1 **a)**

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0.1 & -0.79 & 1.78 \end{bmatrix}$$

$$\implies \det(A - \lambda I) = 0$$

$$\implies -\lambda(-\lambda(1.78 - \lambda) + 0.79) = 0$$

$$\implies \lambda_1 = 0.844 \wedge \lambda_2 = 0.936$$

Since $|\lambda_i| \leq 1$ for $i \in \{1, 2\}$ the system is stable.

b) $n_x = 3$ since x_t is a vector multiplied by A which is a 3×3 matrix.
$n_u = 1$ since u_t is a vector multiplied by B which is a 3×1 matrix.
Q and R is defined as:

$$Q = 2\begin{bmatrix} C & 0 \\ 0 & 0 \end{bmatrix} = 2\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$R = 2r = 2$$

The multiplication by 2 comes because the objective function is divided by 2 in the formulation.

c) Since Q is positive semidefinite and R is positive definitive this is a convex QP problem. The convexity depends on Q, R and C (because Q depends on C).

d) The state space model is:

$$x_{t+1} = Ax_t + Bu_t$$

$$\implies x_1 - Ax_0 - Bu_0 = 0 \Leftrightarrow Ix_1 - Ax_0 - Bu_0 = 0$$

$$x_2 - Ax_1 - Bu_1 = 0 \Leftrightarrow Ix_2 - Ax_1 - Bu_1 = 0$$

$$\vdots$$

$$x_N - Ax_{N-1} - Bu_{N-1} = 0 \Leftrightarrow Ix_N - Ax_{N-1} - Bu_{N-1} = 0$$

With $z = \left[x_1^\top, x_2^\top, ..., x_N^\top, u_0^\top, u_1^\top, ..., u_{N-1}^\top\right]$ and moving Ax_0 to the other side of the equation since x_0 isn't in z:

$$b_{eq} = \begin{bmatrix} Ax_0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$A_{eq} = \begin{bmatrix} I & 0 & \dots & \dots & 0 & -B & 0 & \dots & \dots & 0 \\ -A & I & \ddots & & \vdots & 0 & \ddots & \ddots & & \vdots \\ 0 & -A & \ddots & \ddots & \vdots & 0 & \ddots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & 0 & \vdots & & \ddots & \ddots & 0 \\ 0 & \dots & 0 & -A & I & 0 & \dots & \dots & 0 & -B \end{bmatrix}$$

The KKT system is then (from the book):

$$\begin{bmatrix} G & -A_{eq}^\top \\ A_{eq} & 0 \end{bmatrix} \begin{bmatrix} z^\star \\ \lambda^\star \end{bmatrix} = \begin{bmatrix} 0 \\ beq \end{bmatrix}$$

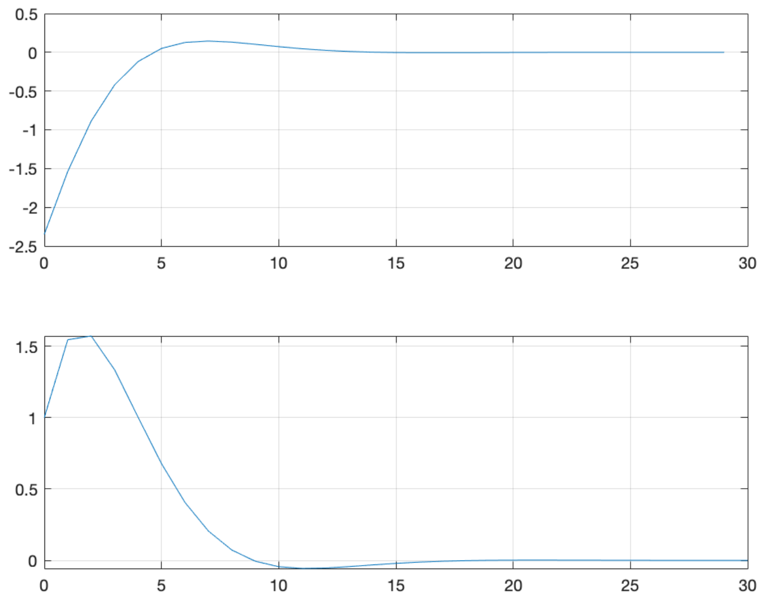

Figure 1: Solution by solving the KKT conditions.

e) The plots in Figure 2, 3 and 4 shows the results with the `quadprog` function with different values of r. As we can see the solution is the same as in Task d when $r = 1$. When we make r smaller we see that the control inputs are steeper, this is because we allow for larger values of u_i in the objective function, when we make r larger the control inputs are smoother since larger control inputs are punished more.

