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Plastic CD containers as cylindrical acoustical resonators

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An empty cylindrical CD container is a ready-made acoustical resonant cavity. It requires only two additional holes: one in the base for a small speaker and one for a small microphone in the side. Because the top can rotate with respect to the base, the angular behavior of each resonant mode can be observed in addition to its frequency. The frequency is easily changed, and signals are readily displayed on an oscilloscope. The measured frequencies and angular dependence are in good agreement with theory.
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A plastic cylindrical container that holds 50 CDs is well suited for measuring acoustic cylindrical cavity resonances. (A bigger container for 100 or more CDs will work, but as the height L increases, the frequencies become more crowded together, and more modes nearly overlap.) The 50 CD container's approximate dimensions are $L=79$ mm, inner column radius $a_{\rm in}=7$ mm, and outer radius $a=62$ mm. There are minor irregularities (about 1% variation in either radius and small indentations at the bottom and top), but a satisfying number of resonances can be measured close to their calculated values. The beauty of such a container is that the top can be rotated with respect to the base so that as various resonant frequencies are measured, the angular dependence of the frequencies can be checked to see if they correspond to the expected mode.

The CD container is shown in Fig. 1, adapted for use as a resonator with the addition of two holes: one for a microphone[1] attached close to where the side and top surfaces meet and the other for a small internal speaker[2] mounted on the base. The microphone signal is led through an audio amplifier[3] to an oscilloscope. Resonances are found by slowly changing the frequency using a digital signal generator[4] to drive the internal speaker. To see a mode's angular dependence, the top is rotated by hand, while watching the peaks and valleys of the oscilloscope signal. The azimuthal angles θ were measured with a protractor resting on top. The two light lines in Fig. 1 indicate that the angle θ is measured from an imaginary line from the center to the speaker (in the base) and a line from the center to the microphone (in the rotatable top).

The microphone was attached where the curved side meets the flat top surface in order to pick up both the radial and axial pressure peaks on the oscilloscope (both maxima and minima appear as peaks in the oscillating scope signal). The peaks occur close to the top and to the sides because air molecules near a fixed boundary can have no velocity component perpendicular to the boundary surface and the acoustic velocity is proportional to the gradient of the acoustic pressure. This boundary condition requires the pressure at the top or bottom of a right circular cylinder to be a maximum or minimum with respect to the axial direction. Likewise, next to the curved side surface the pressure must be a maximum or minimum in the radial direction.

To excite modes within the cavity, a thumbnail-sized hole was made in the base, about 45 mm from the axis. This hole was for a small speaker,[2] which is not robust and needs careful handling. Poster wax was used to hold the microphone in place, and the speaker was sealed with poster putty.[5] The juncture between the curved side and the base was left free to rotate, and no attempt was made to seal it.

The peak-to-peak angular values in Table I are averages over at least three repeated trials. Care was required because rotation of the top by hand produces some jitter on the oscilloscope, and extra pressure on the top causes the amplitude on the oscilloscope to vary. All peak-peak angles had an uncertainty of 2°, except the very broad peaks separated by 180°. The top has three locking tabs, 120° apart, which slide into three corresponding raised areas on the base. The fit between the top and base is snug, but not airtight, and about 0.3 mm of lateral play exists between the curved surface of the top and each of the three raised areas on the base. The tabs were left on for peak-peak angles of 90°–30° and were removed for the 180° peak separation to improve the precision of the angle measurement.

Frequency and angle measurements were also attempted using an external speaker, letting the sound come through the CD container walls. This arrangement gave the same frequencies as with the internal speaker, but the angle measurements were not satisfactory possibly because the internal speaker established a definite angle reference and the external speaker did not. Because of problems with wall flexing at low frequencies, a different cavity with thicker walls was also used, and sound entered through the walls from an external speaker.

Among the simplest cavity resonances are those where the pressure is constant in r and θ, varying only in the axial direction z. Because the z derivative of the pressure must vanish at $z=0$ and $z=L$, the resonant pressure wave at angular frequency ω is

$$p(z,t) = A\cos(\ell\pi z/L)\cos(\omega t), \quad \ell = 0,1,2,\ldots . \tag{1}$$

These modes are like those in a long pipe with either both ends open or both ends closed, corresponding to an integer number of half-waves in the length L. (For $c=345$ m/s and $L=79$ mm, the lowest pipe mode frequency is 2184 Hz.)

