In this file, we verify that the graph H_{4, 6} (on 26 vertices) has lower occupancy fraction than the graph G_{32} when lambda *is sufficiently large.*
For this, the occupancy fractions of the two graphs are inserted (both are rational functions) **and** *the difference is computed.*
The difference is a rational function whose denominator is always positive, **and** *whose numerator is positive* **for** *x sufficiently large.*
A plot of the difference is shown **and** *the exact threshold (root of the numerator) is computed. Since*

$$\frac{1}{36\,x^{17}}\left(\left(748\,x^{16}+16188\,x^{15}+126234\,x^{14}+580310\,x^{13}+1816977\,x^{12}+4121781\,x^{11}+6940678\,x^{10}+8724027\,x^{9}+8173710\,x^{8}+5692710\,x^{7}+2934351\,x^{6}+1109451\,x^{5}+302397\,x^{4}+57594\,x^{3}+7248\,x^{2}+540\,x+18\right)\right)$$

is a decreasing function **for** $x > 0$, *it is also clear that there is a unique positive root* **for** *the numerator.*
Alternatively, this could be concluded **from** *Descartes' rule of signs as well.*

```
> p26(x) := (x^12 + 12*x^11 + 66*x^10 + 230*x^9 + 576*x^8 + 1128*x^7 + 1890
      *x^6 + 2316*x^5 + 1725*x^4 + 736*x^3 + 174*x^2 + 21*x + 1)*x/(2*x^13
      + 26*x^12 + 156*x^11 + 598*x^10 + 1664*x^9 + 3666*x^8 + 7020*x^7
      + 10036*x^6 + 8970*x^5 + 4784*x^4 + 1508*x^3 + 273*x^2 + 26*x + 1)
```

$$p26 := x \mapsto \left(\left(x^{12}+12\cdot x^{11}+66\cdot x^{10}+230\cdot x^{9}+576\cdot x^{8}+1128\cdot x^{7}+1890\cdot x^{6}+2316\cdot x^{5}+1725\cdot x^{4}+736\cdot x^{3}+174\cdot x^{2}+21\cdot x+1\right)\cdot x\right)\Big/\left(2\cdot x^{13}+26\cdot x^{12}+156\cdot x^{11}+598\cdot x^{10}+1664\cdot x^{9}+3666\cdot x^{8}+7020\cdot x^{7}+10036\cdot x^{6}+8970\cdot x^{5}+4784\cdot x^{4}+1508\cdot x^{3}+273\cdot x^{2}+26\cdot x+1\right) \tag{1}$$

```
> p32(x) := (x^15 + 15*x^14 + 105*x^13 + 468*x^12 + 1509*x^11 + 3795*x^10
      + 7910*x^9 + 14490*x^8 + 21549*x^7 + 22610*x^6 + 15513*x^5 + 6780*x^4
      + 1851*x^3 + 303*x^2 + 27*x + 1)*x/(2*x^16 + 32*x^15 + 240*x^14 + 1152
      *x^13 + 4024*x^12 + 11040*x^11 + 25312*x^10 + 51520*x^9 + 86196*x^8
      + 103360*x^7 + 82736*x^6 + 43392*x^5 + 14808*x^4 + 3232*x^3 + 432*x^2
      + 32*x + 1)
```

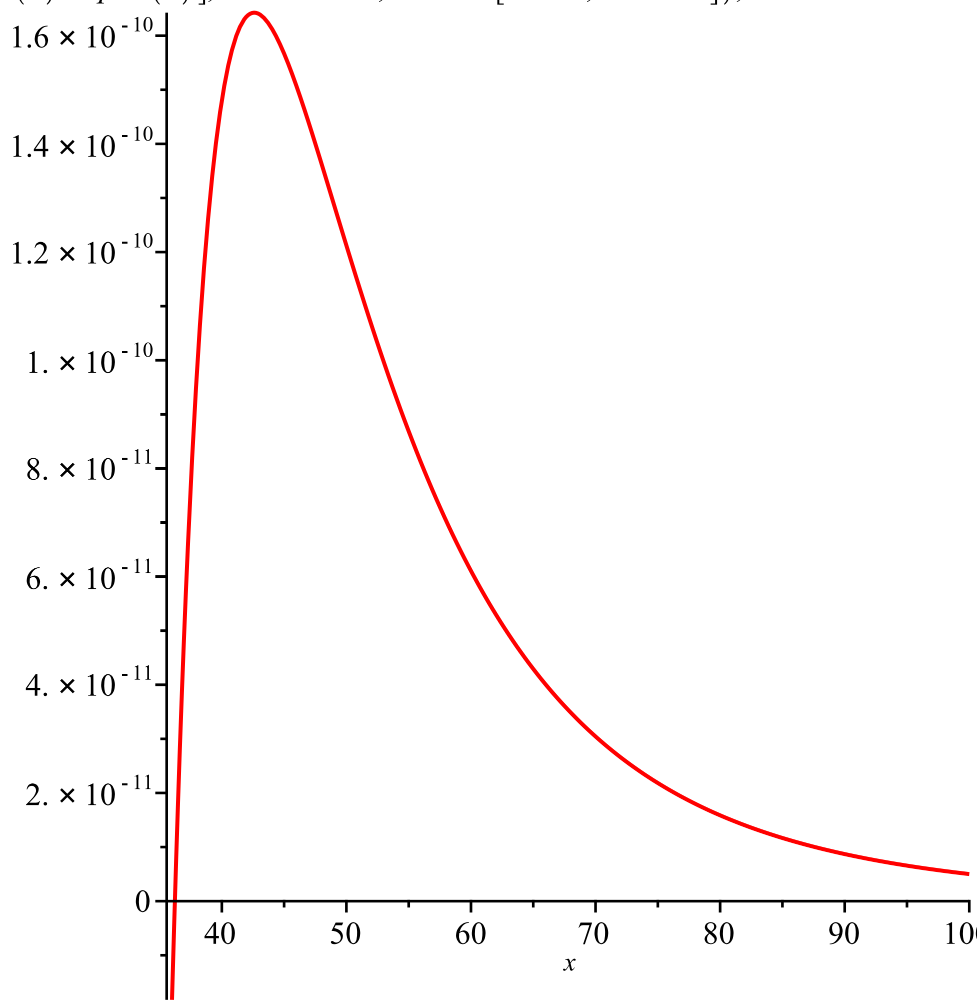
$$p32 := x \mapsto \left(\left(x^{15}+15\cdot x^{14}+105\cdot x^{13}+468\cdot x^{12}+1509\cdot x^{11}+3795\cdot x^{10}+7910\cdot x^{9}+14490\cdot x^{8}+21549\cdot x^{7}+22610\cdot x^{6}+15513\cdot x^{5}+6780\cdot x^{4}+1851\cdot x^{3}+303\cdot x^{2}+27\cdot x+1\right)\cdot x\right)\Big/\left(2\cdot x^{16}+32\cdot x^{15}+240\cdot x^{14}+1152\cdot x^{13}+4024\cdot x^{12}+11040\cdot x^{11}+25312\cdot x^{10}+51520\cdot x^{9}+86196\cdot x^{8}+103360\cdot x^{7}+82736\cdot x^{6}+43392\cdot x^{5}+14808\cdot x^{4}+3232\cdot x^{3}+432\cdot x^{2}+32\cdot x+1\right) \tag{2}$$

```
> simplify(p32(x) - p26(x))
```

