Evaluation of the Hooghoudt drainage equation

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January 14, 2010

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Abstract

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1 Introduction

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The model applied in the present studies, DAISY (Hansen et al., 1991; Abrahamsen and Hansen, 2000) is used for describing the crop production as well as water and nitrogen balances in the root zone.

DAISY have in numerous studies (e.g. Styczen and Strorm, 1993; Refsgaard et al., 1999; Thorsen et al., 2001; Boegh et al., 2004; Hansen et al., 2007; van der Keur et al., 2008) been applied for estimation of water balances or simulation of pollution of non-point sources arising from agricultural production on a catchment scale. In many of the studies the processes in the root zone are handled by a described 1D with DAISY whereas the hydrological catchment model MIKE SHE (Abbott et al., 1986) has been applied for 2D modelling of the surface flow and 3D modelling of the groundwater flow.

Common for all the mentioned studies is that they required a very large number of DAISY simulations due to different soil, crop ,management, etc within the catchment. Thus a fast computation of the processes in the unsaturated zone is crucial. For agricultural fields with parallel placed pipe drains the flow towards the drains is a two dimensionally process. In the 1D DAISY is the estimation of the drain fluxes based on the theory by Hooghoudt which assumes steady state conditions (and?). DAISY has recently been developed so the model also can be used for 2D simulations with a more precise calculation of drain fluxes. But compared with 1D simulation, 2D (or 3D) simulations of water and solute movement in the unsaturated zone are from a computational point of view very expensive.

The question is, can the 2D drain simulations with adequate precision be replaced by 1D simulations for estimation of water and solute drain fluxes on the field scale. Here both water movement as well as the transport of a conservative tracer (bromide) to the drain are considered.

XXXX More Hooghoudt studies XXXX

Fipps et al. (1986)

2 Theory

In the simulations, the water flow is described with Richards' equation

$$\frac{\partial \theta}{\partial t} = \nabla \cdot (\mathbf{K}(\psi)\nabla(\psi + z)) - \Gamma_{\mathrm{w}} \tag{1}$$

where θ is the volumetric water content, ψ is the pressure potential. Γ_{w} is the sink term for water. The sinks can be root uptake as well as contributions from tile drains. $\mathbf{K}(\psi)$ is the hydraulic conductivity matrix. In the present case, the soil is regarded as isotropic and as a consequence the conductivity can be described with a scalar ($K(\psi)$). The x-axis is chosen in horizontal direction and the z-axis is positive upwards.

In the presented numerical examples, the soil-water retention model by van Genuchten (1980) is applied

$$\theta = \begin{cases} \theta_r + \frac{\theta_s - \theta_r}{[1+|\alpha h|^n]^m} & \text{for } \psi < 0 \\ \theta_s & \text{for } \psi \geq 0 \end{cases} \tag{2}$$

where α, n and m are empirical parameters, θ_s and θ_r are the saturated and the residual water content, respectively. By combination with the hydraulic conductivity model by Mualem (1976) and choosing $m = 1 - 1/n$, the hydraulic conductivity can be calculated as

$$K = K_s S_e^l [1 - (1 - S_e^{1/m})^m]^2 \tag{3}$$

where K_s is the hydraulic conductivity at saturation, and $S_e = (\theta - \theta_r)/(\theta_s - \theta_r)$ is the effective saturation. l is a form parameter

When the solution to Richards' equation is found, the water flux can be determined using Darcy's equation $\mathbf{q} = -\mathbf{K}(\psi)\nabla \cdot (\psi + z)$. The movement of an non-absorbing, inert solute can be described with the advection-dispersion equation.

$$\frac{\partial(\theta C)}{\partial t} = -\nabla \cdot (C\mathbf{q} - \theta \mathbf{D} \nabla C) - \Gamma_{\mathrm{s}} \tag{4}$$

where C is the concentration in the liquid phase and Γ_{s} is the sink for solute. $\mathbf{D}$ is the dispersion matrix. The consequence is that the solute tries to move from areas with high concentration to areas with lower concentration. The elements in $\mathbf{D}$ are calculated as:

$$\begin{aligned} D_{xx} &= \alpha_L \frac{v_x^2}{|\mathbf{v}|} + \alpha_T \frac{v_z^2}{|\mathbf{v}|} + D^* \\ D_{zz} &= \alpha_L \frac{v_z^2}{|\mathbf{v}|} + \alpha_T \frac{v_x^2}{|\mathbf{v}|} + D^* \\ D_{xz} &= D_{zx} = (\alpha_L - \alpha_T) \frac{v_x v_z}{|\mathbf{v}|} \end{aligned} \tag{5}$$

where $\mathbf{v} = \mathbf{q}/\theta$ is mean velocity in the pores (Darcy velocity) and v_x and v_z is the component in the x- and z-direction, respectively. α_L and α_T are called the longitudinal and transversal dispersion respectively. The molecular diffusion D^* can be expressed as $D^* = \tau D_0$ where D_0 is the diffusion coefficient and τ is the tortuosity factor which is computed with the model by Millington and Quirk (1961).

