# Research project meeting summary: Trajectory Module for Launcher MDAO


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# Plan

# Aims and objectives

## Aim

To develop and integrate an efficient and accurate 3D trajectory optimization module for a launcher into the LAST tool.

## Objectives

- To implement trajectories of several level of complexity , including 2D and 3D
- To include different command laws for expendable and reusable launchers under different stage recovery scenarios
- To provide the analytic derivatives required for the MDAO for each case

# Literature review

## Quick review of key aspects

- Equations of motion:
  - 2D polar coordinates.
  - Strategies to deal with singularity of 3D spherical EOM.
- Forces:
  - Data interpolation for drag coefficient and atmospheric models must be $C_2$ continuous for Non Linear Programming solvers.
- Guidance laws:
  - Guidance program for expendable vehicles.
  - Recovery scenarios.
- Optimal control in MDAO:
  - Direct Vs. Indirect.
  - Direct pseudospectral methods; LGL and LGR .

# Single shooting method with COBYLA

Features:

- 2D polar coordinates for a Single-stage to orbit launcher
- U.S. standard atmosphere 1976. Variation of Cd with Mach number.
- Variation of thrust as a function of atmospheric pressure. Constant throttle.
- Newton's gravity model.
- 8 design parameters.
- Remark: Each iteration has physical sense.

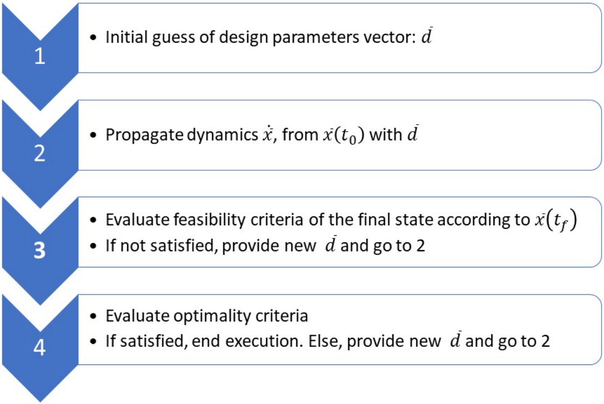
# Single shooting method with COBYLA

Objectives:

- Provide a reference method to compare with the pseudospectral method.
- Provide an initial guess for the pseudospectral method.

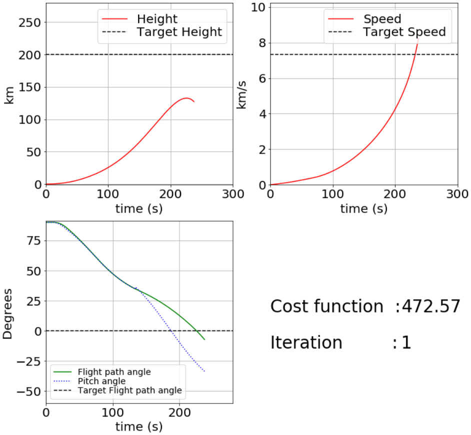
# Single shooting method with COBYLA

Convert the two-point boundary value problem




- Step 2: Runge Kutta 5(4) numerical integration method
- Steps 3 and 4: COBYLA gradient-free optimizer

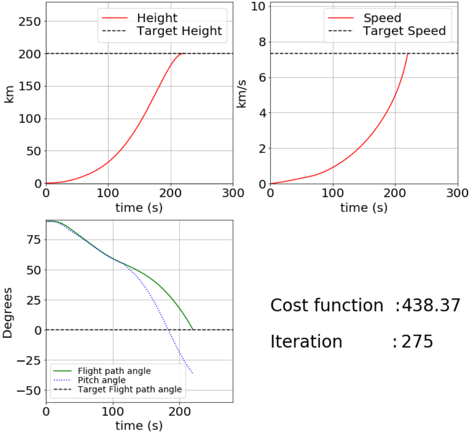
# Single shooting method - Initial guess




Cost function :472.57

Iteration : 1

# Single shooting method - Opt. results




Cost function : 438.37

Iteration : 275

Features:

- Transcription: Legendre-Gauss-Lobatto of order 3
- MDAO environment allows to refine models progressively
- Gradient based optimization with analytic derivatives with SLSQP
- Iterations not necessarily have a physical sense. Advantage for singularities.
- Simulate method allows to compare results with the IVP solution using RK45

