Methods in Artificial
Intelligence
TDT4171
Spring 2021

Jørgen Rosager
Exercise 1

Norwegian University of Science and
Technology
Department for Computer Science

1 **a)**

$$P(Siblings \leq 2) = P(Siblings = 0) + P(Siblings = 1) + P(Siblings = 2)$$
$$= 0.15 + 0.49 + 0.27$$
$$= 0.91$$

b)

$$P(Siblings > 2 | Siblings \geq 1) = \frac{P(Siblings > 2 \land Siblings \geq 1)}{P(Siblings \geq 1)}$$
$$= \frac{P(Siblings > 2)}{P(Siblings \geq 1)}$$
$$= \frac{1 - P(Siblings \leq 2)}{1 - P(Siblings = 0)}$$
$$= \frac{0.09}{0.85} = \frac{9}{85} \approx 0.1059$$

c) Let S_1, S_2 and S_3 denote the siblings for each of the friends.

There are 3 permutations for who can have 3 siblings and the other 0 siblings, only 1 permutation where each have 1 sibling and $3 \cdot 2 = 6$ permutations for who can have 2 siblings, 1 sibling and 0 sibling (we first have 3 choices for the friend with 2 siblings, then 2 choices for the one with 1 sibling). This gives us:

$$P(S_1 + S_2 + S_3 = 3) = 3P(S_1 = 3)P(S_2 = 0)P(S_3 = 0)$$
$$+ P(S_1 = 1)P(S_2 = 1)P(S_3 = 1)$$
$$+ 6P(S_1 = 2)P(S_2 = 1)P(S_3 = 0)$$
$$= 3 \cdot 0.06 \cdot 0.15^2 + 0.49^3 + 6 \cdot 0.27 \cdot 0.49 \cdot 0.15$$
$$= 0.240769$$

d) Let S_E and S_J denote siblings for Emma and Jacob respectfully. We have:

$$P(S_E = 0 | \ S_E + S_J = 3) = \frac{P(S_E = 0 \land S_E + S_J = 3)}{P(S_E + S_J = 3)}$$
$$P(S_E = 0 \land S_E + S_J = 3) = P(S_E = 0)P(S_J = 3)$$
$$P(S_E + S_J = 3) = 2P(S_E = 0)P(S_J = 3) + 2P(S_E = 1)P(S_J = 2)$$
$$\implies P(S_E = 0 | \ S_E + S_J = 3) = \frac{P(S_E = 0)P(S_J = 3)}{2P(S_E = 0)P(S_J = 3) + 2P(S_E = 1)P(S_J = 2)}$$
$$= \frac{0.15 \cdot 0.06}{2 \cdot 0.15 \cdot 0.06 + 2 \cdot 0.49 \cdot 0.27}$$
$$= \frac{5}{157} \approx 0.031847$$

2   **a)** Every node can be represented by 2^k numbers where k is the number of parent nodes when each variable has a Boolean state. This gives us the following:

Variable	Numbers needed
A	1
B	2
C	2
D	2
E	4
F	4
G	2
H	1
Sum	18

As we can see the sum is 18 numbers needed and the statement is thus **true**.

b) **False**, since both G and A have E as an descendant and we are not given any evidence they are not independent.

c) **False**, since E is not given both its parents it can't be conditionally independent from H.

d) **True**, since E is given both its parents it is conditionally independent from H.

3   **a)**

     b)

     c)

     d)

4   **a)**

     b)

     c)