2.1 Hooghoudt drainage equation

When the groundwater table is located above the pipe drain water will flow towards the drain. According to Hooghoudt (1940) the drain flux q can be computed as:

$$q = \frac{8K_b D_e h + 4K_a h^2}{L^2} \tag{6}$$

where K_a is the (vertical) hydraulic conductivity of the saturated layer above drain level and K_b is the (horizontal) hydraulic conductivity of the layer below the drain level. L is the distance between the drains, D is the vertical distance between drain depth and aquitard and h is the midpoint water table height above the drain. The drain flux can be divided into two parts. A part that flows towards the drain above the drain level (q_a) and a part that flows towards the drain below the the drain level (q_b):

$$q_a = \frac{4K_a h^2}{L^2}, \quad q_b = \frac{8K_b D h}{L^2} \tag{7}$$

van der Molen and Wesseling (1991)

$$D_e = \frac{1}{8} \cdot \frac{\pi L}{\ln(\frac{L}{\pi r}) + F(y)} \tag{8}$$

where

$$y = \frac{2\pi D}{L} \tag{9}$$

and

$$F(y) = \begin{cases} \frac{\pi^2}{4y} + \ln\left(\frac{y}{2\pi}\right) & \text{for } y < 0.5 \\ \sum_{j=1}^{\infty} \frac{4\exp((-4j+2)y)}{(2j-1)(1-\exp((-4j+2)y))} & \text{for } y \geq 0.5 \end{cases} \tag{10}$$

$$K_a = \frac{\sum_{i=0}^{N} f_{a,i} \Delta z_i K_{s,i}}{D_a}, \quad D_a = \sum_{i=0}^{N} f_{a,i} \Delta z_i \tag{11}$$

$$K_b = \frac{\sum_{i=0}^{N} f_{b,i} \Delta z_i K_{s,i}}{D_b}, \quad D_b = \sum_{i=0}^{N} f_{b,i} \Delta z_i \tag{12}$$

$$S_i = \frac{f_{a,i} K_{s,i}}{K_a D_a} + \frac{f_{b,i} K_{s,i}}{K_b D_b} \tag{13}$$

2.2 Finite Volume model

2.2.1 Aquitard boundary condition

An aquitard can in DAISY be implemented as a boundary condition with a user defined, constant hydraulic conductivity.

2.2.2 Implementation of 2D drain

XXX Should be modified XXX

In DAISY it is possible to simulate a (user defined) number of tile drains. Tile drains removes water when the matrix pressure potential in the soil around the drain is positive. The actual pressure in a drain pipe depends on position in the drain system, the hydraulic radius, etc, etc. An often applied simplification codes for variably saturated flow is to regard the pressure in the drain pipe as atmospheric. When the soil in the drain point is unsaturated ($\psi < 0$) the solution corresponds to the solution for an undrained soil. If the soil is saturated ($\psi > 0$) the drains removes water from the soil matrix hence $\psi = 0$.

In the numerical model, the drain pipe is described as a point. The drain points shall be placed in the interior of a cell and cannot be placed at cell edges.

For obtaining a numerical stable solution it is in the beginning of a new iteration in the time step tested if the mean value of the matrix pressure in the drain cell and its eastern and western neighbors (if they exists) exceeds 0. If the mean value is positive the pressure in the drain cell is forced to zero. After each time step a mass balance for each of the drain cells is made to calculate the amount of drained water.

Test simulations show that the code both is able to turn on the drain when the soil is getting wetter and turn of the drain when the soil is getting drier. Figure XXXX shows the results from a simulation with an aquitard boundary condition and a drain. The upper boundary has a no flux condition, thus the only supply of water is through the aquitard. As it can be observed, the matrix pressure potential in the drain is 0.

3 Comparative study

3.1 DAISY setup

Besides the models for water and solute movement, DAISY have build-in models for transport of heat, evapotranspiration, crop production and nitrogen dynamics.

In the present work we use meteorological data obtained at Taastrup, situated 20 km west of Copenhagen, Denmark. The meteorological data includes hourly values of global radiation, air temperature precipitation, vapor pressure and wind speed. Due to larger periods with failure in 1997 all the meteorological data at that year where replaced with data from 1998.

As soil data is used is used the JB6 soil parameterization, defined in the standardization project (see Styczen et al., 2006). The JB6 soil is representing typical soil conditions for drained agricultural fields in Denmark. According to the USDA system is the applied JB6 soil characterized as a sandy loam. For the present simulations is the soil divided into XXXX layers with the van Genuchten soil parameters as shown in Table 1. At a depth of 200 cm is placed an aquitard of 200 cm thickness. The pressure potential below the aquitard is 200 cm corresponding to a groundwater depth at 200 cm for a system in equilibrium. The simulations are conducted for three drainage levels: low, moderate and high. For the low drainage level is approximately in average 25% of the net precipitation drained corresponding to a percolation of 75%. For the simulations with moderate and high drainage levels, 50% and 75% of the net precipitation are drained respectively. The different drainage levels are obtained by adjusting the hydraulic conductivity of the aquitard. The obtained conductivities are also shown in Table 1. The drain pipes are placed at a depth of 110 cm with a horizontal spacing of 16 m.

On the the field is simulated growth of spring Barley. Typical management practise for non-organic farming has been applied in the simulations.

For investigating the fate of non-absorbing, inert solute in the simulations Bromide (1.0 kg/ha) was sprayed to the soil surface at on 15. September 1989.

Except the mesh and the methods for calculating and removing the water by the drains, the setup for the 1D and 2D simulations are in every context, including the numerical methods similar.

3.1.1 Mesh

For the 1D simulations is the column discretizised into XXXX cells with the depth ranging from XXXX to XXXX. The finest discretization near the surface.

For the 2D simulations is the same discretization in z-direction applied as for the 1D simulations. The rectangular domain is divided into XXXX parts in the x-direction with the finest discretization near the drains where the largest potential gradients is expected. Thus the 2D domain consists of XXXX volumes.

