```
> a := evalf(25^(1/8))
```

$$a := 1.495348781 \quad (1)$$

```
> a^8
```

$$24.99999997 \quad (2)$$

```
> fl := x->x·(1 − x)
```

$$fl := x \mapsto x \cdot (1 - x) \quad (3)$$

```
> fl(0.5)
```

$$0.25 \quad (4)$$

```
> fl(1)
```

$$0 \quad (5)$$

```
> fl(0)
```

$$0 \quad (6)$$

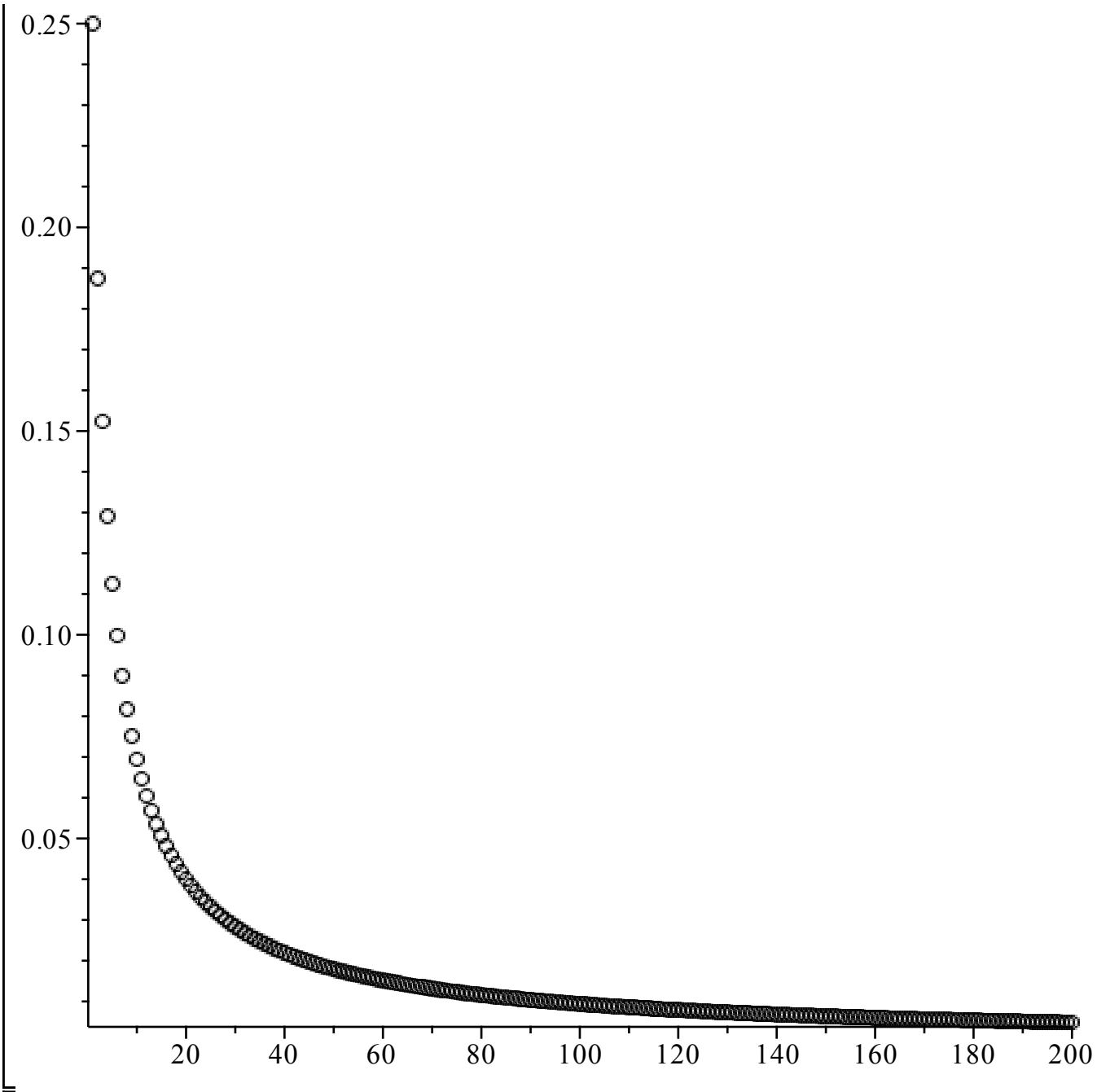
```
> x := 0.5
```

$$x := 0.5 \quad (7)$$

```
> for i from 1 to 200 do x := fl(x) : psi(i) := x; od:
> with(plots)
```

