732A91 - Lab 1

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09 April 2020

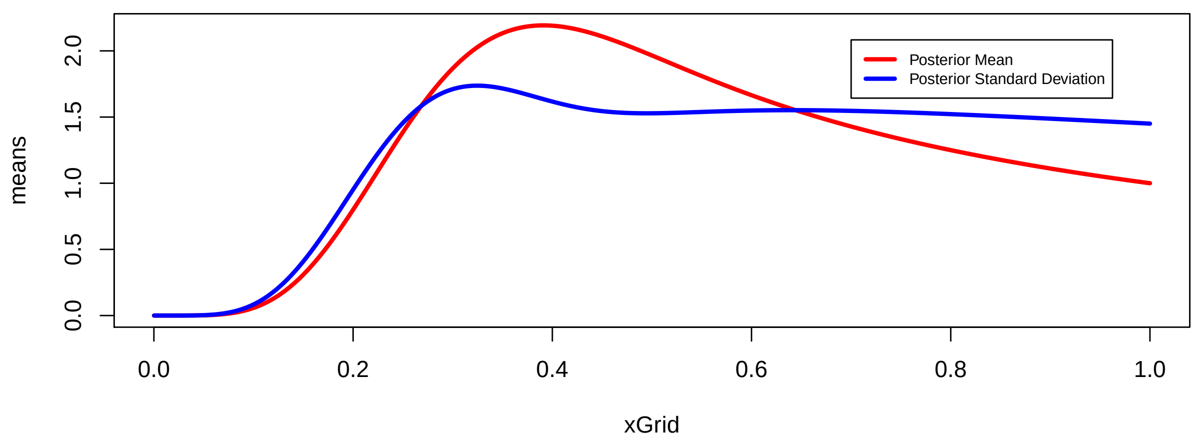
1. Bernoulli . . . again.

Let $y_1, ..., y_n|\theta$ $Bern(\theta)$, and assume that you have obtained a sample with s = 5 successes in n = 20 trials. Assume a $Beta(\alpha_0, \beta_0$ prior for θ and let $\alpha_0 = \beta_0 = 2$

a.

Draw random numbers from the posterior $\theta|y$ $Beta(\alpha_0 + s, \beta_0 + f), y = (y_1, ..., y_n)$, and verify graphically that the posterior mean and standard deviation converges to the true values as the number of random draws grows large.

```
## Number of Draws:  10000
## Posterior Mean:  1
## Posterior Standard Deviation:  1.450244
```



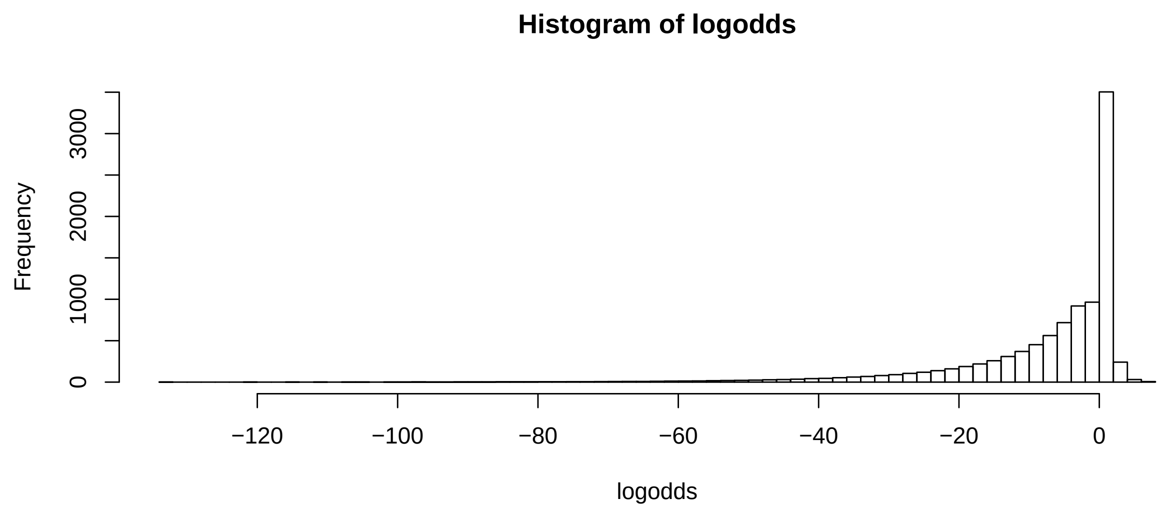
b.

