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Exam Introduction to modeling, 0LEB1, 11 April, 2015, version A
MC-questions with answers and some explanations

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1. Below, applications of modeling purposes (underlined) to concrete cases are listed. In one of those applications the purpose is incorrectly used. Which one?

   A. Analyze how the profit of a food company depends on the prices of ingredients

   B. Optimizing the profit of a food company by choosing the cheapest suppliers

   C. Describing the turnover of a food company while abstracting over the different kinds of cookies

   D. Predicting the turnover of a food company based on its turnover in the past years

   E. Verifying in which way the turnover of the food company is related to the occurence of obesitas in its clientèle

   Answer E

   When verification is the purpose of a model, it is used to answer the question whether a certain statement about the modeled system is true or not. For instance the question whether or not there is a linear relationship between the turnover of the company and the occurence of obesitas. When a model is used to determine *what kind of relationship* exists (not a yes/no-question), the purpose is analysis.

2. To predict if a new kind of drug has side effects, it is administered to a group of lab animals. These animals are observed for changes in their health and behaviour. From these observations, conclusions are drawn regarding side effects. Which categorization of this model is correct?

   A. Immaterial - deterministic - dynamic

   B. Static - material - deterministic

   C. Material - stochastic - black box

   D. Black box - deterministic - dynamic

   E. Stochastic - immaterial – static

   Answer C

   The lab animals are material beings that are part of the model, so the model is material (eliminating answers A and E). The drugs are given to a group of animals because different individuals may react differently to the drug. This makes the model stochastic, rather than deterministic (eliminating answers A, B and D). The model is black box, because it tries to establish side effects of the drug on the basis of data collected from the animals, rather than by reasoning about the fysiology and biochemistry of the animals using causal mechanisms (glass-box approach).

3. In which stage(s) of the modelling process do you use expressions such as: `amAnDetDmp = (nrAnWshs - nrCarefulWshs) * detPWsh + amSpilledDet`?

   A. Definition stage

   B. Conceptualization stage

   C. Definition and conceptualization stage

   D. Formalization stage

   E. Conceptualization and formalization stage

   Answer D

   Expressions of this kind are used for the first time in the modeling process during the constructing of the formal model (chain of dependencies) in the Formalization stage. Hence they cannot belong to the Definition or Conceptualization stage.

4. A modeling team is modeling an animal shelter. In their conceptual model, they define the concept cat as follows:`cat = [animal: TRUE, walksOnFourLegs: TRUE; furry: TRUE]`. The animal shelter currently has three inhabitants: Tim, a pretty Siamese cat; Tom, a hairless cat that lost a leg in a tragic accident; and Pim, a healthy Labrador dog. Given the conceptual model, which of the following claims is true?

   A. Only Tim is a `cat`

   B. Tim and Tom are a `cat`

   C. Pim and Tim are a `cat`

   D. Pim, Tim and Tom are a `cat`

   E. A `dog` is a `cat`

   Answer C

   Tim and Pim have the properties required by the definition of cat in the model, hence they are a `cat`. Tom clearly does not have these properties. Although Pim is a dog in real life and a `cat` according to the model, that does not mean that a `dog` is a `cat` as proposed in answer E: for such a statement to be true, we need a definition of the concept `dog` in the conceptual model, and such a definition has not been given.

5. The Beaufort scale is a measure that relates wind force to observed conditions on land or at sea. It ranges from wind force 0 to 12. The first few steps of the scale are defined as follows:

Force	**Land conditions**	**Sea conditions**
0	smoke rises verically	flat
1	smoke drift indicates wind direction, leaves and wind vanes are stationary	ripples without crests
2	wind felt on exposed skin, leaves rustle, wind vanes begin to move	small wavelets, crests of glassy appearance not breaking

What kind of scale is this?

A. An interval scale

B. A nominal scale

C. An ordinal scale

D. A ratio scale

E. A product scale

Answer C

The scale assigns a number to wind force, hence we have at least an ordinal scale and not a nominal scale. However, given the definitions in terms of observed conditions, we cannot meaningfully perform addition: from adding the conditions for windforce 1 to those for windforce 2 we do not get the conditions for windforce 3. The same holds for substraction, division and multiplication, and hence the Beaufort scale is not an interval or ratio scale (product scale is a fake answer).

6. Suppose that a model contains the expression: `amAnDetDmp = (nrAnWshs - nrCarefulWshs) * detPWsh + amSpilledDet` where `amAnDetDmp` stands for the amount of detergent spilled per year, `nrAnWshs` stands for the number of washes per year, `nrCarefulWshs` for the number of washes per year without spilled detergent, `detPWsh`for the amount of detergent used per wash, and `amSpilledDet` for the amount of detergent spilled via other routes than washes. Which of the following claims is true?