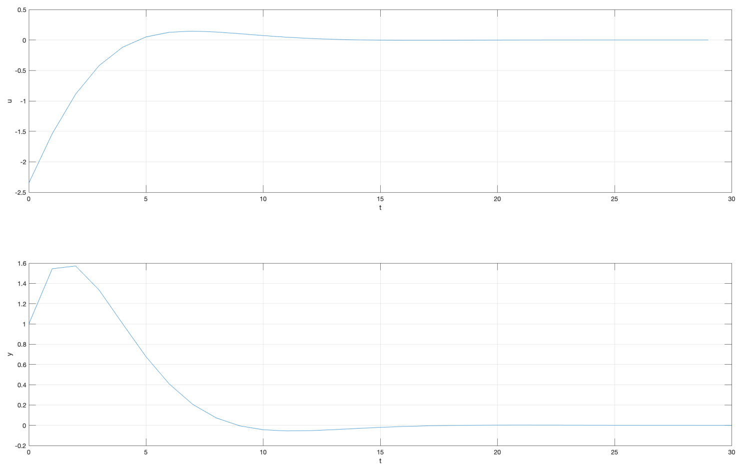

Figure 2: u_t and y_t with $r = 1$.

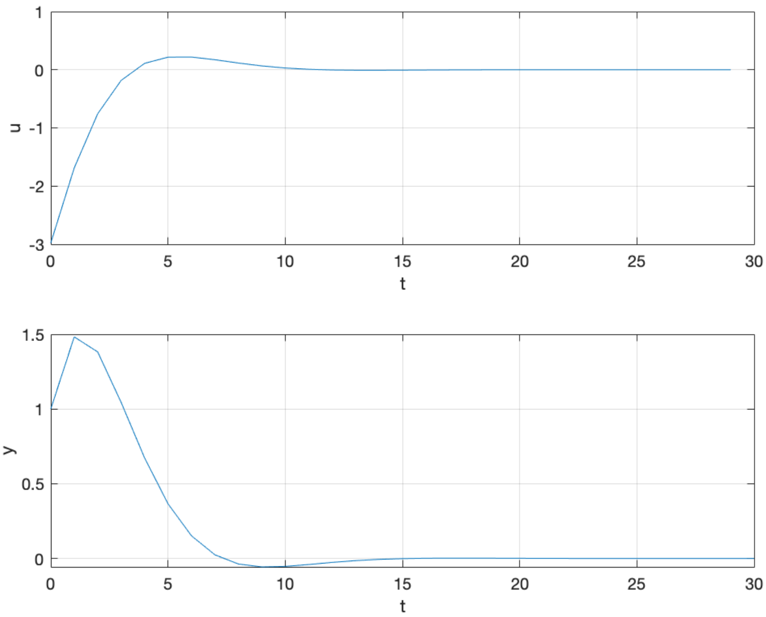

Figure 3: u_t and y_t with $r = 0.5$.

