Faraday Rotation - Teach Spin

2-28-11

Plane of polarization rotates when light passes through matter with a static B parallel to the direction of propagation $\vec{k}$.

A. Malu's Law

- set with 1KΩ resistor. Max output is ~150mV, well in the linear region. <300mV.

- No polarizer at the laser. But Vout does go to a min of 0.2 mV, which suggests the light is highly polarized.

Angle	Voltage (mV)
345.0	0.3mV
340.0	1.6mV
335.0	5.3mV
330.0	11.6 mV
325.0	19.2
320.0	29.2
315.0	40.6
310.0	54.0
305.0	66.3
300.0	81.5

Angle	Voltage (mV)
295.0	94.7
290.0	106.4
285.0	118.3
280.0	129.4
275.0	138.6
270.0	147.4
265.0	151.6
260.0	155.7
255.0	156.4
250.0	155.8

Angle	Voltage (mV)
245.0	153.0
240.0	147.2
235.0	139.2
230.0	129.6
225.0	118.3
220.0	106.0
215.0	92.6
210.0	78.9
205.0	64.6

Angle	Voltage
200.0	51.8
195.0	39.2
190.0	28.0
185.0	18.8
180.0	10.7
175.0	5.1

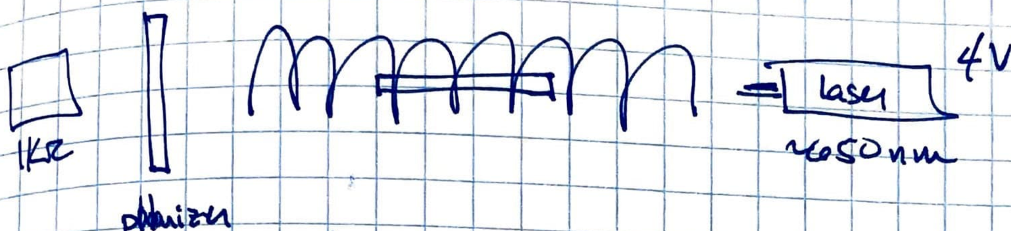
Angle	Voltage
170.0	1.7
165.0	0.3
160.0	1.4
155.0	5.0
150.0	10.2

no polarizer

252.3mV

2-28-11

sample is in place and centered.



- Adust laser alignment to get max signal.
- Rotate polarizer for max output.
  116.0mV at 257.0°.
- Rotate polarizer for min output.
  0.2mV at 345°.
- Rotate polarizer to 345° − 45° = 300°. Output = 59.5mV
- This will be the set point. Now I'll apply magnetic field and adjust polarizer so output returns to this value.

B (A)	Angle	Vout
0	300.0	~~59.8~~ 55.5
.495	300.0+	55.5
0	300.0	55.4
1.018	301.0	55.4
0	300.0	55.5
1.503	302.0°	55.5
0	300.0	56.5
2.007	302.0°	56.5
0	300.0°	56.1
2.510	303.0°	56.1
0	300.0°	56.1
2.999	304.0°	56.1

B (A)	Angle	Vout
0	345.0°	0.4mV
3.014	348.5°	0.4mV

This is not very sensitive. 4°/3A roughly 1° per Amp.

$\theta = VBL$ $\qquad V = \frac{\theta}{BL} = \frac{(4)(\pi)}{(180)(.1m)(3A)(.0111\,\frac{T}{A})} = 20.96$

$\Delta I = I_0 - I = I_0 (1 - \cos^2(\phi - \theta))$

To maximize signal means to maximize ΔI with respect to ϕ.

$$\left.\frac{\partial \Delta I}{\partial \phi}\right|_{\phi_{best}} = \frac{\partial}{\partial \phi}\left(I_0 - I_0 \cos^2(\phi - \theta)\right) = 0$$

$$= 2 I_0 \cos(\phi - \theta)\sin(\phi - \theta) = 0$$

$$= \frac{2 I_0}{2} \sin(2(\phi - \theta)) = 0$$

$\sin\theta$ is small $\theta << \phi$.

signal is maximum when $\phi = 45°$

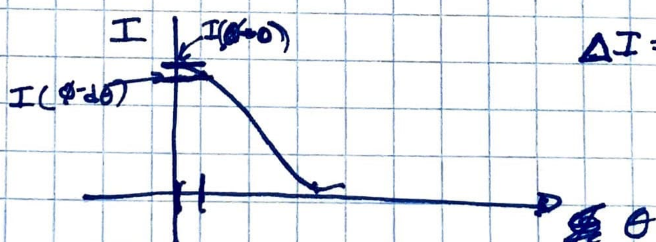
$\approx \sin(2 \cdot 45) = 1$

Redefine

$I_0 = A_0^2 \cos^2 \phi$

$I = A_0^2 \cos^2(\phi - \theta)$ θ is small.

