

Combining climate and remote sensing data in species distribution models

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Preface

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Summary

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Chapter 1

Introduction

Species distribution modelling (SDM)¹ concerns the practice of modelling the distribution of a species by use of explanatory variables. Applications include predicting the effect of climate change (Pearson and Dawson 2003; Pearson, Dawson, and Liu 2004), predicting the impact of invasive species (Strubbe and Matthysen 2008), predicting the occurrence of wildfires (Parisien and Moritz 2009), ...

Some fundamental ecological concepts are introduced in Chapter 2. Firstly, the concept of an ecological niche is introduced and the connection with SDMs is made. Secondly, to enhance the understanding of the niche concept some of the assumptions underlying the practice of SDM are discussed.

The data-sets and variables that are used in this thesis are described in Chapter 3. These variables describe either the climate, human influence, elevation, ... at a certain location. Climate data is nearly always used to model the occurrence probability of species. When the goal of a study is to obtain coarse grain predictions over a large spatial extent the use of climate data is certainly justified by ecological theory (Pearson and Dawson 2003). However, if there is interest in predictions over a relatively small extent, e.g. when selecting the location of a new national park, fine grain remote sensing data might be useful to distinguish between suitable and unsuitable habitat.

Chapter 4 introduces a number of modelling techniques that are often used to model the distribu-

¹The abbreviation SDM will be used for both the verb, species distribution modelling, and the noun, species distribution model.

tion of species. In Section 4.2 we focus on classical binary classification methods. These methods are directly applicable if there is access to presence-absence data. However, often the data-sets only includes occurrence locations, for example data-sets from natural history musea or citizen science projects are usually of this type. To use presence-only data the classical classification algorithms from Section 4.2 can be adapted, this is done in Section 4.3.2. Another approach is to use one of the algorithms specifically constructed for presence-only data, one of these is introduced in Section 4.3.3.

In practice a large part of constructing SDMs consists of variable selection. The goal of this thesis is to investigate the performance of multiple model selection methods. An overview of often used methods to deal with large magnitudes of predictors is given in Chapter 5. More particularly, we will introduce:

- Regularization.
- Step-wise selection.
- Dimensionality reduction of the explanatory variables.

Although we will introduce the most important concepts and some applications of species distribution modelling, it is not the goal of this thesis to describe every aspect in detail. Instead we refer to Franklin and Miller 2009 who gave an overview of the field of species distribution modelling. Other introductory material include Guisan and Zimmermann 2000, Guisan and Thuiller 2005, and Elith and Leathwick 2009. An introduction to most of the statistical methodology can be found in Hastie, Tibshirani, and Friedman 2009.

Chapter 2

The ecological niche concept

2.1 Introduction

In this chapter the concept of the niche of a species is introduced. The ecological niche of a species can, non-rigorously, be defined as the set of environmental conditions where its reproduction rate is larger than or equal to its mortality rate. Although we will speak of the ecological niche, there are in fact at least three different “definitions” that are often used: the Grinnellian niche, the Eltonian niche, and the Hutchinsonian niche. Only a sketch of the niche concept will be given in this section. For a more rigorous description we refer the interested reader to Soberón 2007 and, Soberón and Nakamura 2009.

2.2 Ecological versus geographical space

In most databases that contain data about species only the location of a presence or absence record is available. Hence, these databases include information about the occurrences or absences in the so-called geographical space. Usually the range of a species’ distribution is determined by environmental conditions. We will say that the corresponding variables span the environmental space. It is clear that for each point in the geographical space there is a point in the environmental space. This relation between environmental and geographical space is often called Hutchinson’s duality (Colwell and Rangel 2009). A graphical representation of this relation is given in Figure 2.1. This duality relation is fundamental in SDMs, namely the predictors included in the model are usually assumed to be direct or indirect measures of the variables that span the environmental space. Once a model in the environmental space is constructed the duality relation allows us to make maps of the distribution in the geographical space.

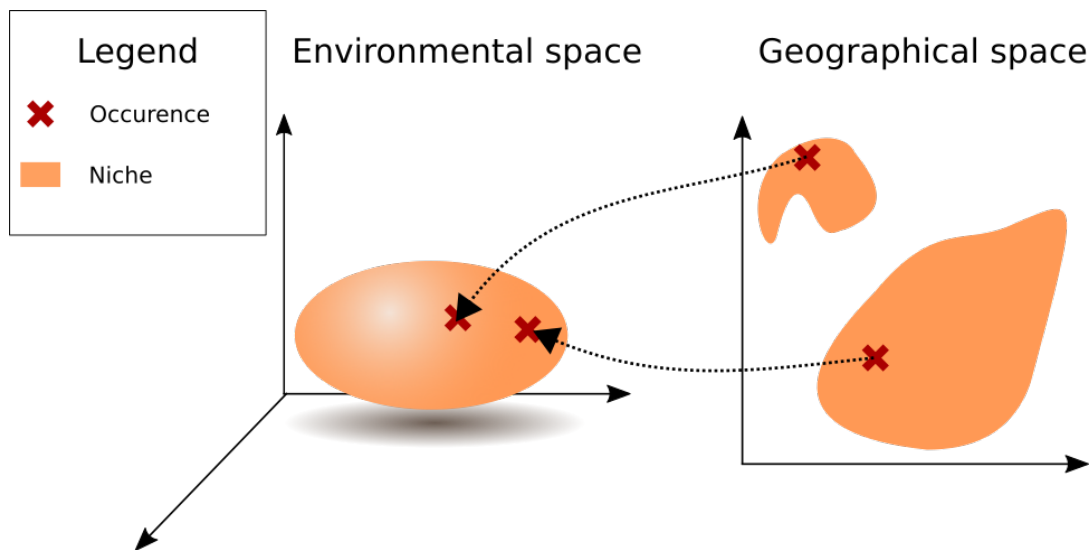


Figure 2.1: Visualization of the duality between environmental and geographical space.

In practice a species will often not occur in certain parts of its niche. This can happen because of limited dispersal capabilities of the species, biotic interactions, etc. Such incomplete occupation of the niche leads to the concepts of a fundamental niche and the realized niche. The fundamental niche does not take into account whether or not the species is present, it only represents the suitable conditions. The realized niche is the subset of the fundamental niche where the species is present. These two concepts are depicted in Figure 2.2.

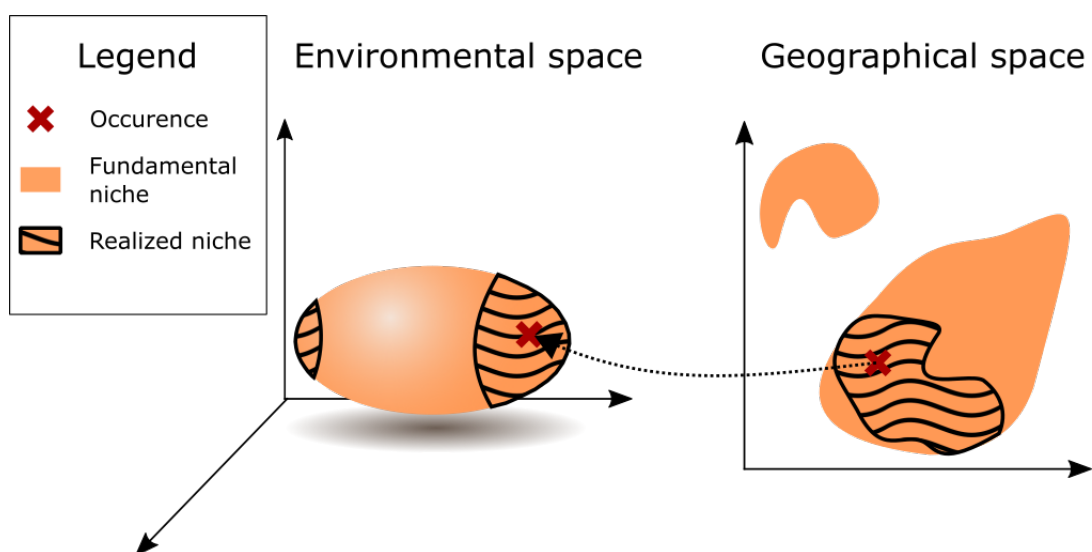


Figure 2.2: Visualization of the difference between the fundamental and realized niche.

2.3 Implicit assumptions when building and using species distribution models

Before applying SDMs in practice one has to realize that there are some important underlying assumptions. A few of these assumptions are described below. This is done in order to connect the theoretical niche concept to some more practical scenarios and to make the reader aware of the limitations of species distribution modelling. For a more complete overview of the underlying assumptions we refer to Wiens et al. 2009.

Every observed occurrence belongs, by definition, to the realized niche. SDMs are therefore models of the realized niche. If a SDM is used to e.g. predict areas prone to invasive species, it is implicitly assumed that, a part of, the realized niche is a good approximation of, a part of, the fundamental niche. Whether this assumption is realistic or not depends on: the species, whether the whole niche has to be approximated or only a part thereof, etc.

When data is used to build a model of the realized niche it is assumed that the observed data-points are representative of the niche. In practical settings this is often not the case. We give two examples:

1. Due to climate change tree species might be found in regions where the current environmental conditions are not included in its niche (Woodward, Fogg, and Heber 1990).
2. The niche of a species usually evolves over time, therefore older observations might not be representative of the current niche (Pearson and Dawson 2003).

Another assumption that is implicitly made in most SDMs is that the effect of biotic interactions is negligible or indirectly captured by other environmental variables. However, in some applications explicitly including biotic interactions has been shown to improve the predictive capabilities (e.g. Heikkinen et al. 2007).

Chapter 3

Data

3.1 Introduction

To study the performance methods to reduce the number of dimensions in SDMs we need data-sets containing on explanatory and outcome data. The different explanatory variables that are used are described in Section 3.2. Given the goals of the thesis we opted for using 33 different variables. This is more than used in the average SDM and it should be expected that some of them are quite redundant for the species of interest. The species of interest and data-sets that consist of locations where the species was observed to be present or absent are introduced in Section 3.3.