A. The unit of `nrAnWshs` must be 'wash'

B. The unit of `amSpilledDet` must contain 'kilogram'

C. The dimensions of `amSpilledDet` and `detPWsh` must be different

D. The unit for each of the quantities in the expression must contain 'per year'

E. There is no consistent attribution of units to the quantities in the expression

Answer C

By reasoning about the expression, we see that the dimensions of `amSpilledDet` and `detPWsh` must be different. As follows: suppose that the unit of `amAnDetDmp` is U. Then the unit of `(nrAnWshs - nrCarefulWshs) * detPWsh` must be U and the unit of `amSpilledDet` must be U in order to be able to add the two. Given that the unit of `(nrAnWshs - nrCarefulWshs) * detPWsh` is U, and that `nrAnWshs` and `nrCarefulWshs` have some unit V (which must be the same for both, otherwise we can't subtract), the dimension of `detPWsh` is U/V, which is different from U. An example of a consistent attribution of units would be to take $U=$ `kg` and $V =$ `wash` (eliminating answer E and D). But we could also have used another unit for U, for instance liter instead of kilogram (eliminating answer B). The number of washes could also have been expressed as the number of kilograms of clothes washed per year, so the unit of `nrAnsWshs` doesn't have to be 'wash' (eliminating answer A).

7. We have a system with P states ($P > 1$). I extend the system with a property that can have 3 values. What do you know about the extended system?

   A. The new system has at most $P + 3$ states

   B. The new system has at least $3 * P$ states

   C. The new system has at most $3 * P$ states

   D. The new system has at least 3^P states

   E. The new system has at least $P + 3$ states

   Answer C For any state in the original system, there could be three new states in the extended system, each with a different value for the new property. Hence there is an upper limit of $3 * P$ states for the extended system. If the new property is somehow dependent on the properties already present in the system, not all possible combinations of values for the properties may occur and the number of states can be smaller.

8. A newsletter of a club can be seen as model of life within the club, with communication as the purpose. There are different ways of publishing a newsletter: *regularly*, for instance every month, with a varying number of pages, but also *irregularly* with a fixed number of pages, whenever a sufficient amount of news has been collected for a next edition. We compare a newsletter that is published regularly with one that is published irregularly, with respect to their flavour of time. Which of the following statements is correct?

   A. The regular newsletter has partially ordered time, the irregular newsletter totally ordered time

   B. Time is partially ordered for both newsletters

   C. The regular newsletter has totally ordered time, the irregular newsletter partially ordered time

   D. The regular newsletter samples with equal intervals, the irregular newsletter samples with infinitesimal intervals

   E. The regular newsletter samples with equal intervals, the irregular newsletter with unequal intervals

   Answer E

   For both newsletters, time is ordered: we always know which issue came before/after which other issues (eliminating answers A,B and C). Also, for neither newsletter, the time interval between two issues will be infinitesimally small (eliminating D).

For the regular newsletter the time elapsed between two issues will always be the same, hence *equal intervals*, and for the irregular newsletter this time can vary, hence *unequal intervals*.

9. The formal model produced by a modelling team has the structure of a cyclic graph. Is this a problem?

   A. No, as long as they do not want to expand their formal model

   B. No, as long as the formal model does not contain any category III quantities

   C. Yes, because some quantities have no definition

   D. Yes, because the corresponding To-Do list cannot be empty

   E. This depends on the models purpose.

   Answer D

   Cycles in the formal model are always a problem. Like in a model that does have the required structure for a chain of dependencies (DAG), all quantities in cyclic model are defined in terms of other quantities (eliminating answer C). However, following dependencies along the chain, starting from a category II quantity, does not end because of the cycles. This problem manifests itself in the To Do list. Quantities appear on the list when they are introduced in the model, and are removed from the list when they are defined in terms of other quantities on which they depend. When there is a cycle in the model, quantities will reappear on the list as you go through the cycle. Suppose that somewhere in the model there is a quantity $A = B * 4$. After adding this equation to the model, we take A off the To Do list, and add B to it. If we now define B in terms of A, $B = A + 3$, we can take B off the To Do list but have to put A back on it. We already know that $A = B * 4$, so we can take A off the list again, but then we have to put B on it, and so on.

10. Suppose that a model contains the following expression: $a = (b * c)/(d * e)$ where the quantities b, c, d and e have the following values and error margins: $b = 10 \pm 2$; $c = 20 \pm 2$; $d = 4 \pm 1$; $e = 10 \pm 4$. Given these values for b, c, d, and e, a has value 5 with a certain error margin. How can the smallest possible value for a be found?

