Norwegian University of Science and Technology
Department for Computer Science

Methods in Artificial Intelligence TDT4171 Spring 2021

Jørgen Rosager
Exercise 1

$\boxed{1}$ **a)**

$$\begin{aligned} P(Siblings \leq 2) &= P(Siblings = 0) + P(Siblings = 1) + P(Siblings = 2) \\ &= 0.15 + 0.49 + 0.27 \\ &= 0.91 \end{aligned}$$

b)

$$\begin{aligned} P(Siblings > 2 | Siblings \geq 1) &= \frac{P(Siblings > 2 \wedge Siblings \geq 1)}{P(Siblings \geq 1)} \\ &= \frac{P(Siblings > 2)}{P(Siblings \geq 1)} \\ &= \frac{1 - P(Siblings \leq 2)}{1 - P(Siblings = 0)} \\ &= \frac{0.09}{0.85} = \frac{9}{85} \approx 0.1059 \end{aligned}$$

c) Let S_1, S_2 and S_3 denote the siblings for each of the friends.
There are 3 permutations for who can have 3 siblings and the other 0 siblings, only 1 permutation where each have 1 sibling and $3 \cdot 2 = 6$ permutations for who can have 2 siblings, 1 sibling and 0 sibling (we first have 3 choices for the friend with 2 siblings, then 2 choices for the one with 1 sibling). This gives us:

$$\begin{aligned} P(S_1 + S_2 + S_3 = 3) &= 3P(S_1 = 3)P(S_2 = 0)P(S_3 = 0) \\ &+ P(S_1 = 1)P(S_2 = 1)P(S_3 = 1) \\ &+ 6P(S_1 = 2)P(S_2 = 1)P(S_3 = 0) \\ &= 3 \cdot 0.06 \cdot 0.15^2 + 0.49^3 + 6 \cdot 0.27 \cdot 0.49 \cdot 0.15 \\ &= 0.240769 \end{aligned}$$

d) Let S_E and S_J denote siblings for Emma and Jacob respectfully. We have:

$$\begin{aligned} P(S_E = 0 | \; S_E + S_J = 3) &= \frac{P(S_E = 0 \wedge S_E + S_J = 3)}{P(S_E + S_J = 3)} \\ P(S_E = 0 \wedge S_E + S_J = 3) &= P(S_E = 0)P(S_J = 3) \\ P(S_E + S_J = 3) &= 2P(S_E = 0)P(S_J = 3) + 2P(S_E = 1)P(S_J = 2) \\ \implies P(S_E = 0 | \; S_E + S_J = 3) &= \frac{P(S_E = 0)P(S_J = 3)}{2P(S_E = 0)P(S_J = 3) + 2P(S_E = 1)P(S_J = 2)} \\ &= \frac{0.15 \cdot 0.06}{2 \cdot 0.15 \cdot 0.06 + 2 \cdot 0.49 \cdot 0.27} \\ &= \frac{5}{157} \approx 0.031847 \end{aligned}$$

2 **a)** Every node can be represented by 2^k numbers where k is the number of parent nodes when each variable has a Boolean state. This gives us the following:

Variable	Numbers needed
A	1
B	2
C	2
D	2
E	4
F	4
G	2
H	1
Sum	18

As we can see the sum is 18 numbers needed and the statement is thus **true**.

b) **False**, since both G and A have E as an descendant and we are not given any evidence they are not independent.

c) **False**, since E is not given both its parents it can't be conditionally independent from H.

d) **True**, since E is given both its parents it is conditionally independent from H.

3 **a)**

$$\begin{aligned} P(b) &= P(b|a)P(a) + P(b|\neg a)P(\neg a) \\ &= 0.44 \\ \implies P(\neg b) &= 1 - P(b) = 0.56 \end{aligned}$$

b)

$$\begin{aligned} P(d) &= P(d|b)P(b) + P(d|\neg b)P(\neg b) \\ &= 0.712 \end{aligned}$$

c)

$$\begin{aligned} P(B|\neg d) &= \alpha\langle P(\neg d|b)P(b), P(\neg d|\neg b)P(\neg b)\rangle \\ &= \alpha\langle \frac{22}{125}, \frac{14}{125}\rangle = \langle \frac{11}{18}, \frac{7}{18}\rangle \\ \implies P(c|\neg d) &= P(c|b)P(b|\neg d) + P(c|\neg b)P(\neg b|\neg d) \\ &= \frac{11}{180} + \frac{7}{60} = \frac{8}{45} \approx 0.178 \end{aligned}$$

α is the normalizing factor.

d)

$$\begin{aligned} P(A|\neg c, d) &= \alpha\langle P(a)\sum_b P(\neg b|a)P(\neg c|b)P(d|b), P(\neg a)\sum_b P(\neg b|\neg a)P(\neg c|b)P(d|b)\rangle \\ &= \alpha\langle \frac{11}{25}, \frac{139}{1250}\rangle = \langle \frac{550}{689}, \frac{139}{689}\rangle \approx \langle 0.7983, 0.2017\rangle \end{aligned}$$

α is the normalizing factor.

4 **a)**

b)

c)