To process the spatial data R (R Core Team 2013) is used as a geographic information system (GIS). To do this we rely on the RASTER (Hijmans 2015) and SP (Pebesma et al. 2015) packages.

3.2 Predictor data

3.2.1 Vegetation Continuous Fields

The Vegetation Continuous Fields (VCF; DiMiceli et al.) data-set contains values between 0 and 100 which are proportional estimates of the tree cover in the cell. Some cells also contain values larger than 100 which indicate that the cell contains water or that no data is available. Since R has a NA value all the cells with values larger than 100 are set to NA. The raster is provided in the geographical coordinate system combined with the World Geodetic System 1984 datum (GCS_WGS84). The resolution of the VCF raster is 0.00208 decimal degrees.

3.2.2 Bioclimatic variables

The bioclimatic variables are a set of variables that describe ecologically relevant climate patterns.

find a citation

The definition of each of the 19 bioclimatic variables can be found in Table 3.1. The bioclimatic variables can be derived from monthly minimum, maximum, and average temperature and precipitation data. Furthermore, it is clear that the BIO05, BIO6, and BIO7 variables are linear dependent. This linear dependence can be problematic when using classification methods. It is interesting that for some of the methods that we introduce this will lead to no problems while it will for others, see Section 5.6.

Variable name	Definition
BIO1	Annual mean temperature
BIO2	Mean diurnal range (mean of monthly $(temp_{max} - temp_{min})$)
BIO3	Isothermality $\left(100 \times \frac{BIO2}{BIO7}\right)$
BIO4	Temperature seasonality $\left(SD(temp_{avg}) \times 100\right)$
BIO5	Max temperature of warmest month
BIO6	Min temperature of coldest month
BIO7	Temperature annual range (BIO5 – BIO6)
BIO8	Mean temperature of wettest quarter
BIO9	Mean temperature of driest quarter
BIO10	Mean temperature of warmest quarter
BIO11	Mean temperature of coldest quarter
BIO12	Annual precipitation
BIO13	Precipitation of wettest month
BIO14	Precipitation of driest month
BIO15	Precipitation seasonality $\left(\frac{SD(percipitation)}{mean(percipitaiton)}\right)$
BIO16	Precipitation of wettest quarter
BIO17	Precipitation of driest quarter
BIO18	Precipitation of warmest quarter
BIO19	Precipitation of coldest quarter

Table 3.1: Definition of the bioclimatic variables.

The monthly temperature and precipitation data was obtained from the PRISM database (Daly et al. 2002, PRISM Climate Group). The PRISM rasters have a grid cell size of 0.00833 decimal degrees and the rasters are provided in the GCS_WGS84 spatial reference system.

To calculate the bioclimatic variables we adapted the BIOVARS function from the DISMO package

(Hijmans et al. 2015). The BIOVARs function from the DISMO package does not allow the user to provide a layer of the mean temperature. Instead of using the mean temperature in the calculations it uses the average of the minimum and maximum temperature. Our adaptation does use the mean temperature layers and should be slightly more accurate.

3.2.3 Normalized Difference Vegetation Index

The Normalized Difference Vegetation Index (NDVI) is an index of the amount of vegetation. It is based on measurements of the reflectance of the infra-red and the near infra-red region. The NDVI takes on values between -1 and 1 . High values correspond with live green vegetation. The NDVI raster used in this thesis originates from the Global Inventory Modelling and Mapping Studies (GIMMS) and is provided by the University of Maryland Global Land Cover Facility (Pinzon, Brown, and Tucker 2005; Tucker et al. 2005). This database contains semi-monthly rasters of the NDVI value for the period 1983-2006. The original rasters had a cell-size of 0.07266 decimal degrees. These rasters were originally

cite paper Brody / Svenning on greenes in the US

resampled to 0.04166 decimal degree rasters and cells with values corresponding to water or with no data available were set to NA. The semi-monthly rasters were then combined such that monthly rasters were obtained. To obtain an average monthly NDVI raster for each month the 24 NDVI rasters of the different years were averaged. The 12 resulting monthly NDVI rasters can then be used to calculate a minimum, maximum, and mean NDVI raster.

3.2.4 Digital elevation model

The digital elevation model (DEM) raster (*CIAT-CSI SRTM*) contains data on the elevation throughout the US. It is the raster with the highest resolution, more specifically the cell size is 0.000833 decimal degrees and the raster is provided in the GCS_WGS84 spatial reference system. The use of elevation data in SDM is somewhat contested, see e.g. Hof, Jansson, and Nilsson 2012 and Oke and Thompson 2015. However, since our main purpose is to test model selection techniques adding a potentially irrelevant predictor should not matter. Furthermore, it is often suggested that other variables directly derived from DEMs, e.g. slope, are more ecological relevant (Franklin and Miller 2009). However, to keep the amount of data in this thesis handleable we will restrict ourselves to the original DEM raster.

3.2.5 Land cover

The land cover data was created from the National Land Cover Databases (NLCD) provided by the US Geological Survey. The NLCD were derived from landsat imagery of 2001 (Vogelmann et al. 2001), 2007 (Homer et al. 2007), and 2011 (Fry et al. 2011). These datasets were then transformed into rasters that utilize the Anderson level 1 classification (Anderson et al. 1976). Eight different land cover classes are used: barren, forest, ice-snow, grassland, urban, water, wetlands, agriculture. For each land cover class-year combination rasters with a cell size of 0.04166 decimal degrees were created. In order to obtain one raster for each land cover class the three corresponding rasters were averaged. The eight final rasters were then rescaled such that the values of the rasters lie within the interval $[0, 1]$. For each land cover class raster the cell values are an estimate of the percentage of the land of this class within the cell. It is interesting that the sum of these rasters equals one, hence there is a linear dependence between the variables.

3.2.6 Human Influence Index

The Human Influence Index (Wildlife Conservation Society - WCS and AU - Center for International Earth Science Information Network) raster contains integer values between 0 and 64. High values indicate a strong human influence and vice versa. The index is derived from measures of the population density, the amount of roads, the amount of light sources during night-time, etc. The raster is reprojected to the GCS_WGS84 spatial reference system and has a cell size of 0.00833 decimal degrees.

3.2.7 Preprocessing of the predictor data

In order to speed up computations and facilitate general GIS operations the rasters were, if necessary, reprojected to the GCS_WGS84 spatial reference system. The extent and resolution of the rasters was set to be equal to those of the DEM layer. When it was necessary bilinear interpolation was used. This procedure makes sure that the cells of the rasters line up nicely. A visualization of the whole process can be found in Figure 3.1. Once all the data was preprocessed the rasters amount to over 300 GB of data.

3.2.8 Exploratory analysis of the predictor data

It might be expected that the relationship between the variables is different in different regions of the contiguous US. To test this we defined four regions, their bounding rectangles can be found

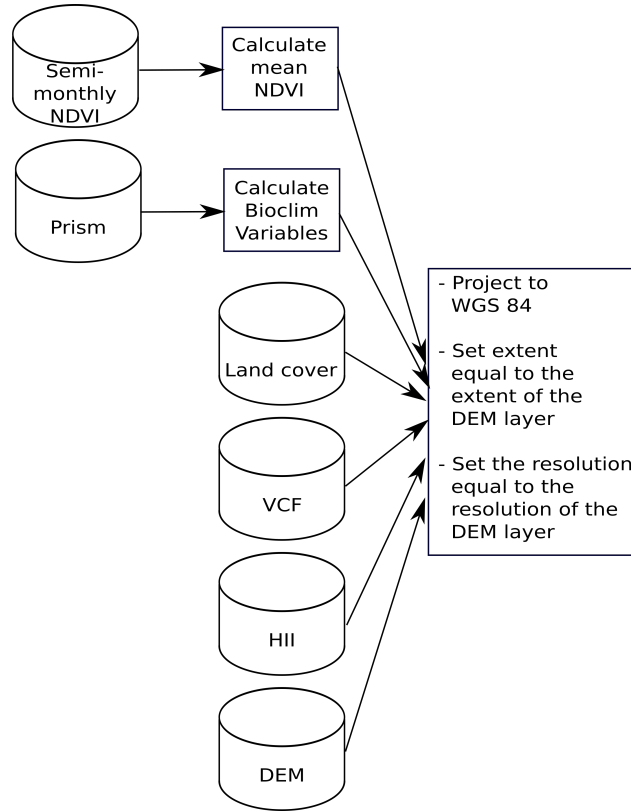


Figure 3.1: Visualization of the preprocessing of the raster data.

in Figure 3.2. To test our suspicions two sets of random points were generated: one with points within the contiguous United States and one that contains points within the West Coast region. For each point the corresponding values of the predictor rasters were extracted. Heat maps of the correlations of the predictor variables can be found in Figures 3.3 and 3.4. A quick inspection of these plots learns that most correlations are approximately equal in the two ranges. There are also some correlations that change quite dramatically, e.g. the correlation between the ice-snow land cover class and the NDVI indices. Even though we only report the heat maps of the correlations for the US and the West Coast region similar changes can be observed for the other regions.

