PHYS460: FARADAY ROTATION

A PREPRINT

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ABSTRACT

In this experiment, material properties of optically transparent material are examined, namely the Verdet constant. The material, SF-59 glass, is placed in a nearly uniform magnetic field generated by a current carrying solenoid surrounding it. Data on the strength of the magnetic field along the length of the sample is collected and used to tabulate the Verdet constant, $\upsilon = (22 \pm 2)\,\mathrm{rad\,T^{-1}\,m^{-1}}$ for partially polarized light from a laser, nominally of $\lambda = 650\,\mathrm{nm}$ wavelength. This value is accurate relative to the accepted value. Additionally, properties of Malus' Law, Faraday Rotation, and Ampere's Law are verified experimentally.

1 Introduction

Faraday rotation (also called the Faraday effect), named after the scientist who discovered it, is an optical phenomenon of electromagnetic waves. The effect is a rotation in the polarization of light.

For the effect to be produced, two requirements must be satisfied: the light measured must pass through a nearly uniform magnetic field, and the light must pass through a glass tube or some other optically transparent medium. In the experiment, the magnetic field is produced by means of a current carrying solenoid wrapped around a glass tube. While the magnetic field is mostly uniform around the center of the solenoid, the edges see a loss of magnetic field flux. This detail is important when considering the β field profile for this experiment.

The rotation angle θ measured as the difference in the alignment of light entering the apparatus and the alignment of light incident on a detector (having passed through the apparatus) is quite small, yet Faraday rotation has some practical use in telecommunications, since a signal passing through the apparatus can be measured and interpreted (i.e., modulated). A large current must be supplied to the solenoid to produce a magnetic field strong enough to result in the rotation in the orientation of the light.

One important optical property of matter, the Verdet constant, is discussed. The Verdet constant is proportional to the rotation angle. In the experiment, this value is determined. Additionally, Malus's Law, and the proportionality of light intensity passing through various polarizers is examined. Ampere's Law is utilized to generate a formula for the β field profile.

The lab is based on a TeachSpin laboratory, but adjustments to the process are made (discussed throughout). The primary objective will be to determine the Verdet constant using two different techniques. The whole process requires the design and building of a custom magnetic field sensing circuit and the sampling of data along a Slink coil to calibrate it. The magnetic field profile along the length of the sample must also be determined from analysis of collected data. Finally, the Verdet constant can be calculated. The details are discussed in this paper.

2 Theory

The phase speeds of circularly polarized light through a medium is different for left v_- and right v_+ polarized light, a phenomenon called circular bifringence. This is the physical mechanism that results in the rotation in the orientation of light sent through the tube. But there are other physical parameters that affect the magnitude of rotation. The phase speed difference, $\Delta v = v_+ - v_-$ is due to a difference in the index of refraction $\Delta n = n_+ - n_-$ for light through a medium with different polarizations (left/right circularly polarized light), and the difference in the index of refraction is proportional to the magnetic field induced by the current running through the solenoid. The linearly polarized light that is examined in this experiment is a result of the superposition of the left and right circularly polarized light:

$$\vec{E}_{linear} = \vec{E}_{+} + \vec{E}_{-} \tag{1}$$

Linearly polarized light is measured as an electric field plane wave at position z and time t. The light propagates as a sinusoidal wave through space, which may take the form $\vec{E} = E_o cos(kz - wt)\hat{i}$ - here the $\hat{i}$ direction was arbitrarily chosen, but $\vec{E}$ can be in any direction.

The electric field rotation angle between the plane wave entering the tube and the plane wave incident on the detector is given by:

$$\theta = \upsilon \beta_o L \tag{2}$$

Where υ is the Verdet constant (material property), β_o is the nearly uniform magnetic field, and L is the length of the tube.