XXX Eventually, Figure of of 2D mesh XXX

Drainage levels

The simulations are conducted for three drainage levels: low, moderate and high. For the low drainage level is approximately in average 25% of the net precipitation drained corresponding to a percolation of 75%. For the simulations with moderate and high drainage levels, 50% and 75% of the net precipitation are drained respectively.

The different drainage levels are obtained by adjusting the hydraulic conductivity of the aquitard. The obtained conductivities are also shown in Table 1.

3.2 Statistical evaluation

Statistical methods based on the difference between the 2D results (regarded as the observation) and the 1D results (regarded as the predicted value) are used to evaluate the performance of the 1D drain model. The methods are the Root Mean Square Error (RMSE), the Coefficient of Determination (CD) and the Modelling Efficiency (ME):

$$\mathrm{RMSE} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(P_i - O_i)^2} \tag{14}$$

$$\mathrm{CD} = \frac{\sum_{i=1}^{N}\left(O_i - \bar{O}\right)^2}{\sum_{i=1}^{N}\left(P_i - \bar{O}\right)^2} \tag{15}$$

$$\mathrm{ME} = \frac{\sum_{i=1}^{N}\left(O_i - \bar{O}\right)^2 - \sum_{i=1}^{N}(P_i - O_i)^2}{\sum_{i=1}^{N}\left(O_i - \bar{O}\right)^2} \tag{16}$$

where O_i is the observed value and P_i the predicted. N is the number of observations. RMSE, CD and ME all have an optimum of zero. According to Loague and Green (1991) is a negative ME an indication of that the mean of the observed value is an better estimate than the predicted value.

4 Results and discussion

4.1 Yearly values

Figure 2

Figure 3

Figure 4

XXX Comparison of drainage flows with rain XXX

XXX Comparison of GW table from 2D sims and from Hooghoudts

4.2 Daily values

Table 2

Figure 5

Figure 6

Figure 7

Figure 9

4.3 Water table

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5 Conclusions

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Tables

Table 1: Hydraulic properties of the horizons.

Depth	θ_s	θ_r	α	n	K_s	l
	cm^3cm^{-3}	cm^3cm^{-3}	cm^{-1}	-	$cm\,hour^{-1}$	-
0-30 cm	0.386	0.000	0.044	1.246	1.469	-2.365
30-80 cm	0.360	0.000	0.054	1.249	0.958	-1.574
80-200 cm	0.338	0.000	0.046	0.625	1.223	-0.983
Aquitard, 25%	-	-	-	-	0.248	-
Aquitard, 50%	-	-	-	-	0.501	-
Aquitard, 75%	-	-	-	-	0.750	-

Table 2: Statistical analysis of daily values.

Simulated	RMSE	CD	ME
Water percolation, 25%	$7.88 \cdot 10^{-3}$ mm/day	0.98	1.00
Water percolation, 50%	$6.15 \cdot 10^{-3}$ mm/day	0.97	1.00
Water percolation, 75%	$3.36 \cdot 10^{-3}$ mm/day	0.95	1.00
Water drainage, 25%	$3.84 \cdot 10^{-2}$ mm/day	1.15	0.98
Water drainage, 50%	$4.78 \cdot 10^{-2}$ mm/day	1.11	0.99
Water drainage, 75%	$5.68 \cdot 10^{-2}$ mm/day	1.09	0.99
Bromide leaching, 25%	$2.40 \cdot 10^{-2}$ g/ha/day	0.94	1.00
Bromide leaching, 50%	$1.79 \cdot 10^{-2}$ g/ha/day	0.91	0.99
Bromide leaching, 75%	$1.26 \cdot 10^{-2}$ g/ha/day	0.98	0.99
Bromide drainage, 25%	$3.37 \cdot 10^{-2}$ g/ha/day	1.31	0.96
Bromide drainage, 50%	$5.69 \cdot 10^{-2}$ g/ha/day	1.22	0.97
Bromide drainage, 75%	$7.27 \cdot 10^{-2}$ g/ha/day	1.11	0.97
Nitrogen leaching, 25%	$1.98 \cdot 10^{-3}$ kg/ha/day	0.97	1.00
Nitrogen leaching, 50%	$1.18 \cdot 10^{-3}$ kg/ha/day	0.95	1.00
Nitrogen leaching, 75%	$5.25 \cdot 10^{-4}$ kg/ha/day	0.94	1.00
Nitrogen drainage, 25%	$9.72 \cdot 10^{-3}$ kg/ha/day	1.20	0.97
Nitrogen drainage, 50%	$9.74 \cdot 10^{-3}$ kg/ha/day	1.15	0.98
Nitrogen drainage, 75%	$9.57 \cdot 10^{-3}$ kg/ha/day	1.12	0.98

Figure captions

Fig. 1. Blabre blabre blabre xxx.

Fig. 2. Blabre blabre blabre xxx.

Fig. 3. Blabre blabre blabre xxx.

Fig. 4. Blabre blabre blabre xxx.

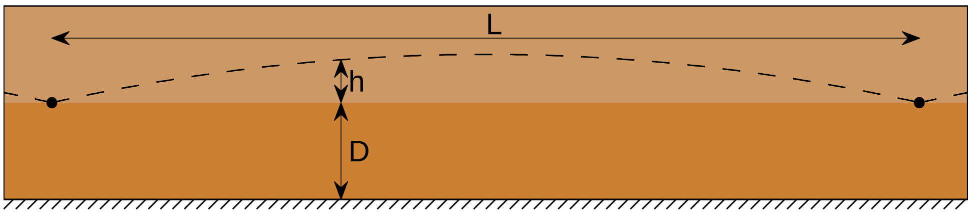


Figure 1: Hooghoudt.