It is interesting to note that the rank of the predictor data matrix of the random points is 32. This is rather surprising since BIO7 is a linear combination of BIO5 and BIO6 and the land cover variables should sum to a constant. Hence one would expect that the rank is smaller or equal to $33 - 2 = 31$. A closer inspection leads to the conclusion that some small rounding errors in the creation of the land cover rasters “remove” the linear dependence. This “near linear dependence” also becomes clear when we look at the singular values of the scaled data matrix. The three smallest are 0.2139771, 0.000002, and 0, for all practical purposes this means that there are two

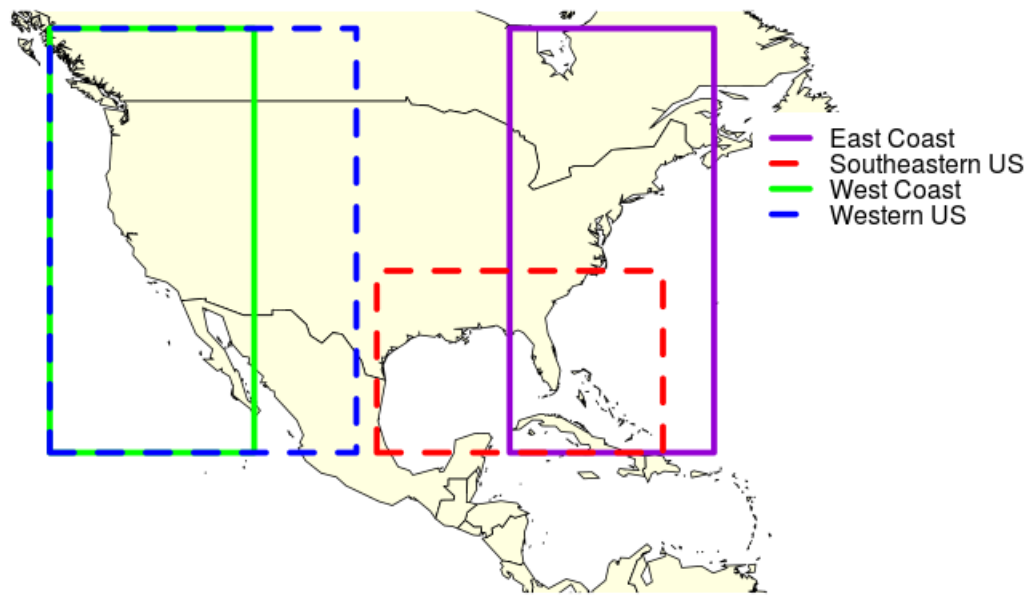


Figure 3.2: Regions of the contiguous US and their bounding rectangles.

redundant variables.

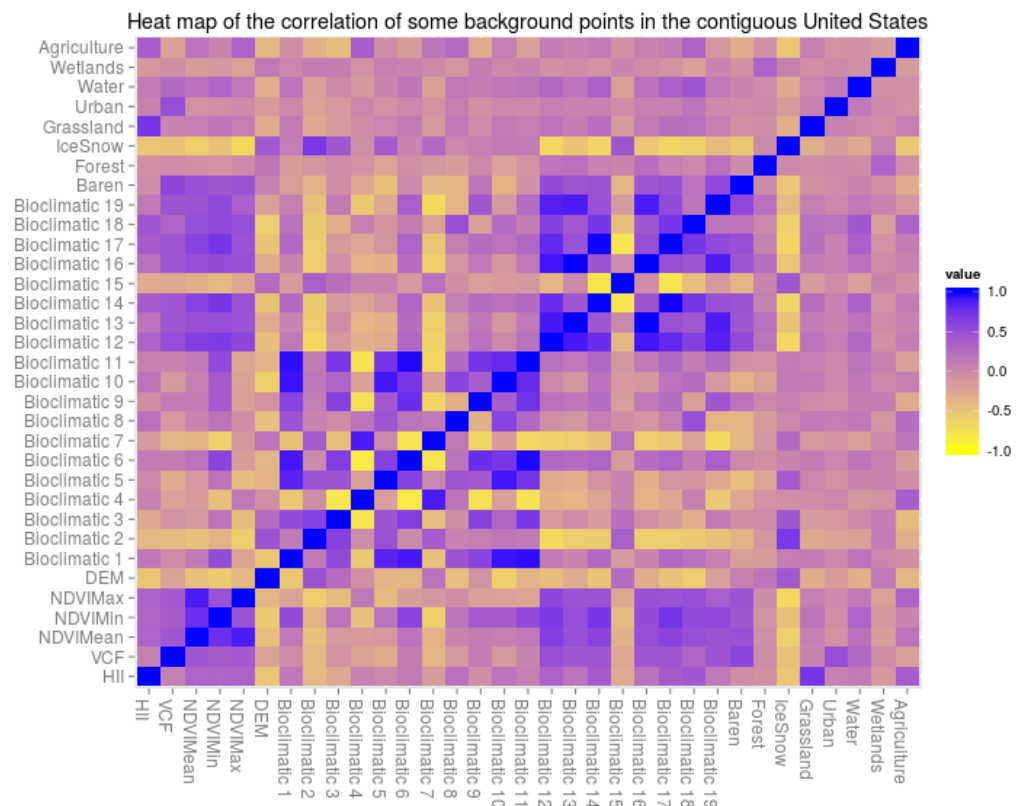


Figure 3.3: Heat map of the correlations in the contiguous US.

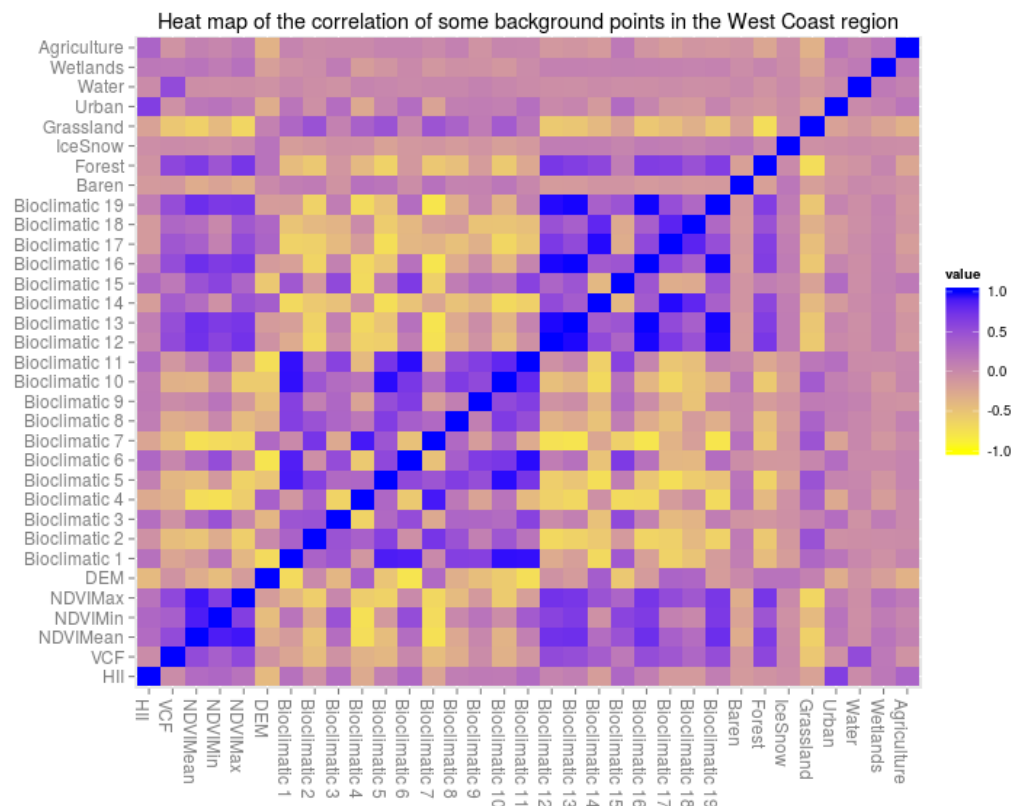


Figure 3.4: Heat map of the correlations in the West Coast region.

3.3 Outcome data

3.3.1 Species considered

The species that will be studied can be found in Table 3.2. These species were selected so that different regions in the US are represented. This was done because, as we saw in Section 3.2.8, the relationship between the different predictors can be different in different regions. The extent of the species distribution is quite different across the selected species. For example, the copperhead snake is spread throughout a large part of the US while the *Sequoia sempervirens* only occurs in a small strip of land stretching from Southern California to Southern Oregon. Finally, the selected species consist of five plant and five animal species. Selecting both plant and animal species was done because it seems reasonable that the including fine grain predictors will lead to a larger increase in predictive performance of the classification models when the species is stationary.

3.3.2 Global Biodiversity Information Facility

The presence-only data originates from the Global Biodiversity Information Facility (GBIF) database. This database contains data from other smaller presence-only databases. Examples include data from citizen science projects (e.g. the iNaturalist project) or herbariums (e.g. The New York Botanical Garden Herbarium). These data sources are quite prone to errors. Citizen science data is usually provided by non-experts and misidentifications are quite likely. Even data collected by experts can be irrelevant for our purposes, for example herbarium data often includes specimens located inside botanical gardens etc. GBIF data tends to contain a lot of duplicated observations. Hence, before using the data from GBIF some data-cleaning was performed. Finally, because the predictors were recorded quite recently we decided to restrict ourselves to observations obtained from the 1980s onward. This somewhat prevents the situation where the current predictor values have changed recently, e.g. due to deforestation.

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Species	Common name	US	West Coast	East Coast	Western US	Southeastern US
<i>Aesculus glabra</i>	Ohio buckeye	✓				
<i>Juniperus osteosperma</i>	Utah juniper			✓		
<i>Quercus ilicifolia</i>	bear oak			✓		
<i>Salix caroliniana</i>	coastal plain willow	✓				
<i>Sequoia sempervirens</i>	coast redwood		✓			
<i>Agkistrodon contortrix</i> Linnaeus	copperhead snake	✓				
<i>Geomys pinetis</i> Rafinesque	southeastern pocket gopher					✓
<i>Pituophis catenifer</i> catenifer	Pacific gopher snake		✓			
<i>Sorex pacificus</i>	Pacific shrew		✓			
<i>Sylvilagus nuttallii</i>	mountain cottontail				✓	

Table 3.2: The different species studied and their study extent.

3.3.3 Forest Inventory and Analysis data

The presence-absence data of the plant species was obtained from the The United States Forest Service Forest Inventory and Analysis (FIA) database. The data from this database consists of plot locations and all the tree species observed within each plot are recorded. The reported coordinates of the plots are, for privacy reasons, slightly distorted.