Since $\Delta n \neq 0$ and because $\beta_o \propto \Delta n$, Equation 2 can be rewritten:

$$\theta = \frac{\omega}{2c}(n_{-} - n_{+})L \tag{3}$$

The intensity of polarized light passing through a polarizer is described by Malus's Law. Here, I_1 is the intensity of the light incident to the detector and I_o is the source intensity. Malus's Law is:

$$I_1 = I_o cos^2(\theta) \tag{4}$$

The curve $I_1(\theta)$ is plotted in Fig 1. Note the derivative of the intensity is maximum at angles θ that are $45°$ between peaks and troughs. This idea is important in the experiment - choosing such an angle θ will maximize sensitivity over a range $\delta\theta$. This concept is important in the second method used to calculate the Verdet constant - see Section 3 for details.

The formula for an ideal magnetic field outside a solenoid is $B = \mu_o in$ (from Ampere's Law), where μ_o is the vacuum permeability, and n is the number of turns per unit length of the solenoid. Because the magnetic field induced by a current carrying solenoid is non-uniform, it is not ideal, and an integral must be used. Then, the magnetic field $d\beta$ for an infinitesimal length of the glass dx is given:

$$\int_{-L/2}^{L/2} \beta(x)\, dx \tag{5}$$

This integral replaces the $\beta_o L$ term from 2. A ratio can be determined - the integral over the maximum B field strength (measured at the center of the solenoid), i.e.

$$\theta = \upsilon \frac{\int_{-L/2}^{L/2} \beta(x)\, dx}{\beta_o L} \beta_o L \tag{6}$$

The fraction in Eq 6 is the ratio of areas - the area under the integrand over the area of the rectangle formed by β_o, both over the whole interval of the length of the solenoid. This ratio is dimensionless, and will be used to make easy calculations in the experiment. Letting this ratio be called f, the rotation angle is $\theta = \upsilon f \beta_o L$.

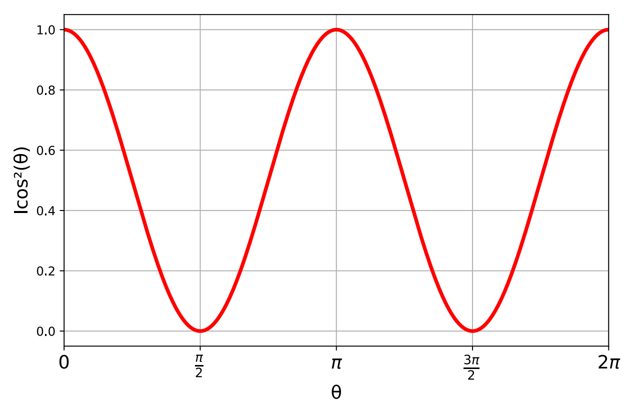


Figure 1: Malus's Law. Note how the slope is greatest between peak and trough. Measurements of θ over intervals of light intensity will be most sensitive at the halfway point between peak and trough.

3 Methods

The appartus used to conduct the experiment consists of four components: the first, a red laser pointer used as the light source, with a nominal wavelength of 650 nm at 3 mW power. The second is a glass tube wrapped in a copper winding. Third, a polarizer, which may be rotated 360°. Finally, a photodiode serves as the detector. Each component is discussed further.

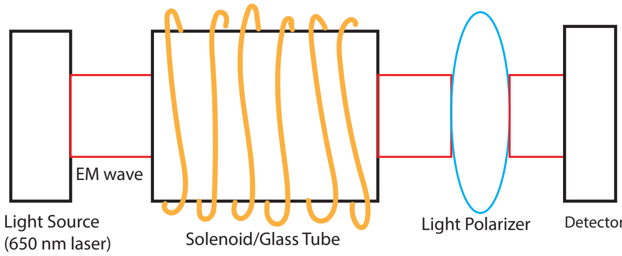


Figure 2: Schematic of the apparatus used in the experiment.

The light from the laser is 60% polarized. It must be passed through a polarizer to increase polarization - up to about 95%. This polarizer is part of the laser, and must be rotated to produce maximum light intensity as the light enters the solenoid. This can be achieved by placing a white piece of paper at the entrance of the solenoid, and dialing the polarizer until peak intensity is achieved. The photodiode detector can also be used to determine peak source intensity angle. The laser produces light nominally as red visible light at $\lambda = 650 \times 10^{-9}\,\mathrm{nm}$. A $4\,\mathrm{V}$ power supply provides power for the laser.