A standing pressure wave in an empty cylindrical cavity (no central column) involves[6] a Bessel function $J_m(x)$ of in-

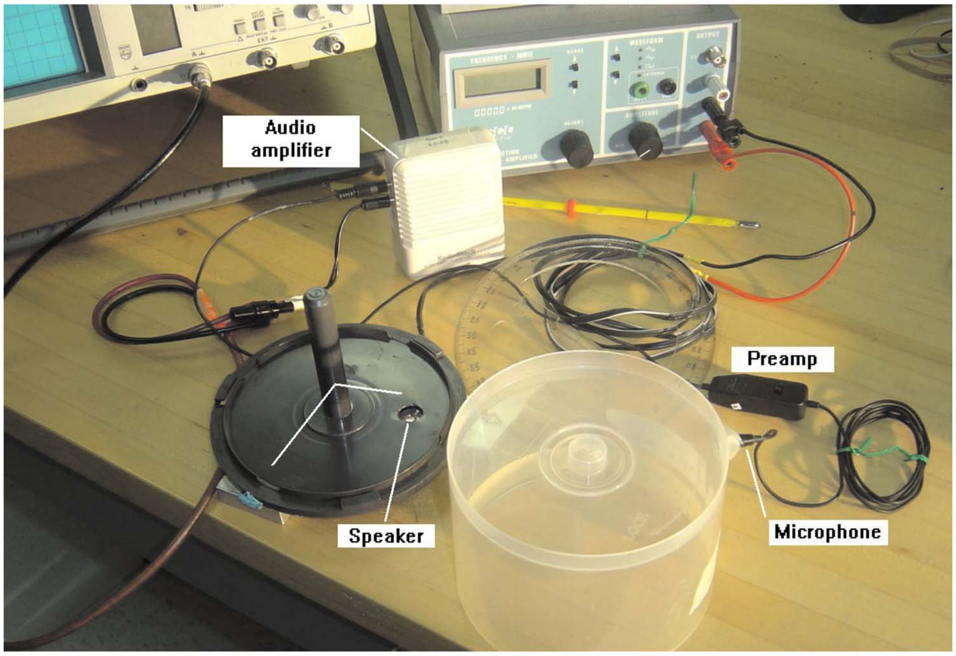

Fig. 1. CD container acoustical resonator setup with a signal generator, small speaker, microphone, audio amplifier, oscilloscope, and protractor.

teger order m, where $x = k_r r$ and the parameter k_r is the radial component of the propagation vector k. For a pressure maximum or minimum at $r = a$, the radial derivative of $J_m(x)$ must vanish. The corresponding values of x are the roots of $J'_m(x)$, with the nth root of $J'_m(x)$ designated as $x'_{mn} = k_{r\ mn} a$.

The pressure wave function[6] in the cavity includes a sine or cosine of $m\theta$, the Bessel function $J_m(k_r r)$, and a cosine function in the z direction, along with a sine or cosine in the angular frequency ω,

$$p(r,\theta,z,t) = AJ_m(k_r r)\cos(m\theta)\cos(\ell\pi z/L)\cos(\omega t), \quad \ell, m = 0,1,2,\ \ldots . \tag{2}$$

For waves in one dimension, $c = \omega/k$ and $\omega = ck$. The generalization to cavity resonances is

$$\omega^2_{mn\ell} = c^2[k^2_{r\ mn} + (\ell\pi/L)^2]. \tag{3}$$

The mode frequency $f_{mn}\ell$ involves the integer m of the Bessel function $J_m(x)$ (the same integer as for cos $m\theta$), the nth root (x'_{mn}) of $J'_m(x)$, and the integer ℓ for the z direction. With $k_{r\ mn} = x'_{mn}/a$, we have

$$f_{mn\ell} = \frac{c}{2\pi}\left[\frac{x'^2_{mn}}{a^2} + \left(\frac{\ell\pi}{L}\right)^2\right]^{1/2}. \tag{4}$$

Following Ref. 6, we designate the 0th root x'_{m0} as the first time $J'_m(x) = 0$, where $J_m(x)$ itself does not vanish. Figure 2 shows that $J_0(0) = 1$ at $r = 0$ and its slope $J'_0(0) = 0$. So the 0th root of $J'_0(x)$, x'_{00} occurs at $x = 0$. When $x'_{00} = 0$ is substituted in Eq. (4), we obtain the pipe modes with $f_{00}\ell = (c/2L)\ell$.