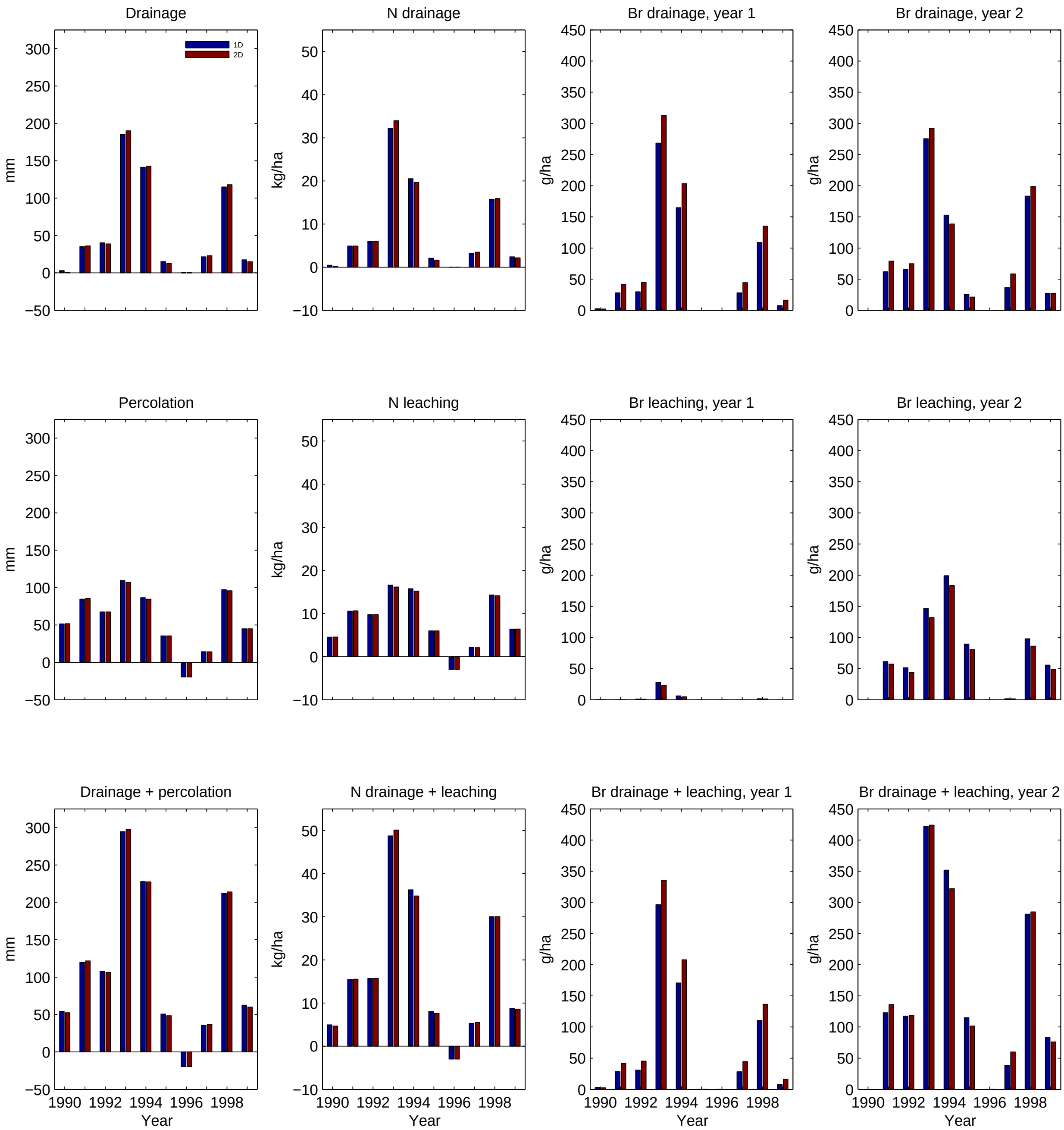
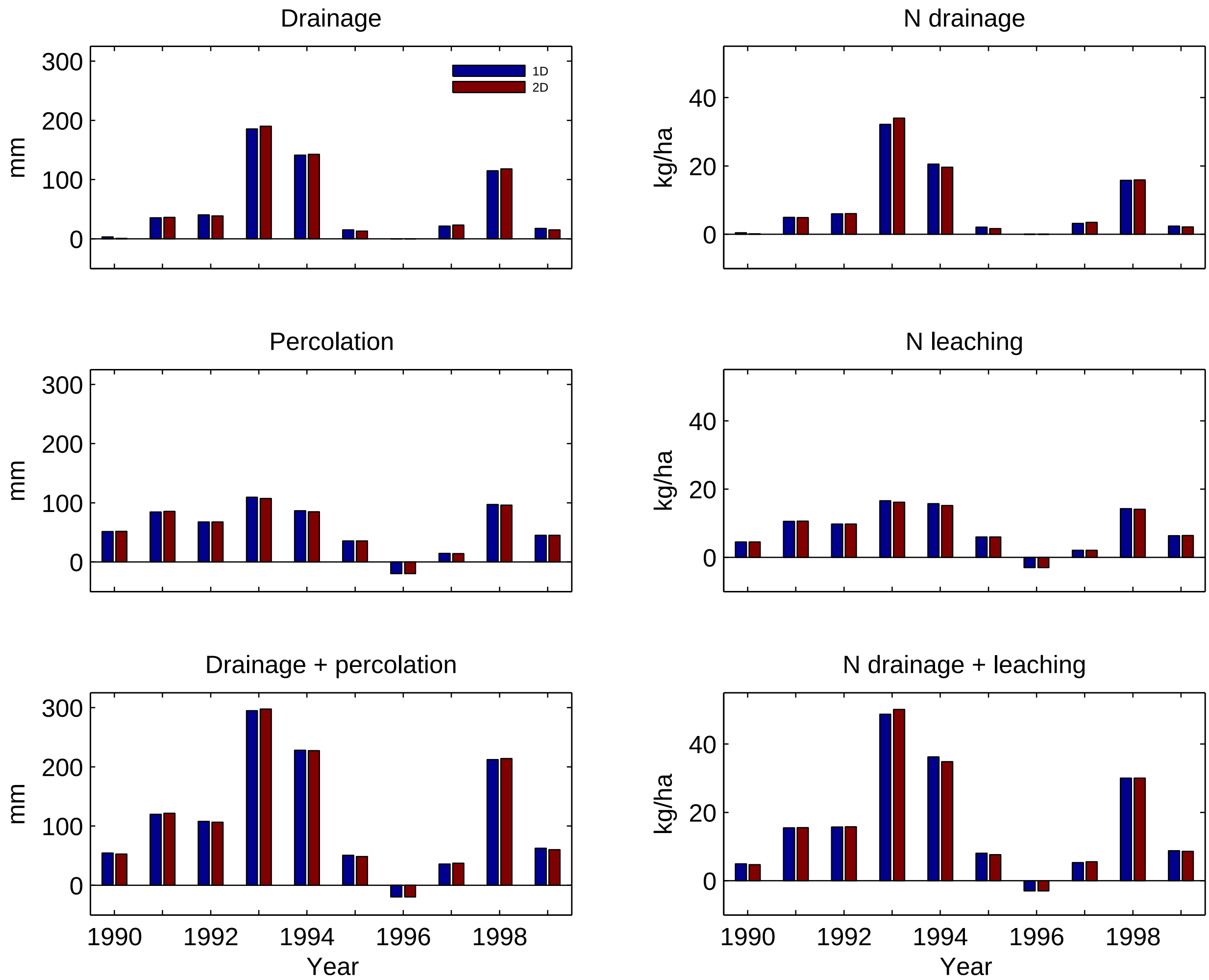