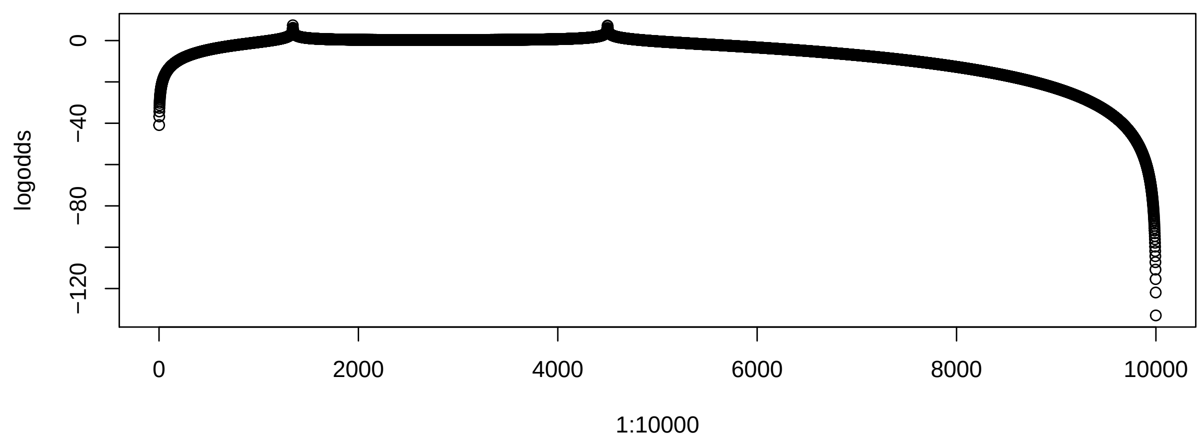
Use simulation (nDraws = 10000) to compute the posterior probability $Pr(\theta > 0.3|y)$ and compare with the exact value

```
## The probability of theta being larger than 0.3 is:  0.7613
```

c.

Compute the posterior distribution of the log-odds ϕ






```
##
## Call:
##  density.default(x = logodds)
##
## Data: logodds (10000 obs.);  Bandwidth 'bw' = 1.154
##
##        x                  y
##  Min.   :-136.47   Min.   :0.000e+00
##  1st Qu.: -99.63   1st Qu.:4.515e-05
##  Median : -62.80   Median :3.945e-04
##  Mean   : -62.80   Mean   :6.783e-03
##  3rd Qu.: -25.96   3rd Qu.:3.620e-03
##  Max.   :  10.88   Max.   :1.309e-01
```

2. Log-normal distribution and the Gini coefficient

Assume that you have asked 10 randomly selected persons about their monthly income (in thousands Swedish Krona) and obtained the following ten observations: 44, 25, 45, 52, 30, 63, 19, 50, 34 and 67. A common model for non-negative continuous variables is the log-normal distribution. The log-normal distribution $log(N(\mu, \sigma^2))$ *has density function:*

$$p(y|\mu, \sigma^2) = \frac{1}{y * \sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(log(y)-\mu)^2}$$

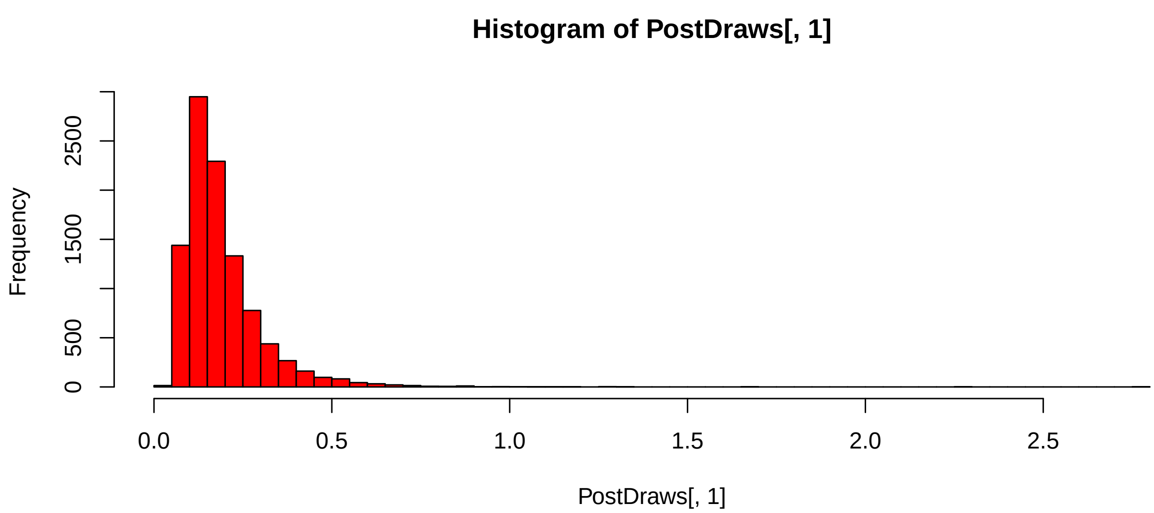
For y>0, $\mu > 0 and \sigma^2 > 0$. *The log-normal distribution is related to the normal distribution as follows: if y* $logN(\mu, \sigma^2)$ *then* $log(y)$ $N(\mu, \sigma^2)$. *Let* $y_1, ..., y_n|\mu, \sigma^2 \, ^{iid} log(N(\mu, \sigma^2))$, *where* $\mu = 3.7$ *is assumed to be known but* σ^2 *is unknown with non-informative prior* $p(\sigma^2) \propto \frac{1}{\sigma^2}$. *The posterior for the* σ^2 *is the* $Inv - \chi^2(n, \tau^2)$ *distribution, where*

