In this file, we verify that the graph H_{3,
8}
(on 30 vertices) has lower occupancy fraction than the graph G_{38} when lambda *is sufficiently large.*
For this, the occupancy fractions of the two graphs are inserted (both are rational functions) **and** *the difference is computed.*
The difference is a rational function whose denominator is always positive, **and** *whose numerator is positive* **for** *x sufficiently large.*
A plot of the difference is shown **and** *the exact threshold (root of the numerator) is computed.*
By Descartes' rule of signs, we know there is only one positive root.

```
> p38(x) := 1/19*(19*x^18 + 342*x^17 + 3230*x^16 + 21280*x^15 + 111015*x^14
    + 504994*x^13 + 1956006*x^12 + 5685108*x^11 + 11535315*x^10 + 16191780
    *x^9 + 15919299*x^8 + 11109260*x^7 + 5543839*x^6 + 1976646*x^5 + 497800
    *x^4 + 86222*x^3 + 9747*x^2 + 646*x + 19)*x/(2*x^19 + 38*x^18 + 380*x
    ^17 + 2660*x^16 + 14802*x^15 + 72142*x^14 + 300924*x^13 + 947518*x^12
    + 2097330*x^11 + 3238356*x^10 + 3537622*x^9 + 2777315*x^8 + 1583954*x
    ^7 + 658882*x^6 + 199120*x^5 + 43111*x^4 + 6498*x^3 + 646*x^2 + 38*x
    + 1);
```

$$p38 := x \mapsto \left(\left(x^{18} + 18\cdot x^{17} + 170\cdot x^{16} + 1120\cdot x^{15} + \frac{111015}{19}\cdot x^{14} + \frac{504994}{19}\cdot x^{13} + \frac{1956006}{19}\cdot x^{12} + \frac{5685108}{19}\cdot x^{11} + \frac{11535315}{19}\cdot x^{10} + \frac{16191780}{19}\cdot x^{9} + \frac{15919299}{19}\cdot x^{8} + \frac{11109260}{19}\cdot x^{7} + 291781\cdot x^{6} + 104034\cdot x^{5} + 26200\cdot x^{4} + 4538\cdot x^{3} + 513\cdot x^{2} + 34\cdot x + 1\right)\cdot x\right) \Big/ \left(2\cdot x^{19} + 38\cdot x^{18} + 380\cdot x^{17} + 2660\cdot x^{16} + 14802\cdot x^{15} + 72142\cdot x^{14} + 300924\cdot x^{13} + 947518\cdot x^{12} + 2097330\cdot x^{11} + 3238356\cdot x^{10} + 3537622\cdot x^{9} + 2777315\cdot x^{8} + 1583954\cdot x^{7} + 658882\cdot x^{6} + 199120\cdot x^{5} + 43111\cdot x^{4} + 6498\cdot x^{3} + 646\cdot x^{2} + 38\cdot x + 1\right) \quad \textbf{(1)}$$

```
>
> p30(x) := (x^14 + 14*x^13 + 104*x^12 + 548*x^11 + 2354*x^10 + 8316*x^9
    + 19827*x^8 + 29660*x^7 + 28161*x^6 + 17322*x^5 + 6956*x^4 + 1802*x^3
    + 289*x^2 + 26*x + 1)*x/(2*x^15 + 30*x^14 + 240*x^13 + 1370*x^12
    + 6420*x^11 + 24948*x^10 + 66090*x^9 + 111225*x^8 + 120690*x^7 + 86610
    *x^6 + 41736*x^5 + 13515*x^4 + 2890*x^3 + 390*x^2 + 30*x + 1)
```