When $\ell = 0$, another important group of frequencies occurs. The f_{nm0} modes depend only on r and θ, the pressure is independent of z. These mode frequencies are given by

$$f_{mn0} = \frac{cx'_{mn}}{2\pi a}. \tag{5}$$

Table I gives the measured and calculated frequencies f_{m00} for $m = 1-6$, as well as their angular dependence. The angular component of f_{100} is cos θ, with twofold symmetry, and 180° between one maximum and the succeeding minimum. This angular displacement is the distance between the observed peaks on the oscilloscope and is referred to in Table I

Table I. Resonant frequencies f_{m00} of a cylindrical cavity (radius of $a = 61.8$ mm at 24 °C) for $m = 1-6$ and the angle between observed peaks, $\Delta\theta_{\text{peak-peak}}$, for each. (Integer values of m come from the solution of the wave equation in cylindrical coordinates.) Values of x'_{m0} occur where the slope of the radial wave function vanishes at $r = a$ (see text). The frequencies f_{m00} are proportional to x'_{m0}.

m	1	2	3	4	5	6
x'_{m0}	1.841	3.045	4.201	5.317	6.415	7.468
f_{m00} calculated Hz	1639	2718	3739	4732	5709	6675
f_{m00} measured Hz	1604	2755	3746	4736	5706	6668
$\Delta\theta_{\text{peak-peak}}$, theory	180°	90°	60°	45°	36°	30°
$\Delta\theta_{\text{peak-peak}}$, observed	184°	88°	58°	44°	37°	30°

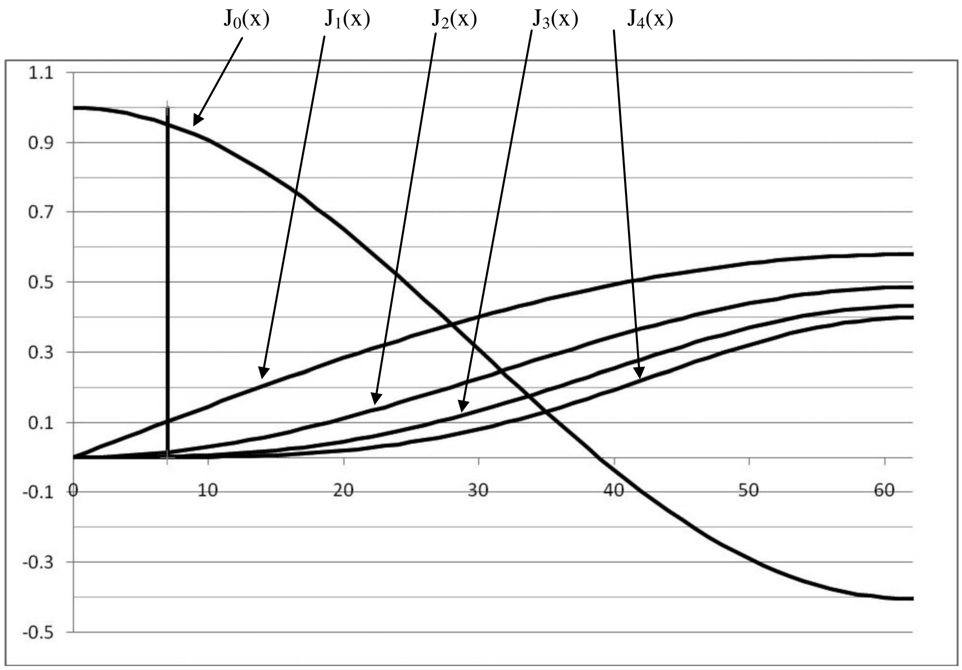

Fig. 2. The lowest five Bessel functions (as they would occur at resonance in a cylinder with a radius of 62 mm) versus radial distance in millimeters. The vertical line represents the 7 mm radius of the inner column in a CD container.