$\Delta I = I_0 - I = A_0^2 \cos^2\phi - A_0^2 \cos^2(\phi - \theta)$



$$\Delta I = \left.\frac{dI}{d\theta}\right|_{\theta=0} d\theta$$

$$\left.\frac{\partial I}{\partial \theta}\right|_{\theta=0} = 2A_0^2 \cos(\phi - \theta)\sin(\phi - \theta)\Big|_{\theta=0}$$

$2A_0^2 \cos\phi \sin\phi = \frac{2}{2} A_0^2 \sin^2(2\phi)$ ← maximum when $\phi = 45°$.

$\phi = 45°$ is the best choice.

For $45°$

$$I = A_0^2 \cos^2\left(\frac{\pi}{4} - \theta\right) = \frac{1}{2}A_0^2\left[\cos\left(2\left(\frac{\pi}{4} - \theta\right)\right) + 1\right]$$

$$= \frac{1}{2}A_0^2\left[\cos\left[\frac{\pi}{2} - 2\theta\right] + 1\right] = \frac{1}{2}A_0^2\left[\sin(2\theta) + 1\right]$$

If θ is small then

1-28-11

$$I = \frac{1}{2}A_0^2\left[\sin(2\theta)+1\right] \approx \frac{1}{2}A_0^2(2\theta+1)$$

$\theta \propto B$ $\quad B = B_0 \sin(2\pi f t)$

$$\theta = \theta_0 \sin(2\pi f t)$$

θ_0 is max angle occuring for max B-field. B_0.

$$I = \frac{1}{2}A_0^2\left(2\theta_0 \sin(2\pi f t) + 1\right)$$

$$= \frac{1}{2}A_0^2 + A_0^2\theta_0 \sin(2\pi f t)$$

$$I_0 = A_0^2\cos^2\phi = A_0^2\cos^2\left(\frac{\pi}{4}\right) = \frac{1}{2}A_0^2$$

$$I = I_0 + A_0^2\theta_0 \sin(2\pi f t)$$

$$\Delta I = I_0 - I = A_0^2\cos^2\phi - A_0^2\cos^2(\phi-\theta)$$

$$= \frac{1}{2}A_0^2 - \frac{1}{2}A_0^2(2\theta+1)$$

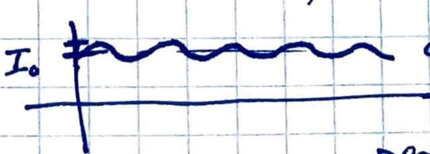
$$\cancel{\frac{1}{2}A_0^2} - A_0^2\theta - \cancel{\frac{1}{2}A_0^2} = -A_0^2\theta_0 \sin(2\pi f t) = \Delta I$$

$$\Delta I \equiv I_0 - A_0^2\theta_0$$

$$I = \frac{1}{2}A_0^2 + A_0^2\theta_0 \sin(2\pi f t)$$

$$I_0 = A_0^2\cos^2\phi = A_0^2\cos^2\left(\frac{\pi}{4}\right) = \frac{1}{2}A_0^2$$

$$I = I_0 + A_0^2\theta_0 \sin(2\pi f t)$$



define $\Delta I \equiv A_0^2\theta_0$ the peak modulation about I_0.

$$I = I_0 + \Delta I \sin(2\pi f t) \qquad \Delta I = \frac{1}{2}A_0^2\theta_0 \cdot 2 = 2I_0\theta_0$$

2-28-11

Relative change in intensity

$$\frac{\Delta I}{I_0} = \frac{2 \cancel{I_0} \Theta_0}{\cancel{I_0}} = 2\Theta_0$$

$$\boxed{\Theta_0 = \frac{1}{2}\frac{\Delta I}{I_0}}$$

$\Theta_0 = V B_0 L$ $\qquad V = \dfrac{\Theta_0}{B_0 L}$ $\qquad$ or $\Theta_0 = V \displaystyle\int_{-L/2}^{L/2} B(z)\,dz$

$$V = \frac{\Delta I}{2 B_0 L I_0}$$

ΔI ← peak in modulated signal with sinusoidal field.

B_0 ← peak B-field

I_0 ← detector signal with 0 applied field.

Polarizer Angle 45° - ~~33~~ 345° - 45° = $\underline{\underline{300°}}$

↑ max extinction

Excitation Freqeuncy = 200.0 Hz
Ampl. = 1.000 V
Phase = 0°
Time Constant = ~~100ms~~ 1 sec.
Sensitivity = 2 mV

Sync. Filter on.
12 dB/oct

- Connected to audio amp.
- Attenuator max = 0.4. 0.4555 Amp AC RMS measured with Fluke 189.
- I_0 = 60.6 mV

ΔI ~~p~~ rms

← Fluke 115 mV (dc)

i_{rms} (A)	R (mV)	θ deg	I_0 (mV)
0.4555	1.249	~~[illegible]~~ -67.76	60.6
0.4004	1.099	-67.83	60.7
0.3503	0.962	-67.78	60.7
0.2999	0.822	-67.75	60.7
0.1998	0.548	-67.73	60.7
0.1010	0.273	-67.77	60.7
0.0503	0.135	-67.7	60.7