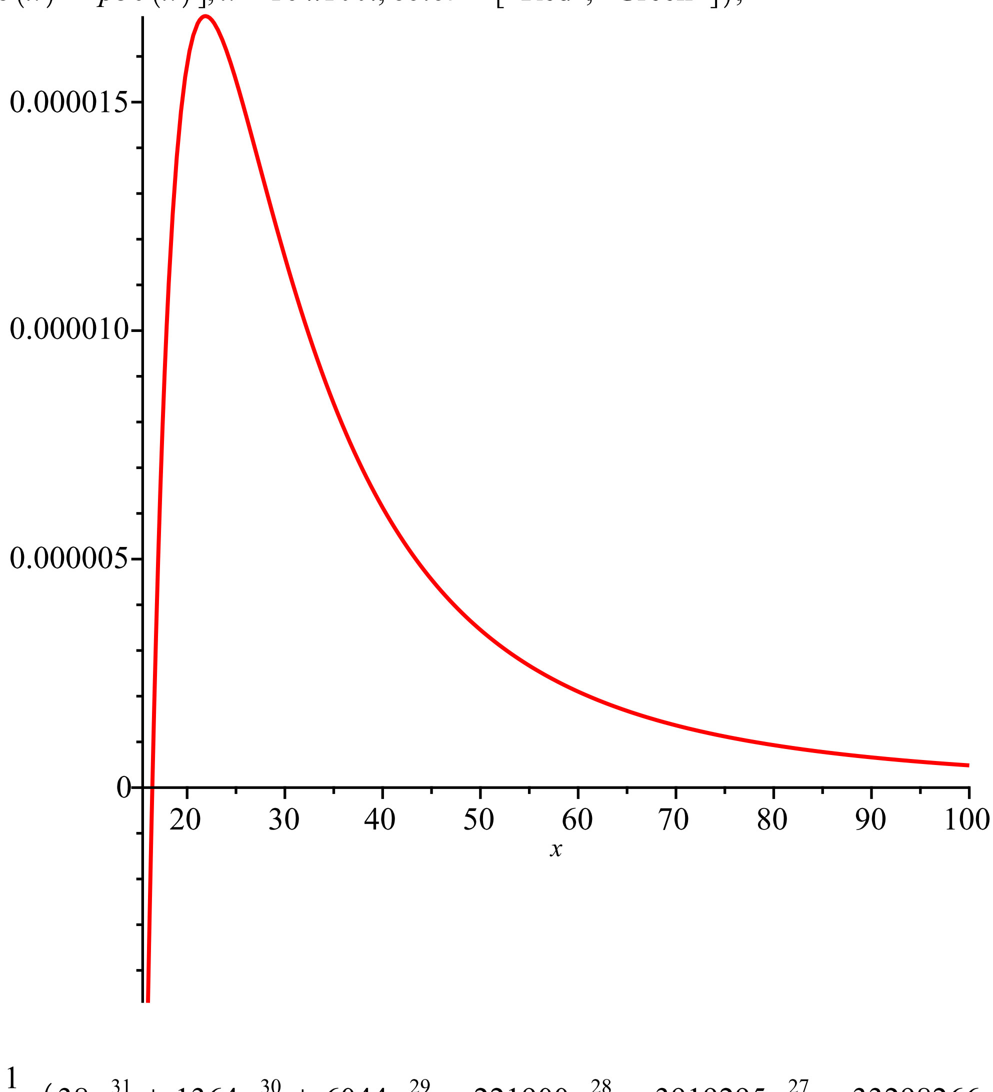
$$p30 := x \mapsto \left(\left(x^{14} + 14\cdot x^{13} + 104\cdot x^{12} + 548\cdot x^{11} + 2354\cdot x^{10} + 8316\cdot x^{9} + 19827\cdot x^{8} + 29660\cdot x^{7} + 28161\cdot x^{6} + 17322\cdot x^{5} + 6956\cdot x^{4} + 1802\cdot x^{3} + 289\cdot x^{2} + 26\cdot x + 1\right)\cdot x\right) \Big/ \left(2\cdot x^{15} + 30\cdot x^{14} + 240\cdot x^{13} + 1370\cdot x^{12} + 6420\cdot x^{11} + 24948\cdot x^{10} + 66090\cdot x^{9} + 111225\cdot x^{8} + 120690\cdot x^{7} + 86610\cdot x^{6} + 41736\cdot x^{5} + 13515\cdot x^{4} + 2890\cdot x^{3} + 390\cdot x^{2} + 30\cdot x + 1\right) \quad \textbf{(2)}$$

```
> simplify(p38(x) - p30(x))
```

$$(38\,x^{31} + 1364\,x^{30} + 6044\,x^{29} - 221900\,x^{28} - 3919295\,x^{27} - 33298266\,x^{26} - 184690988\,x^{25} - 732659816\,x^{24} - 2156516311\,x^{23} - 4799725166\,x^{22} - 8181173524\,x^{21} - 10785471472\,x^{20} \quad \textbf{(3)}$$