A solenoid serves as the source of magnetic field in the experiment. The $\frac{50turns}{52cm}$ Slink solenoid wraps around the glass sample. The approximate calibration constant for the solenoid is given as $B = (11.1\,\mathrm{mT\,A^{-1}})(I)$. As mentioned, the magnetic field is non-uniform. The magnetic field is determined along the length of the sample. More details on this follow. The maximum continuous current (MCC) for the solenoid is $3\,\mathrm{A}$ - running this level of current through the solenoid for any prolonged period will result in overheating and possible damage to the apparatus.

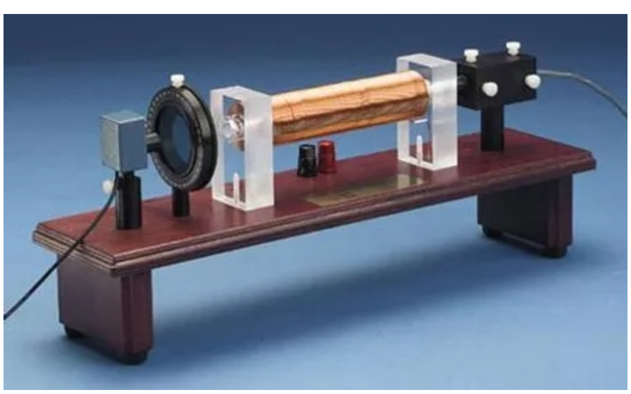


Figure 3: Photograph of the apparatus used in the experiment. *Source: https://www.teachspin.com/faraday-rotation*

A rotatable Polaroid film serves as the polarizer in the experiment. The polarizer is marked with $5°$ increments, with an approximate associated uncertainty of $0.5°$.

Finally, a photodiode serves as the light detector at the end opposite of the source. The photodiode has three resistances to choose from 10, 3, or $1\,\mathrm{k\Omega}$. Values read from the photodiode are taken only if the bias voltage is well below saturation voltage - in this case, $0.3\,\mathrm{V}$. This guarantees that measurements are in the linear regime. The intensity of the light incident on the detector is measured as a voltage. Figure 2 shows the configuration for the apparatus.

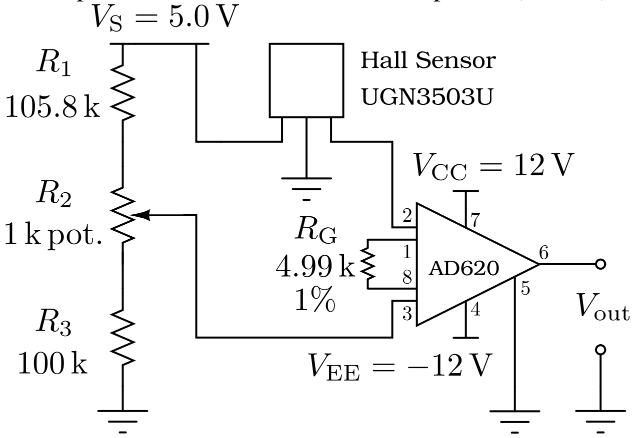
The Faraday rotation angle of the polarized light is used in computing the Verdet constant. Two methods are used to measure the rotation angle - the first is called the extinction method. The first step in this process is to rotate the Polaroid film $90°$ relative to the light incident to the tube from the laser so that maximum extinction of the light occurs at the detector. Then, a current is supplied to the solenoid. When the magnetic field is produced, the polarizer is rotated to find the angle which maximimizes light intensity. This process is repeated for a variety of magnetic field strengths by varying the current supplied to the solenoid. The rotation angle θ is measured over a range of regularly-spaced currents.

The second method exploits the maximally sensitive part of the light intensity curve in Malus' Law. To maximize sensitivty of the photodiode, measurements of voltage are taken when the polarizer is rotated $45°$ past the angle which maximizes light intensity. Since the intensity is a wave function which peaks and troughs at $\pm\frac{n\pi}{2}$, a higher sensitivity is achieved at $\pm 45°$ of the maximum/minimum angle due to the steeper slope. See figure 1 to see this visually.

