



1 a)

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0.1 & -0.79 & 1.78 \end{bmatrix}$$

$$\begin{aligned} \implies \det(A - \lambda I) &= 0 \\ \implies -\lambda(-\lambda(1.78 - \lambda) + 0.79) &= 0 \\ \implies \lambda_1 = 0.844 \wedge \lambda_2 = 0.936 \end{aligned}$$

Since $|\lambda_i| \leq 1$ for $i \in \{1, 2\}$ the system is stable.

- b) $n_x = 3$ since x_t is a vector multiplied by A which is a 3×3 matrix.
 $n_u = 1$ since u_t is a vector multiplied by B which is a 3×1 matrix.
 Q and R is defined as:

$$Q = 2 \begin{bmatrix} C & 0 \\ 0 & 0 \end{bmatrix} = 2 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$
$$R = 2r = 2$$

The multiplication by 2 comes because the objective function is divided by 2 in the formulation.

- c) Since Q is positive semidefinite and R is positive definitive this is a convex QP problem. The convexity depends on Q , R and C (because Q depends on C).
- d) The state space model is:

$$\begin{aligned} x_{t+1} &= Ax_t + Bu_t \\ \implies x_1 - Ax_0 - Bu_0 &= 0 \Leftrightarrow Ix_1 - Ax_0 - Bu_0 = 0 \\ x_2 - Ax_1 - Bu_1 &= 0 \Leftrightarrow Ix_2 - Ax_1 - Bu_1 = 0 \\ &\vdots \\ x_N - Ax_{N-1} - Bu_{N-1} &= 0 \Leftrightarrow Ix_N - Ax_{N-1} - Bu_{N-1} = 0 \end{aligned}$$

With $z = [x_1^\top, x_2^\top, \dots, x_N^\top, u_0^\top, u_1^\top, \dots, u_{N-1}^\top]$ and moving Ax_0 to the other side of the equation since x_0 isn't in z :

$$b_{eq} = \begin{bmatrix} Ax_0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$A_{eq} = \begin{bmatrix} I & 0 & \dots & \dots & 0 & -B & 0 & \dots & \dots & 0 \\ -A & I & \ddots & & \vdots & 0 & \ddots & \ddots & & \vdots \\ 0 & -A & \ddots & \ddots & \vdots & 0 & \ddots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & 0 & \vdots & & \ddots & \ddots & 0 \\ 0 & \dots & 0 & -A & I & 0 & \dots & \dots & 0 & -B \end{bmatrix}$$

The KKT system is then (from the book):

$$\begin{bmatrix} G & -A_{eq}^\top \\ A_{eq} & 0 \end{bmatrix} \begin{bmatrix} z^* \\ \lambda^* \end{bmatrix} = \begin{bmatrix} 0 \\ b_{eq} \end{bmatrix}$$

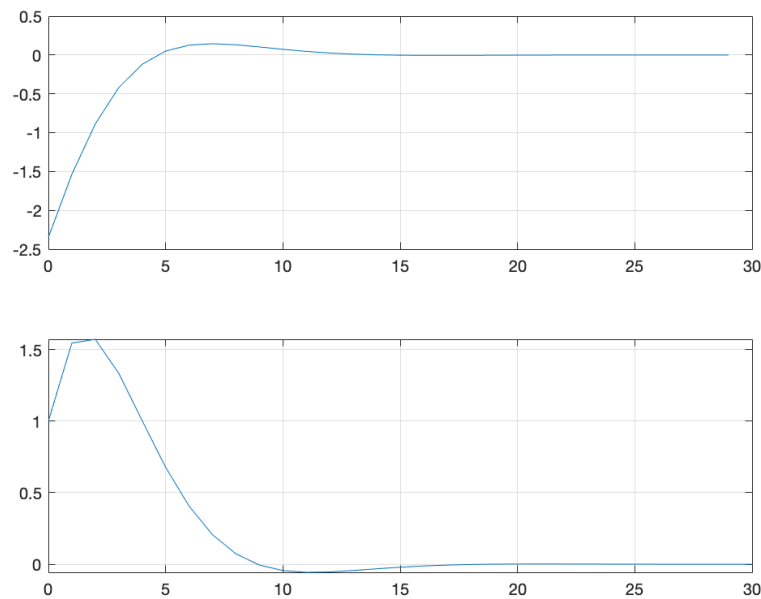


Figure 1: Solution by solving the KKT conditions.

