Quantum Trajectories

Jorrit Hortensius & Bas van 't Hooft

April 29, 2016

Overview

- Introduction
- Bloch sphere
- Time evolution

Overview

- Introduction
- Bloch sphere
- Time evolution
- Method
- Results
- Summary

Introduction

- Goal: Simulate random transitions (quantum jumps) in quantum systems

Introduction

- Goal: Simulate random transitions (quantum jumps) in quantum systems
- Qubit $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$

Introduction

- Goal: Simulate random transitions (quantum jumps) in quantum systems
- Qubit $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$
- Problem: Wave functions are 'deterministic'

Introduction

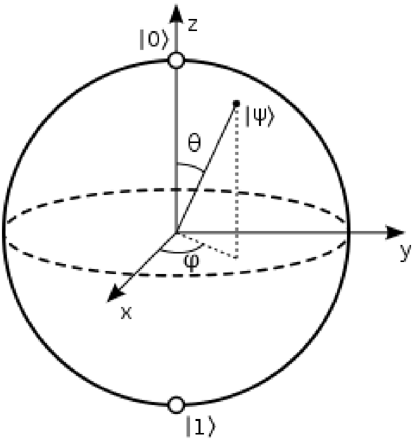
- Goal: Simulate random transitions (quantum jumps) in quantum systems
- Qubit $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{bmatrix}\alpha \\ \beta\end{bmatrix}$
- Problem: Wave functions are ‘deterministic’
- Solution 1: Density matrix

Introduction

- Goal: Simulate random transitions (quantum jumps) in quantum systems
- Qubit $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$
- Problem: Wave functions are 'deterministic'
- Solution 1: Density matrix
- Solution 2: Average over many realisations
  $\rightarrow$ *Quantum Trajectories*

Bloch sphere

- $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = e^{i\delta}(\cos(\theta/2)|0\rangle + e^{i\phi}\sin(\theta/2)|1\rangle)$



Time evolution

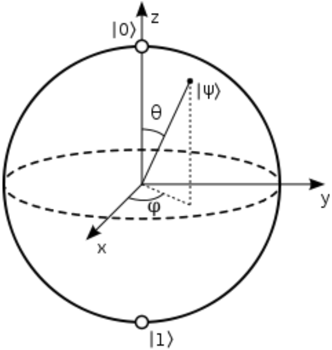
- Energy difference between two states: $\hat{\mathcal{H}} = 0|0\rangle\langle 0| + \omega|1\rangle\langle 1|$

Time evolution

- Energy difference between two states: $\hat{\mathcal{H}} = 0|0\rangle\langle 0| + \omega|1\rangle\langle 1|$
- Time evolution wavefunction $|\psi(t)\rangle = e^{-i\hat{\mathcal{H}}t/\hbar}|\psi(0)\rangle$

Time evolution

- Energy difference between two states: $\hat{\mathcal{H}} = 0|0\rangle\langle 0| + \omega|1\rangle\langle 1|$
- Time evolution wavefunction $|\psi(t)\rangle = e^{-i\hat{\mathcal{H}}t/\hbar}|\psi(0)\rangle$
- Rotation around z-axis
- Energy
- Dipole



Method: Lindblad equation

- Time evolution density matrix (von Neumann equation)

$$\frac{\partial \rho}{\partial t} = -\frac{i}{\hbar}[\hat{\mathcal{H}}, \rho] \tag{1}$$

Method: Lindblad equation

- Time evolution density matrix (von Neumann equation)

$$\frac{\partial \rho}{\partial t} = -\frac{i}{\hbar}[\hat{\mathcal{H}}, \rho] \tag{1}$$

- Add jump operators (Lindblad equation)

$$\frac{\partial \rho}{\partial t} = -\frac{i}{\hbar}[\hat{\mathcal{H}}, \rho] - \frac{1}{2}\sum_k (C_k^\dagger C_k \rho + \rho C_k^\dagger C_k - 2 C_k \rho C_k^\dagger) \tag{2}$$

Method: Lindblad equation

- Time evolution density matrix (von Neumann equation)

$$\frac{\partial \rho}{\partial t} = -\frac{i}{\hbar}[\hat{\mathcal{H}}, \rho] \tag{1}$$

- Add jump operators (Lindblad equation)

$$\frac{\partial \rho}{\partial t} = -\frac{i}{\hbar}[\hat{\mathcal{H}}, \rho] - \frac{1}{2}\sum_k (C_k^\dagger C_k \rho + \rho C_k^\dagger C_k - 2 C_k \rho C_k^\dagger) \tag{2}$$

- Most general non-unitary evolution of the ρ that is trace-preserving and completely positive for any initial condition

Method: Quantum Trajectories

- Time evolution according to effective Hamiltonian $\hat{\mathcal{H}}_{eff} = \hat{\mathcal{H}}_0 - \frac{i\hbar}{2} \sum_k C_k^\dagger C_k$
- Jumps C_k with certain probability dp_k

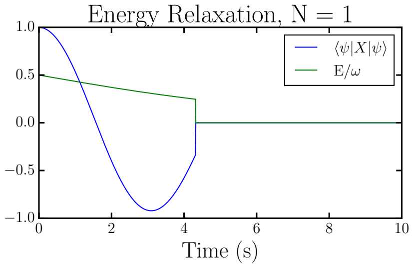
Method: Quantum Trajectories

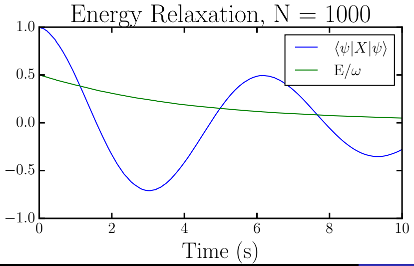
- Time evolution according to effective Hamiltonian $\hat{\mathcal{H}}_{eff} = \hat{\mathcal{H}}_0 - \frac{i\hbar}{2}\sum_k C_k^\dagger C_k$
- Jumps C_k with certain probability dp_k