Figure 2: blabre blabre.



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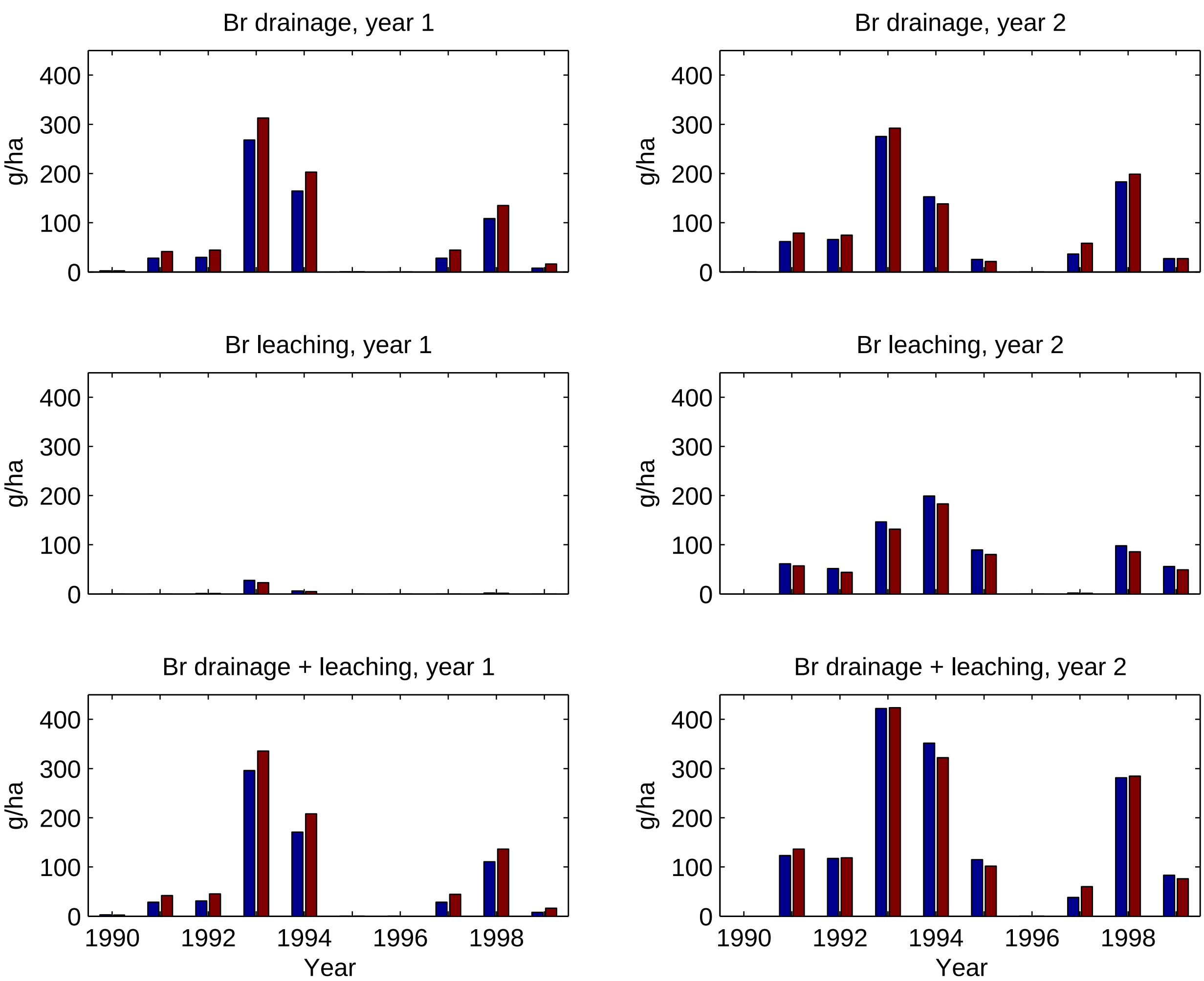