- e) The plots in Figure 2, 3 and 4 shows the results with the `quadprog` function with different values of r . As we can see the solution is the same as in Task d when $r = 1$. When we make r smaller we see that the control inputs are steeper, this is because we allow for larger values of u_i in the objective function, when we make r larger the control inputs are smoother since larger control inputs are punished more.

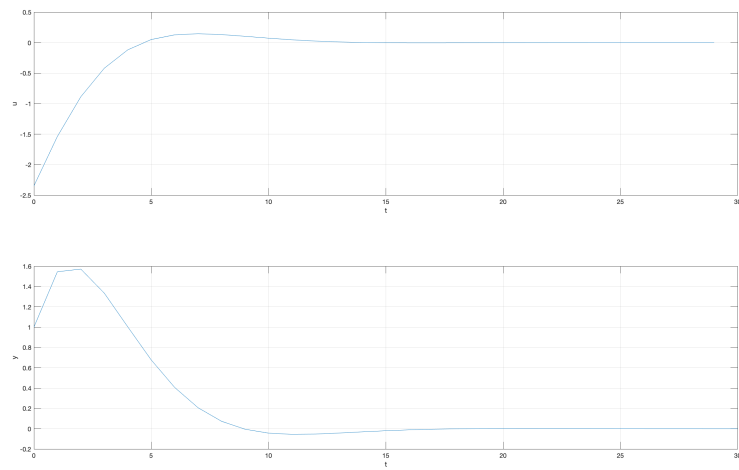


Figure 2: u_t and y_t with $r = 1$.

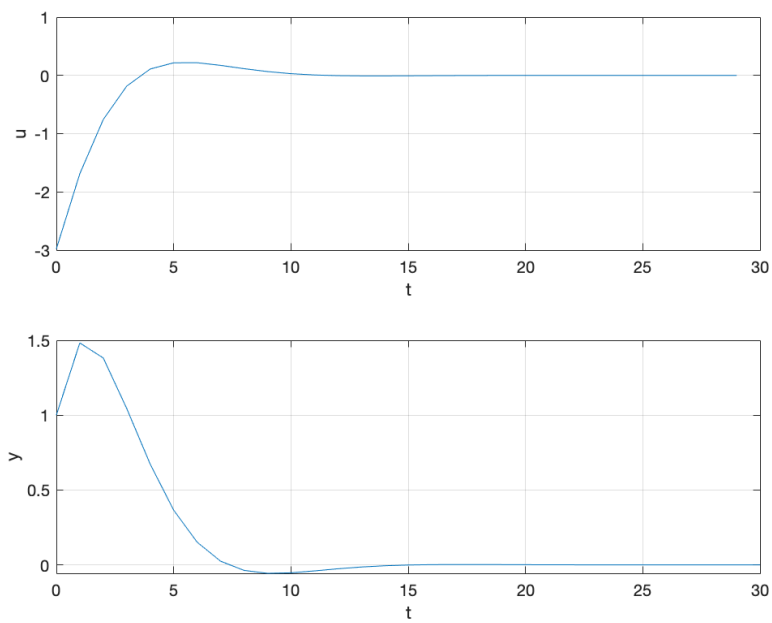


Figure 3: u_t and y_t with $r = 0.5$.

