Contents

1 Definition

Given data $(X_i, Y_i, W_i)_{i=1}^{i}$, the ISLS estimator of c_J is simply

$$\hat{c}_J = \left(\mathbf{\Psi}_J' \mathbf{P}_K \mathbf{\Psi}_J\right)^{-} \mathbf{\Psi}_J' \mathbf{P}_K \mathbf{Y}$$

where
$\mathbf{\Psi}_J = \left(\psi^J(X_1), \ldots, \psi^J(X_n)\right)'$ $(n \times J)$ matrix
$\mathbf{B}_K = \left(b^K(W_1), \ldots, b^K(W_n)\right)'$ $(n \times K)$ matrix
$\mathbf{P}_K = \mathbf{B}_K \left(\mathbf{B}_K' \mathbf{B}_K\right)^{-} \mathbf{B}_K$ is the projection matrix onto the instrument space
$\mathbf{Y} = (Y_1, \ldots, Y_n)'$ is a $n \times 1$ vector.
Sieve NPIV estimators of h_0 and its derivative $\partial^a h_0$ are given by

$$\hat{h}_J(x) = \left(\psi^J(x)\right)' \hat{c}_J, \quad \partial^a \hat{h}_J(x) = \left(\partial^a \psi^J(x)\right)' \hat{c}_J,$$

where $\partial^a \psi^J(x) = \left(\partial^a \psi_{J1}(x), \ldots, \partial^a \psi_{JJ}(x)\right)'$

2 bsplnbasis_rep

2.1 bsplnbasis_rep

This code returns a b-spline basis Ψ of degree m and r additional interior knots adding on both ends of the interval. X is a set of variables, $X = (X_1, ..., X_n)$

The output are:
XX: $\Psi = (\psi(X_1), ..., \psi(X_n)$
DX: $\partial^a \psi^J(x) = (\partial^a \psi_{J1}(x), \ldots, \partial^a \psi_{JJ}(x))'$

3 Bsplnmat

3.1 bsplinemat

Increase the degree by 2 and return the b-spline basis. (mainly serves to the next function)

3.2 bsplinemat_quantile

It calculates the knot vector **kts** based on the quantiles of the data points in X and the specified degree **m**. Using *bsplinemat* to generate the B-spline basis functions and their derivatives with suitable knot vector.

4 Sieve set

4.1 Tset

Corresponding to $\mathcal{T} = \left\{ \left(2^l + r - 1\right)^d : l \in \mathbb{N}_0 \right\}$ in the paper.
It generates two list: Definition
J a set J of possible values of sieve dimensions of $\mathbf{\Psi}_J$ we search over.
K a set of possible values of of sieve dimensions of $\mathbf{B}_K$ we search over.
K is pinned down by J. $l_w = \lceil l + q \rceil$, where l is the resolution level of J. here we pick q=1(Appendix C in the paper).

5 shat

5.1 shat

$\hat{s}_J$ returns the smallest singular value of $\left(\mathbf{B}'_{K(J)} \mathbf{B}_{K(J)}\right)^{-1/2} \left(\mathbf{B}'_{K(J)} \mathbf{\Psi}_J\right) \left(\mathbf{\Psi}'_J \mathbf{\Psi}_J\right)^{-1/2}$

6 UCB matrix

The main purpose of this file is to estimate UCBs for h_0 and its derivatives based on a deterministic J and "undersmoothing". Let $\hat{\mathbf{u}}_J = (\hat{u}_{1,J}, \ldots, \hat{u}_{n,J})'$ denote the $n \times 1$ vector of residuals whose i th element is $\hat{u}_{i,J} = Y_i - \hat{h}_J(X_i)$. Then $\hat{h}_J(x) - h_0(x)$ and $\partial^a \hat{h}_J(x) - \partial^a h_0(x)$ can be estimated by

$$D_J(x) = \left(\psi^J(x)\right)' \mathbf{M}_J \hat{\mathbf{u}}_J, \quad D_J^a(x) = \left(\partial^a \psi^J(x)\right)' \mathbf{M}_J \hat{\mathbf{u}}_J$$