With the polarizer set at the reference angle, the solenoid is powered to produce a magnetic field, the magnitude of which is computed for the supplied current. This produces a voltage in the light detector, which is the associated reference voltage, V_{out}. Then, the polarizer is rotated θ such that the light intensity returns to its maximum, and the difference in voltage at the detector ΔV_{out} is determined. $\delta\theta$ is also noted.

The TeachSpin magnetic field detector was not used in this experiment. Instead, a custom circuit was built to detect the magnetic field (see Fig 4). The circuit uses a Hall Sensor (UGN3503U), which produces a variable resistance change with changing magnetic flux through the sensor, which is then measured as a voltage from the output of the instrumentation amplifier (AD620). This circuit serves as the instrument used to collect data on voltage output from various magnetic flux (resulting from variable current supply to the solenoid).

Figure 4: Circuit diagram for the Hall Sensor B-Field probe with instrumentation amplifier (AD620).



Source: Dr. Hetrick

A calibration constant must be generated for the Hall sensor, so that a voltage can be converted to a magnetic field strength. This was done by placing the sensor in the center of a Slinky solenoid, and measuring the voltage output from the sensor at various currents (see Table I). The slope of this linear relationship serves as the conversion factor.

The magnetic field is a function of position along the length of the solenoid. This means the β field must be profiled. This process included placing the (now calibrated) Hall sensor at the center of the TeachSpin solenoid and measuring the voltage (converted to magnetic field strength by the conversion factor) at many positions along the solenoid. $1\,\text{A}$ was the current supplied for every data point in this collection (Table 4).

4 Results and Discussion

In the calibration of the Hall sensor, the constant of proportionality for the line that best fit the data was found to be $k = 5.292\,\text{mT}\,\text{V}^{-1}$. This parameter were determined over four data points. It is worth noting that the range over which voltage was measured was quite small - $0.066\,\text{V}$. The TeachSpin manual determines the calibration constant over a much different domain - ($2.0202\,\text{V}$ was found for $1.00\,\text{A}$ supplied to the solenoid, versus $0.022\,\text{V}$ at $1.0\,\text{A}$ in the experiment). It is a stretch to assume that the linear regime that was found from the data extrapolates to and agrees with the TeachSpin values, but there is one other factor in determining this value that shows promise for the data. The exact calibration constant quoted by TeachSpin is $11.08\,\text{mT}\,\text{A}^{-1}$. The magnetic field at the center of the solenoid is given as $B = (11.08\,\text{mT}\,\text{m}^{-1})(I)$, and $1\,\text{A}$ supplied to the solenoid resulted in a $2.020\,\text{V}$ potential in the photodiode circuit. If we multiply this voltage times the inverse of the calibration constant determined from the data (to match units), the TeachSpin constant results. This is promising for the experimental calibration constant.

Table I: Data for B-Field calibration using the Hall Sensor circuit. The B column is tabulated using the formula for the magnitude of an ideal magnetic field for the solenoid used in the experiment.

i(A)	V(V)	di(A)	dV(V)	B(mT)
0.0	0.000	0.00	0.000	0.000
1.0	0.022	0.01	0.001	0.121
2.0	0.044	0.01	0.001	0.242
3.0	0.066	0.01	0.001	0.362

Fig 5 shows the best fit line for which the slope/calibration constant k was calculated, using data collected from the magnetic flux passing through the Hall sensor within the Slink solenoid. The two graphs in this figure are placed together to show the direct proportionality that should be expected between current and magnetic field strength for an ideal solenoid.

Fig 6 shows the plots generated from Table 4. The magnetic field can be calculated from the voltage as $\beta = kV$ The abundance of points allowed the integration of the curve generated by them. The scipy integration routine *simps* was used for this purpose. Due to the symmetry of the magnetic field induced around the solenoid, only half of the length of the glass sample required direct measurement of the voltage, i.e. the integral of the magnetic field along half of the length of the sample can be doubled to produce the total integral.

