Physics informed Machine Learning for Celestial Mechanics: The N-body problem

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Key Points:

- Celestial mechanics
- N-body problem
- Physics informed Neural Networks (PINNs)

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Abstract

1 Introduction

2 Related Work

2.1 Three-body problem — From Newton to supercomputer plus machine learning

In their work titled "Three-body problem – from Newton to supercomputer plus machine learning," Shijun Liao, Xiaoming Li, and Yu Yang address the longstanding challenge of identifying periodic orbits within three-body systems. (?, ?) Historically, only a limited number of such orbits were discovered over three centuries. The authors introduce an innovative approach that leverages machine learning, specifically artificial neural networks (ANNs), to systematically uncover planar periodic orbits for three-body systems with arbitrary masses. By starting with a known periodic orbit, their method iteratively expands the set of known orbits, effectively training the ANN to predict accurate periodic orbits across various mass configurations. This approach not only broadens the understanding of three-body dynamics but also underscores the potential of combining high-performance computing with artificial intelligence to tackle complex problems in celestial mechanics.

2.2 Newton vs the machine: solving the chaotic three-body problem using deep neural networks

In their work titled "Newton vs the Machine: Solving the Chaotic Three-Body Problem Using Deep Neural Networks," Breen et al. (2019) address the computational challenges inherent in solving the three-body problem due to its chaotic nature. Traditional numerical methods often demand extensive computational resources and time to achieve accurate solutions. To mitigate this, the authors trained a deep artificial neural network (ANN) on a dataset of solutions generated by high-precision numerical integrators. Their findings demonstrate that the ANN can predict the motions of three-body systems over bounded time intervals with fixed computational costs, achieving speeds up to 100 million times faster than conventional solvers. This approach holds promise for efficiently simulating complex many-body systems, such as those involving black-hole binaries or dense star clusters.

2.3 Physics Informed Deep Learning: Data-driven Solutions of Nonlinear Partial Differential Equations

The work titled "Physics-informed neural networks: A deep learning framework for solving forward and inverse problems involving nonlinear partial differential equations" by Raissi et al. (2017) introduces Physics-Informed Neural Networks (PINNs) as a novel approach to solving problems governed by partial differential equations (PDEs). PINNs incorporate the underlying physical laws, expressed as PDEs, directly into the neural network's loss function, enabling them to learn solutions while respecting the governing equations. This eliminates the need for labeled data, relying instead on the residuals of the PDEs to guide the training. The study demonstrates the application of PINNs to a variety of forward and inverse problems, such as fluid dynamics and heat conduction. The framework is particularly useful for problems with limited observational data or where traditional numerical solvers are computationally expensive. PINNs offer a generalizable and efficient alternative for modeling complex physical systems.

3 Problem statement

On a closed system, the governing laws are described by the minimal action principle. The current work adheres to this principle as it derives the Euler-Lagrange equations, from which we can induce a Legendre transform that turns into the hamiltonian formulation:

$$\mathcal{H}(\mathbf{q},\mathbf{p}) = K(\mathbf{p}) + U(\mathbf{q}) \tag{1}$$
$$\dot{\mathbf{q}} = \nabla_{\mathbf{p}}\mathcal{H} \tag{2}$$
$$\dot{\mathbf{p}} = -\nabla_{\mathbf{q}}\mathcal{H} \tag{3}$$

In this case, we adhere to the classical gravitational potential function for the N bodies and kinetic energy transformed to the desired phase space:

$$U(\mathbf{q}) = -G \sum_{1\leq i\leq n\leq N} \frac{m_i m_n}{\|\mathbf{r}_i(\mathbf{q}) - \mathbf{r}_n(\mathbf{q})\|_2}$$
$$K(\mathbf{p}) = \sum_{i=1}^{N} \frac{\mathbf{p}_i^2(\mathbf{p})}{2m_i}$$

G being the gravitational constant, for each cartesian $\mathbf{p}_i$ and $\mathbf{r}_i$ as functions of the generalized coordinates $\mathbf{p}$ and $\mathbf{q}$ respectively within our phase space $\mathcal{P}$. We also remind the reader of the properties of this system in the appendix section.

Hence, we are looking for a universal approximator that can resemble the properties and mechanics described by this system:

$$\mathcal{H}_\theta \colon \mathcal{P} \to \mathbb{R}$$
$$\dot{\mathbf{q}} = \frac{\partial \mathcal{H}}{\partial \theta}\frac{\partial \theta}{\partial \mathbf{p}} = \nabla_{\mathbf{p}}\nabla_\theta \mathcal{H}_\theta$$
$$\dot{\mathbf{p}} = -\frac{\partial \mathcal{H}}{\partial \theta}\frac{\partial \theta}{\partial \mathbf{q}} = -\nabla_{\mathbf{q}}\nabla_\theta \mathcal{H}_\theta$$

We finally define the following loss function to solve the system:

$$\mathcal{L}_{PDE} = \left\| \int_{\mathcal{P}} \dot{\mathbf{p}} + \nabla_{\mathbf{q}}\nabla_\theta \mathcal{H}_\theta d(\boldsymbol{\lambda}(t)) \right\|^2 + \left\| \int_{\mathcal{P}} \dot{\mathbf{q}} - \nabla_{\mathbf{p}}\nabla_\theta \mathcal{H}_\theta d(\boldsymbol{\lambda}(t)) \right\|^2$$

(4)

for some $\boldsymbol{\lambda}(t) \colon \mathbb{R}^+ \to \mathbb{R}^N$ defining the paths of the N bodies.

The initial conditions will be defined randomly for each iteration. Finally, the loss function is:

$$\mathcal{L} = \alpha_0 \mathcal{L}_{PDE} + \alpha_1 \mathcal{L}_{Boundary}$$

The present work's hypothesis is the existence of some function that embodies the whole dynamics of the system.

4 Architecture

5 Results

6 Discussion

7 Conclusion

8 Appendix

Appendix A The Principle of Least Action

Appendix B The Hamiltonian Formulation