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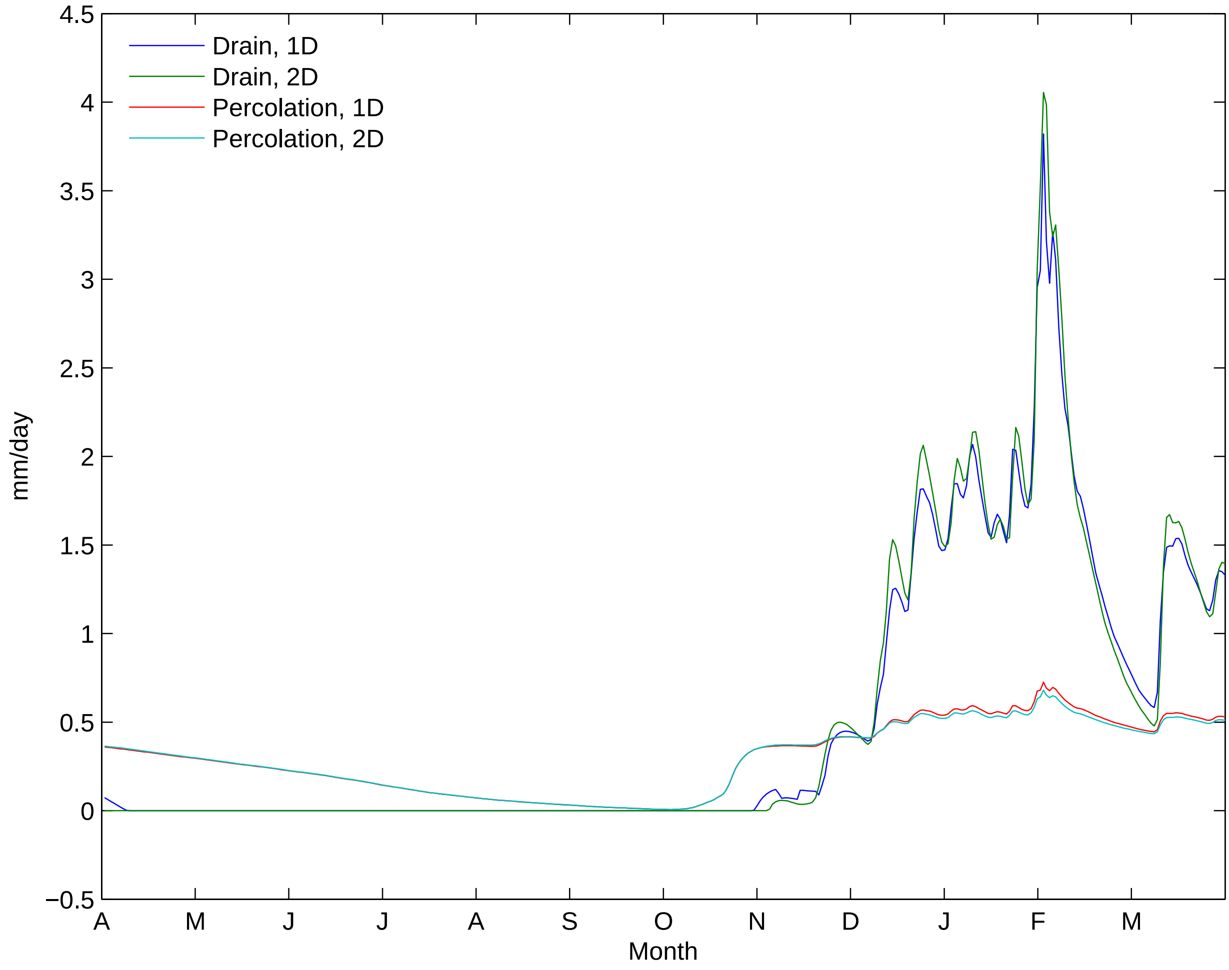

Figure 5: Water drainage and percolation for the 1D and 2D simulations in the hydrological year 1993.