$$\left(36\,x^{23}-748\,x^{22}-16188\,x^{21}-126234\,x^{20}-580310\,x^{19}-1816977\,x^{18}-4121781\,x^{17}-6940678\,x^{16}-8724027\,x^{15}-8173710\,x^{14}-5692710\,x^{13}-2934351\,x^{12}-1109451\,x^{11}-302397\,x^{10}-57594\,x^{9}-7248\,x^{8}-540\,x^{7}-18\,x^{6}\right)\Big/\Big(4\,\Big(x^{16}+16\,x^{15}+120\,x^{14}+576\,x^{13}+2012\,x^{12}+5520\,x^{11}+12656\,x^{10}+25760\,x^{9}+43098\,x^{8}+51680\,x^{7}+41368\,x^{6} \tag{3}$$

$$+ 21696\,x^5 + 7404\,x^4 + 1616\,x^3 + 216\,x^2 + 16\,x + \frac{1}{2}\Big) \Big(x^{13} + 13\,x^{12} + 78\,x^{11} + 299\,x^{10} + 832\,x^9 + 1833\,x^8 + 3510\,x^7 + 5018\,x^6 + 4485\,x^5 + 2392\,x^4 + 754\,x^3 + \frac{273}{2}\,x^2 + 13\,x + \frac{1}{2}\Big)\Big)$$

> *plot*([*p32*(*x*) − *p26*(*x*)], *x* = 36 ..100., *color* = ["Red", "Green"]);



> *solutions* := [*solve*(36 *x*^23 − 748 *x*^22 − 16188 *x*^21 − 126234 *x*^20 − 580310 *x*^19 − 1816977 *x*^18 − 4121781 *x*^17 − 6940678 *x*^16 − 8724027 *x*^15 − 8173710 *x*^14 − 5692710 *x*^13 − 2934351 *x*^12 − 1109451 *x*^11 − 302397 *x*^10 − 57594 *x*^9 − 7248 *x*^8 − 540 *x*^7 − 18 *x*^6 = 0, {*x*}, *useassumptions*) assuming *x* > 0];

$$solutions := [\{x = RootOf(36\,\_Z^{17} - 748\,\_Z^{16} - 16188\,\_Z^{15} - 126234\,\_Z^{14} - 580310\,\_Z^{13} - 1816977\,\_Z^{12} - 4121781\,\_Z^{11} - 6940678\,\_Z^{10} - 8724027\,\_Z^9 - 8173710\,\_Z^8 - 5692710\,\_Z^7 - 2934351\,\_Z^6 - 1109451\,\_Z^5 - 302397\,\_Z^4 - 57594\,\_Z^3 - 7248\,\_Z^2 - 540\,\_Z - 18, index = 1)\}] \quad (4)$$

>

evalf(*solutions*)

$$[\{x = 36.23052034\}] \quad (5)$$

> $simplify\left(-\frac{1}{x^6}\left(36x^{23} - 748x^{22} - 16188x^{21} - 126234x^{20} - 580310x^{19} - 1816977x^{18} - 4121781x^{17} - 6940678x^{16} - 8724027x^{15} - 8173710x^{14} - 5692710x^{13} - 2934351x^{12} - 1109451x^{11} - 302397x^{10} - 57594x^9 - 7248x^8 - 540x^7 - 18x^6\right)\right)$

$$-36x^{17} + 748x^{16} + 16188x^{15} + 126234x^{14} + 580310x^{13} + 1816977x^{12} + 4121781x^{11} + 6940678x^{10} + 8724027x^9 + 8173710x^8 + 5692710x^7 + 2934351x^6 + 1109451x^5 + 302397x^4 + 57594x^3 + 7248x^2 + 540x + 18 \qquad \textbf{(6)}$$