as $\Delta\theta_{\text{peak-peak}}$. Observing modes higher than $m=6$ was increasingly difficult. The internal speaker was 45 mm from the central axis and the diameter of its moving surface was about 20 mm, so the speaker subtended an angle of 25°. Higher-m peaks are more and more closely spaced. For example, the $m=7$ peaks are separated by 26°, $m=8$ peaks by 22.5°, so the internal speaker was not able to effectively excite modes much above $m=6$.

There are discrepancies between calculated and measured frequencies for $m=0$, 1, and 2. To find out why these occur, the following measures were taken. First, the internal speaker was blocked by a coin sealed with poster putty, and sound from an external speaker was allowed to enter through the cavity walls. This procedure left the frequencies unchanged, so the internal speaker appeared not to cause the problems. The central column was then removed and its hole sealed. The frequencies changed, but the agreement with theory was not any better. The juncture between the top and the base was sealed, but this seal did not appreciably change the frequencies. A more general theory (see the Appendix) was applied to account for the central column. This theory altered the calculated low-m frequencies, but agreement with theory was unimproved. That left the possibility that the low-m frequency discrepancies were due to slight flexing of the container walls. To test this idea, several 20 g masses were attached to the curved sides at various points, and it was found that the resonant frequency near 1600 Hz could be changed by 20 Hz.

The CD container walls are about 1 mm thick and are made of a relatively soft plastic. A thicker (3.5 mm) cylinder of stiffer plastic (one of a set of stackable storage units) was used to reduce or eliminate wall flexing. A single hole for the microphone was made in the side where it meets the top. An external speaker was placed a few inches outside the cylinder, and its sound waves penetrated the walls. This arrangement was satisfactory for determining mode frequencies inside the cylinder, but the observer had to be quite still because movement outside the cylinder changed the pattern of reflections and altered the amplitude seen on the oscilloscope. Of 16 frequencies ($m=0$ through $m=4$, and various ℓ values), one measurement differed from the theory by 0.9% and another by 0.5%. The others averaged within 0.3% of the theory.

A section of 1/4 in. pvc pipe was then cut to serve as a central column and put in place with poster putty, carefully centered by eye. The low-m frequencies which had been somewhat off for the CD container, agreed within 1% with the calculations based on a linear combination of Bessel functions (see the Appendix). A piece of 1 in. pvc pipe was cut and served as the central column in a second trial. Again, there was agreement to 1% or better for the low-m modes, while the higher-m modes continued to agree well.

An empty cylindrical CD container works well as an acoustic resonator, allowing measurement of both frequency and angular dependence of acoustic modes. Its radius increases by about 1% from top to bottom so that roughly 1% agreement between measurement and theory might be expected (ignoring minor irregularities in the base and top). Table I shows better than 0.2% agreement for the highest four modes listed, which is surprisingly good. The 2% disagreement for the lowest two listed modes is not entirely satisfactory. Part of the discrepancy in the lower modes is due to the presence of the central column. (All slopes of J_m should vanish at a_{in}, but Fig. 2 shows a pronounced slope there for J_1 and smaller slopes for J_0 and J_2. See the Appendix for further discussion.) A bigger effect appears to be the flexing of the relatively thin container walls in the lower modes. To avoid wall flexing, another cylinder with much thicker walls was employed, having a short section of pvc pipe serving as a central column. Data were taken on this thicker container with and without a central column, and in all cases agreement of 1% or better was obtained for the lower modes. (The theory for cylindrical resonances with an inner cylinder requires a linear combination of Bessel functions, as discussed in the Appendix.)

Before working with a cylindrical cavity like the one we have discussed, students might initially study a rectangular cavity. The modes in such a cavity would be a scaled-down version of those for an empty rectangular room (see Ref. 6, p. 402), and the functions describing the modes would be the more familiar sine and cosine functions. If the walls were not too thick (perhaps 1/8 in. plastic), only a single hole might be needed for a microphone, with sound from an external speaker entering the cavity through its walls.