    A. Calculate a for the largest possible values of d and e and the smallest possible values of b and c

    B. Calculate a for the smallest possible values of d en e and the largest possible values of b and c

    C. Calculate a for the smallest possible values of b, c, d and e

    D. Calculate a for the largest possible values of b and d and the smallest possible values of c and e

    E. Calculate a for the smallest possible values of b en d and the largest possible values of c and e

    Answer A

    The value of a depends on a fraction, so it is smallest when the value of the numerator $(b * c)$ is as small as possible and the value of the denominator $(d * e)$ as large as possible. The smallest value of $(b * c)$ is obtained when the values of b and c are as small as possible. The largest value of $(d * e)$ is obtained when the values of d and e are as large as possible.

11. Suppose a model is constructed to determine the optimal design of a cellphone, in terms of the cellphone's weight, battery life and production cost. The ATBD must have the value `TRUE` for the predicate `isLessThan1kg(weight)`. The model also contains a penalty function that yields `FALSE` if `batterylife` is lower than 120 minutes. Furthermore, the user of the model likes the production cost to be as small as possible. What is true about this model?

    A. It contains only wishes

    B. It contains both wishes and desires, but no requirements

    C. It contains both wishes and requirements, but no desires

    D. It contains only desires

    E. It contains wishes, desires and requirements

    Answer E

    A requirement is expressed by means a predicate applied to the ATBD that should have the value `True`. A predicate applied to the ATBD that may have the value `FALSE` is a desire. A condition that cannot be expressed as a predicate, like 'production cost should be as small as possible' is a wish. Hence in this case we have wishes, desires and requirements.

12. Suppose that in a formal model, the relation between quantities P, Q, S and T is expressed as: $P^2Q^2 = 3S + T$, in which P is the desired output of the model. Three statements are made about this model:

    C1. At least one of the quantities Q, S, T must be a category I quantity

    C2. Not all quantities Q, S, T can be category III quantities

    C3. All of the quantities Q, S, T can be category IV quantities

    Which of the following statements is correct?

    A. Only statement C1 is true

    B. Only statement C3 is true

    C. Both statement C2 and statement C3 are true

    D. Both statement C1 and statement C2 are true

    E. None of the statements C1,C2 and C3 is true

    Answer B

    P is the desired output of the model, in other words: it is a category II quantity, depending on the quantities Q, S, and T (rewriting the given equation: $P = \sqrt{(\frac{3S+T}{Q^2})}$. What roles could Q, S and T have in the model? They could all be intermediate quantities, category-IV, depending on further quantities in the model. Hence statement C3 is true and statement C1 is false (there doesn't have to be a category-I quantity). It also possible that they are all quantities given by the environment, category III, hence C2 is false. In fact Q, S, T could be any mix of category I, III and IV quantities.

13. Suppose the police wants a penalty-function $f(carSpeed)$ to calculate the fines for driving faster than the legal maximum speed. What could such a function look like?

    A. $f(carSpeed) = (max(maxSpeed, carSpeed) - maxSpeed)) * fineFactorPerKmh$

    B. $f(carSpeed) = (min(maxSpeed, carSpeed) + maxSpeed)) * fineFactorPerKmh$

    C. $f(carSpeed) = (min(|maxSpeed - carSpeed|, maxSpeed)) * fineFactorPerKmh$

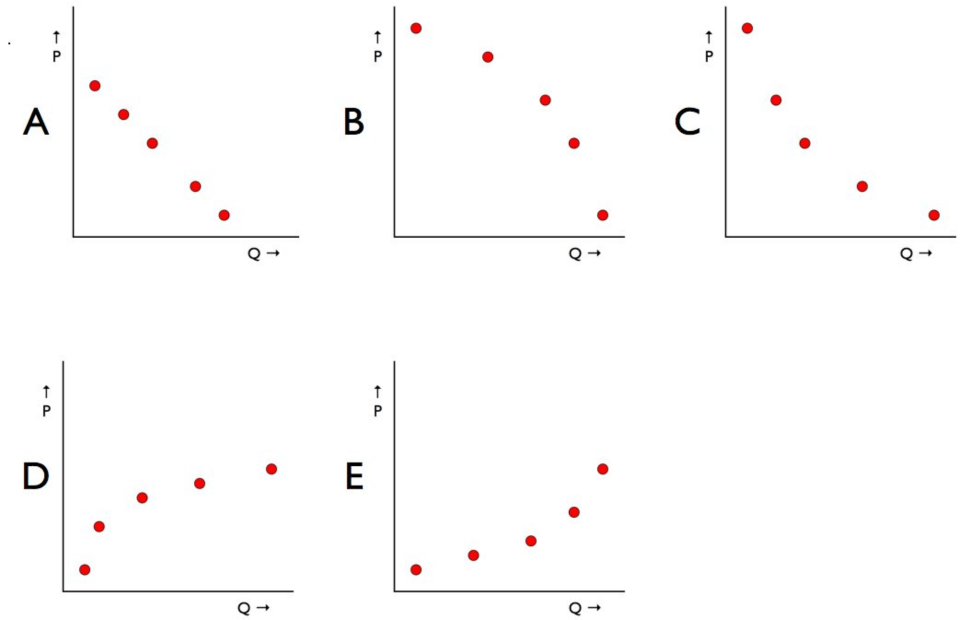
    D. $f(carSpeed) = (max(|maxSpeed - carSpeed|, maxSpeed)) * fineFactorPerKmh$

