PHYS460: Faraday Rotation

A Preprint

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Abstract

1 Introduction

Faraday rotation (also called the Faraday effect), named after the scientist who discovered it, is an optical phenomenon of electromagnetic waves. The effect is a rotation in the polarization of light.

For the effect to be produced, two requirements must be satisfied: the light measured must pass through a nearly uniform magnetic field, and a glass tube or some other transparent optical medium. In the experiment, the magnetic field is produced by means of a current carrying solenoid wrapped around a glass tube. While the magnetic field is more uniform near the center of the solenoid, the gradient near the edges and just outside the tube are negligible.

The rotation angle θ measured as the difference in the alignment of light entering the apparatus and the alignment of light incident on a detector (having passed through the apparatus) is quite small, yet Faraday rotation has some practical use in telecommunications, since a signal passing through the apparatus can be measured and interpreted (i.e., modulated).

One important optical property of matter, the Verdet constant, is discussed. The Verdet constant is proportional to the rotation angle. In the experiment, this value is calibrated. Additionally, Malus's Law, and the proportionality of light intensity passing through various polarizers is examined.

2 Theory

The phase speeds of circularly polarized light through a medium is different for left v_- and right polarized v_+ light, a phenomenon called circular bifringence. This is the physical mechanism that results in the rotation in the orientation of light sent through the tube. The phase speed difference, $\Delta v = v_+ - v_-$ is due to a difference in the index of refraction $\Delta n = n_+ - n_-$ for light through a medium with different polarizations (left/right circularly polarized light), and the difference in the index of refraction is proportional to the magnetic field induced by the current running through the solenoid. This phenomenon is known as circular birefringence. The linearly polarized light that is examined in this experiment is a result of the superposition of the left and right circularly polarized light:

$$\vec{E}_{linear} = \vec{E}_+ + \vec{E}_- \tag{1}$$

Linearly polarized light is measured as a electric field plane wave at position z and time t. The light propagates as a sin wave through space, which may take the form $\vec{E} = E_o cos(kz - wt)\hat{\imath}$ - here the $\hat{\imath}$ direction was arbitrarily chosen, but $\vec{E}$ can be in any direction.

The electric field rotation angle between the plane wave entering the tube and the plane wave incident on the detector is given by:

$$\theta = \upsilon \beta_o L \tag{2}$$

Where υ is the Verdet constant (material property), β_o is the nearly uniform magnetic field, and L is the length of the tube.

Since $\Delta n \neq 0$ and because $\beta_o \propto \Delta n$, Equation 2 can be rewritten to

$$\theta = \frac{\omega}{2c}(n_- - n_+)L \tag{3}$$

The intensity of polarized light passing through a polarizer is described by Malus's Law. Here, I_1 is the intensity of the light incident to the detector and I_o is the source intensity. Malus's Law is:

$$I_1 = I_o cos^2(\theta) \quad (4)$$

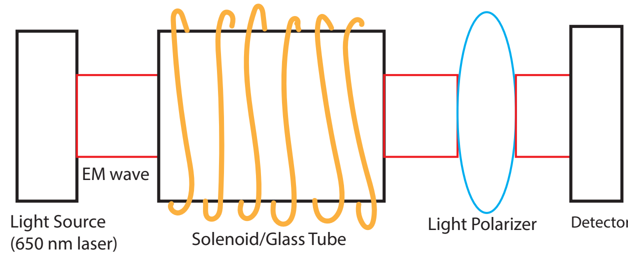

Figure 1: The apparatus used in the experiment.

3 Methods

The appartus used to conduct the experiment consists of four components: the first, a red laser pointer used as the light source, with a nominal wavelength of 650 nm at 3 mW power. The second is a glass tube wrapped in a copper winding. Third, a polarizer, which may be rotated $360°$. Finally, a photodiode serves as the detector. Each component is discussed further.

The light from the laser is not entirely polarized. It must be passed through a polarizer to increase polarization. This polarizer is part of the laser, and must be rotated to produce maximum light intensity as the light enters the solenoid. This can be achieved by placing a white piece of paper at the entrance of the solenoid, and dialing the polarizer until peak intensity is achieved (this can be done visually). The laser produces light nominally as red visible light at $\lambda = 650 \times 10^{-9}\,\mathrm{nm}$. A $4\,\mathrm{V}$, 40 power supply provides power for the laser.