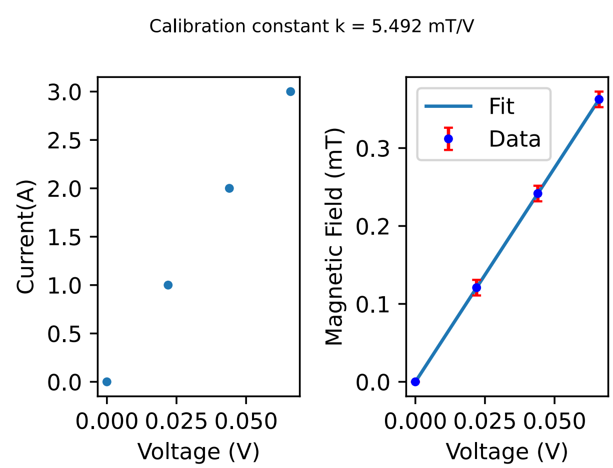


Figure 5: Data used to calibrate the Hall sensor from Table I. The calibration constant, $k = 5.492\,\text{mT}\,\text{V}^{-1}$ is determined from the slope of the best fit line generated from the data points. The right graph shows the strength of the magnetic field as a function of the current supplied to the solenoid (shown by the left graph).

In section 2, we discussed the ratio of total integral area of β field profile over the maximum β field magnitude at the center of the solenoid extrapolated to all points x along the glass. In Fig 6, this can be easily seen. The measured half of the length of the glass sample runs from $3.3\,\mathrm{cm}$ on the x axis (the center of the sample/solenoid) to $8.8\,\mathrm{cm}$ on the axis - which is why the β field drops sharply after this point. The product of the potential measured at the center of the tube and the length $\frac{L}{2} = 5\,\mathrm{cm}$ is $10.337\,\mathrm{V}$. The integral over the potential for the same interval is $9.936\,\mathrm{V}$. If the voltage is converted to β (using k) and integrated over again, then the result is $109.143\,\mathrm{mT}$. The ratio f discussed in 2 is $\frac{9.9365}{10.3377} = 0.96119$.

Table II: A sample (near center and near end of sample) of the data collected for building B-field profile through the sample.

z(cm)	V(V)	dz(cm)	dV(V)
3.3	2.027	0.1	0.001
3.4	2.027	0.1	0.001
3.6	2.027	0.1	0.001
3.8	2.027	0.1	0.001
8.2	1.702	0.1	0.001
8.4	1.646	0.1	0.001
8.6	1.559	0.1	0.001
8.8	1.447	0.1	0.001

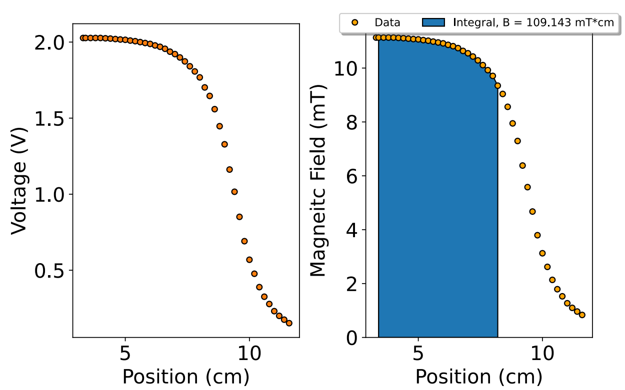


Figure 6: The calibrated Hall sensor was used to measure the voltage induced by magnetic field flux at various positions z along the solenoid (left graph). The calibration constant k was used to convert the voltage to a magnetic field magnitude for all points z (right graph). The right graph also shows the area under the curve - the total magnetic field strength along the length of the glass sample, $B = 109.143\,\mathrm{mT\,cm}$. The blue area under the curve represents the magnitude of magnetic flux through the length of the sample. Here measurement started at the center of the rod - 3.3 cm. The B field is symmetric about this origin.

