order chosen for the method.





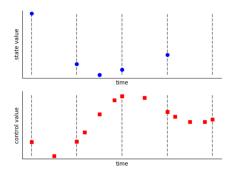
Legendre-Gauss-Lobatto transcription III

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LGL Transcription

3 Input: Initial guess for states at discretization nodes (Even LGL nodes). Initial guess for control at all nodes.





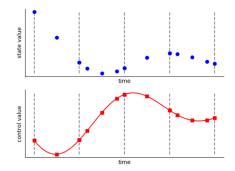
Legendre-Gauss-Lobatto transcription IV

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LGL Transcription

4 Control rate interpolation: Lagrange interpolating polynomial. This allows to provide derivative values of the dynamic controls as inputs for the ODE





Legendre-Gauss-Lobatto transcription V

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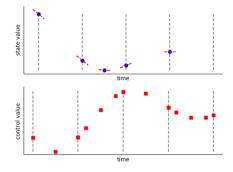
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Equations of motion Shooting method **5** Evaluation of the ODE at the state discretization nodes:





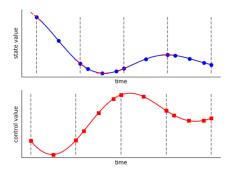
Legendre-Gauss-Lobatto transcription VI

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LGL Transcription

6 State interpolation: Hermite polynomial. Calculate rate at collocation nodes (odd LGL nodes)





Legendre-Gauss-Lobatto transcription VII

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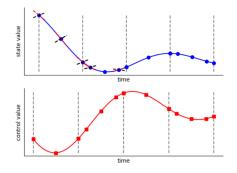
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7 Evaluation of the ODE at the collocation nodes:





Legendre-Gauss-Lobatto transcription VIII

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LGL Transcription

- 8 Evaluation of the collocation defects: Compare 6 and 7.
- Iterate: Steps 3 to 8 are repeated until the solution reaches feasibility and optimality criteria.

⁹Dymos: Open-Source Optimal Control for Multidisciplinary Systems Dymos 0.15.0 Documentation.



N2 diagrams I

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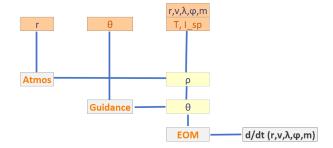
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Lift-off:





N2 diagrams II

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NZ Diagrams

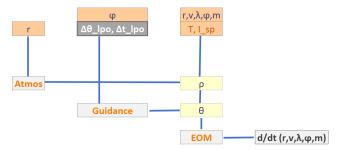
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Shooting me pseudospec Linear pitch over:





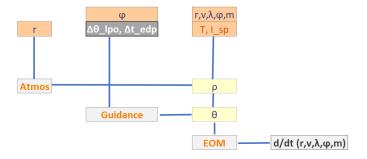
N2 diagrams III

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Exponential decay of pitch:





N2 diagrams IV

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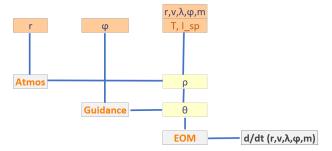
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Gravity turn:





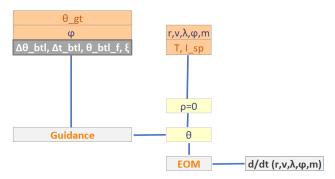
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Bilinear tangent law:





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■ Gravity: Axisymmetric planet model including up to the fourth Jeffery constant $(J_4)^{1011}$

Atmosphere: U.S Standard atmosphere 1976, exponential atmosphere

■ Aerodynamics: Drag coefficient data (C_d) data must be C_2 continuous for non-linear programming $(NLP)^{12}$

¹⁰Castellini 2012.

¹¹Tewari 2007.

¹² Betts 2010.



Atmospheric models

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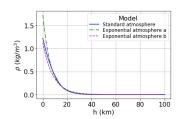
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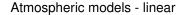
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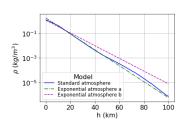
Optimal control formulation
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Equations of motion Shooting method pseudospectral Atmospheric model based on the 1976 and 1962 U.S. Standard Atmospheres¹³

 Exponential atmosphere based reference height and reference pressure. Model "a" is from Tewari and model "b" from Falck.







Atmospheric models - log

¹³ Tewari 2007.



Thrust model

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$$T = T_{vac} - A_e \rho_a$$

$$T_{\it vac} = g_0 I_{\it sp} \mu$$



Optimal control formulation

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Minimize

 $\mathbf{J} = f_{obj}(\bar{x}, t, \bar{u}, \bar{d})$

Subjected to the dynamics

 $\dot{ar{x}} = \mathit{f}_{\mathit{ode}}(ar{x}, t, ar{u}, ar{d})$

Bounded as

Time : $t_{lb} \leq t \leq t_{ub}$

State Variables : $\bar{x}_{lb} \leq \bar{x} \leq \bar{x}_{ub}$

Dynamic Controls : $\bar{u}_{lb} \leq \bar{u} \leq \bar{u}_{ub}$

Design Parameters : $\bar{d}_{lb} \leq \bar{d} \leq \bar{d}_{ub}$



Optimal control formulation

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Also bounded by the non-linear constraints

Initial Boundary Constraints : $ar{g}_{0,\textit{lb}} \leq g_0 \left(ar{x}_0, t_0, ar{u}_0, ar{d}\right) \leq ar{g}_{0,\textit{ub}}$

Final Boundary Constraints : $\bar{g}_{f,lb} \leq g_f(\bar{x}_f, t_f, \bar{u}_f, \bar{d}) \leq \bar{g}_{f,ub}$