    E. $f(carSpeed) = (max(maxSpeed, carSpeed) + maxSpeed)) * fineFactorPerKmh$

    Answer A

    A penalty function has value 0 in the desired situation: when $carSpeed \leq maxSpeed$ the fine should be zero (no speeding ticket). This requirement is not met by the functions in answers B, D and E, because their value is always at least $maxSpeed$. The function of answer C only yields zero in case $carSpeed = maxSpeed$, it returns positive values when $carSpeed < maxSpeed$, hence it would give you a fine for driving below the speed limit. The function of answer A only has non-zero values when $carSpeed > maxSpeed$, and these values are equal to the difference in kilometers between the two speeds, as desired.

14. A modeling team has made a model in which quantities P and Q have been optimized with the SPEA algorithm. P was minimized en Q was maximized. After a large number of steps, it turns out that the Pareto front consists of 5 solutions. In the following graphs, the value for P is plotted on the y-axis and the value for Q on the x-axis. Which graph depicts the shape of the Pareto front resulting from the optimization?



Answer E

If solutions are on the Pareto-front, they are not dominated by any other solutions, so in particular not by each other. In the optimization, P was minized (the y-coordinate should be as small as possible) and Q maximized (the x-coordinate should be as large as possible). Hence figures A, B and C cannot depict a Pareto-front for this case, since the rightmost solution in these pictures has *both* the largest x-coordinate *and* the smallest y-coordinate of all solutions. Hence, this point dominates the other four, and the five points cannot form a Pareto-front together. During optimization, the Pareto-front moves in the direction of absolute improvement, which is to down and to the right in this case. Hence after a large number of steps, the front should bulge in this direction. This is the case in E but not in D where the direction of absolute improvement is up and to the left.

15. Which of the following questions should you ask yourself when validating a model?

    A. Are the obtained values for category II quantities conclusive given the modeling purpose?
    B. Do the values obtained by execution of the model fall with the admitted sets of values (types) specified during conceptualization?
    C. Does the model contain cycles?
    D. Is the model correct in terms of units and dimensions?
    E. Do all properties in the conceptual model occur as quantities in the formal model?

    Answer A

    The questions under B-E all concern the internal structure of the formal model or the relationship between the formal model and the conceptual model. Such questions are part of the verification of a model. In the validation of a model we are concerned with the relation between the model and the modeled system and purpose. The only question addressing this is A.

16. Suppose we use the wisdom of the crowds to find the value of a quantity. The bigger the crowd ...

    A. ... the more precise the value
    B. ... the less precise the value
    C. ... the more accurate the value
    D. ... the less accurate the value
    E. ... the less certain the value

    Answer A

    We cannot say that the average of a larger group will be closer to the actual value than the average of a smaller group. Taking the average of more guesses will improve precision. Precision can typically be improved by repeating measurements; accuracy only gets better is sources of systematic deviation are removed.

17. Which of the following characterizations best captures what is meant by the asymptotic analysis of a model?

    A. Checking whether its behaviour for extreme input values conforms to expectations

    B. Setting all input values to zero to check whether any of the output quantities becomes infinite

    C. Determining the asymptotes of its output quantities, in order to find limitations to the applicability of the model

    D. Executing the model a very large number of times, to discover systematic biases

    E. A specific form of sensitivity analysis, which uses very large relative uncertainties in the input quantities

    Answer A

    In asymptotic analysis, we check whether the model behaves as expected for extreme values of the inputs (outside the normal working range or regime of the model). So it is not about extreme values of the output quantities (eliminating B and C), nor about uncertainties in output or input quantities (eliminating E), nor about (repeated) execution of the model (eliminating D).