The present internal speaker has a finite diameter and a limited ability to excite higher angular modes. A worthwhile student project would be to change the speaker diameter and to study the effect on higher-m cavity modes. A rigid baffle could effectively reduce the diameter, or a speaker of different diameter might be used.

When c is taken as the speed of light rather than the speed of sound in Eq. (4), we obtain the formula for TE mode resonances of an empty conducting circular cylindrical cavity.[7] Such a cavity has often been employed as a wavemeter in 2–20 GHz waveguide systems.[8] For students of electromagnetic theory, a cylindrical acoustic cavity could be an instructive demonstration or laboratory experiment.

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APPENDIX: A CENTRAL COLUMN IN A CYLINDRICAL RESONATOR

When a central rigid column of radius a_{in} exists inside a cylindrical cavity of outer radius a, the radial slope of the pressure function must vanish at both a_{in} and a. Figure 2 shows a plot of Bessel functions J_0–J_4 versus r. Each function is plotted with a maximum or minimum at $r=62$ mm, as would occur at a resonance. A vertical line denotes the location of the central column at $a_{\text{in}}=7$ mm, the approximate column radius for a CD container. The slope of each J_m should vanish at a_{in}, but Fig. 2 shows a pronounced slope there for J_1 and smaller slopes for J_0 and J_2, while slopes at a_{in} for J_3 and J_4 are close to zero. When the slope at a_{in} is noticeably different from zero, the mode figures to be affected by the presence of the inner column. But the slopes of J_3 and J_4 are already very close to zero at $r=a_{\text{in}}$ and satisfy the zero-slope condition there for all practical purposes, and thus the central column would barely change their resonances. The slope of J_5 at a_{in} will be smaller yet, so this and higher modes should be practically unaffected by the central column.

To find the resonant frequencies in the presence of the central column, a linear combination of both types of Bessel functions is needed

$$p(r,\theta,z,t) = (AJ_m(x) + BY_m(x))\cos(m\theta)\cos\left(\frac{\ell\pi z}{L}\right)\cos(\omega t). \tag{A1}$$

The radial slope of $AJ_m(x)+BY_m(x)$ must vanish at $x_1 = k_r a_{\text{in}}$ and at $x_2=k_r a$, so we must find the ratio A/B and also the value of k_r. Both J_m and Y_m satisfy the recursion relation (Ref. 7, p. 113) $\mathrm{d}\Omega_m(x)/\mathrm{d}x=(\Omega_{m-1}(x)-\Omega_{m+1}(x))/2$, so the ratio A/B for vanishing radial slope can be expressed as

$$A/B = -Y'_m(x)/J'_m(x) = -\frac{Y_{m-1}(x) - Y_{m+1}(x)}{J_{m-1}(x) - J_{m+1}(x)}. \tag{A2}$$

The A/B ratios were set up in a spreadsheet at x_1 and again at x_2. (Bessel functions are not supported in versions of Excel before 2007 but are supported by a freeware spreadsheet.[9]) A scroll bar[10] was used to adjust the value of k_r until the A/B values agreed at x_1 and x_2.

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[1]Radio Shack Tie-Clip microphone, Catalog No. 33-3013.
[2]Radio Shack 8-Ohm minispeaker, Catalog No. 273-092.
[3]Radio Shack Mini Audio Amplifier, Catalog No. 277-1008.
[4]PASCO PI-9587C function generator (now superseded by model PI-8127) (www.pasco.com).
[5]The author's blue putty (as well as an equivalent wax product) came from Office Max.
[6]P. M. Morse, *Vibration and Sound*, 2nd ed. (McGraw-Hill, New York, 1948), p. 399.
[7]J. D. Jackson, *Classical Electrodynamics*, 3rd ed. (Wiley, New York, 1999), p. 370.
[8]E. L. Ginzton, *Microwave Measurements* (McGraw-Hill, New York, 1957), Chap. 7.
[9]OpenOffice is available from (www.openoffice.org/).
[10]D. L. Hatten and M. J. Moloney, "User-defined scroll bars in spreadsheets," Phys. Teach. **42**, 166–170 (2004).