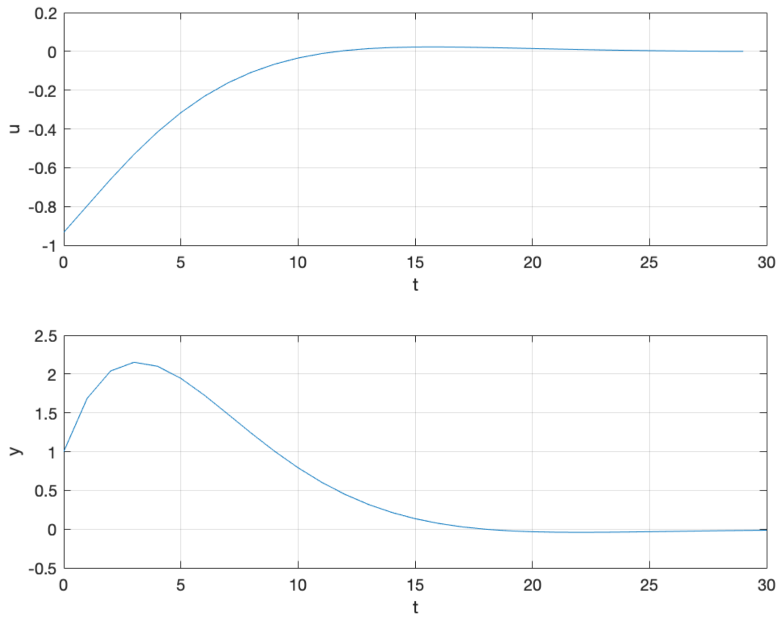

Figure 4: u_t and y_t with $r = 10$.

f) The resulting plot is in Figure 5. The number of iterations with `quadprog` was 5, this increase comes because the QP is now inequality constrained, which is harder to solve.

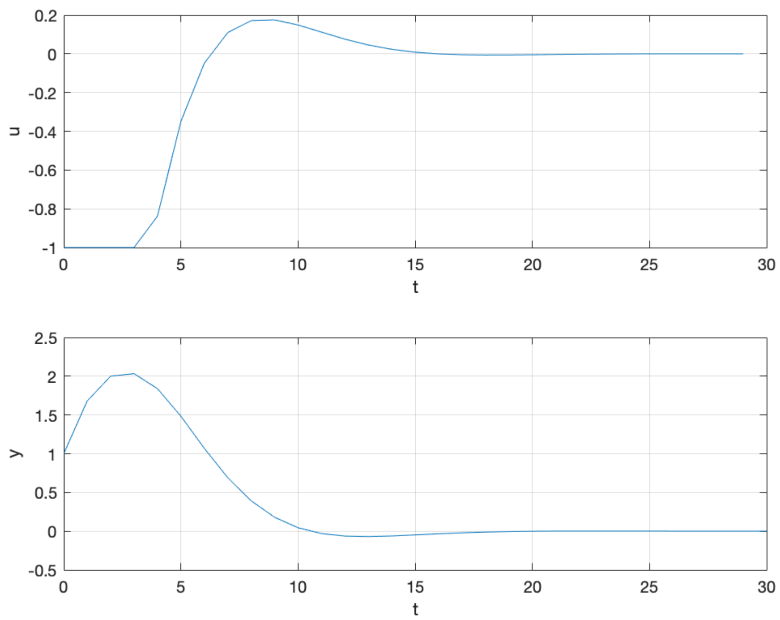

Figure 5: u_t and y_t for inequality constrained control input.

$\boxed{2}$ **a)** Model predictive control is a regulation method for finding the optimal inputs of control at an instant. MPC does this by using finite-horizon open-loop control at a certain sampling. The main advantage of MPC is that it will keep the future in mind when optimizing (thus "predictive"). Since MPC is open-loop it doesn't directly involve feedback, but MPC works by sampling at a certain rate and is thus able to include feedback at every sampling step.

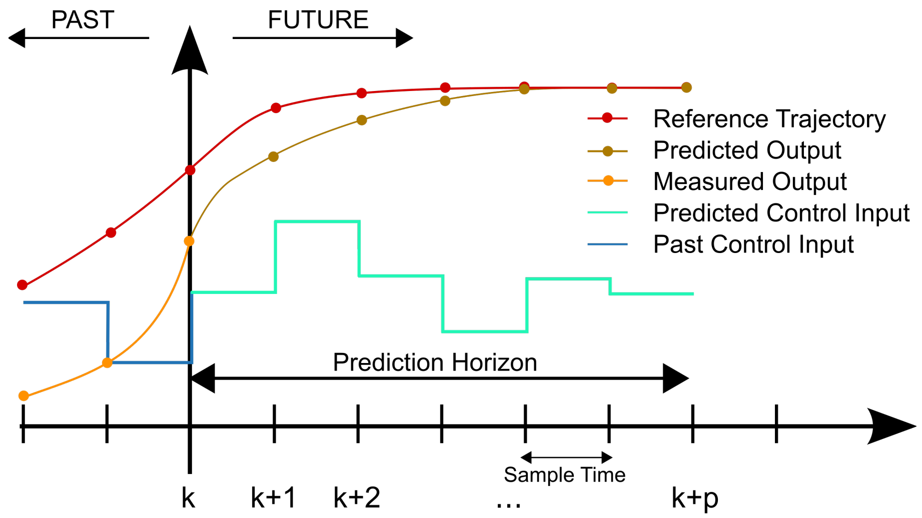

Figure 6: This figure shows how the MPC will optimize over a finite-horizon timeline and predicting the outcome of the control inputs to achieve an optimal sequence of inputs.