# Pseudospectral method with DYMOS

Objectives - Potential degree of novelty:

- Assess the use of pseudospectral methods in launcher MDAO

# Progressive modeling in Dymos

## 1st model

- SSTO
- Exponential atmosphere. Easy derivatives.
- Constant $g$
- Constant $C_d$
- Direct ascent to circular orbit
- Own launcher parameters

# Progressive modeling in Dymos

## 2nd model

- SSTO
- U.S Standard atmosphere. Akima interpolator.
- Newton's gravity model.
- $C_d$ Vs. *Mach* curve with Pchip interpolator
- Direct ascent to circular orbit
- Own launcher parameters

# Progressive modeling in Dymos

## 3rd model

- DSTO with fairing jettisoning
- U.S Standard atmosphere. Akima interpolator.
- Newton's gravity model.
- $C_d$ Vs. *Mach* curve with Pchip interpolator
- Hohmann transfer ascent up to elliptical transfer orbit. Analytic circularization
- Own launcher parameters

# Progressive modeling in Dymos

## 3rd model for Falcon 9 type launcher

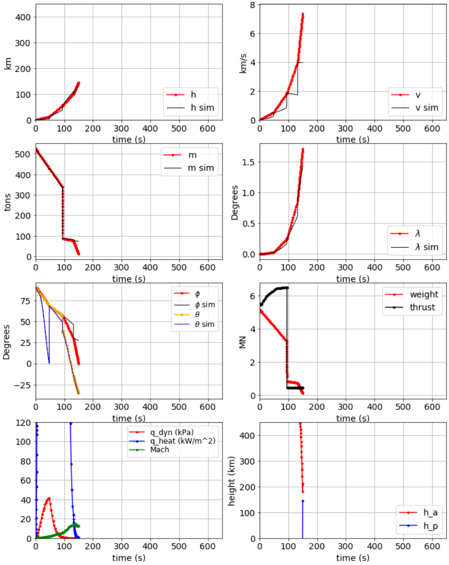
- DSTO with fairing jettisoning
- U.S Standard atmosphere. Akima interpolator.
- Newton's gravity model.
- $C_d$ Vs. *Mach* curve with Pchip interpolator
- Hohmann transfer ascent up to elliptical transfer orbit. Analytic circularization
- Falcon 9 launcher parameters

# Progressive modeling in Dymos

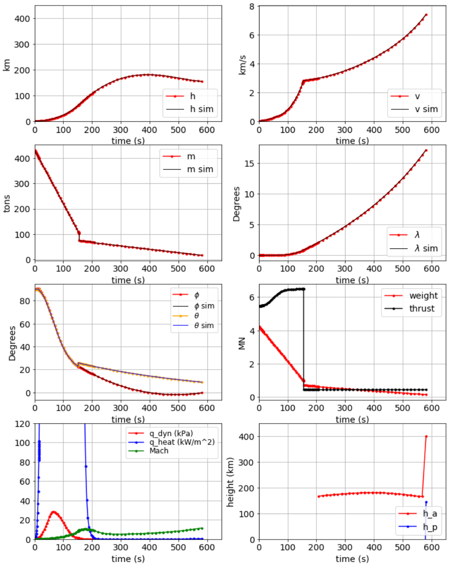
## Guidance program

- Lift off
- Pitch over; linear and exponential
- gravity turn
- exoatmospheric a; first stage flight with fairing
- exoatmospheric b; second stage flight with fairing
- exoatmospheric c; second stage flight without fairing

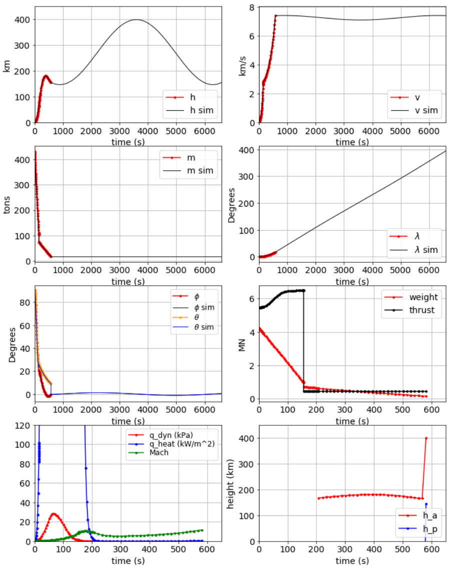
# Initial guess in Dymos



# Results Falcon 9. 11 Ton to 400km in Dymos



# Results Falcon 9. 11 Ton to 400km in Dymos



# Progressive modeling in Dymos

## Main challenges

- Debugging is slow
- Discontinuities in states were difficult to define
- Many possibilities to model the same features. Not all of them are efficient