18. A collection of data points is given in which each point represents a person who has died between the age of 40 and 80. The x-coordinate of such data point is the amount of french fries that persons has consumed on average per year; the y-coordinate is the age at which this person has died. I draw the best possible straight line through all these points; the slope of the line shows the expected shortening of life per kilo/year of consumed fries. This connection is more convincing when:

    A. . . . slope of the line is positive

    B. . . . the line passes (almost) through the origin

    C. . . . the values for the x-coordinates of the points are close to each other

    D. . . . the values for the y-coordinates of the points are close to each other

    E. . . . the points are close enough to the line

    Answer E

    Sketching the graph according to the description (this example was also discussed in video lecture 31), you can immediately see that the slope of the line must be negative (eliminating answer A), and that it does not (almost) pass through the origin (eliminating B). When the values for the y-coordinates are closer together, the size of the effect is smaller (slope of the line can be less steep), making the connection less convincing (eliminating D). If the x-coordinates are closer together the data concerns a smaller range of french fries consumption, making the connection less convincing (eliminating C). If the points are close (enough) to the fitted line the residue is small, the straight line represents more of the information present in the data, making the connection more convincing.

19. In addition to basic principles of physics, climate models usually contain a number of built-in optimizations to improve their computational efficiency. This makes these models:

    A. Less surprising

    B. More distinctive

    C. Less convincing

    D. Less generic

    E. None of the above

    Answer D

    The built-in optimizations make us of assumptions or approximations that are correct for a particular modeled system, but could prevent the model from being applied to other modeled systems. A climate model could for instance be optimized for one geometrical situation, because it is always applied to the whole of the earth atomsphere, making it impossible to apply in situations with a different geometry, say modeling the climate inside Studyhub 2.

20. 'Agent-based' models simulate the behaviour of large groups of entities by means of simple interaction rules. A well-known example is Conways Game of Life, in which the state of an agent ('on' or 'off') in fase $t+1$ is determined by the states of the eight agents directly surrounding it in fase t. Suppose that a modeling team wants to replace this interaction rule with a rule in which the state of an agent at $t+1$ depends on the states of all agents in the system at t. What happens to the scalability of this model?

    A. The new model is less scalable than the old one

    B. The new model scales just as well as the old one

    C. The new model is more scalable than the old one

    D. This is impossible to say without knowing the purpose of the model

    E. The two models are too different to compare in terms of scalability

    Answer A

    For this model, the size or scale is characterized by the number of agents in the simulation. Whenever an agent is added to the simulation under the orginal interaction rule, a constant number of additional calculations is needed to compute the new state: the new agent has to inspect the states of his 8 neighbours. Under the new interaction rule, the number of additional calculations needed per additional agent increases as more agents are added: the new agent has to inspect the states of all old agents, and all old agents now also have to inspect the state of the new agent. Hence, assuming limitations on available computing power and time, the system with the original rule scales better: it can be extended to a larger number of agents than the new system.

21. Which of the following functions has the two properties $f_x = 2x\sin(y) + 7x^6y^7$ and $f_y = x^2\cos(y) + 7x^7y^6 - 3$?

A. $f(x, y) = x^2\sin(y) + x^8y^8 - 3xy$

B. $f(x, y) = x^2\sin(y) + x^7y^7 - 3y$

C. $f(x, y) = -x^2\cos(y) + x^7y^7 - 3xy$

D. $f(x, y) = -x^2\cos(y) + x^8y^8 - 3y$

E. $f(x, y) = x^2\sin(y) + x^7y^8 - 3y$

Answer B

22. A correct equation for the tangent plane to $z = xy-x-y$ at the point $(x, y) = (-2, 5)$ is

A. $4x - 3y - z = 0$

B. $4(x + 2) - 3(y - 5) - (z + 13) = -13$

C. $-4(x + 2) + 3(y - 5) - z = 0$

D. $-4(x + 2) + 3(y - 5) + z = +13$

E. $4x - 3y - z = -10$

Answer E
If we write $f(x, y) = xy - x - y$, then we have $f(-2, 5) = -13$, $f_x(x, y) = y - 1$ and $f_x(-2, 5) = 4$, $f_y(x, y) = x - 1$ and $f_y(-2, 5) = -3$.

Therefore, the tangent plane is (see the formula in the slides)

$$0 = 4(x + 2) - 3(y - 5) - (z + 13).$$

This gives $z = 4x - 3y + 10$.

23. Consider the function $f(x, y) = \sqrt{x^2 + y^2}$. Let $L(x, y)$ be the linear approximation of the function $f(x, y)$ at the point $(-3, 0)$. This function is used to approximate the value of $f(x, y)$ at the point $(-3.1, -0.1)$. The approximation is equal to

A. -3.2

B. -3.1

C. 3.1

D. 3.2

E. 3.15

Answer C
We have $f_x(x, y) = x/\sqrt{x^2 + y^2}$, $f_x(-3, 0) = -1$, $f_y(x, y) = y/\sqrt{x^2 + y^2}$ and $f_y(-3, 0) = 0$.

Therefore, the linear approximation is

$$L(x, y) = -(x + 3) + 3.$$

This gives $L(-3.1, -0.1) = -(-3.1 + 3) + 3 = 3.1$.