The sampling design that is used in the construction of this database changed in 1999 and details can be found in O’Connel et al. 2015. By 2004 the new sampling design was implemented in nearly all the states of the contiguous US. The exceptions to this are New Mexico, Oklahoma, and Wyoming for which the new design was implemented in 2005, 2008-2009, and 2011. In each state at least 10% of the plots are sampled each year. By using a time-frame of 10 years we ensure that each plot site was sampled. More particularly, the time-frame that we use is 2004-2014. Since the sampling in New Mexico, Oklahoma, and Wyoming started later than 2004 these states are under-sampled. States in the Eastern US tend to have a sample intensity larger than 10% and some plots were sampled multiple times. Plots that were sampled multiple times were replaced by new observations. If a plot contained the species of interest at least once the species is said to have been present, otherwise it was absent in the plot. It might be interesting to use modelling methods that allow for a sampling design correction. However, this would lead us astray and the sampling design will not be corrected for in this thesis.

Finally, all of the sampled plots are contained within forested areas. This implies that lone standing trees were not observed.

3.3.4 Data preparation

In order to build the necessary models the predictor values corresponding to the presence or absence locations need to be extracted. For certain locations some of the rasters contain a NA value. These locations were removed before the models were constructed. The number of occurrences for each species can be found in Table ???. The total number of plots contained within each region can be found in Table ???.

Species	GBIF	FIA
<i>Aesculus glabra</i>	126	177
<i>Juniperus osteosperma</i>	230	4131
<i>Quercus ilicifolia</i>	98	62
<i>Salix caroliniana</i>	68	
<i>Sequoia sempervirens</i>		206
<i>Agkistrodon contortrix</i> Linnaeus	1426	
<i>Geomys pinetis</i> Rafinesque	53	
<i>Pituophis catenifer catenifer</i>	232	
<i>Sorex pacificus</i>	141	
<i>Sylvilagus nuttallii</i>		

Table 3.3: Number of occurrence observations.

Region	Number of plots
US	287860
West Coast	82184
East Coast	42029
Western US	144436

Table 3.4: Number of plots within the regions.

3.4 Spatial scale

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Chapter 4

Classification techniques

4.1 Introduction

This chapter deals with the statistical foundations of species distribution modelling. Since it is impractical to list all the available methods that are used we restrict ourselves to the most popular or fundamental ones. In the last 10 years a lot of research has focused on the performance of these different algorithms (e.g. Elith* et al. 2006; Segurado and Araújo 2004). The results of these studies were taken into account when the methods that are used were selected.

4.2 Presence-absence data

In this section some important and often used methods to classify binary data are reviewed. First of all, the outcome, y_i , of observation i indicates whether a species occurs, $y_i = 1$, or is absent, $y_i = 0$. We denote the vector of explanatory variables as \mathbf{X} . The general form of the models used in this section is:

$$P(Y = 1|\mathbf{X} = \mathbf{x}) = f(\mathbf{x}; \boldsymbol{\gamma}). \quad (4.1)$$

In this representation $f(\cdot; \cdot)$ is a function parametrized by $\boldsymbol{\gamma}$. The main differences between the techniques introduced below are the functional form of $f(\cdot; \cdot)$ and the loss function that is minimized.

4.2.1 Logistic regression

Perhaps the most fundamental modelling technique for binary data is logistic regression. In logistic regression the log odds ratio of the probability of an occurrence is modelled as a linear function of the covariates. Hence, the model can be depicted as

$$\log \left(\frac{P(Y = 1 | \mathbf{X} = \mathbf{x})}{P(Y = 0 | \mathbf{X} = \mathbf{x})} \right) = \gamma_0 + \mathbf{x}^t \boldsymbol{\gamma}.$$

It is easy to show that this model can be written in the form used in Equation 4.1. More specifically, if we define $\text{expit}(\cdot) = \frac{\exp(\cdot)}{1 + \exp(\cdot)}$ we get

$$f(\mathbf{x}; \boldsymbol{\gamma}) = \text{expit}(\gamma_0 + \mathbf{x}^t \boldsymbol{\gamma}).$$

Usually the coefficients of a logistic regression model are obtained by using maximum likelihood estimation (MLE). Obtaining the MLE $\hat{\boldsymbol{\gamma}}$ corresponds with solving the following maximization problem:

$$\hat{\boldsymbol{\gamma}} = \arg \max_{\boldsymbol{\gamma}} \sum_{i=1}^N \{y_i \log f(\mathbf{x}_i; \boldsymbol{\gamma}) + (1 - y_i) \log f(\mathbf{x}_i; \boldsymbol{\gamma})\}.$$

When one multiplies this log-likelihood function by minus one we get a loss function that is often called the cross-entropy. For more information about logistic regression and numerical optimization techniques for obtaining the MLE we refer to Agresti 2013 and McCullagh and Nelder 1999.

The main advantages of logistic regression models are that they are relatively simple to implement, interpret, etc. This simplicity is also its greatest disadvantage. In particular, when modelling the distribution of a species there is often no a priori knowledge of the shape of the response curves.

4.2.2 Generalized additive models

In standard logistic regression a linear systematic component is used. It is easy to extend logistic regression models to include non-linear systematic components. However, the functional form of the log odds ratio might not be known by the researcher and hence a non-parametric (or semi-parametric) modelling technique can be useful. When the distribution of the outcome belongs to the exponential family generalized additive models (GAMs) are one possible class of non-parametric (or semi-parametric) models. In the case of a Bernoulli distribution the resulting

GAM is sometimes called an additive logistic regression model and has the form:

$$\log \left(\frac{P(Y = 1 | \mathbf{X} = \mathbf{x})}{P(Y = 0 | \mathbf{X} = \mathbf{x})} \right) = \gamma_0 + f_1(x_1) + \cdots + f_p(x_p). \quad (4.2)$$

In this representation the $f_k(\cdot)$'s are certain smooth functions. Again one can write this model in the form of Equation 4.1:

$$f(\mathbf{x}; \boldsymbol{\gamma}) = \gamma_0 + f_1(x_1) + \cdots + f_p(x_p)$$

There is a multitude of popular ways to specify the $f_k(\cdot)$'s (Hastie and Tibshirani 1990; Wood 2006). We will follow Wood 2006 and Wood and Augustin 2002 and focus on using cubic smoothing splines to represent the $f_k(\cdot)$'s.

GAMs are most often fitted by using MLE. In order to restrict the “wiggleness” of the smoothing functions in model 4.2 one can add a penalization term to the likelihood function. An example of such a “wiggleness” penalty is:

$$\sum_{j=1}^p \lambda_j \int_{x_{j(1)}}^{x_{j(n)}} \{f_j^{(2)}(x)\}^2 dx.$$

In this penalization term the $x_{j(1)}$ (resp. $x_{j(n)}$) is the smallest (resp. largest) value of the j 'th covariate. Furthermore, it can be shown that, in a class of sufficiently smooth functions, the minimizer of

$$-\sum_{i=1}^N \{y_i \log f(\mathbf{x}_i; \boldsymbol{\gamma}) + (1 - y_i) \log f(\mathbf{x}_i; \boldsymbol{\gamma})\} + \sum_{j=1}^p \lambda_j \int_{x_{j(1)}}^{x_{j(n)}} \{f_j^{(2)}(x)\}^2 dx$$

is a natural cubic spline with knots at the n covariate values. The two limiting cases, $\lambda = 0$ and $\lambda = \infty$, are interesting. If $\lambda = 0$ an interpolating spline is optimal, while when $\lambda \rightarrow \infty$ the solution converges to the linear logistic regression solution. Finally, fitting GAMS with natural cubic splines as smoothing functions is, compared to most other smoothers used in GAMS, computationally efficient.

In practice we have to select the penalization parameters, this is usually done by using cross-validation. Because the fitting procedures for GAMs are often computationally intensive usually

the closely related generalized cross-validation (GCV) is used (Wood and Augustin 2002). Finally, since $\lambda = \infty$ still allows a first order fit, it can be interesting to perform additional model selection. Although variable selection is the topic of Chapter 5 we mention that it is possible to introduce an extra penalization term that leads to an automatic variable selection technique (Marra and Wood 2011), i.e. might remove a predictor from the model. This approach is implemented in the `MGCV` R package (Wood 2015).

Finally, up till now only univariate splines have been considered, if interaction terms are to be included one can use e.g. thin plate splines. However, using multidimensional splines usually results in a computationally intensive fitting procedure. Finally, for a more applied review of the use of GAMs in species distribution modelling we refer to Guisan, Edwards Jr, and Hastie 2002.

4.2.3 Artificial neural networks

Artificial neural networks (ANNs) are a non-linear modelling technique. We refer to Bishop 1995 for an introduction to the general methodology and some of the technical details. The terminology used in the ANN literature is slightly different than in the standard statistical literature. More particularly, the explanatory variables are usually called the input features. Furthermore, an ANN consists of so-called “layers” of “neurons”. The first layer is called the input layer and consists of one neuron for each variable. In each successive layer the output of the corresponding neurons is the result of applying an activation function, $g(\cdot)$, to a linear combination of the values from the previous layer. The coefficients of these linear combinations are called the weights of the ANN. These weights are the parameters one can tune. The process of feeding the values of the previous layer into the next is repeated up to the last layer which is called the output layer. The layers that are neither the input nor output layer are called hidden layers. A graphical representation of an ANN with one hidden layer can be found in Figure 4.1.