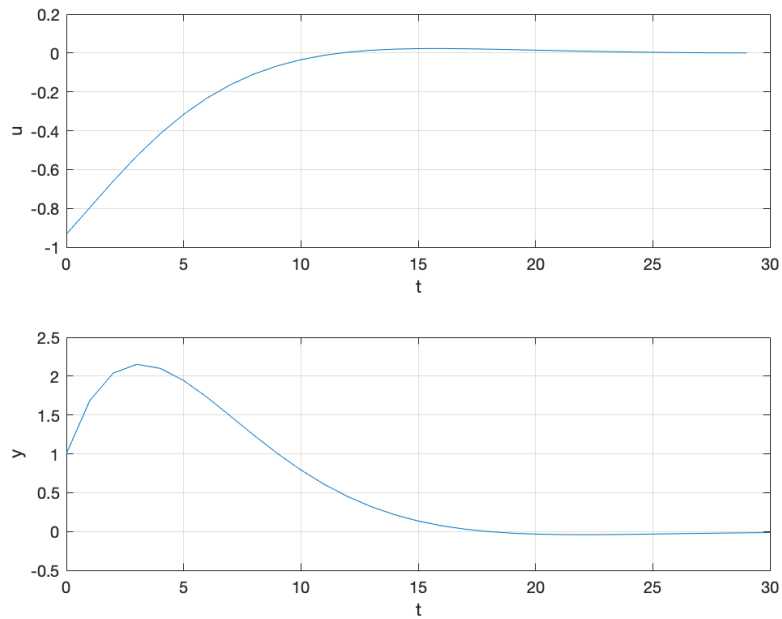


Figure 4: u_t and y_t with $r = 10$.

- f) The resulting plot is in Figure 5. The number of iterations with `quadprog` was 5, this increase comes because the QP is now inequality constrained, which is harder to solve.

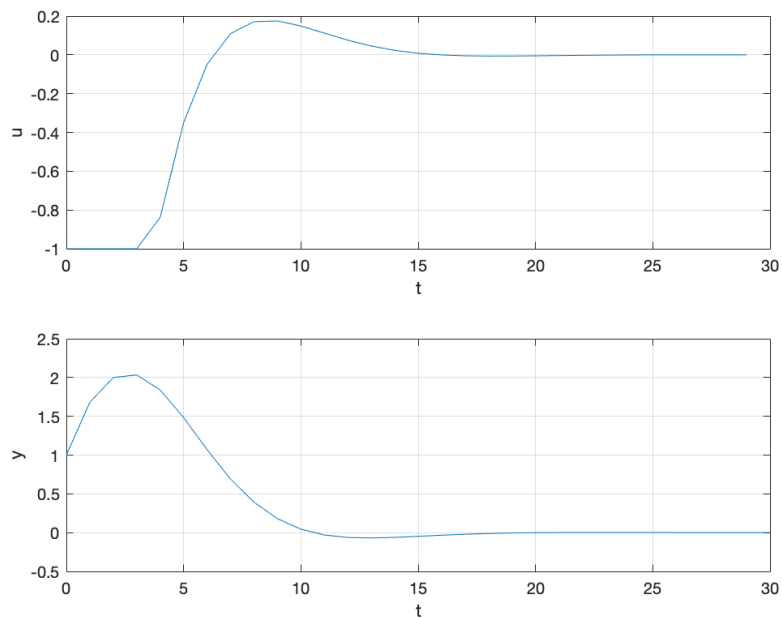


Figure 5: u_t and y_t for inequality constrained control input.

- 2 a) Model predictive control is a regulation method for finding the optimal inputs of

control at an instant. MPC does this by using finite-horizon open-loop control at a certain sampling. The main advantage of MPC is that it will keep the future in mind when optimizing (thus "predictive"). Since MPC is open-loop it doesn't directly involve feedback, but MPC works by sampling at a certain rate and is thus able to include feedback at every sampling step.

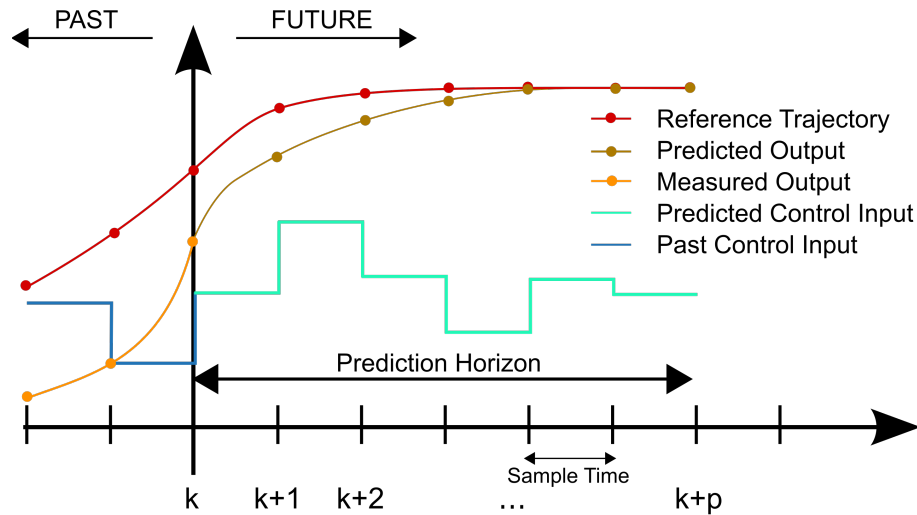


Figure 6: This figure shows how the MPC will optimize over a finite-horizon timeline and predicting the outcome of the control inputs to achieve an optimal sequence of inputs.