Table III: Data collected in Verdet constant tabulation.

i(A)	Ang(°)	di(a)	dAng(°)	B(mT)
0.000	0.0	0.002	0.5	0.000000
0.495	0.0	0.002	0.5	5.510481
1.018	1.0	0.002	0.5	11.332665
1.503	2.0	0.002	0.5	16.731823
2.007	2.0	0.002	0.5	22.342494
2.510	3.0	0.002	0.5	27.942033
2.999	4.0	0.002	0.5	33.385720

The current measured in Table III is converted to β by simply scaling the current i by the magnitude of the magnetic field when current supplied to the solenoid was $1\,\mathrm{A}$. The relation is given $\beta_o(i) = (i)(\beta_o(i = 1\,\mathrm{A}))$. This allows for the construction of Fig 7, which maps the Faraday rotation angle θ by the magnetic field strength. The slope of the best fit line to this set of data provides the extra information needed to calculate the Verdet constant, υ. The slope was found to be $m = (12.4 \pm 0.1) \times 10^{-3}\,^{\circ}\,\mathrm{mT}^{-1}$. By rearranging Eq 2:

$$\upsilon = \frac{(12.4 \pm 0.1) \times 10^{-3}\,^{\circ}\,\mathrm{mT}^{-1}}{fL} \tag{7}$$

Calculating this value and converting units yields $(22 \pm 2)\,\mathrm{rad\,T^{-1}\,m^{-1}}$. The accepted value is set as $23\,\mathrm{rad\,T^{-1}\,m^{-1}}$. By this uncertainty, the experimental result agrees with the accepted value for the Verdet constant υ of SF-59 glass for $\lambda = 650\,\mathrm{nm}$ incident light.

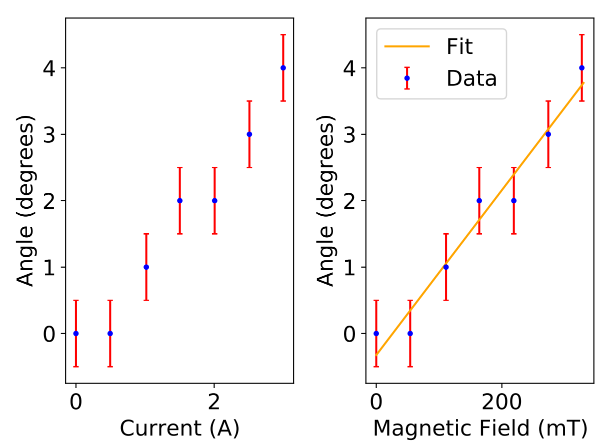


Figure 7: Data collected from table III is plotted here. The left graph shows the current supplied to the solenoid vs. the Faraday rotation angle. The right graph shows the magnitude of the magnetic field vs. the Faraday rotation angle. The best fit parameters for the right graph are slope $m = (12.4 \pm 0.1) \times 10^{-3}\,^{\circ}\,\mathrm{mT}^{-1}$ and y-int $y = (32 \pm 22) \times 10^{-2}$

5 Conclusion

The Verdet constant υ for the SF-59 glass sample for incident light of $\lambda = 650\,\mathrm{nm}$ was found to be $(22 \pm 2)\,\mathrm{rad\,T^{-1}\,m^{-1}}$. The accepted value is set as $23\,\mathrm{rad\,T^{-1}\,m^{-1}}$, giving the measured value a -4.34% error. However, the experimental result falls well within the uncertainty derived from propagation of error, making this an acceptable result. From simple physics, namely Ampere's Law applied to a solenoid, and Faraday Rotation, material properties such as the Verdet constant can be determined for various optically transparent materials, such as the SF-59 glass in this experiment. Malus' Law is also utilized, and the increased sensitivity predicted by it is verified. The proportionality predicted by Ampere's Law holds under the scrutiny of the experiment.

References

[1] "Faraday Rotation." TeachSpin, www.teachspin.com/faraday-rotation.

[2] Lab book. Hetrick. 2011, pp. 93–109, Faraday Rotation - TeachSpin.