From here on we will denote the vector of all the weights as γ . As usual, the estimated weights are obtained by minimizing a loss function. When ANNs are applied to classification problems often either the squared loss, $L(\mathbf{y}; \gamma) = \sum_i (y_i - f(\mathbf{x}_i; \gamma))^2$ or the cross-entropy, $L(\mathbf{y}; \gamma) = -\sum_i [y_i \log\{f(\mathbf{x}_i; \gamma)\} + (1 - y_i) \log\{1 - f(\mathbf{x}_i; \gamma)\}]$ is used. We will use the cross-entropy since it is closely related to the likelihood of a Bernoulli distribution and hence more in line with the other techniques. To minimize the loss criterion, often backpropagation (Rumelhart, Hinton,

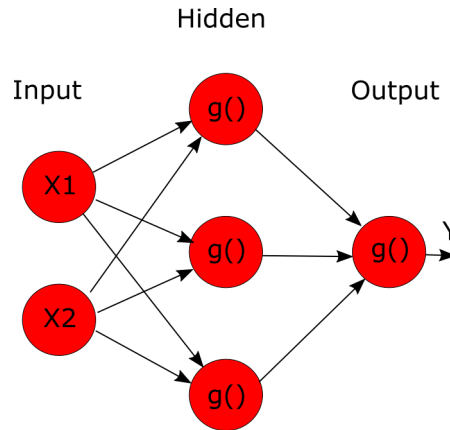


Figure 4.1: Visualization of a feed-forward neural network with one hidden layer.

and Williams 1986) is used in combination with a numerical optimization algorithm, e.g. steepest descent.

It is easy to see that logistic regression can be seen as an ANN with: no hidden layers, the expit function as the activation function of the output layer, and the cross-entropy loss.

The biggest strength of ANNs is that, under some regularity constraints, they can approximate any continuous function arbitrarily well (Hornik, Stinchcombe, and White 1989). Some disadvantages include:

- Selecting an optimal number of layers and neurons is far from trivial.
- The loss-function often has multiple local minima.
- Different numerical optimization methods often lead to different solutions.
- Fitting large neural network architectures can be computationally infeasible.
- The backpropagation algorithm cannot be used in combination with non-differentiable penalty functions, e.g. the lasso penalty (see Section 5.3.2).
- The fitted parameters can be sensitive to the initial weights.
- The obtained model seems like a “black-box”, i.e. there is often no easy way to interpret the parameters and the effect of different predictors.

Some of these disadvantages can be, partially, overcome by using e.g. weight decay, averaging networks, early stopping, pruning, ... Weight decay is also referred to as L_2 regularization and is

described in Section 5.3.1. Averaging networks boils down to using different sets of initial values, fitting the same network structure for each set, and then combining these into one model (Ripley 2009). The CARET package (Kuhn et al. 2015) provides an implementation of averaged neural networks based upon the NNET package (Ripley and Venables 2016).

4.2.4 Tree based methods

In this section some tree based methods are introduced. The section starts with a short explanation of decision trees. Afterwards boosting is introduced as a method to deal with some of the shortcomings of decision trees.

Decision trees

Tree based methods are a class of algorithms that partition the input space into rectangular regions. The same predicted value is then assigned to all observations within a certain region. In the context of SDMs we can interpret this as partitioning the environmental space into rectangles. Each of these rectangles is then labelled as being part of the niche or not. To obtain such rectangles we start by splitting the input space into two regions along one variable. The variable and the value that is used to obtain this split is selected in such a way that the loss function of interest is minimized. In the following steps either the algorithm stops if some stopping criterion is met or the obtained sub-regions are split into smaller sub-regions. After the tree is grown usually a winner takes all approach is applied to obtain the label for each region. Thus, if a region contains mainly presences its predicted class will be the presence class, otherwise it will be labelled as a region containing absences. Finally, the number of nodes J of a tree is defined as the number of splits $+1$. A visualization of a partitioning obtained by using a classification tree is given in Figure 4.2.

Unless a stopping criteria is specified this approach would lead to over-fitting. More particularly we would end up with as many regions as there are observations. Hence, the algorithm is usually stopped when no new splits can be found that decrease the loss by some pre-specified amount. Another possible stopping criterion is to stop the algorithm once a certain number of splits is reached. As usually we opt for using the cross-entropy as the loss function. Finally, we note that there are many variations to the algorithm sketched above.

The most important advantages of decision trees include:

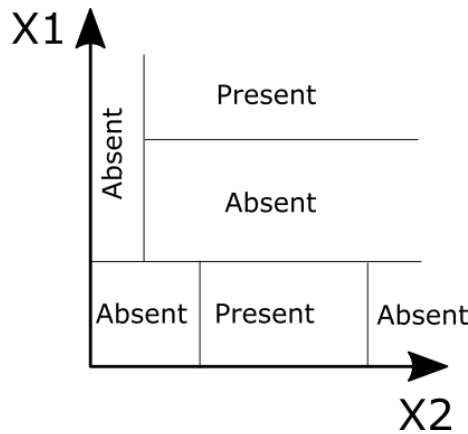


Figure 4.2: Visualization of a classification tree in a two dimensional input space.

- Complex interaction effects can easily be modelled.
- Small decision trees are easily visualized.
- Decision trees usually perform relatively well even when no variable selection was applied.
- The idea underlying decision trees is quite intuitive.
- Decision trees are invariant under monotonic transformations of the predictors.

Some of the disadvantages of using decision trees include:

- Decision trees usually have a large variance
- Categorical variables with a lot of classes can lead to computational problems.

Boosting

In order to combat the large variance of decision tree boosting was introduced. The overview of boosting presented in this section is largely based on Elith, Leathwick, and Hastie 2008 and Friedman, Hastie, and Tibshirani 2000. Elith, Leathwick, and Hastie 2008 gave an applied working guide to boosted regression trees while Friedman, Hastie, and Tibshirani 2000 gave a theoretical explanation of boosted regression trees.

An ensemble of different classifiers is sometimes useful to obtain improved classifiers. This is what is done in boosted classification trees. Boosting can be described as creating a sequence of models such that the i 'th model focusses on correctly classifying observations that were misclassified by the $i - 1$ previous models. The corresponding algorithm then takes the following form:

1. Construct a classifier.
2. Classify the observations.
3. Assign large weights to wrongly classified observation and vice versa for correctly classified observations.
4. Fit a classifier on the weighted data-set.
5. Create a new classifier by adding the the new and old classifiers, usually the new classifier is shrunk by multiplying it with a regularization parameter.
6. Stop if some stopping criteria is met, otherwise use the new classifier obtained in step 5. and repeat from step 2. onwards.

In order to avoid over-fitting one can use a new subsample of the full data-set in each iteration of the algorithm. When combining boosting with classification trees it is recommended that the subsamples have a sample size in between 0.5 and 0.75 times the size of the full dataset (Elith, Leathwick, and Hastie 2008).

Another popular ensemble method that is used in combination with decision trees is the Random Forest (RF) method. In most cases RFs perform slightly worse than boosted classification trees (Hastie, Tibshirani, and Friedman 2009) and we will not consider them in this thesis.

A disadvantage of boosted classification trees is that they are not as easily visualized / interpreted as normal classification trees. Secondly, there are quite a few tuning parameters, namely the depth of the trees J , the regularization term, and the number of trees. However, Hastie, Tibshirani, and Friedman 2009 note that using $J \in \{4, \dots, 8\}$ usually leads to the best performing models, additionally they observed that the specific value of J in this set has little effect on the performance of the classifier. Hence, cross validation will be used to obtain optimal values for the learning rate and the number of trees.

Finally, although we described boosting for classification trees the same idea can be readily applied to other classification algorithms. Furthermore, boosted classification trees seem to include a form of internal variable selection, i.e. the performance of this method usually doesn't

degrade a lot when irrelevant predictors are present. We will use the implementation provided by the GBM package (Ridgeway 2015).

4.3 Presence-only data

Instead of having access to presence-absence data it happens quite often that only presence data is available. Since the previously described classification techniques need binary data they can not immediately be used with presence-only data. In Section 4.3.1 the inhomogeneous Poisson process (IPP) is introduced. Presence-background classification is introduced in Section 4.3.2. Finally, in Section 4.3.3 Maximum Entropy (MaxEnt; Phillips, Anderson, and Schapire 2006; Phillips and Dudík 2008) is concisely described.

Although we will not consider it we note that there is another interesting way to use regression models in combination with presence-only data. Ward et al. 2009 used the EM algorithm (Dempster, Laird, and Rubin 1977) in combination with regression models. Although this leads to an elegant and rigorously motivated method of fitting regression models, the prevalence of the species needs to be known, or estimable, which is nearly never the case.

4.3.1 Poisson point processes

First of all, all the measure theoretic machinery involved in point processes is blatantly ignored in this section. We denote the study area of interest by S . Usually S corresponds to an area preserving projection of the earth and therefore our focus is on $S \subset \mathbb{R}^2$. A point process is a random variable \mathbf{X} that consists of points $u_i \in S$, hence $\mathbf{X} = (u_1, \dots, u_n)$. In the presence-only scenario the \mathbf{X} variable equals the locations where an occurrence was reported. One popular point process is the inhomogeneous Poisson point process. Before giving a non-rigorous definition of the IPP we define the intensity of a point process as a function $\lambda(\cdot) : S \rightarrow \mathbb{R}_+$ for which $\int_B \lambda(u) du < \infty$, \forall bounded $B \subset S$. The random variable \mathbf{X} is said to be an IPP if \forall bounded $B \subset S$:

1. The number of observed points within B , $N(B) = \#(X \cap B)$, follows a Poisson distribution with mean $\Lambda(B) = \int_B \lambda(u) du$.
2. Given the number of observed points within B , $N(B)$, the elements of X within B are i.i.d. distributed with density $\frac{\lambda(u)}{\Lambda(B)}$.

The first part of the definition implies that the the expected number of observations in a small area surrounding a point is approximately equal to the intensity, assuming it is a continuous function, multiplied by size of the area.