[*animate*, *animate3d*, *animatecurve*, *arrow*, *changecoords*, *complexplot*, *complexplot3d*, *conformal*, *conformal3d*, *contourplot*, *contourplot3d*, *coordplot*, *coordplot3d*, *densityplot*, *display*, *dualaxisplot*, *fieldplot*, *fieldplot3d*, *gradplot*, *gradplot3d*, *implicitplot*, *implicitplot3d*, *inequal*, *interactive*, *interactiveparams*, *intersectplot*, *listcontplot*, *listcontplot3d*, *listdensityplot*, *listplot*, *listplot3d*, *loglogplot*, *logplot*, *matrixplot*, *multiple*, *odeplot*, *pareto*, *plotcompare*, *pointplot*, *pointplot3d*, *polarplot*, *polygonplot*, *polygonplot3d*, *polyhedra_supported*, *polyhedraplot*, *rootlocus*, *semilogplot*, *setcolors*, *setoptions*, *setoptions3d*, *shadebetween*, *spacecurve*, *sparsematrixplot*, *surfdata*, *textplot*, *textplot3d*, *tubeplot*] **(8)**

```
> points := [[n, psi(n)]$n = 1..200] : pointplot(points, symbol = circle)
```



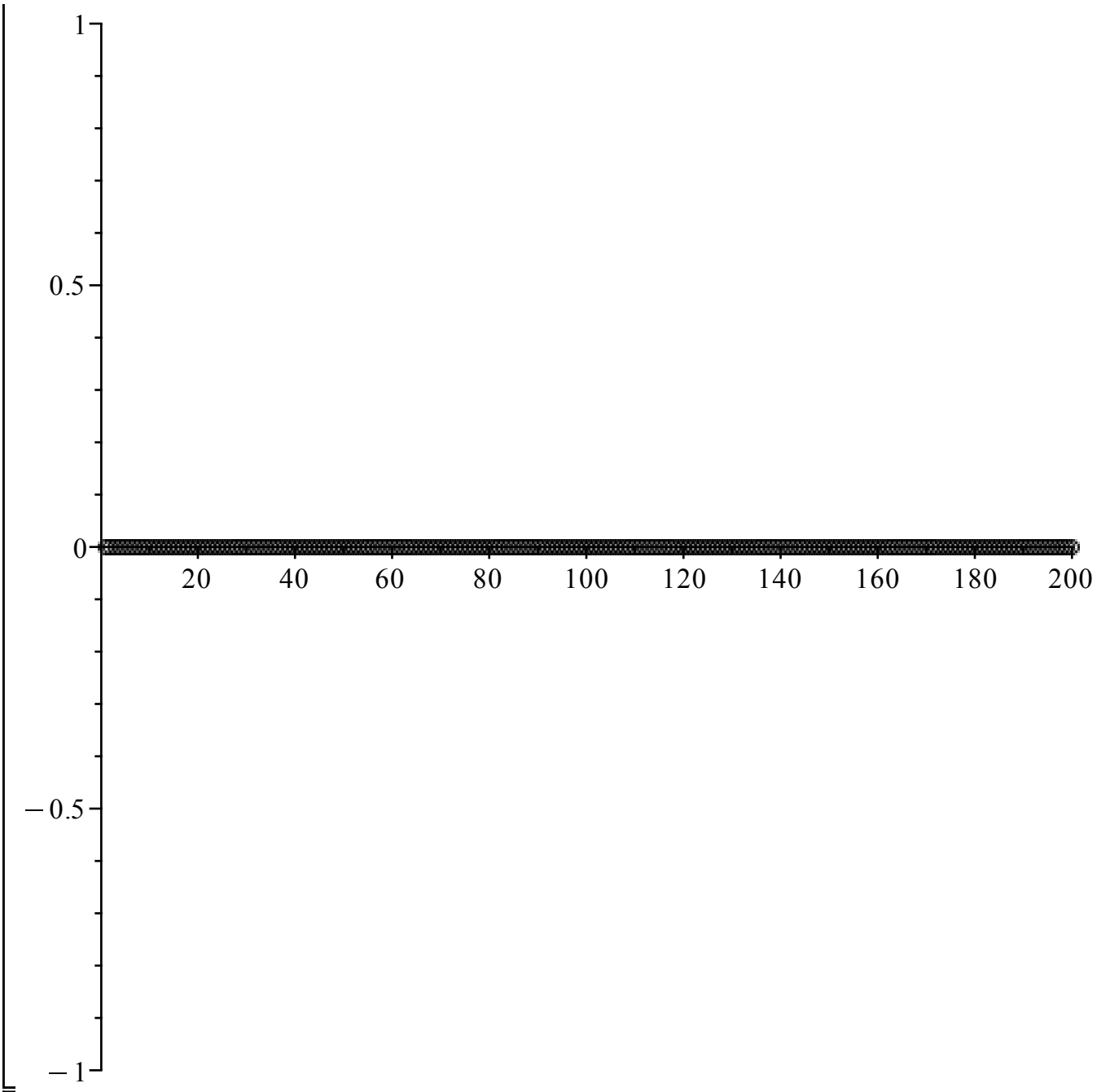

> **The function converges to 0**

Error, missing operator or `;`

> $x := 0$

$$x := 0 \quad \textbf{(9)}$$

```
> for i from 1 to 200 do x := fl(x) : psi(i) := x; od:
> points := [[n, psi(n)]$n = 1..200] : pointplot(points, symbol = circle)
```

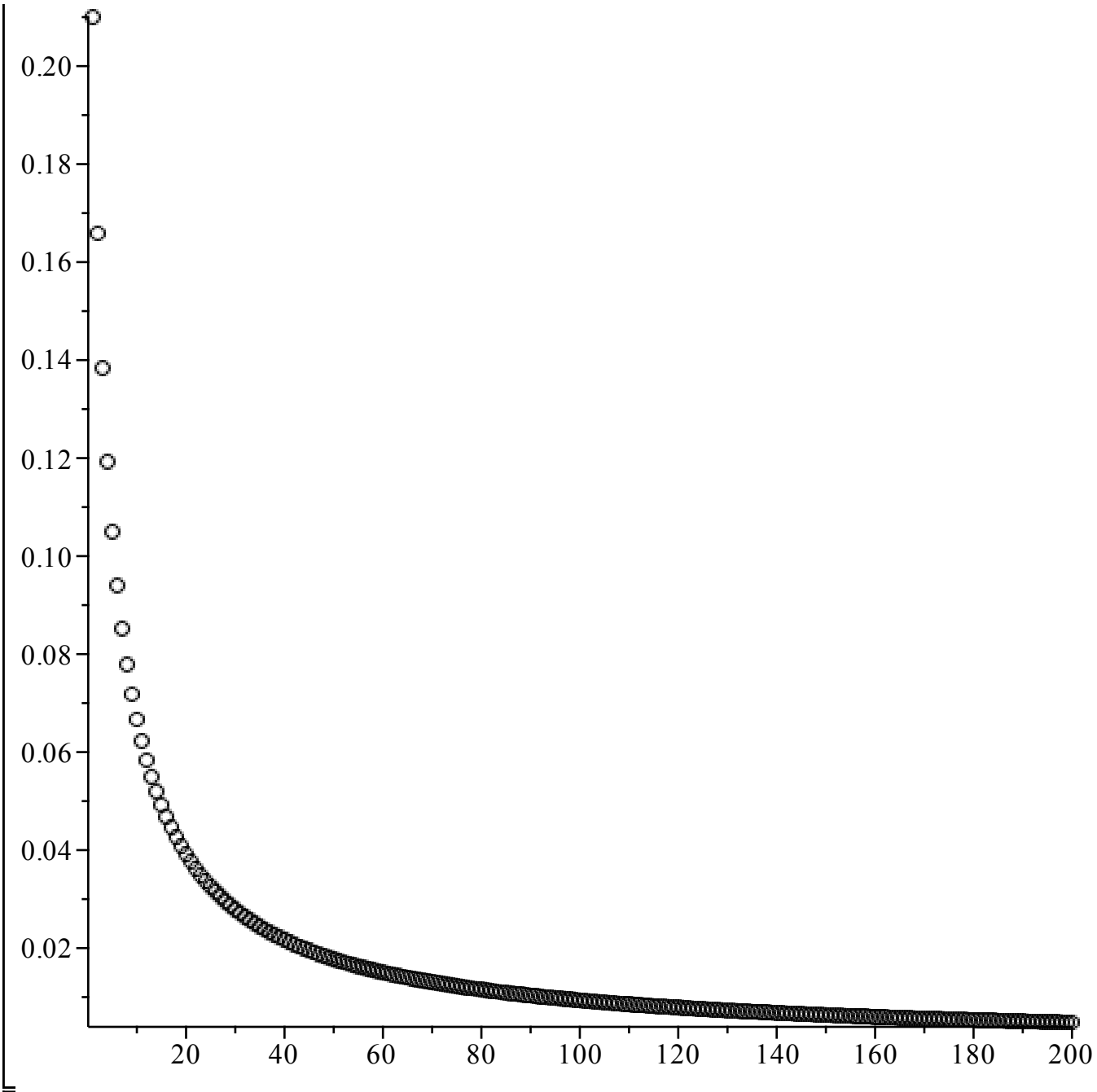



```
> The function is constant, since 0 is a fixed point
```