- b) The resulting plot is found in Figure 7. As we can see this solution is very similar to our open-loop prediction over the whole horizon in Task f.

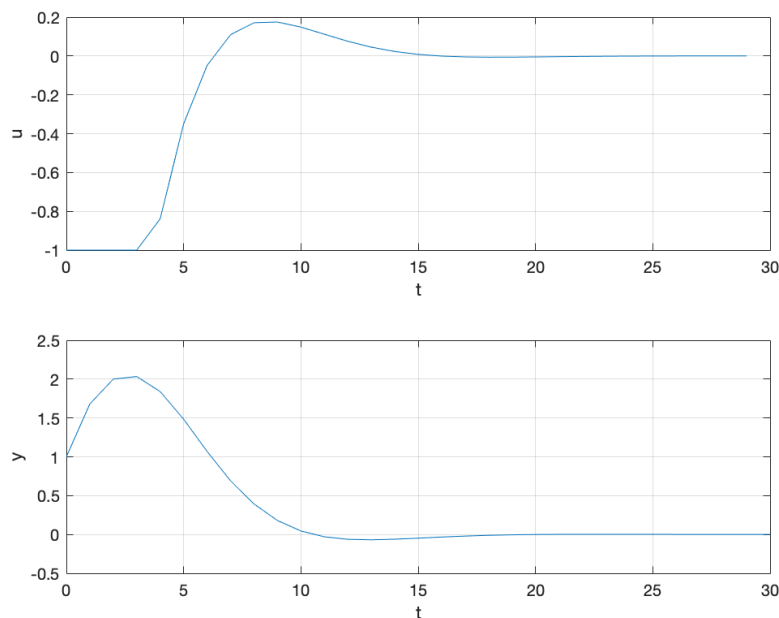


Figure 7: u_t and y_t for MPC with horizon of 30.

- c) The resulting plot is found in Figure 8. As we can see the MPC must use more input into the system and it doesn't get the y_t to zero before $t = 30$, this

happens because the differences between the model in MPC and the real plant makes the MPC's control inputs non-optimal for the real plant.

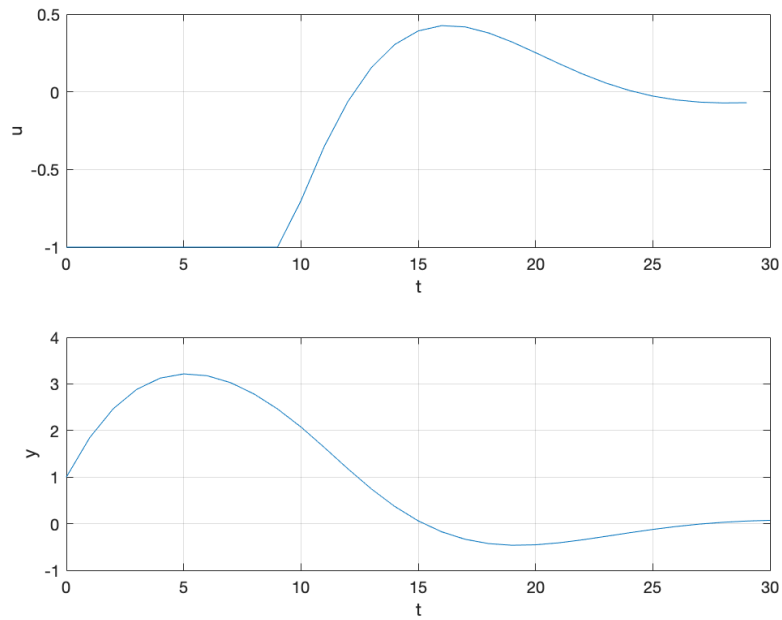


Figure 8: u_t and y_t for MPC with horizon of 30 and different plant between MPC model and simulation.