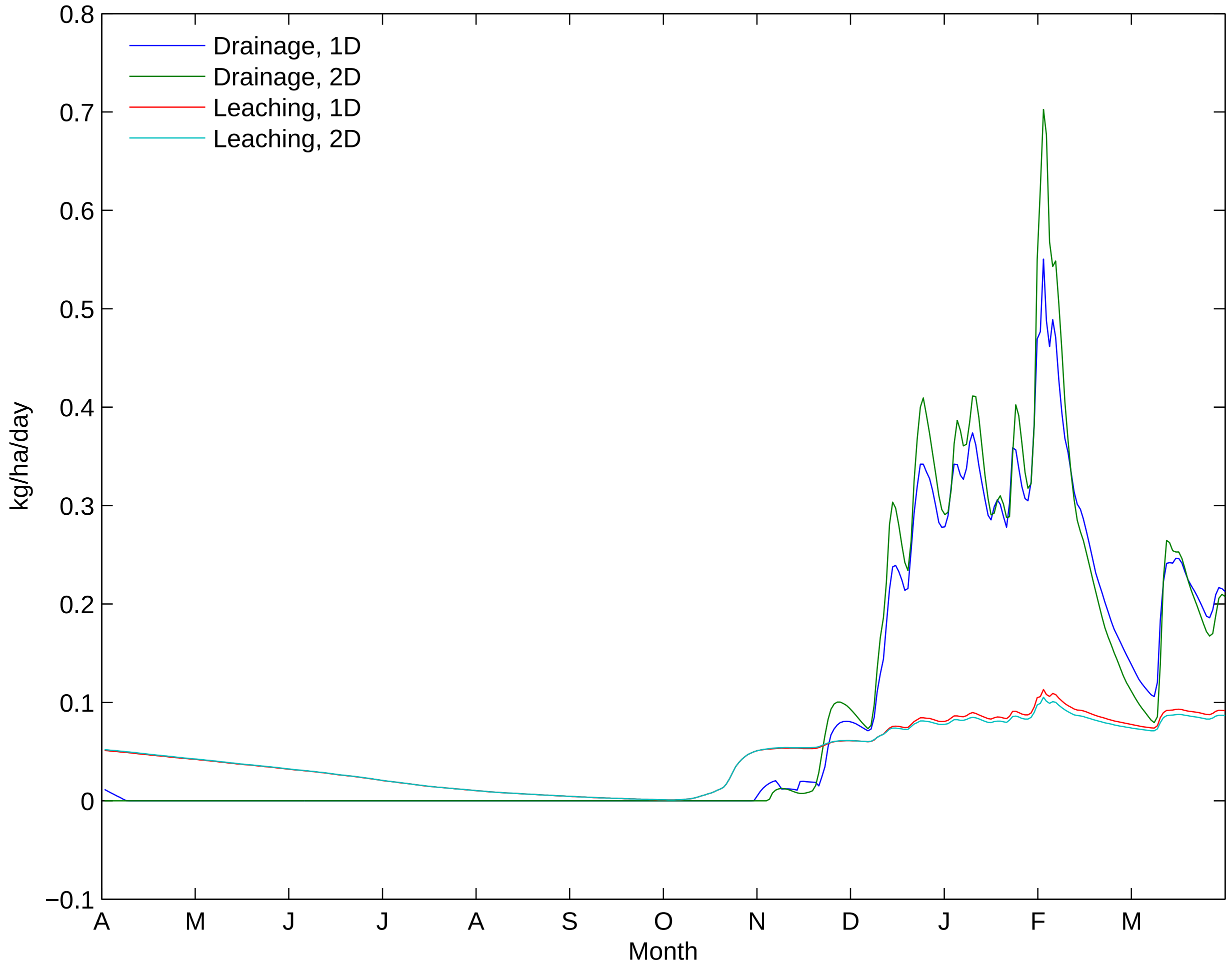

Figure 6: Nitrogen drainage and leaching for the 1D and 2D simulations in the hydrological year 1993.