24. Consider the function $f(x, y) = 2x^2 + y^3 - x^2y - 3y$. The critical points of this function are

A. $(0, 1), (0, -1), (-3, 2), (3, 2)$

B. $(0, 1), (0, -1)$

C. $(0, 1), (0, -1), (2, -3), (2, 3)$

D. $(0, 1), (3, 2)$

E. $(0, 2), (0, -1), (0, 1)$

Answer A
We have $f_x(x, y) = 4x - 2xy = 2x(2 - y)$ and $f_y(x, y) = 3y^2 - x^2 - 3$.

Therefore, $f_x(x, y) = 0$ gives $x = 0$ and $y = 2$. Substituting this is in $f_y(x, y) = 0$, we find as critical points $(0, 1), (0, -1), (3, 2), (-3, 2)$.

25. Consider the function

$$f(x, y) = \frac{x^2 + y^2 + 6}{2x + 6y}.$$

The circle $(x-1)^2 + (y-3)^2 = 4$ is

A. the level curve of $f(x, y)$ with $f(x, y) = -2$

B. the level curve of $f(x, y)$ with $f(x, y) = 2$

C. the level curve of $f(x, y)$ with $f(x, y) = -1$

D. the level curve of $f(x, y)$ with $f(x, y) = 1$

E. not a level curve of $f(x, y)$

Answer D
The point $(3, 3)$ is a point on the circle. So, if it is a level curve, the function value must be 1. Indeed it is a level curve because $\frac{x^2+y^2+6}{2x+6y} = 1$ gives $x^2 + y^2 + 6 = 2x + 6y$ and $(x-1)^2 + (y-3)^2 = 4$.

26. Consider the function

$$f(x, y) = x^3 - 3xy + y^3.$$

It is given that the function has exactly one local minimum. The value of this local minimum is

A. 0

B. $-1/3$

C. -1

D. -3

E. -5

Answer C
We must have $f_x(x, y) = 3x^2 - 3y = 0$ and $f_y(x, y) = 3y^2 - 3x = 0$. This gives $x^4 - x = 0$, so $x = 0$ or $x = 1$.
Critical points are $(0, 0)$ and $(1, 1)$. It is given that there is one local minimum. This must be in one of these points. Given the function value in these points, the minimum value must be -1.

27. Consider the function $f(x, y) = 4x - x^2 + y^2$. What is the maximal value of $f(x, y)$ on the rectangle where x is between 0 and 3 and y is between 0 and 1?

A. 1

B. 3

C. 4

D. 5

E. 9

Answer D
We must have $f_x(x, y) = 4 - 2x = 0$ and $f_y(x, y) = 2y = 0$. This gives the critical point $(2, 0)$.

So the maximum value is attained at the border.
For $x = 0$ and $0 \leq y \leq 1$ we have $f(0, y) = y^2$ and the maximum value is 1.
For $x = 3$ and $0 \leq y \leq 1$ we have $f(3, y) = 3 + y^2$ and the maximum value is 4.
For $y = 0$ and $0 \leq x \leq 3$ we have $f(x, 0) = 4x - x^2$ and the maximum value is 4, attained for the value $x = 2$.
For $y = 1$ and $0 \leq x \leq 3$ we have $f(x, 1) = 4x - x^2 + 1$ and the maximum value is 5, attained for the value $x = 2$.

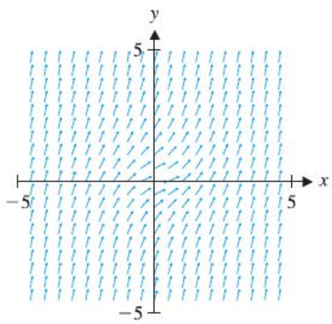
28. Match each differential equation ((a),(b),(c),(d)) to the correct direction field (A-D)

(a) $y' = 2 - xy$

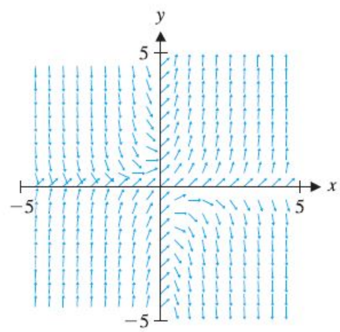
(b) $y' = 1 + 2xy$

(c) $y' = x\cos 3y$

(d) $y' = \sqrt{x^2 + y^2}$



FIELD A



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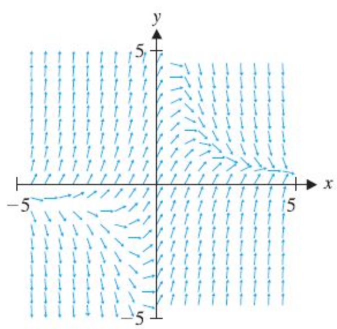
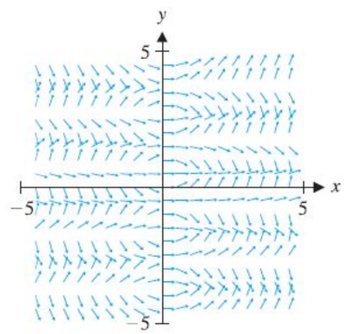