In order to obtain a parametric model of an IPP the intensity is usually modelled as a function of the covariates $\mathbf{z}(u)$ observed at the location u . A popular model is the log linear model:

$$\lambda(u) = \exp(\gamma_0 + \boldsymbol{\gamma}^t \mathbf{z}(u)).$$

It is interesting to note that the intercept term, γ_0 , only affects the expected number of points but not the configuration of the points in S . When we use C to denote a rest term that is independent of the parameters, the likelihood function becomes:

$$\begin{aligned} l(\mathbf{x}; \boldsymbol{\gamma}) &= \log \left(\prod_{i=1}^n \frac{\lambda(u_i)}{\Lambda(S)} \frac{\Lambda(S)^n \exp(-\Lambda(S))}{n!} \right) \\ &= \sum_{i=1}^n \left\{ \gamma_0 + \boldsymbol{\gamma}^t \mathbf{z}(u_i) \right\} - \int_S \exp(\boldsymbol{\gamma}^t \mathbf{z}(u)) du + C. \end{aligned}$$

The difficulty of using MLE in combination with IPP models is approximating the integral. For an overview of different methods to obtain the MLEs and a more complete treatment of spatial point processes we refer to Møller and Waagepetersen 2007.

4.3.2 Classification with pseudo-absences

discuss effect of using PCA on background vs occurrence data

One of the older techniques to deal with presence-only data consists of generating so-called pseudo-absence or background points. The original motivation of this technique is that by uniformly sampling n_0 points in the geographical space S one obtains a representation of the available habitat in the study region (Pearce and Boyce 2006). The available habitat is then contrasted with the locations where a presence was observed by using standard classification methods. In practice one has to decide on the number of pseudo-absences that should be used and whether or not the pseudo-absences should be weighted (Barbet-Massin et al. 2012).

Another, more rigorous, way to justify the pseudo-absence method is by using the IPP model. Warton and Shepherd 2010 were the first to note the connections between fitting an IPP model

and fitting logistic regression models with pseudo-absences. Fithian and Hastie 2013 further refined these equivalences and also considered the situation when the model is misspecified. We follow the derivation given by Fithian and Hastie 2013.

First of all, we condition on the number of presence points, n_1 , and the number of background points, n_0 , and view this as a case-control sampling scheme. It is clear that the probability of outcome Y_i being a presence point equals $P(Y_i = 1) = \frac{n_1}{n_0 + n_1}$ and hence $P(Y_i = 0) = \frac{n_0}{n_0 + n_1}$. Now suppose that the presence points are generated by a IPP with intensity $\lambda_1(u) = \exp(\gamma_0 + \gamma^t \mathbf{z}(u))$. Since the background points are generated uniformly over S the intensity of this process is $\lambda_0(u) \propto 1$. Conditional on n_1 and n_0 it is now easy to derive the expression of a logistic model:

$$\begin{aligned}
P(Y_i = 1|U = u) &= \frac{f(u|Y_i = 1)P(Y_i = 1)}{P(Y_i = 1)f(u|Y_i = 1) + P(Y_i = 0)f(u|Y_i = 0)} \\
&= \frac{\frac{\lambda_1(u)}{\Lambda_1(S)} \frac{n_1}{n_0 + n_1}}{\frac{\lambda_1(u)}{\Lambda_1(S)} \frac{n_1}{n_0 + n_1} + \frac{\lambda_0(u)}{\Lambda_0(S)} \frac{n_0}{n_0 + n_1}} = \frac{\frac{\lambda_1(u)n_1 \int_S du}{\Lambda_1(S)\lambda_0(u)n_0}}{\frac{\lambda_1(u)n_1 \int_S du}{\Lambda_1(S)n_0} + 1} \\
&= \frac{\exp\left(\gamma_0 + \log\left(\frac{n_1 \int_S du}{n_0 \Lambda_1(S)}\right) + \gamma^t \mathbf{z}(u)\right)}{\exp\left(\gamma_0 + \log\left(\frac{n_1 \int_S du}{n_0 \Lambda_1(S)}\right) + \gamma^t \mathbf{z}(u)\right) + 1}
\end{aligned} \tag{4.3}$$

This derivation implies that, with the exception of the intercept, the coefficients of the IPP model can be estimated by using the standard fitting procedures for logistic regression. Furthermore, it shows that combining logistic regression together with back-ground data is quite a natural approach to the presence-only problem. The fact that the intercept of the IPP model cannot be obtained is not problematic. As we saw in Section 4.3.1 the intercept does not affect the configuration of the observed points within S . Instead it basically rescales the expected number of observed presences. Since the number of observed presences is partially determined processes of which we have no data, e.g. the sampling intensity, the intercept is usually not of interest,

4.3.3 Maximum Entropy modelling

Perhaps the most popular method to create models from presence-only data is Maximum Entropy modelling. Phillips, Anderson, and Schapire 2006 considered a gridded study area \mathcal{X} . An occurrence of a species then occurs in a cell based manner, hence instead of an exact location in S it is only known that the species was present inside the corresponding cell in \mathcal{X} . They then

went on to find a distribution $\pi(x) : \mathcal{X} \rightarrow \mathbb{R}$ which minimizes the entropy:

$$-\sum_{x \in \mathcal{X}} \pi(x) \log\{\pi(x)\},$$

under the constraint that the observed mean of the features, $\mathbf{z}(x) \in \mathbb{R}^p$, is “close” to the expected mean. If we denote the set of cells for which an occurrence was observed by \mathcal{X}_1 the constraint is:

$$\left| \sum_{x_i \in \mathcal{X}_1} \mathbf{z}(x_i) - \sum_{x \in \mathcal{X}} \pi(x) \mathbf{z}(x) \right| < \boldsymbol{\lambda}, \quad \boldsymbol{\lambda} \in \mathbb{R}_+^p,$$

where the inequality is applied component wise. It can be shown that the solution to this problem has the following form:

$$\pi(x) \propto \exp(\boldsymbol{\gamma}^t \mathbf{z}(x)), \quad \boldsymbol{\gamma} \in \mathbb{R}^p.$$

It is interesting to note that this is exactly the form of a log linear IPP model. Renner and Warton 2013 showed that when the grid size becomes small the MaxEnt solution converges towards the penalized IPP solution. The standard implementation of MaxEnt uses as features a combination of products between covariates, hinge features, step functions, quadratic terms and linear terms (Phillips and Dudík 2008). Hence, in the end MaxEnt modelling is equivalent with presence-background logistic regression in an extended covariate space.

4.4 Taking the scale hierarchy into account

In Section 3.4 the influence of spatial scale was discussed. There have been some attempts to incorporate this hierarchical structure into the classification techniques. E.g. Pearson, Dawson, and Liu 2004 combined two models, one for coarse scale processes and one that introduces fine grain variables. More specifically their approach was as follows:

1. An initial model is obtained by solely using climate variables as predictors.
2. The predicted values are saved as a new variable.
3. The predicted values and remote sensed variables are combined and used as the input for a second model.

4. The predicted values of the second model are the final predictions.

Graphically this can be depicted as in Figure 4.3.

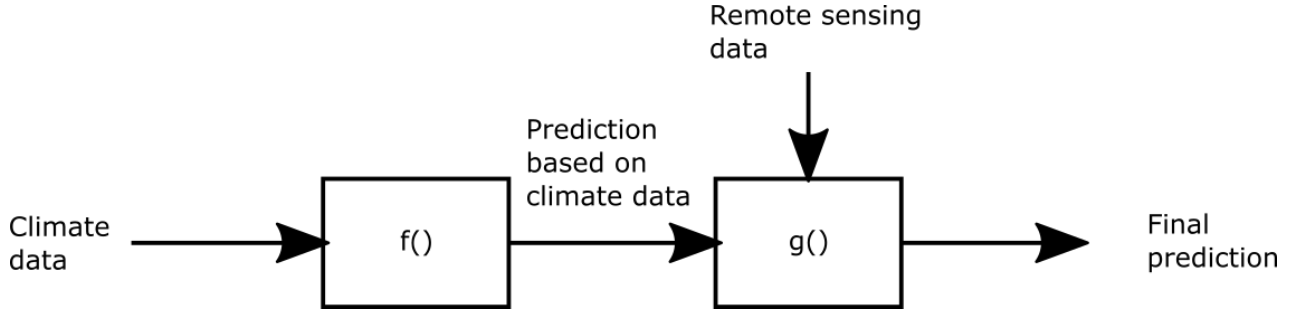


Figure 4.3: Visualization of the hierarchical model.

This hierarchical model has since been applied in combination with e.g. stacked SDMs (Cord et al. 2014). However, from a statistical point of view this approach is not well motivated. The main drawback of this approach is that interaction effects between the climate and remotely sensed variables cannot be taken into account. Furthermore, since there is usually quite some correlation between the climate and remotely sensed variables model selection is hampered. For example, if a remotely sensed variable is fundamental in determining the niche and highly correlated with a climate variable that is not as relevant for defining the niche the climate variable will usually end up in the model instead of the remotely sensed variable. Because of these drawbacks this approach will not be investigated.

Chapter 5

Reducing the number of explanatory variables

5.1 Introduction

The explanatory variables used in SDMs are often correlated. Such high correlation can be an indication of redundant information in the data-set. Moreover, since there is usually no knowledge about which variables make up the niche of a species and irrelevant predictors might be included into the model. It is well known that this can lead to over-fitting and unstable predictions. In this chapter we describe some methods to deal with large sets of correlated predictors. More specifically, in Section 5.2 we introduce methods that transform the input space. In Section 5.3 techniques that penalize “large” models are introduced. Finally, Section 5.4 deals with step-wise variable selection methods.

elaborate on problems when using models for prediction vs interpretable models, see Dormann et al. and Harel 2001 (see paper references Dormann)

For a review of methods to deal with correlated covariates in ecology we refer to Dormann et al. 2013. Finally, these automatic selection procedures are of course not meant to replace a well founded motivation of why certain predictors should be selected. A discussion of how the available data, scale of the predictors, etc. should influence the decision of using a complex or a simple model can be found in Merow et al. 2014.

5.2 Dimensionality reduction

Discuss the effect of the number of background observations ?