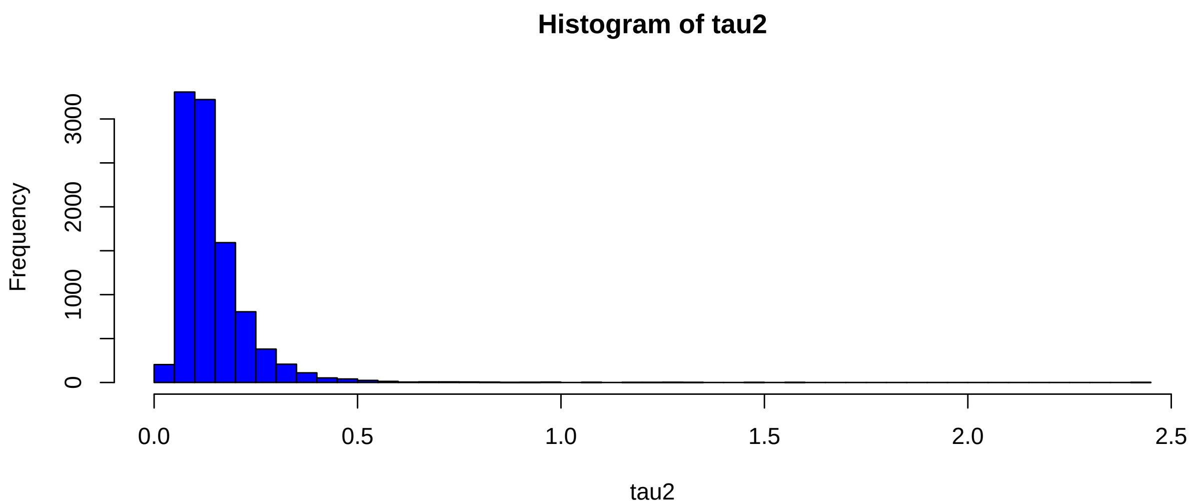
$$\tau^2 = \frac{\sum_{i=1}^{n}(log(y_i) - \mu)^2}{n}$$

a.

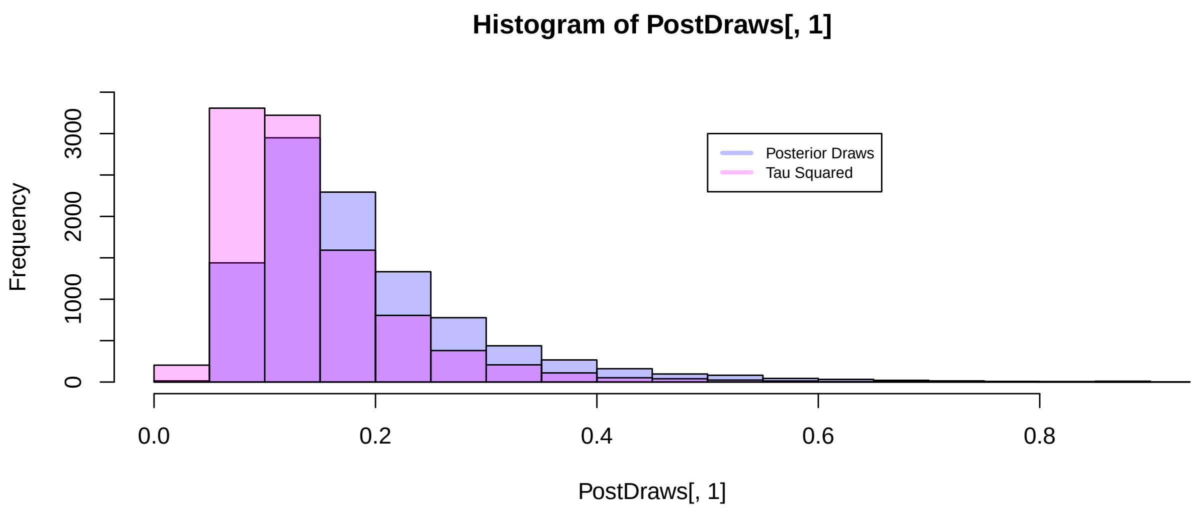
Simulate 10,000 draws from the posterior of σ^2 *(assuming* $\mu = 3.7$*) and compare with the theoretical* $Inv - \chi^2(n, \tau^2)$ *posterior distribution.*