FIELD C



FIELD D

	(i)	(ii)	(iii)	(iv)
A.	Field C	Field B	Field D	Field A
B.	Field C	Field D	Field B	Field A
C.	Field B	Field C	Field D	Field A
D.	Field B	Field C	Field A	Field D
E.	Field A	Field B	Field D	Field C

Answer A
One can find the answer by checking some points. If one chooses for example $x = 0$, then we see

(a) $y' = 2$

(b) $y' = 1$

(c) $y' = 0$

(d) $y' = y$

29. Consider the differential equation $y' = y^2 - 7y + 6$. Find the equilibrium solutions and determine if they are stable or unstable.

A. $y = -1$ (stable), $y = -6$ (unstable)

B. $y = -1$ (unstable), $y = -6$ (stable)

C. $y = 1$ (unstable), $y = 6$ (stable)

D. $y = 1$ (stable), $y = 6$ (stable)

E. $y = 1$ (stable), $y = 6$ (unstable)

Answer E
The equation $y^2 - 7y + 6 = 0$ gives $y = 1$ and $y = 6$. For $y < 1$ we find $y' > 0$, so the arrows of the direction field point towards the line $y = 1$, for $1 < y < 6$ we find $y' < 0$, so the arrows of the direction field point towards the line $y = 1$, and away from the line $y = 6$. For $y > 6$ we find $y' > 0$, so the arrows of the direction field point away from the line $y = 6$. Therefore the correct answer is E.

30. Consider the differential equation $y' = 4x + 2y$. The method of Euler is used to approximate the solution. The stepsize is 0.5 and we have $y(1) = 1$. What is the missing value in the table below?

x	y
1	1
1.5	4
2	

A. 9

B. 11

C. 7

D. 14

E. 2

Answer B
We have $y_{i+1} = y_i + (4x_i + 2y_i) \cdot 0.5$.
So $y_2 = y_1 + 6 \cdot \frac{1}{2} = 1 + 3 = 4$ and $y_3 = y_2 + (3 \cdot \frac{3}{2} + 2 \cdot 4) \cdot \frac{1}{2} = 11$.

31. Consider the function $f(x) = x^3/(x^2 - 1)$.
Which of the following statements is true?

A. The function has two horizontal asymptotes and a slant asymptote $y = x$

B. The function has an inflection point in $x = 0$

C. There are exactly two critical points

D. The function has two vertical asymptotes and has no inflection points

E. The function has a local maximum in $x = 0$

Answer B
The derivative is

$$f'(x) = \frac{x^2(x^2-3)}{(x^2-1)^2}.$$

So there are more than two critical points; there are no horizontal asymptotes, but two vertical asymptotes; there is no local maximum in $x = 0$ since the derivative is positive close to $x = 0$. So the correct answer must be B or D. The second derivative is

$$f''(x) = \frac{2x(x^2+3)}{(x^2-1)^3}.$$

So there is one inflection point, the correct answer is B.
Remark: by inspecting the first derivative one can see (because of the factor x^2 that there is an inflection point.

32. Consider the function $f(x) = (x^2 - 1)^3$.
The number of inflection points is equal to

A. 0

B. 1

C. 2

D. 3

E. 4

Answer E
The first derivative is

$$f'(x) = 3(x^2 - 1)^2 \cdot 2x.$$

The second derivative is

$$f''(x) = 6(x^2 - 1)(5x^2 - 1).$$

Therefore, there are 4 inflection points.

33. An output quantity z is proportional to the square root of an input quantity x and inversely proportional to the square of an input quantity y. We know that $z = 3$ if $x = 3$ and $y = 4$. What is the relation between z and x and y if we assume multiplicativity?

A. $z = -1 + \sqrt{3x} + 16/y^2$

B. $z = 47/16 - \sqrt{3} + \sqrt{x} + 1/y^2$

C. $z = 16\sqrt{3x}/y^2$

D. $z = 2x^2/(3\sqrt{y})$

E. $z = 3 + \sqrt{x}/y^2 - \sqrt{3}/16$

Answer C
Because of multiplicativity we have that for some value of c the following holds

$$z = \sqrt{x} \cdot \frac{1}{y^2} \cdot c.$$

Substituting $x = 3$ and $y = 4$ we find $c = 16\sqrt{3}$.