A solenoid serves as the source of magnetic field in the experiment. The $140\frac{turns}{layer}$ solenoid wraps around the glass sample, with 10 layers total. The approximate calibration constant for the solenoid is given as $B = (11.1\,\mathrm{mT\,A^{-1}})(I)$. It is important to note that the magnetic field produced by the current carrying solenoid is not uniform throughout the sample - this is vitally important to the experiment. The magnetic field is determined along the length of the sample. More details on this follow. The maximum continuous current (MCC) for the solenoid is $3\,\mathrm{A}$ - running this level of current through the solenoid for any prolonged period will result in overheating and possible damage to the apparatus.

A rotatable Polaroid film serves as the polarizer in the experiment. The polarizer is marked with $5°$ increments, with an approximate associated uncertainty of $1 - 2°$.

Finally, a photodiode serves as the light detector at the end opposite of the source. The photodiode has three resistances to choose from 10, 3, or $1\,\mathrm{k\Omega}$. Values read from the photodiode are taken only if the bias voltage is well below saturation voltage - in this case, $0.3\,\mathrm{V}$. This guarantees that measurements are in the linear regime. The intensity of the light incident on the detector is measured as a voltage. Figure **??** shows the configuration for the apparatus.

The Faraday rotation angle of the polarized light is used in computing the Verdet constant. There are two primary methods of measuring this angle - one is called the extinction method. The first step in this process is to rotate the Polaroid film $90°$ relative to the light incident to the tube from the laser so that maximum extinction of the light occurs at the detector. Then, a current is supplied to the solenoid. After the solenoid is powered on and the magnetic field is produced, the polarizer is rotated to find the angle which maximimizes light intensity. This process is repeated for a variety of magnetic field strengths by varying the current supplied to the solenoid.

As noted previously, the induced magnetic field is not uniform along the length of the solenoid. The magnetic field is then a function of position along the length of the solenoid. This mandates the necessity to calibrate and profile the B field.

To calibrate, first a set point angle is determined. To maximize sensitivty of the photodiode, measurements of voltage are taken when the polarizer is rotated $45°$ past the angle which maximizes light intensity. Since the intensity is a wave function which peaks and troughs at $\pm\frac{n\pi}{2}$, a higher sensitivity is achieved at $\pm 45°$ of the maximum/minimum angle due to the steeper slope. See figure 2 to see this visually.

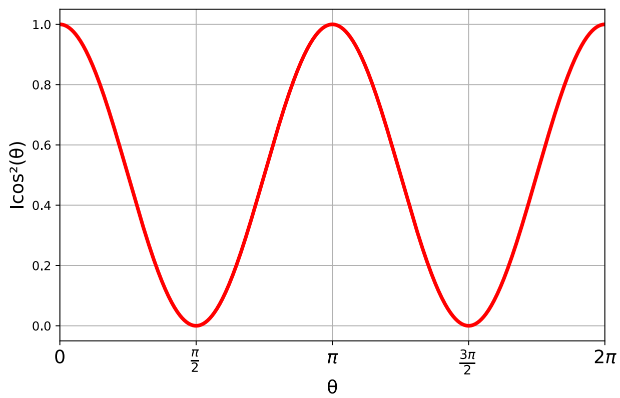

Figure 2: Malus's Law. Note how the slope is greatest between peak and trough.

With the polarizer set at the reference angle, the solenoid is powered to produce a magnetic field, the magnitude of which is computed for the supplied current. This produces a voltage in the light detector, which is the associated reference voltage, V_{out}. Then, the polarizer is rotated θ, and the difference in voltage at the detector ΔV_{out} is determined. This process is repeated over various currents to generate a calibration constant in $\frac{V}{mT}$. This constant was determined to be $0.1823\frac{V}{mT}$. This value agrees with the TeachSpin manual's given calibration constant for the magnetic field, $11.08\,\mathrm{mT}$. It is worth mentioning that the calibration constant which was generated experimentally was found over a smaller voltage range than Teachspin's - smaller by two orders of magnitude. While the calibration constants were found to agree with each other, it is a stretch to assume that the experimental result extends into the voltage regime used by TeachSpin.

The non-uniform magnetic field was profiled by measuring its strength along various points of the solenoid. A custom magnetic field sensing circuit was built specifically for this using a resistive sensor, as the TeachSpin probe was not compatible with the sample. The magnetic field approximates a gaussian curve over the length of the solenoid, with the peak at the center. To determine a theoretical Faraday rotation, an integral must be taken over the length of the solenoid

$$\Delta\theta = \upsilon \int_{-5cm}^{5cm} \beta \, dx$$

4 Results and Discussion

5 Conclusion

References