$$- 11082570066\, x^{19} - 8924901258\, x^{18} - 5649654982\, x^{17} - 2811504912\, x^{16}$$
$$- 1096265343\, x^{15} - 332404964\, x^{14} - 77346870\, x^{13} - 13517816\, x^{12} - 1713829\, x^{11}$$
$$- 148596\, x^{10} - 7872\, x^{9} - 192\, x^{8}\Big) \Big/ \Big(76 \Big(x^{19} + 19\, x^{18} + 190\, x^{17} + 1330\, x^{16} + 7401\, x^{15}$$
$$+ 36071\, x^{14} + 150462\, x^{13} + 473759\, x^{12} + 1048665\, x^{11} + 1619178\, x^{10} + 1768811\, x^{9}$$
$$+ \frac{2777315}{2}\, x^{8} + 791977\, x^{7} + 329441\, x^{6} + 99560\, x^{5} + \frac{43111}{2}\, x^{4} + 3249\, x^{3} + 323\, x^{2} + 19\, x$$
$$+ \frac{1}{2}\Big) \Big(x^{15} + 15\, x^{14} + 120\, x^{13} + 685\, x^{12} + 3210\, x^{11} + 12474\, x^{10} + 33045\, x^{9} + \frac{111225}{2}\, x^{8}$$
$$+ 60345\, x^{7} + 43305\, x^{6} + 20868\, x^{5} + \frac{13515}{2}\, x^{4} + 1445\, x^{3} + 195\, x^{2} + 15\, x + \frac{1}{2}\Big)\Big)$$

> $plot([p38(x) - p30(x)], x = 16..100., color = ["Red", "Green"]);$



>

> $simplify\Big(\frac{1}{x^8}\big(38\, x^{31} + 1364\, x^{30} + 6044\, x^{29} - 221900\, x^{28} - 3919295\, x^{27} - 33298266\, x^{26}$
$- 184690988\, x^{25} - 732659816\, x^{24} - 2156516311\, x^{23} - 4799725166\, x^{22} - 8181173524\, x^{21}$
$- 10785471472\, x^{20} - 11082570066\, x^{19} - 8924901258\, x^{18} - 5649654982\, x^{17}$

$- 2811504912\, x^{16} - 1096265343\, x^{15} - 332404964\, x^{14} - 77346870\, x^{13} - 13517816\, x^{12}$
$- 1713829\, x^{11} - 148596\, x^{10} - 7872\, x^{9} - 192\, x^{8}))$

$$38\, x^{23} + 1364\, x^{22} + 6044\, x^{21} - 221900\, x^{20} - 3919295\, x^{19} - 33298266\, x^{18} - 184690988\, x^{17} - 732659816\, x^{16} - 2156516311\, x^{15} - 4799725166\, x^{14} - 8181173524\, x^{13} - 10785471472\, x^{12} - 11082570066\, x^{11} - 8924901258\, x^{10} - 5649654982\, x^{9} - 2811504912\, x^{8} - 1096265343\, x^{7} - 332404964\, x^{6} - 77346870\, x^{5} - 13517816\, x^{4} - 1713829\, x^{3} - 148596\, x^{2} - 7872\, x - 192 \quad \textbf{(4)}$$

>

> $solutions := [solve(38\, x^{23} + 1364\, x^{22} + 6044\, x^{21} - 221900\, x^{20} - 3919295\, x^{19} - 33298266\, x^{18} - 184690988\, x^{17} - 732659816\, x^{16} - 2156516311\, x^{15} - 4799725166\, x^{14} - 8181173524\, x^{13} - 10785471472\, x^{12} - 11082570066\, x^{11} - 8924901258\, x^{10} - 5649654982\, x^{9} - 2811504912\, x^{8} - 1096265343\, x^{7} - 332404964\, x^{6} - 77346870\, x^{5} - 13517816\, x^{4} - 1713829\, x^{3} - 148596\, x^{2} - 7872\, x - 192 = 0, \{x\}, useassumptions)\ \text{assuming}\ x > 0];$

$$solutions := [\{x = RootOf(38\, \_Z^{23} + 1364\, \_Z^{22} + 6044\, \_Z^{21} - 221900\, \_Z^{20} - 3919295\, \_Z^{19} - 33298266\, \_Z^{18} - 184690988\, \_Z^{17} - 732659816\, \_Z^{16} - 2156516311\, \_Z^{15} - 4799725166\, \_Z^{14} - 8181173524\, \_Z^{13} - 10785471472\, \_Z^{12} - 11082570066\, \_Z^{11} - 8924901258\, \_Z^{10} - 5649654982\, \_Z^{9} - 2811504912\, \_Z^{8} - 1096265343\, \_Z^{7} - 332404964\, \_Z^{6} - 77346870\, \_Z^{5} - 13517816\, \_Z^{4} - 1713829\, \_Z^{3} - 148596\, \_Z^{2} - 7872\, \_Z - 192, index = 1)\}] \quad \textbf{(5)}$$

>

$evalf(solutions)$

$$[\{x = 16.44248149\}] \quad \textbf{(6)}$$