Dimensionality reduction techniques can be used to obtain a new, often lower-dimensional, representation of important structures in the input space. This can be particularly useful when two or more explanatory variables are proxies of one underlying latent variable. For example, there are multiple indices that indirectly measure the amount of vegetation. A combination of these indices might be a better indicator of the amount of vegetation than the individual indices. The dimensionality reduction techniques that are used in this thesis use only the input space and ignore the outcome values. It should however be noted that there are other dimensionality reduction techniques that do take into account the relationship between input and output variables, e.g. partial least squares (see e.g. Marx 1996).

5.2.1 Principal component analysis

Often principal component analysis (PCA) is introduced as a method which constructs uncorrelated linear combinations of the variables, these new variables are called principal components. However, for our purposes it might be more interesting to view PCA as a way to find a low dimensional affine subspace such that when the original data is projected onto this subspace the “information loss” is minimal. One possible characterization of PCA is that a set of K orthonormal vectors \mathbf{u}_i and an offset vector \mathbf{b} are constructed so that

$$\sum_{i=1}^N \sum_{j=1}^K \|\mathbf{x}_i - \mathbf{x}_i' \mathbf{u}_j \mathbf{u}_j' - \mathbf{b}\|_2^2$$

is minimized. It can be shown that for a certain K this sum is minimized when the \mathbf{u}_i are the eigenvectors corresponding with the K largest eigenvalues of the covariance matrix. A prototype scenario is shown in Figure 5.1.

Once we have calculated the principal components they can be used as the explanatory variables in one of the models of Chapter 4. Since the new variables are uncorrelated and some irrelevant noise in the predictors might be removed the resultant models often exhibit less variance.

There are multiple criteria based upon which the number of principal components, K , can be chosen. One possibility is to plot the so-called “explained” variance versus the number of compo-

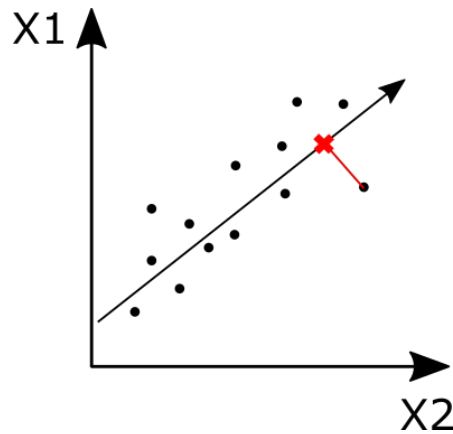


Figure 5.1: Visualization of the a typical scenario where PCA is useful.

nents and look for either a kink or a point where a certain percentage of variance is explained. If PCA is combined with a classification method it is usually more appropriate to use cross-validation to select the number of principal components. We will use the cross-validation approach.

It should be clear that the main disadvantage of PCA is that it only allows for linear representations of the data. Hence, PCA will often be useless if the observations are scattered around a non-linear manifold. For example in Figure 5.2 there is a clear underlying space that is one dimensional. However, using the first principal component of this fictional data-set would lead to as big an “information” loss as using just one of the original axes. For the sake of completeness we mention that there are extensions of PCA that use non-linear manifolds instead of linear ones, e.g. principal curves and surfaces (Hastie and Stuetzle 1989).

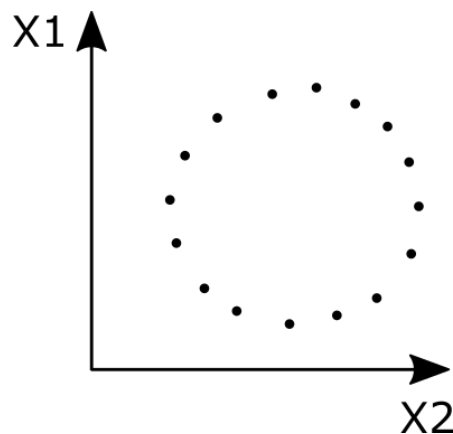


Figure 5.2: Visualization of the a scenario where PCA is useless.

5.2.2 Kernel principal component analysis

A popular and computationally efficient non-linear dimensionality reduction technique is kernel PCA (Schölkopf, Smola, and Müller 1997). In kernel PCA the elements of the input space \mathcal{X} , in our case we have $\mathcal{X} = \mathbb{R}^p$, are mapped to a Hilbert space F . We will denote the map as:

$$\phi(\cdot) : \mathcal{X} \rightarrow F.$$

In this new vector space a PCA is conducted and new coordinates are obtained. A visual presentation of these steps is given in Figure 5.3.

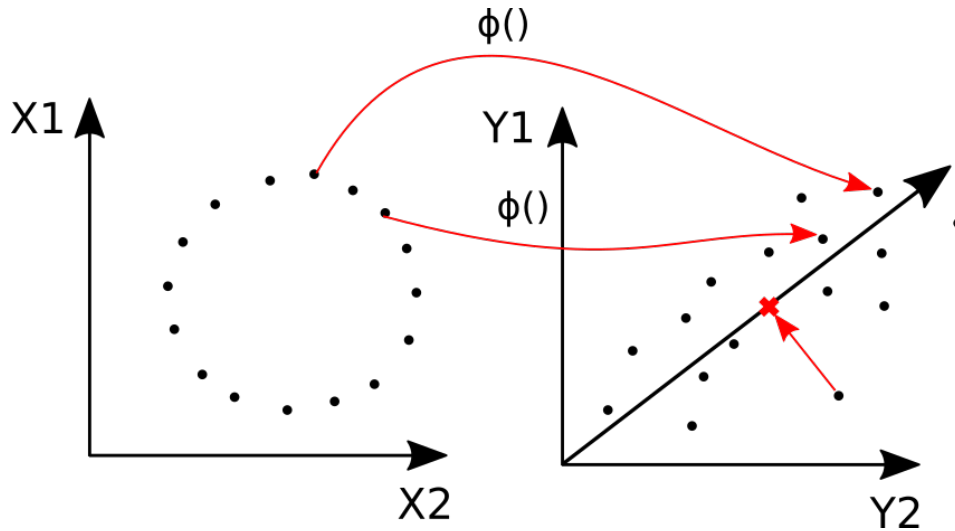


Figure 5.3: Visualization of kernel PCA.

One attractive property of kernel PCA is that we do not need to actually compute $\phi(\mathbf{x})$. More specifically we only need to compute the so-called kernel matrix \mathbb{K}

$$\mathbb{K}_{ij} = \langle \phi(\mathbf{x}_i), \phi(\mathbf{x}_j) \rangle = k(\mathbf{x}_i, \mathbf{x}_j).$$

In general it can be shown that, under some regularity conditions, for any kernel function

$$k(\cdot, \cdot) : \mathcal{X} \times \mathcal{X} \rightarrow \mathcal{R}$$

there exists a corresponding $\phi(\cdot)$ and Hilbert space F . Hence usually one specifies the kernel function instead of the map $\phi(\cdot)$.

Popular choices of the kernel function include:

- The polynomial kernel $k(\mathbf{x}, \mathbf{y}) = (\mathbf{x}'\mathbf{y} + c)^d$.
- The radial basis kernel (RBF kernel) $k(\mathbf{x}, \mathbf{y}) = \exp(-\frac{\|\mathbf{x}-\mathbf{y}\|^2}{\sigma})$, with $\sigma > 0$.
- The hyperbolic tangent kernel, $k(\mathbf{x}, \mathbf{y}) = \tanh(\sigma\mathbf{x}'\mathbf{y} + c)$.

In these representations the σ , resp. c , parameter can be seen as a scale, resp. offset, parameter. The value of these parameters is in general selected by using cross validation. We will focus on the RBF kernel, this kernel is usually used when no prior information is available (Karatzoglou et al. 2004).

A disadvantage of kernel PCA is that it is not always clear which kernel should be used. For our goals, combining kernel PCA and a classification method, we can use cross-validation and treat the type of kernel as a tuning parameter. Furthermore, it is not always particularly clear what the new features are supposed to represent (we might not even know in which Hilbert space we are working). An R implementation of kernel pca is provided as part of the KERNLAB package (Karatzoglou et al. 2004).

5.2.3 Presence versus background data

5.3 Regularization

When one uses regression methods, e.g. logistic regression or ANNs, in combination with a large set of correlated predictors the obtained coefficients are often excessively large and unstable. To combat this large coefficients can be penalized such that the solution consists of shrunken coefficients. To do this the standard minimization problem

$$\hat{\gamma} = \arg \min_{\gamma} L(\mathbf{y}, \gamma)$$

is adjusted to

$$\hat{\gamma} = \arg \min_{\gamma} L(\mathbf{y}, \gamma) + J(\gamma, \boldsymbol{\lambda}).$$

In this representation the function $J(\gamma, \boldsymbol{\lambda})$ is usually a monotonically increasing function in γ . Furthermore, the $\boldsymbol{\lambda}$ parameters are used to control “the amount of regularization” and are often

selected by using cross-validation. Finally, the described regularization methods can be used in conjunction with logistic regression by using the GLMNET (Friedman et al. 2015) package.

5.3.1 Ridge regression / L_2 regularization

Ridge regression (also called L_2 regularization) is obtained when the Euclidean norm of the coefficients is used as the penalization function:

$$J(\gamma, \lambda) = \lambda \|\gamma\|_2^2.$$

We immediately see that small λ 's correspond to a small amount of regularization and the non-penalized solution is obtained when $\lambda = 0$. Furthermore, when $\lambda \gg$ the coefficients are shrunk to zero. A typical evolution of the coefficients in function of the regularization parameter can be found in Figure 5.4.