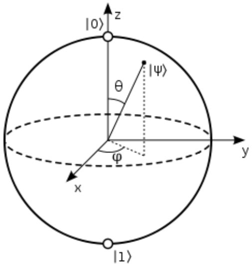
Algorithm 2 Quantum Trajectory Method

for $i = 1$ to $N_{timesteps}$ **do**
    Calculate $dp_k = dt\langle\psi|C_k^\dagger C_k|\psi\rangle$ and $dp = \sum_k dp_k$
    Accept jump with probability dp
    if jump **then**
        Choose k with probability dp_k/dp
        $|\psi(i+1)\rangle = C_k|\psi(i)\rangle\sqrt{dt/dp_k}$
    else
        Time evolution according to $\hat{\mathcal{H}}_{eff}$, normalize state
    end if
end for

Results 1: Relaxation







Results 2: Boltzmann relaxation

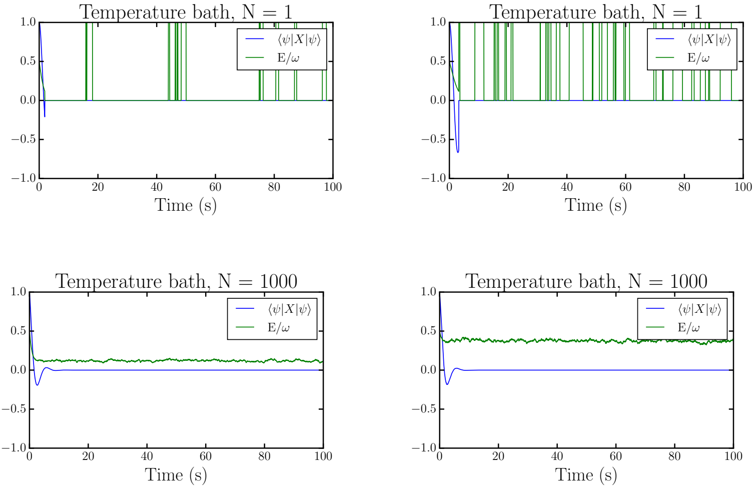
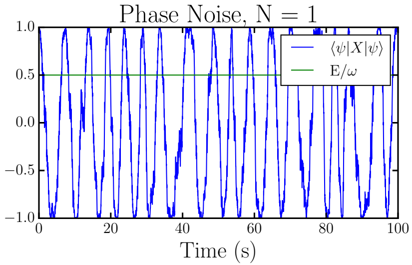
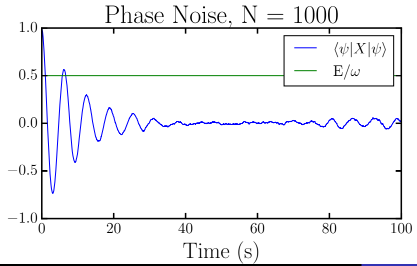
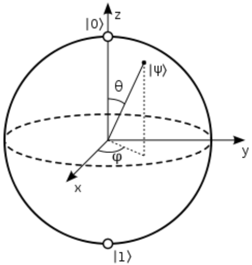


Figure: Low Temperature, Right: High Temperature

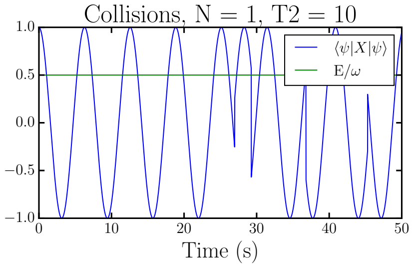
Results 3: Dephasing

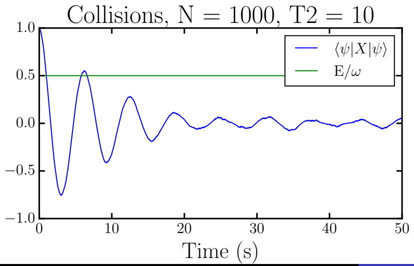


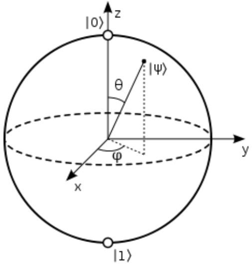




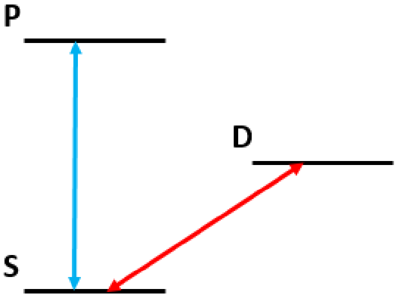
Results 4: Elastic collision







Results 5: Three level system



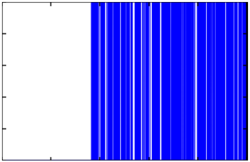


Figure: Number of transitions from P to S state in time

Summary

- The quantum trajectory method is an alternative approach to determine the time evolution of open quantum systems
- We have looked into relaxation, dephasing and other processes