Path Constraints : $\bar{p}_{f,lb} \leq p_f(\bar{x},t,\bar{u},\bar{d}) \leq \bar{p}_{f,ub}$



Guidance laws

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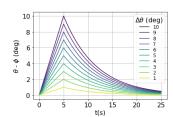
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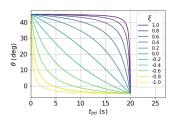
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Equations of motion Shooting method pseudospectral Control of pitch angle θ for a Single-Stage to Orbit launcher. Definition of phases

- 1 Lift-off: $\theta = 90^{\circ} (h_{lo})$
- **2** Linear pitch over $(\Delta \theta_{lpo}, \Delta t_{lpo})$
- **3** Exponential decay of pitch (Δt_{edp})
- **4** Gravity turn: $\theta = \phi$
- **5** Bilinear tangent law $(\xi, \Delta \theta_{gt}, \theta_{btl_f})$







Guidance laws

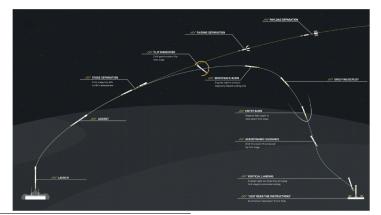
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Recovery scenarios¹⁴

 Return to launch site Downrange landing (Image below. Credits¹⁵)



¹⁴Sagliano et al. 2019.

¹⁵SpaceX 2020.



Equations of motion in 2D

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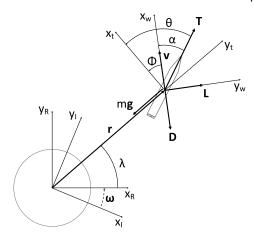
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Reference frames

- F_R := {x_R, y_R, z_R}, Rotating Earth Centered reference frame
- $F_I := \{x_I, y_I, z_I\}$, Inertial Earth Centered reference frame
- F_t := {x_t, y_t, z_t}, Local Vertical - Local Horizontal reference frame
- $F_w := \{X_w, y_w, Z_w\}$, Wind reference frame



Equations of motion in 2D

X_t\

mg

λ

∤ω Xi

 X_R

y_R

 X_{w}

θ

α

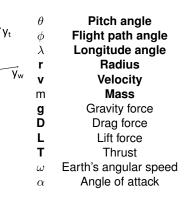
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Equations of motion Shooting method

$\dot{r} = v \sin{(\phi)}$
$\dot{\lambda} = \frac{v\cos(\phi)}{1}$
$\dot{ extbf{v}} = rac{-D + T\cos{(heta - \phi)}}{m} + \left(-g + \omega^2 r ight)\sin{(\phi)}$
• • • • • • • • • • • • • • • • • • • •
$\dot{\phi} = \frac{L}{mv} + \frac{T\sin(\theta - \phi)}{mv} + \frac{(\omega^2 r - g)\cos(\phi)}{v} + 2\omega + \frac{v\cos(\phi)}{r}$

θ	Pitch angle	g	Gravity force
ϕ	Flight path angle	D	Drag force
λ	Longitude angle	L	Lift force
r	Radius	Т	Thrust
٧	speed	ω	Earth's angular speed
m	Mass	α	Angle of attack



Direct single-shooting - Reference method

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Initial B.C.	Value	Units
$r(t_0)$	6378.135	km
$\lambda(t_0)$	0	deg
$v(t_0)$	1	m/s
$\phi(t_0)$	90	deg
$m(t_0)$	$m_e + m_p = 507.6$	ton

Initial boundary conditions



Direct single-shooting - Reference method

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Equations of moti Shooting method

Design parameter	Value	lb	up	Units	Description
h _{lo}	222.45	150	300	m	Height end of lift-off
$\Delta \theta_{lpo}$	1.47	0.1	8	deg	Change in pitch angle
Δt_{lpo}	20.32	1	40	S	Phase duration
Δt_{edp}	18.4	1	25	S	Phase duration
ξ	-0.01	-1	1		Shape coefficient btl
$\Delta heta_{btl}$	-1.66	-60	60	deg	Change in pitch angle
$ heta_{btl_{f}}$	-33.56	-50	60	deg	Final pitch angle
m_{p}	472.6	370	800	ton	Mass of propellants

Initial guess - Design parameters for shooting method



Direct single-shooting method - Initial guess

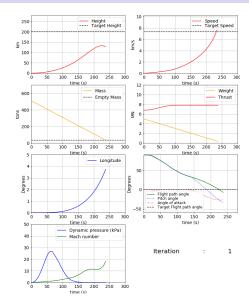
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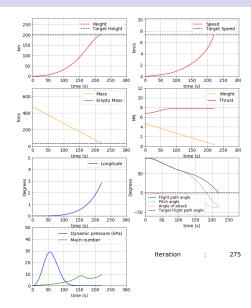
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Initial guess - Design parameters for pseudospectral method

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Design parameter	Value	lb	up	Units	Description
h _{lo}	212.8	150	300	m	Height end of lift-off
$\Delta \theta_{lpo}$	1.77	0.1	8	deg	Change in pitch angle
Δt_{lpo}	25.33	1	40	S	Phase duration
Δt_{edp}	20.0	1	80	S	Phase duration
ξ	-0.01	-1	1		Shape coefficient btl
$\Delta heta_{btl}$	0.9	-60	60	deg	Change in pitch angle
$ heta_{btl_f}$	-36.4	-50	60	deg	Final pitch angle
m _p	438.4	$-\infty$	∞	ton	Mass of propellants

Initial guess - Design parameters for pseudospectral method

These were taken from the optimization results of the shooting method