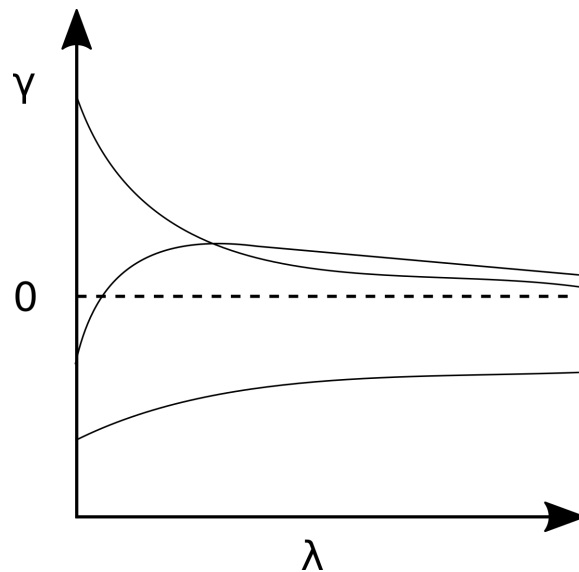


Figure 5.4: A stereotypical evolution of the regression parameters in combination with the ridge parameter.

It is clear that rescaling the covariates will usually lead to different penalized coefficients. It is therefore recommended to standardize the covariates before applying ridge regression.

Advantages of using an L_2 penalty include:

- The penalty is differentiable and hence compatible with e.g. backpropagation.

- For most regression problems it is easy to adopt the standard algorithms to include the L_2 penalty.
- It is usually no problem if the number of variables is larger than the number of observations.

The biggest disadvantage is that all the predictors are kept in the model, i.e. usually no coefficients are equal to zero.

Finally, we note that L_2 regularization is also called weight decay when it is used in combination with neural networks.

5.3.2 Lasso / L_1 regularization

Another popular option was proposed by Tibshirani 1996. He suggested to use the absolute norm (also called the L_1 norm or Manhattan distance) as penalty function, or thus the minimization problem becomes:

$$J(\gamma, \lambda) = \lambda \|\gamma\|_1 = \lambda \sum_{i=1}^p |\gamma_j|.$$

The L_1 penalty term is also called the least absolute shrinkage and selection operator (lasso). Unlike L_2 regularization, the lasso solution will usually contain some coefficients that are equal to zero. This implies that, in addition to the parameter shrinkage, the lasso performs some form of automatic variable selection. A typical parameter trace can be found in Figure 5.5.

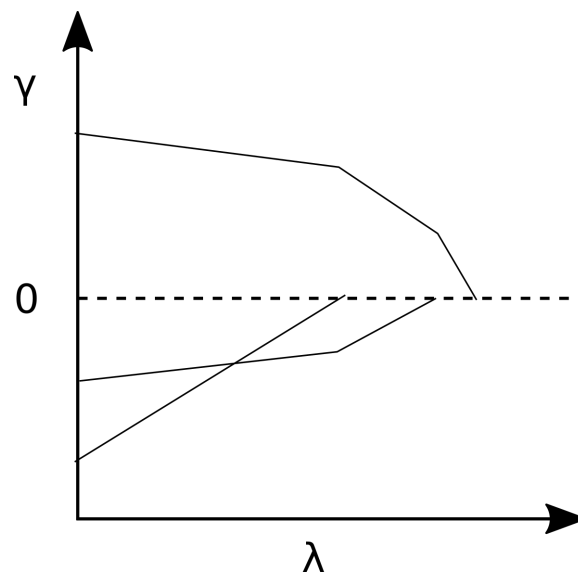


Figure 5.5: Visualization of the typical evolution of the regression parameters in combination with the lasso parameter.

Usually the shrinkage parameter is selected by using cross-validation. The main advantage is that the lasso performs both shrinkage and parameter selection at once. Disadvantages include:

- The lasso penalty is not differentiable and hence one cannot use algorithms like backpropagation.
- At most n variables are included in the model.
- If there is a group of highly correlated variables usually only one will be selected. Sometimes the variable that is selected is rather arbitrary.

5.4 Subset selection methods

Subset selection methods are a set of older methods to reduce the number of explanatory variables. Although subset selection methods are often criticised for ignoring problems with bias, multiple testing, etc. (Whittingham et al. 2006) they are still popular. In this section three different subset selection techniques are discussed.

5.4.1 Best subset selection

The most elementary technique in this set of methods is best-subset selection. Best subset selection consists of fitting a model for each combination of predictors and then selecting the best model from these. What constitutes the best model depends on the goals of the study but often used selection criteria include: AIC, missclassification error, p-values, etc. The biggest limitation of best-subset selection is that when the number of predictors increases there is an exponential increase in computational complexity. In particular, if there are p potential terms to be included we need to fit 2^p models. Since we have 33 variables this method is infeasible for our purposes.

5.4.2 Stepwise subset selection

A second subset selection method is backward-stepwise selection. This method starts with the model containing all p predictors. In the second step of the algorithm we try removing each predictor from the full model and select the most optimal model from the models with $p - 1$ covariates. In the third step we remove each predictor from the model obtained in the second step, fit a model

with $p - 2$ predictors, and select the best model from these. This process is repeated until there is no improvement possible. One of the most important disadvantages of this method is that it is quite variable. Furthermore, for some algorithms (e.g. logistic regression) we need to have $p < n$.

Forward-stepwise selection is basically the reverse of the backward-stepwise method. More particularly, one starts with the model containing only an intercept. Then the variable that leads to the largest improvement is added. And this process is repeated until the performance criterion can not be improved any further. The main advantage of this method over backward-stepwise selection is that it is generally less variable. On the other hand, this method usually leads to a bigger bias. Another limitation of forward-stepwise subset selection is that it fails if, in order to minimize some selection criterion, two predictors need to be included at the same time.

5.4.3 Univariate pre-screening

Underlying idea

Univariate pre-screening is somewhat related to forward-stepwise regression. Just as in forward-stepwise selection we start by fitting the p models including an intercept and one predictor. In the second step a final model is fitted by using all predictors for which the corresponding model from the first step met a certain criteria, e.g. a significant p-value.

An advantage of this method is that, compared to the other subset selection methods that were discussed, it is computationally efficient. An important disadvantage of univariate pre-screening is that, because of its univariate nature, the correlation between predictors is ignored. Hence, highly correlated predictors will often be included in the final model. It is interesting to note that the multiple testing problem is particularly clear when using this method. However, since we know the exact number of tests it is quite easy to control some multiple testing error rate instead of the type 1 error. Since we will not use standard univariate pre-screening we will not discuss methods to control the multiple error rate. For two methods to control a multiple testing error rate we refer the interested reader to e.g. Holm 1979 or Benjamini and Hochberg 1995.

Taking the correlation into account

In ecological research a variation on univariate pre-screening that tries to take into account the correlation is regularly used, e.g. Cord et al. 2014. The method, which we will call select07, selects one variable from each set of highly correlated variables. The algorithm works as follows:

1. Make a set A of all variables.
2. Calculate all correlations.
3. For each pair of variables with $|r| > 0.7$ we fit a univariate model.
4. For each pair selected in step 3. we remove the worst¹ performing variable from A .
5. Fit a final model that includes all variables left in the set A .

An implementation of this method was provided by Dormann et al. 2013. In this implementation both GAMs and logistic regression can be used and the performance of the univariate models is measured by the AIC value. Finally we note that using $|r| > 0.7$ is quite arbitrary, however this threshold seems quite popular in SDM.

5.5 Taking the scale hierarchy into account

double check which kind of models / residuals the used in the paper

Thuiller, Araújo, and Lavorel 2004 used a hierarchical model selection approach. More specifically, they investigated the effect of adding land-cover data to models build with climate data. To do this they used the following three steps:

1. A model with only climate variables is constructed by using a stepwise selection method.
2. Regress the residuals of the climate model on the land-cover variables and use a stepwise selection method to select the most influential variables.
3. Build a new model with the selected climate and land-cover variables.

Although the focus in the article was on stepwise regression techniques, the same procedure can be applied in combination with other variable selection methods. However, this hierarchical approach has a clear disadvantage, it cannot consider interactions between climate and land-cover

¹As usual, different performance measures can be used, e.g. AIC, p-values, etc.

variables. Furthermore, the obtained solution will most often be sub-optimal compared to using a selection procedure with all the variables. Hence we see little reason to consider this approach any further.

5.6 Meaningful combinations of classification and

For each method of Chapter 4 a “vanilla” model will be considered. The vanilla logistic regression and ANN model use all predictors except the agriculture land cover class and BIO7. These two predictors are removed to avoid computational problems associated with design matrices that are not of full rank. For the vanilla GAM all predictors are used together with GCV to select the smoothness parameters. The vanilla MaxEnt model is the standard MaxEnt model with the default parameters (Phillips and Dudík 2008). Finally, the vanilla version of boosted regression trees is just the standard implementation.

Next to the vanilla models ten combinations of classification algorithms and methods to reduce the number of variables will be considered, see Table 5.1. Combinations that are not included were often not computationally feasible to implement, e.g. combining kernel PCA and GAMs, or not meaningful, e.g. L_1 regularization combined with boosted regression trees. Furthermore, following the remarks in Section 5.2.3 when PCA or kernel PCA are used in combination with presence-only data three different version will be considered:

1. a version where the PCA is performed on the predictor values associated with background and presence locations.
2. a version where the PCA is performed only on the predictors values associated with background locations.
3. a version where the PCA is performed only on the predictors values associated with presence locations.

In total 20 methods are used when presence-only data is considered and 14 when presence-absence data is considered.

²Regularization of the “wiggleness” instead of the coefficients, see Section 4.2.2

³Using five fold CV to select the L_1 regularization parameter instead of using the default parameter.

	PCA	kernel PCA	L_2 penalty	L_1 penalty	stepwise selection	select07
Logistic regression	✓	✓	✓	✓	✓	✓
Additive logistic regression			✓ ²			✓
Artificial neural networks			✓			
Boosted regression trees						
MaxEnt				✓ ³		

Table 5.1: Table with the combinations of classification and dimensionality reduction techniques that are considered.

Chapter 6

Applications

Chapter 7

Simulation study

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