

1 a)

$$\begin{aligned}P(\text{Siblings} \leq 2) &= P(\text{Siblings} = 0) + P(\text{Siblings} = 1) + P(\text{Siblings} = 2) \\&= 0.15 + 0.49 + 0.27 \\&= 0.91\end{aligned}$$

b)

$$\begin{aligned}P(\text{Siblings} > 2 | \text{Siblings} \geq 1) &= \frac{P(\text{Siblings} > 2 \wedge \text{Siblings} \geq 1)}{P(\text{Siblings} \geq 1)} \\&= \frac{P(\text{Siblings} > 2)}{P(\text{Siblings} \geq 1)} \\&= \frac{1 - P(\text{Siblings} \leq 2)}{1 - P(\text{Siblings} = 0)} \\&= \frac{0.09}{0.85} = \frac{9}{85} \approx 0.1059\end{aligned}$$

c) Let S_1 , S_2 and S_3 denote the siblings for each of the friends.

There are 3 permutations for who can have 3 siblings and the other 0 siblings, only 1 permutation where each have 1 sibling and $3 \cdot 2 = 6$ permutations for who can have 2 siblings, 1 sibling and 0 sibling (we first have 3 choices for the friend with 2 siblings, then 2 choices for the one with 1 sibling). This gives us:

$$\begin{aligned}P(S_1 + S_2 + S_3 = 3) &= 3P(S_1 = 3)P(S_2 = 0)P(S_3 = 0) \\&\quad + P(S_1 = 1)P(S_2 = 1)P(S_3 = 1) \\&\quad + 6P(S_1 = 2)P(S_2 = 1)P(S_3 = 0) \\&= 3 \cdot 0.06 \cdot 0.15^2 + 0.49^3 + 6 \cdot 0.27 \cdot 0.49 \cdot 0.15 \\&= 0.240769\end{aligned}$$

d) Let S_E and S_J denote siblings for Emma and Jacob respectfully. We have:

$$\begin{aligned}P(S_E = 0 | S_E + S_J = 3) &= \frac{P(S_E = 0 \wedge S_E + S_J = 3)}{P(S_E + S_J = 3)} \\P(S_E = 0 \wedge S_E + S_J = 3) &= P(S_E = 0)P(S_J = 3) \\P(S_E + S_J = 3) &= 2P(S_E = 0)P(S_J = 3) + 2P(S_E = 1)P(S_J = 2) \\ \implies P(S_E = 0 | S_E + S_J = 3) &= \frac{P(S_E = 0)P(S_J = 3)}{2P(S_E = 0)P(S_J = 3) + 2P(S_E = 1)P(S_J = 2)} \\&= \frac{0.15 \cdot 0.06}{2 \cdot 0.15 \cdot 0.06 + 2 \cdot 0.49 \cdot 0.27} \\&= \frac{5}{157} \approx 0.031847\end{aligned}$$

- 2 a) Every node can be represented by 2^k numbers where k is the number of parent nodes when each variable has a Boolean state. This gives us the following:

Variable	Numbers needed
A	1
B	2
C	2
D	2
E	4
F	4
G	2
H	1
Sum	18

As we can see the sum is 18 numbers needed and the statement is thus **true**.

- b) **False**, since both G and A have E as an descendant and we are not given any evidence they are not independent.
- c) **False**, since E is not given both its parents it can't be conditionally independent from H .
- d) **True**, since E is given both its parents it is conditionally independent from H .

- 3 a)

$$\begin{aligned}
 P(b) &= P(b|a)P(a) + P(b|\neg a)P(\neg a) \\
 &= 0.44 \\
 \implies P(\neg b) &= 1 - P(b) = 0.56
 \end{aligned}$$

- b)

$$\begin{aligned}
 P(d) &= P(d|b)P(b) + P(d|\neg b)P(\neg b) \\
 &= 0.712
 \end{aligned}$$

- c)

$$\begin{aligned}
 P(B|\neg d) &= \alpha \langle P(\neg d|b)P(b), P(\neg d|\neg b)P(\neg b) \rangle \\
 &= \alpha \langle \frac{22}{125}, \frac{14}{125} \rangle = \langle \frac{11}{18}, \frac{7}{18} \rangle \\
 \implies P(c|\neg d) &= P(c|b)P(b|\neg d) + P(c|\neg b)P(\neg b|\neg d) \\
 &= \frac{11}{180} + \frac{7}{60} = \frac{8}{45} \approx 0.178
 \end{aligned}$$

α is the normalizing factor.

- d)

$$\begin{aligned}
 P(A|\neg c, d) &= \alpha \langle P(a) \sum_b P(\neg b|a)P(\neg c|b)P(d|b), P(\neg a) \sum_b P(\neg b|\neg a)P(\neg c|b)P(d|b) \rangle \\
 &= \alpha \langle \frac{11}{25}, \frac{139}{1250} \rangle = \langle \frac{550}{689}, \frac{139}{689} \rangle \approx \langle 0.7983, 0.2017 \rangle
 \end{aligned}$$

α is the normalizing factor.

- 4 c) The Monty-Hall problem can be described by a bayesian network where *OpenedByHost* is conditional based on *Prize* and *ChosenByGuest*:

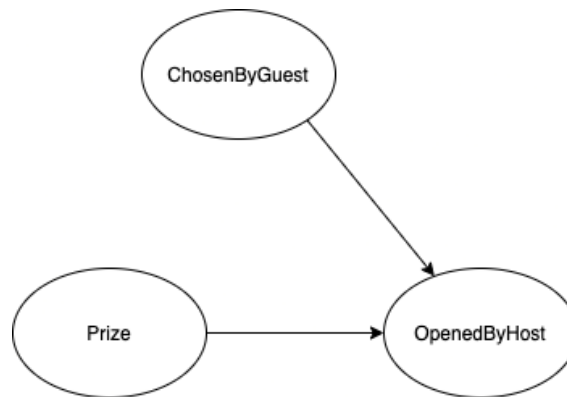


Figure 1: Bayesian network for the Monty Hall-problem.

The conditional probability tables are given by:

	$P(Prize)$
<i>Door1</i>	0.333
<i>Door2</i>	0.333
<i>Door3</i>	0.333

Table 1: Probability distribution for *Prize*.

	$P(ChosenByGuest)$
<i>Door1</i>	0.333
<i>Door2</i>	0.333
<i>Door3</i>	0.333

Table 2: Probability distribution for *ChosenByGuest*.

<i>Prize</i>	0	1	2	0	1	2	0	1	2
<i>ChosenByGuest</i>	0	0	0	1	1	1	2	2	2
<i>Door1</i>	0	0	0	0.5	0.5	1	0	1	0.5
<i>Door2</i>	0.5	0	1	0	0	0	1	0	0.5
<i>Door3</i>	0.5	1	0	0.5	0.5	0	0	0	0

Table 3: Probability distribution for *ChosenByGuest*.

$$P(Prize|ChosenByGuest = 1, OpenedByHost = 3) = \langle 0.3333, 0.6667, 0.0000 \rangle$$

Based on this it is of the interest of the guest to switch their choice.