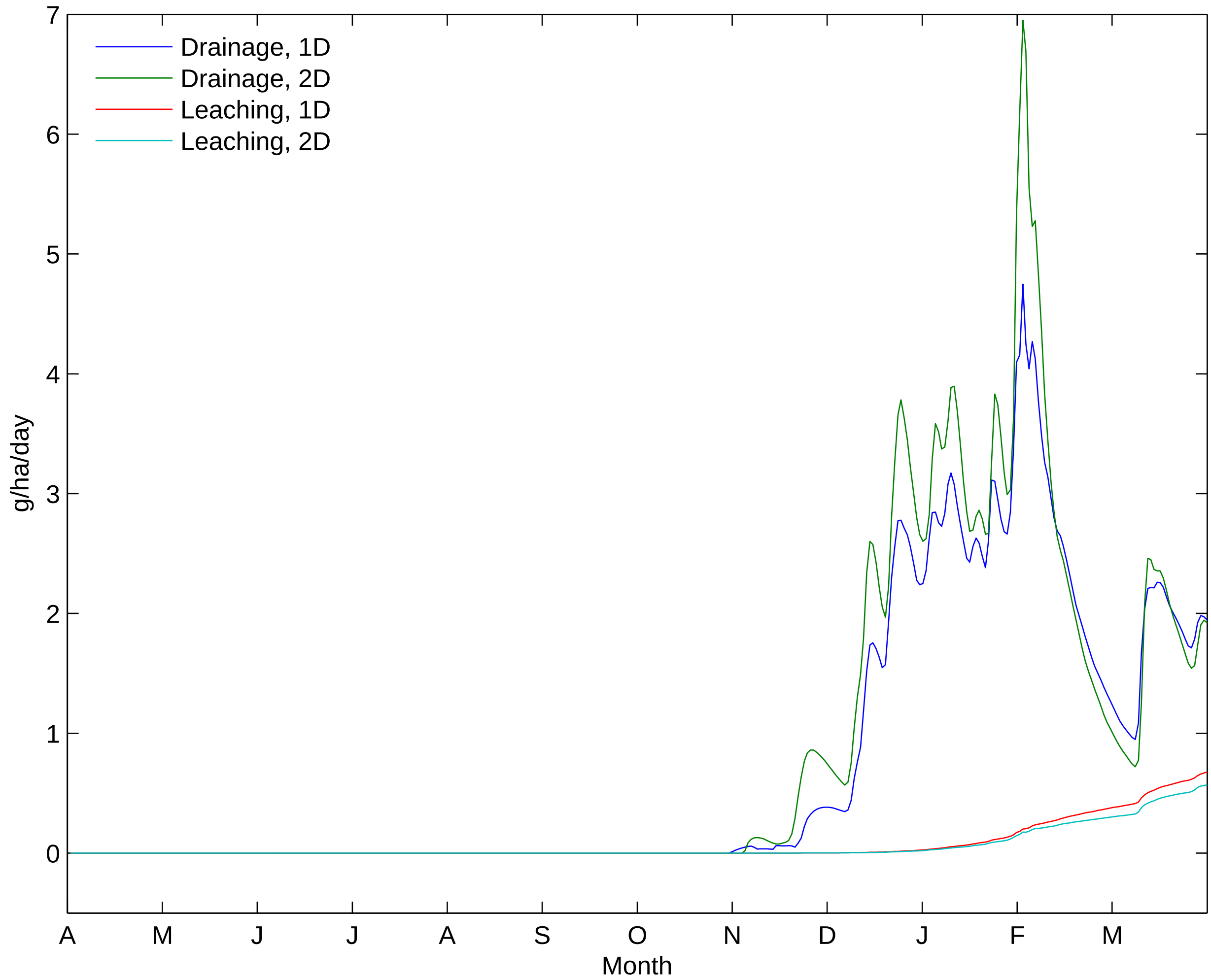

Figure 7: Bromide drainage and leaching for the 1D and 2D simulations in the hydrological year 1993. The Bromide was applied at XXXXXX.

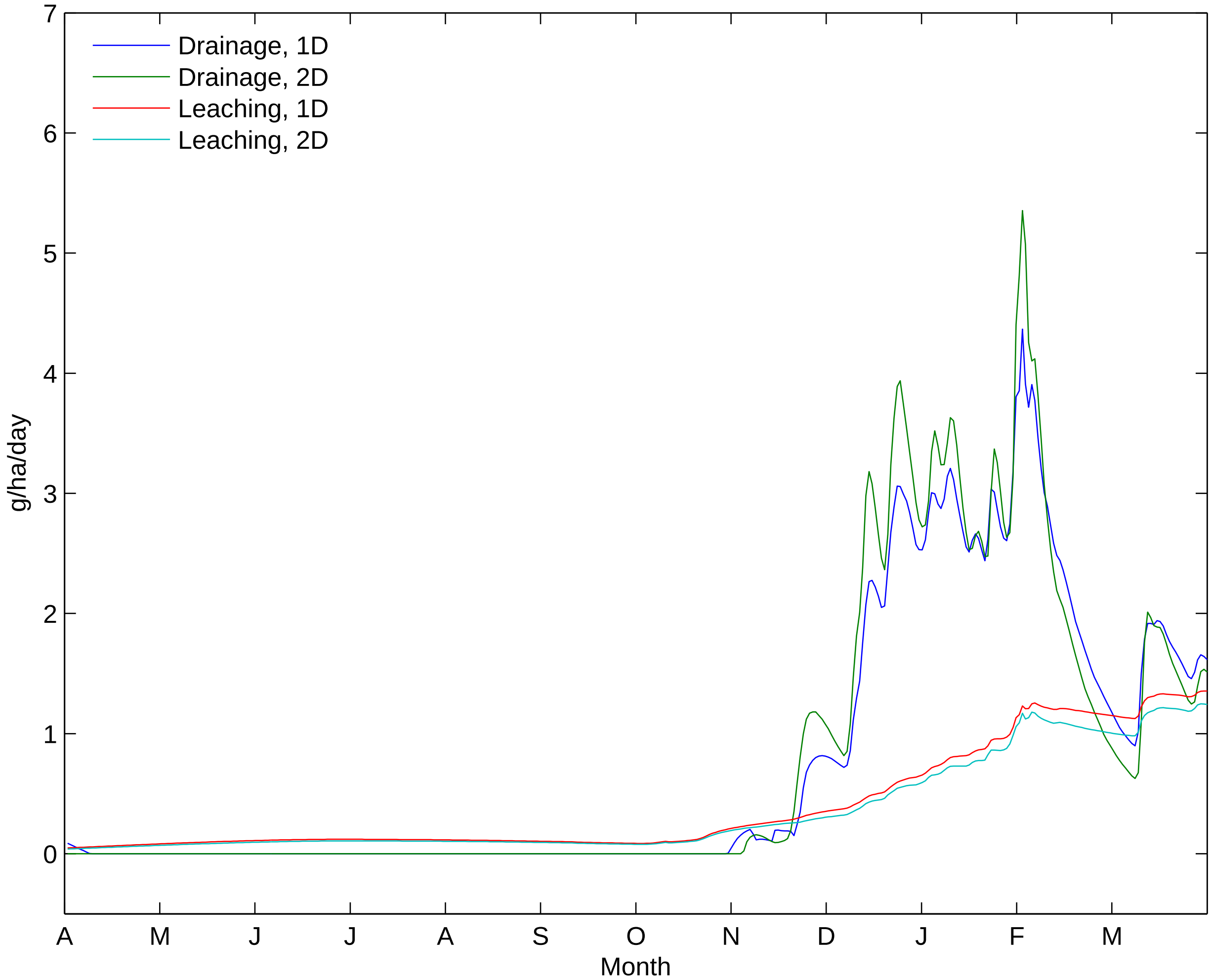

Figure 8: Bromide drainage and leaching for the 1D and 2D simulations in the hydrological year 1993. The Bromide was applied at XXXXXX.

Figure 9: blabre blabre.