b) The resulting plot is found in Figure 7. As we can see this solution is very similar to our open-loop prediction over the whole horizon in Task f.

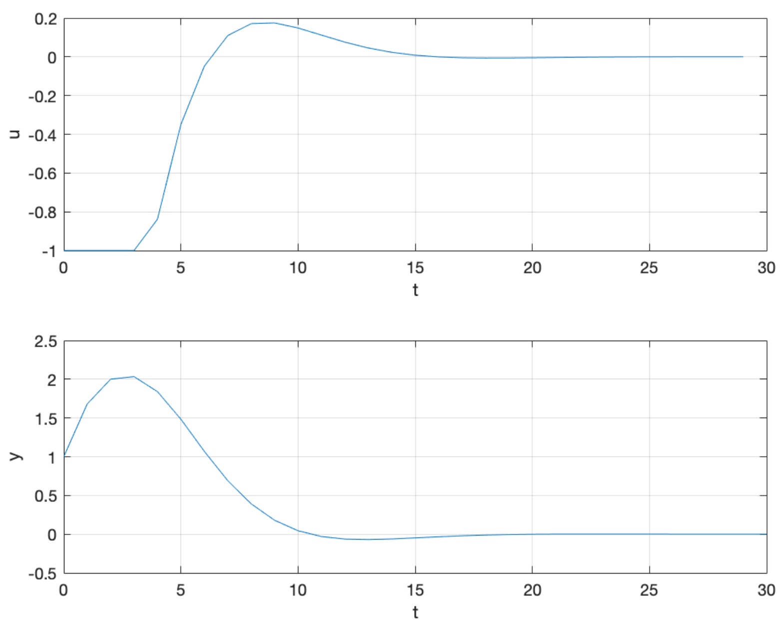

Figure 7: u_t and y_t for MPC with horizon of 30.

c) The resulting plot is found in Figure 8. As we can see the MPC must use more input into the system and it doesn't get the y_t to zero before $t = 30$, this happens because the differences between the model in MPC and the real plant makes the MPC's control inputs non-optimal for the real plant.

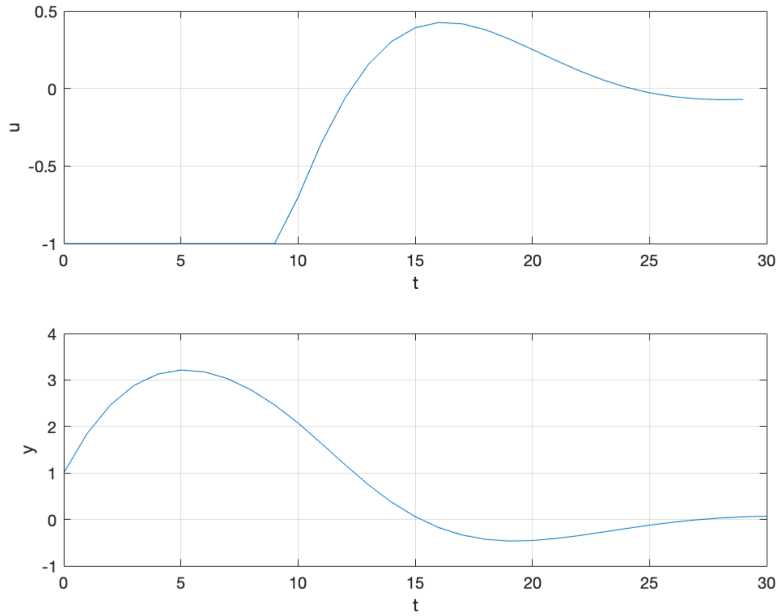

Figure 8: u_t and y_t for MPC with horizon of 30 and different plant between MPC model and simulation.