34. We want to construct a box and maximize the volume of the box and minimize the surface used. Suppose that the width, depth and length of the box are all between 1cm and 10cm and suppose that, for some reason, we want the length l of the box to be at least 5cm ($l \geq 5$) and the width w of the box at most 5cm ($w \leq 5$). Which of the following functions is a proper penalty function?

    A. $|\min(l, 5)| + |\max(w, 5)|$

    B. $|5 - \max(l, 5)| \cdot |\min(w, 5) - 5|$

    C. $|5 - \max(l, 5)| + |\min(w, 5) - 5|$

    D. $|5 - \min(l, 5)| + |\max(w, 5) - 5|$

    E. $|5 - \min(l, 5)| \cdot |\max(w, 5) - 5|$

    Answer D
    See the slides of Chapter 5. Forming mathematical expressions. Example: penalty functions.

35. Consider the function $f(x) = x^2 + 2x + 3$.
    The condition number of $f(x)$ in the point $x = -2$ equals

    A. 2

    B. 12/11

    C. 4

    D. -4

    E. 4/11

    Answer: 4/3
    Here was a mistake. The correct answer is not given. So, all answers were considered good.
    The correct answer is 4/3. The condition number equals

$$\frac{|-2|}{|f(-2)|} \cdot |f'(-2)| = \frac{|-2|}{|4-4+3|} \cdot |2 \cdot (-2) + 2| = \frac{2}{3} \cdot 2 = \frac{4}{3}.$$

36. Consider the relation between the maximal speed v of a cyclist given a force F

$$v = g(F) = \frac{1}{\sqrt{c \cdot \rho \cdot A}} \sqrt{F}.$$

Suppose we know the force with 2% uncertainty. Which of the following is true?

A. The percentage uncertainty cannot be determined since we need to know the value of F

B. The percentage uncertainty cannot be determined since we need to know the value of ρ, c, A

C. We know the maximal speed with 0.5% uncertainty

D. We know the maximal speed with 1% uncertainty

E. We know the maximal speed with 2% uncertainty

Answer D
The condition number is 1/2. See Example: the speed of a cyclist from the slides of Chapter 6. Therefore, the maximal speed is known with 1% uncertainty.

37. Consider the function $f(x, y) = \frac{x}{y}$.
The absolute uncertainty in $f(x, y)$ at the point $(x, y) = (4, 2)$ equals

A. 3/2

B. 1/2

C. $-1/2$

D. 2

E. -2

Answer A
We have $f_x(x, y) = \frac{1}{y}$ and $f_y(x, y) = -\frac{x}{y^2}$.
The absolute uncertainty equals

$$|f_x(4, 2)| + |f_y(4, 2)| = \left|\frac{1}{2}\right| + \left|\frac{-4}{2^2}\right| = \frac{1}{2} + 1 = \frac{3}{2}.$$

38. Consider the function $f(x, y) = x^2 + 4xy^2 - y^5$. What is the direction in which the minimum rate of change occurs at the point $(2, 1)$?

A. $(-8, -11)$

B. A direction orthogonal to $(8, 11)$

C. $(14, 7)$

D. $(8, -11)$

E. $(8, 11)$

Answer A
We have $f_x(x, y) = 2x + 4y^2$ and $f_y(x, y) = 8xy - 5y^4$.
This gives $f_x(2, 1) = 8$ and $f_y(2, 1) = 11$. The direction with the minimum rate of change is $(-8, -11)$.

39. What is the directional derivative of $f(x, y) = x^2y+2y^2$ at the point $(x, y) = (2, -1)$ in the direction of $v = (-1, \sqrt{3})$?

A. $-4 + 4\sqrt{3}$

B. $2 + 4\sqrt{3}$

C. -2

D. 4

E. 2

Answer E
We have $f_x(x, y) = 2xy$ and $f_y(x, y) = x^2 + 4y$.
This gives $f_x(2, -1) = -4$ and $f_y(2, -1) = 0$. The unit vector in the direction of v is $(-\frac{1}{2}, \frac{1}{2}\sqrt{3})$.
The directional derivative is $(-4, 0) \cdot (-\frac{1}{2}, \frac{1}{2}\sqrt{3}) = 2$.

40. Find the value of the minimum rate of change of $f(x, y) = xy^2 - 3xy$ at the point $(x, y) = (-3, 1)$.

A. -1

B. $-\sqrt{13}$

C. $\sqrt{13}$

D. 1

E. $\sqrt{5}$

Answer B
We have $f_x(x, y) = y^2 - 3y$ and $f_y(x, y) = 2xy - 3x$.
This gives $\nabla f(-3, 1) = (-2, 3)$. The value of the minimum rate of change equals $-\sqrt{(-2)^2 + 3^2} = -\sqrt{13}$.