732A91 - Lab 3

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Normal model, mixture of normal model with semi-conjugate prior.

The data rainfall.dat consist of daily records, from the beginning of 1948 to the end of 1983, of precipitation (rain or snow in units of 1/100 inch, and records of zero precipitation are excluded) at Snoqualmie Falls, Washington. Analyze the data using the following two models.

a.

Assume the daily precipitation $y_1, ..., y_n$ *are independent normally distributed,* $y_1, ..., y_n|\mu, \sigma^2 \sim N(\mu, \sigma^2)$ *where both* μ *and* σ^2 *are unknown. Let* $\mu \sim N(\mu_0, \tau_0^2)$ *independently of* $\sigma^2 \sim Inv - \chi^2(\nu_0, \sigma_0^2)$.

i.

Implement (code!) a Gibbs sampler that simulates from the joint posterior $p(\mu, \sigma^2|y_1, ..., y_n)$. *The full conditional posteriors are given on the slides from Lecture 7.*

We have the following full conditional posteriors:

$$\mu|\sigma^2, x \sim N(\mu_n, \tau_n^2)$$

and

$$\sigma^2|\mu, x \sim Inv - \chi^2(\nu_n, \frac{\nu_0\sigma_0^2 + \sum_{i=1}^{n}(x_i - \mu)^2}{n + \nu_0})$$

where

$$\mu_n = w\bar{x} + (1 - w)\mu_0$$

$$w = \frac{\frac{n}{\sigma^2}}{\frac{n}{\sigma^2} + \frac{1}{\tau_0^2}}$$

$$\tau_n^2 = \frac{\sigma^2}{n} + \tau_0^2$$

2. Metropolis Random Walk for Poisson regression.

Consider the following Poisson regression model

$$y_i|\beta \sim Poisson[exp(x_i^T\beta)], i = 1, ..., n$$

where yi is the count for the ith observation in the sample and x_i is the p-dimensional vector with covariate observations for the ith observation. Use the data set eBayNumberOfBidderData.dat. This dataset contains observations from 1000 eBay auctions of coins. The response variable is nBids and records the number of bids in each auction. The remaining variables are features/covariates (x):

- **bla1** item 1
- **bla2** item 2
- **bla3** item 3

Appendix

```
knitr::opts_chunk$set(echo = TRUE)
knitr::opts_chunk$set(fig.width=9, fig.height = 4.1)
library(tidyverse)
library(dplyr)
library(knitr)
library(mvtnorm)
set.seed(12345)
data0 <- read.table("rainfall.dat",header = F)
```