We worked with the log of the data, assuming that is approximately normally distributed with $N(\mu, \sigma^2)$





Histogram of tau2
Frequency
0
1000
2000
3000
0.0
0.5
1.0
1.5
2.0
2.5
tau2



Histogram of PostDraws[, 1]
Frequency
0
1000
2000
3000
Posterior Draws
Tau Squared
0.0
0.2
0.4
0.6
0.8
PostDraws[, 1]

Appendix

```
knitr::opts_chunk$set(echo = TRUE)
knitr::opts_chunk$set(fig.width=9, fig.height = 4.1)
library(tidyverse)
library(dplyr)
library(knitr)
library(LaplacesDemon)
RNGversion("3.6.2")
set.seed(12345)
# ----------------------------------------------------------
# 1a

a = b = 2
n = 20
s = 5
nDraws = 10000
#xGrid <- seq(0.001, 0.999, by=0.001)
#posterior = dbeta(xGrid, a+s, b+(n-s))

means <- c()
sds <- c()

set.seed(12345)

for(i in 1:nDraws){
  xGrid <- seq(1/nDraws, i/nDraws, by=1/nDraws)
  posterior = dbeta(xGrid, a+s, b+(n-s))
  means[i] <- mean(posterior)
  sds[i] <- sd(posterior)

  #at("\nNumber of Draws: ", i , "\nMean: ", mean(posterior), "\nStandard Deviation: ", sd(posterior))
}

cat("Number of Draws: ", nDraws , "\nPosterior Mean: ", means[nDraws], "\nPosterior Standard Deviation:

plot(xGrid, means, type = 'l', lwd = 3, col = "red")
lines(xGrid, sds, lwd = 3, col = "blue")
legend(x = max(xGrid)*0.70, y = 0.95*max(means), legend = c("Posterior Mean", "Posterior Standard Devia
# ----------------------------------------------------------
# 1b

xGrid <- seq(1/nDraws, nDraws/nDraws, by=1/nDraws)
posterior = pbeta(xGrid, a+s, b+(n-s)) # Ask for the difference between pbeta & dbeta

prob_03 <- posterior[posterior > 0.3]
prob <- length(prob_03)/nDraws

cat("The probability of theta being larger than 0.3 is: ", prob)
# ----------------------------------------------------------
# 1c

xGrid <- seq(1/nDraws, nDraws/nDraws, by=1/nDraws)
```

```r
posterior = dbeta(xGrid, a+s, b+(n-s))

logodds <- c()

for(i in 1:length(posterior)){
  logodds[i] <- log(abs(posterior[i]/(1-posterior[i])))
}

hist(logodds, breaks = 100)
plot(1:10000,logodds)
density(logodds)
# -------------------------------------------------------
# 2a
y <- c(44,25,45,52,30,63,19,50,34,67)
n <- length(y)
logy <- log(y)
nDraws <- 10000
mu <- 3.7

LogNormalNonInfoPrior <- function(nDraws, data, mu){
  datamean <- mean(data)
  n <- length(data)
  tau2 <- sum((data-mu)^2) / n
  PostDraws <- matrix(0,nDraws,2)
  PostDraws[,1] <- ((n-1)*tau2)/rchisq(nDraws,n-1)
  PostDraws[,2] <- datamean+rnorm(nDraws,0,1)*sqrt(PostDraws[,1]/n)
  return(PostDraws)
}

#tau2 <- sum((logy-mu)^2) / n
tau2<-rinvchisq(nDraws, n-1, scale=1/(n-1))

PostDraws<-LogNormalNonInfoPrior(10000,logy, mu)

p1<-hist(PostDraws[,1], breaks = 50, col = "red")           # Plotting the histogram of mu-draws
p2<-hist(tau2, breaks = 50, col = "blue")
plot(p1, col=rgb(0,0,1,1/4), ylim = c(0,3500), xlim = c(0,0.90))
plot(p2, col=rgb(1,0,1,1/4), add=T)
legend(x = 0.5, y = 3000, legend = c("Posterior Draws", "Tau Squared"), col = c(rgb(0,0,1,1/4), col=rgb
```