$\mathbf{P}_K = \mathbf{B}_K \left(\mathbf{B}_K' \mathbf{B}_K\right)^{-} \mathbf{B}_K$
$\mathbf{M}_J = \left(\mathbf{\Psi}_J' \mathbf{P}_{K(J)} \mathbf{\Psi}_J\right)^{-} \mathbf{\Psi}_J' \mathbf{P}_{K(J)}$
$\hat{\mathbf{u}}_J^* = \left(\hat{u}_{1,J}\varpi_1, \ldots, \hat{u}_{n,J}\varpi_n\right)'$ denote a multiplier bootstrap version of $\hat{\mathbf{u}}_J$
$\widehat{\mathbf{U}}_{J,J}$ is a $n \times n$ diagonal matrix whose i th diagonal entry is $\hat{u}_{i,J}\hat{u}_{i,J}$
$D_J(x) = \left(\psi^J(x)\right)' \mathbf{M}_J \hat{\mathbf{u}}_J$
$D_J^*(x) = \left(\psi^J(x)\right)' \mathbf{M}_J \hat{\mathbf{u}}_J^*$

7 J

7.1 J_max

Compute

$$\hat{J}_{\max} = \min\left\{J \in \mathcal{T} : J\sqrt{\log J}\hat{s}_J^{-1} \leq 10\sqrt{n} < J^+\sqrt{\log J^+}\hat{s}_{J^+}^{-1}\right\}$$

7.2 TJ_hat

Compute an index set

$$\hat{\mathcal{J}} = \left\{J \in \mathcal{T} : 0.1\left(\log \hat{J}_{\max}\right)^2 \leq J \leq \hat{J}_{\max}\right\}$$

It returns a list containing two value namely **j_hat** and **k_hat**. **k_hat** is the $\hat{K} \in K$(K indicated in 4.1) corresponding to the $\hat{J} \in \mathcal{T}$

8 ucb_quantile

8.1 sigmahat_sq, sigmatilda, sigmajj2

These three functions respectively compute:

$$\hat{\sigma}_J^2(x) = \left(\psi^J(x)\right)' \mathbf{M}_J \widehat{\mathbf{U}}_{J,J} \mathbf{M}_J' \psi^J(x)$$

$$\tilde{\sigma}_{J,J_2}(x) = \left(\psi^J(x)\right)' \mathbf{M}_J \widehat{\mathbf{U}}_{J,J_2} \mathbf{M}_{J_2}' \psi^{J_2}(x)$$

$$\hat{\sigma}_{J,J_2}^2(x) := \hat{\sigma}_J^2(x) + \hat{\sigma}_{J_2}^2(x) - 2\tilde{\sigma}_{J,J_2}(x),$$

The definition of $\mathbf{M}_J$ and $\widehat{\mathbf{U}}_{J,J}$ is given in UCB Matrix

8.2 sup_abs

This function compute the value of

$$\sup_{\left\{(x,J,J_2)\in\mathcal{X}\times\hat{\mathcal{J}}\times\hat{\mathcal{J}}:J_2>J\right\}} \left|\frac{D_J^*(x) - D_{J_2}^*(x)}{\hat{\sigma}_{J,J_2}(x)}\right|. \tag{1}$$

for each independent draw of $(\varpi_i)_{i=1}^n$

8.3 statquantile

Let $\hat{\alpha} = \min\left\{0.5, \left(\log\left(\hat{J}_{\max}\right)/\hat{J}_{\max}\right)^{1/2}\right\}$. This function returns the (1-$al\hat{p}ha$) quantile of equation 1 across t draws.

9 Sieve Diension

9.1 sup_J

This function returns the $\hat{J}$ satisfying the follwoing equation:

$$\hat{J} = \min\left\{J \in \hat{\mathcal{J}} : \sup_{(x,J_2)\in\mathcal{X}\times\hat{\mathcal{J}}:J_2>J}\left|\frac{D_J(x) - D_{J_2}(x)}{\hat{\sigma}_{J,J_2}(x)}\right| \leq 1.1\theta^*_{1-\hat{\alpha}}\right\}.$$

9.2 Jtilda

Return

$$\tilde{J} = \min\left\{\hat{J}, \hat{J}_n\right\}$$

where $\hat{J}$ is generated by the function above and $\hat{J}_n$ is given by $\hat{J}_n = \max\left\{J \in \hat{\mathcal{J}} : J < \hat{J}_{\max}(7.1)\right\}$