1 Kalman Filter Equations

The Kalman filter is a mathematical algorithm used to estimate the state of a system based on a series of noisy measurements. The filter operates on a set of equations that describe the dynamics of the system being measured, as well as the noise that affects the measurements. The filter uses a recursive algorithm to estimate the state of the system, based on a sequence of measurements and predictions.

The basic idea behind the Kalman filter is to:

- Combine a prediction of the system state based on its past behavior with a measurement of the system state based on noisy sensor data.
- Calculate an optimal estimate of the true state of the system based on the relative weights of the prediction and the measurement.

The Kalman filter equations can be divided into two main steps:

1.1 Prediction Step

In the prediction step:

- We predict the state of the system at the current time step using the state transition matrix and the previous estimate of the state.
- We predict the error covariance matrix at the current time step using the process noise covariance matrix and the previous estimate of the error covariance matrix.

The equations for the prediction step are:

$$\hat{x}_k = A\hat{x}_{k-1}$$
$$P_k = AP_{k-1}A^T + Q$$

Here, $\hat{x}_k$ is the estimated state of the system at time step k, P_k is the estimated error covariance matrix at time step k, A is the state transition matrix, Q is the process noise covariance matrix, and $\hat{x}_{k-1}$ and P_{k-1} are the previous estimates of the state and error covariance matrix, respectively.

1.2 Update Step

In the update step:

- We use the measurement matrix and the measurement noise covariance matrix to calculate the Kalman gain K_k.
- We update our estimate of the state and error covariance matrix using the measured value, the estimated state, and the Kalman gain.

The equations for the update step are:

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\begin{aligned}
K_k &= P_k C^T (C P_k C^T + R)^{-1} \\
\hat{x}_k &= \hat{x}_k + K_k (y_k - C \hat{x}_k) \\
P_k &= (I - K_k C) P_k
\end{aligned}
$$

Here, C is the measurement matrix, R is the measurement noise covariance matrix, y_k is the measured value at time step k, and $\hat{x}_k$ and P_k are the updated estimates of the state and error covariance matrix, respectively.

These equations form the core of the Kalman filter algorithm, and they are used to estimate the state of the system at each time step based on the available measurements and the dynamics of the system.