Accepted value

$23 \dfrac{\text{rad}}{\text{T m}}$

$$\underbrace{\frac{\Delta I}{I_0}}_{y} = \underbrace{2LV}_{\text{slope}}\ \underbrace{B_0}_{x}$$

$V = \dfrac{\text{slope}}{2L} \times \sqrt{2}$

$14.5 \times \sqrt{2} = \boxed{20.5}$

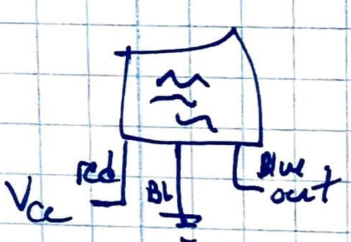
slope = 2.8921 T^{-1} = 2LV

rms

$V = 2.8921/(2 \times .1) = 14.5$ 1/T·m

Repeat at 100 Hz. 2-29-11

i_{rms} (A)	R (mV)	Θ (deg)	I_0 (mV)
0.7579	1.98	-48.8	57.5
.6959	1.79	-48.1	56.8
.6527	1.67	-48.4	56.5
.6007	1.54	-48.5	56.4
.5508	1.40	-48.1	56.2
.4982	1.26	-48.0	55.9
.4499	1.14	-48.0	55.8
.3992	1.004	-48.2	55.6
.3496	0.877	-48.3	55.6
.2994	0.752	-48.2	55.5
.2497	0.629	-48.4	55.7
.1995	0.504	-48.0	55.6
0.1488	0.378	-48.5	55.9
.0998	0.251	-47.8	56.0
0.0513	0.127	-49.7	56.0



overestimate. slope should be a bit steeper.

Slope $\frac{\Delta I}{I_0}$ vs. B_0 $= 2.8922 \frac{1}{T}$

$$V = \frac{2.89224 T}{(2)(0.1m)} = 14.46 \frac{1}{T\cdot m} \times \sqrt{2} = 20.45$$

In good agreement with the result at 200 Hz.

$$G = 1 + \frac{49.4K}{R_G}$$

$$G - 1 = \frac{49.4k}{R_G}$$

$$R_G = \frac{49.4K}{G-1} = \frac{49.4K}{100-1} = \frac{49.4K}{99} = 499\,\Omega$$

3A
0.33 mT
$\mu_0 = 4\pi \times 10^{-7}$ Tm/A

$n = \frac{50 \text{ turns}}{.52 m}$

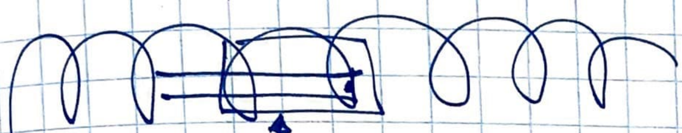
$$B = \frac{(4\pi \times 10^{-7})(3.00A)(50)}{(0.52)}$$

$B = 0.362$ mT

3-2

$$\frac{\Delta I}{I_0} = 2V \int_{-L/2}^{L/2} B_0 \, dz$$

Inside the Slinky Solenoid.



$n = \frac{50 \text{ turns}}{52 \text{ cm}}$

I(A)	V
1.00	0.022V
2.00	0.044V
3.00	0.066V

3A

B = 0.362 mT

$$\frac{0.066V}{0.362mT} = \frac{0.1823V}{mT}$$

TeachSpin

Max = 2.020 V

I = 1.00 A

$$\frac{2.020V}{} \times \frac{1mT}{0.1823V} = 11.08mT$$

exactly what TeachSpin calibration constant is quoted at.

2.020 at 2.40 cm

2.020 at 4.20

$$\frac{4.20 + 2.40}{2} = \frac{6.60}{2} = 3.30$$

z	V
3.30	2.027
3.4	2.027
3.6	2.027
3.8	2.027
4.0	2.027
4.2	2.025
4.4	2.023
4.6	2.020
4.8	2.017
5.0	2.015

← center

5.2	2.010
5.4	2.006
5.6	2.000
5.8	1.994
6.0	1.988
6.2	1.978
6.4	1.969
6.6	1.956
6.8	1.937
7.0	1.921

7.2	1.899
7.4	1.873
7.6	1.841
7.8	1.807
8.0	1.768
8.2	1.702
8.4	1.646
8.6	1.559
8.8	1.447
9.0	1.328

9.2	1.162
9.4	1.016
9.6	.851
9.8	.691
10.0	.569
10.2	.477
10.4	.389
10.6	.326
10.8	.278
11.0	.231

11.2	.200
11.4	.175
11.6	.152
11.8	
12.0	

3-14-2011

• Hall probe amplifier for Faraday Rotation exp.