# Plan

1. Review of work from March up to September 25
2. Review of work from September 25 up to now
   - Using saved results as initial guess
   - Scaling of state variables
   - Exploring with 3D trajectories
3. Key points to discuss
4. Clarifications on the presentation
5. Future actions

# Initial guess

## Previous method

- manual input of initial guess for states, phase durations and design parameters
- Linear interpolation from initial and final value of each state.

## New method

- Save all the iterations of the optimization in a text file
- Read the only the last result, corresponding to the optimal value
- I have converged results that can be used as initial guess for missions at 400 km for payload mass of $6.8 ton$, $9 ton$, $11 ton$ and $13 ton$

# Scaling of state variables

Scaling must be performed for the following variables

- States
- Design parameters
- Initial time and duration of each phase
- Defects

The methods ”ref0” and ”ref” take the values to be scaled to ”0” and ”1” respectively.

Ex. if ”ref0” = 0 and ”ref” = 10 for

- $x = 0 \rightarrow x_{scaled} = 0$
- $x = 5 \rightarrow x_{scaled} = 0.5$
- $x = 10 \rightarrow x_{scaled} = 1$

# Scaling of state variables

Methods used to scale state variables

1. Experience based method:
   - Manually tune *ref*0 and *ref* for 7 states and 7 phases
2. Auto scaling phase based:
   - For each state of each phase of the initial guess take the initial and final values to define ”ref” and ”ref0”
   - Big deviations for short duration phases
3. Auto scaling trajectory based:
   - For each state of the trajectory of the whole trajectory take the initial and final values to define ”ref” and ”ref0”
   - Some variables as ”v” can vary 2 orders of magnitude from one phase to another

# Scaling of state variables

| Initial guess | Scaling method | Opt. Duration | Gradient eval. | Function eval. |
|---|---|---|---|---|
| **F9 6.8Ton** | Experience based | 7min10s | 289 | 584 |
| | Phase based | 8min57s | 382 | 712 |
| | Trajectory based | 22min9s | 400 | 984 |
| **F9 9.0Ton** | Experience based | 10min2s | 232 | 487 |
| | Phase based | 18min0s | 356 | 800 |
| | Trajectory based | 17min46s | 360 | 777 |

Table: Results for a F9 mission to 400 km orbit with 11Ton payload

# Scaling of state variables

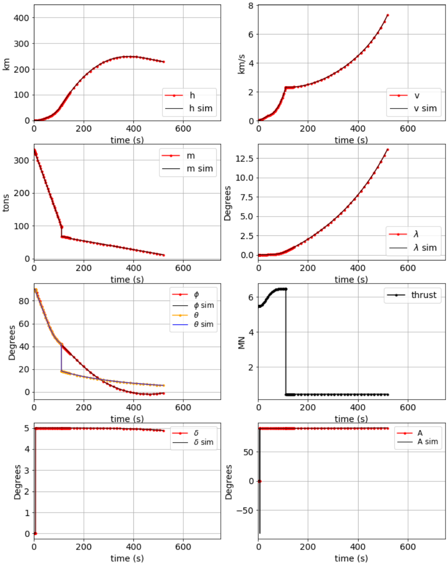
| Mission | Scaling method | Opt. Duration | Gradient eval. | Function eval. |
|---|---|---|---|---|
| **F9 11.0Ton** | Experience based | 7min10s | 289 | 584 |
| | Phase based | 8min57s | 382 | 712 |
| | Trajectory based | 22min9s | 400 | 984 |
| **F9 9.0Ton** | Experience based | 7min44s | 173 | 302 |
| | Phase based | 17min10s | 338 | 600 |
| | Trajectory based | 4min55s | 122 | 208 |

Table: Results from initial guess of F9 6.8 ton to 400 km orbit

# Exploring with 3D trajectories

- Lift off phase using 2D EOM in equatorial plane
- No yaw guidance yet. I'm letting it free.
- Gravity of oblate planet including $J_4$

# F9 6.8Ton to 400km from 5deg latitude



# Plan

1. Review of work from March up to September 25
2. Review of work from September 25 up to now
3. Key points to discuss
4. Clarifications on the presentation
5. Future actions

# Key points to discuss

## How to improve efficiency

- Parallel processing
- Scaling of remaining variables
- Better management of data transfer between subsystems
- Dymos grid refinement
- Transcription type and order

# Key points to discuss

## How to improve the model

- Dynamic pressure path constraints
- Load factor path constraints
- Coast phase after jettisoning first stage
- 3D model?

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# Clarifications on the presentation

**1. Variation of thrust with atmospheric pressure:**
The thrust, $T$, is modeled as function of the thrust at vacuum, $T_{vac}$, the nozzle expansion area, $A_e$, and the local atmospheric pressure, $p_a$, as expressed by

$$T = T_{vac} - A_e p_a$$

The thrust at vacuum is modeled as

$$T_{vac} = g_0 I_{sp} \mu$$

Where $I_{sp}$ is the specific impulse and $\mu$ is the mass flow rate.

# Clarifications on the presentation

For a Falcon 9 first stage $T_{vacuum} = 6.47MN$ and I estimated $A_e = 10m^2$, thus $T_{sealevel} = 5.46MN$. This meant 15.6% less thrust at sea level.

In reality $A_e = 5.98m^2$ as the diameter of the Merlin 1D engine is $0,92m$ and there are 9 engines. Thus the correct value of $T_{sealevel} = 5.87MN$, meaning 9.3% less thrust at sea level.

# Clarifications on the presentation

**2. Calculation of apogee and perigee:**

- The model for orbit insertion considers a Hohmann transfer ascent (HTA).
- First Second Engine cut-off happens at an optimal point near perigee of the elliptical transfer orbit determined by the optimizer.
- The coast phase is not considered in the optimization but it is simulated using a Runge Kutta method.
- Second burn of HTA is calculated with analytic expressions

# Clarifications on the presentation

The calculation step by step.
Ref: *Fundamentals of Astrodynamics* - Wakker:

- Departing from speed ($v$), flight path angle ($\phi$), and radius ($r$) in a relative frame.
- Transform speed to inertial frame
- Calculate energy ($E$) and angular momentum per unit mass($H$)

$$E = \frac{v^2}{2} - \frac{\mu}{r}$$

$$H = r * v * \cos(\phi)$$

Where $\mu$ is the product of the gravitational constant and the mass of Earth

# Clarifications on the presentation

- Calculate eccentricity ($e$) and semi-major axis ($a$)

$$e = \sqrt{1 + 2 * \frac{H^2 * E}{\mu^2}}$$

$$a = -\frac{\mu}{2 * E}$$

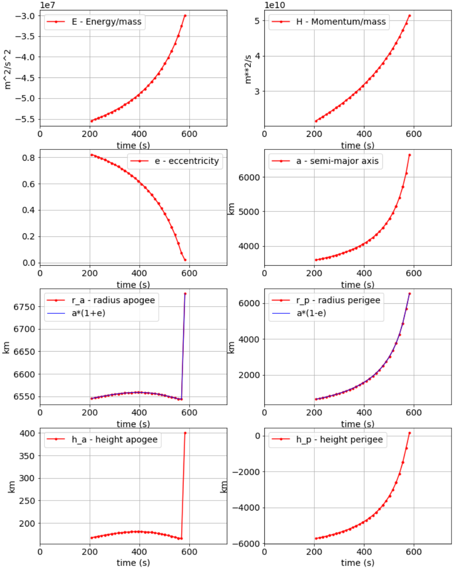
- Calculate radius at apogee ($r_a$) and radius at perigee ($r_p$)

$$r_p = a * (1 - e)$$

$$r_a = a * (1 + e)$$

- Calculate height at perigee ($h_p$) and apogee($h_a$)

# Clarifications on the presentation



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# Future actions

- Start merging propulsion and trajectory modules
- Take the propulsion module from LAST and evaluate if data coming from external modules can be tabulated and interpolated