$x := 0.7$

$$x := 0.7 \qquad \textbf{(10)}$$

```
> for i from 1 to 200 do x := fl(x) : psi(i) := x; od:
> points := [[n, psi(n)]$n = 1..200] : pointplot(points, symbol = circle)
```




```
> lambda := 2
```

$$\lambda := 2 \tag{11}$$

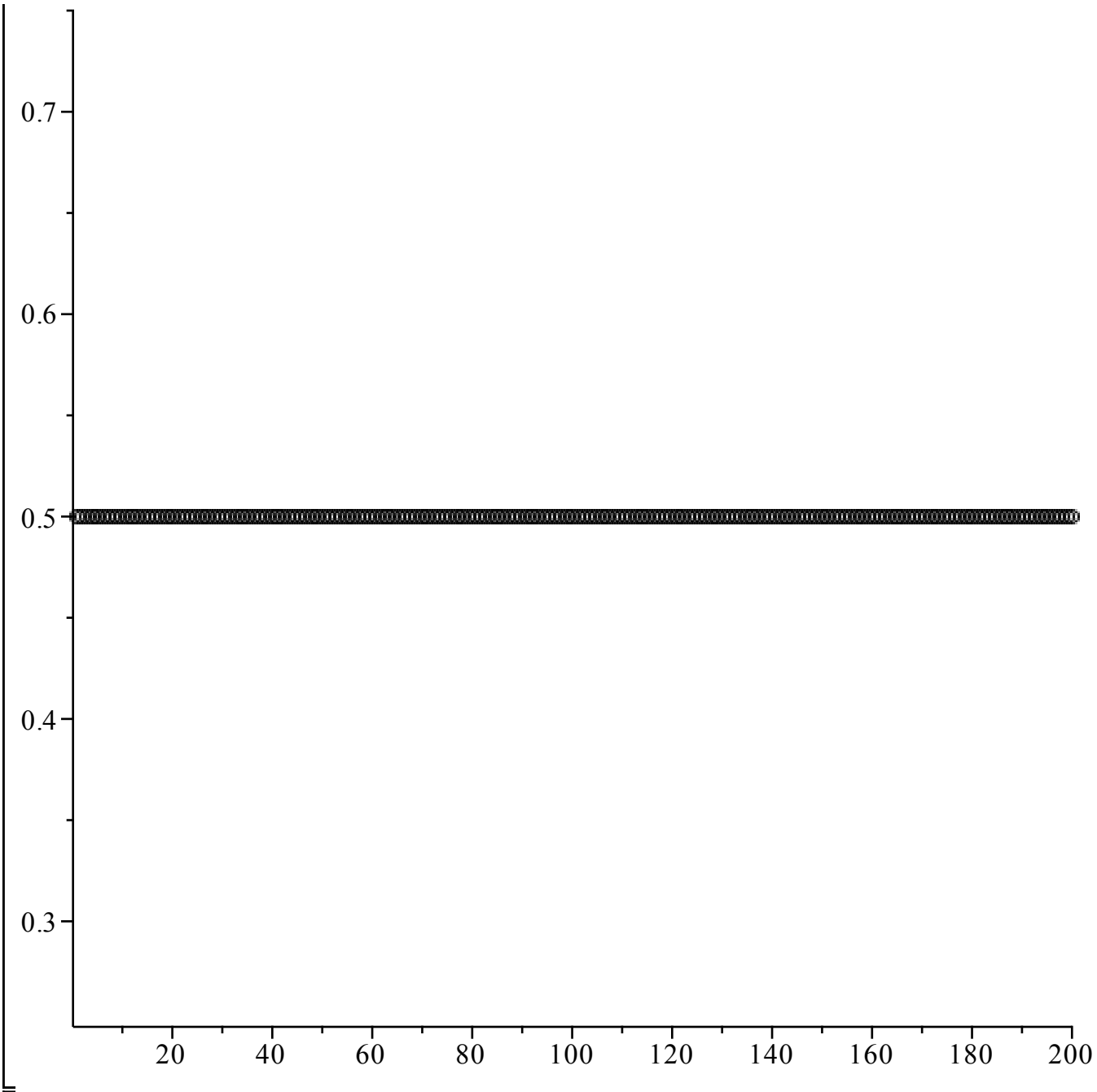
```
> fl := x→lambda · x· (1 − x)
```

$$fl := x \mapsto \lambda \cdot x \cdot (1 - x) \tag{12}$$

```
> x := 0.5
```

$$x := 0.5 \tag{13}$$

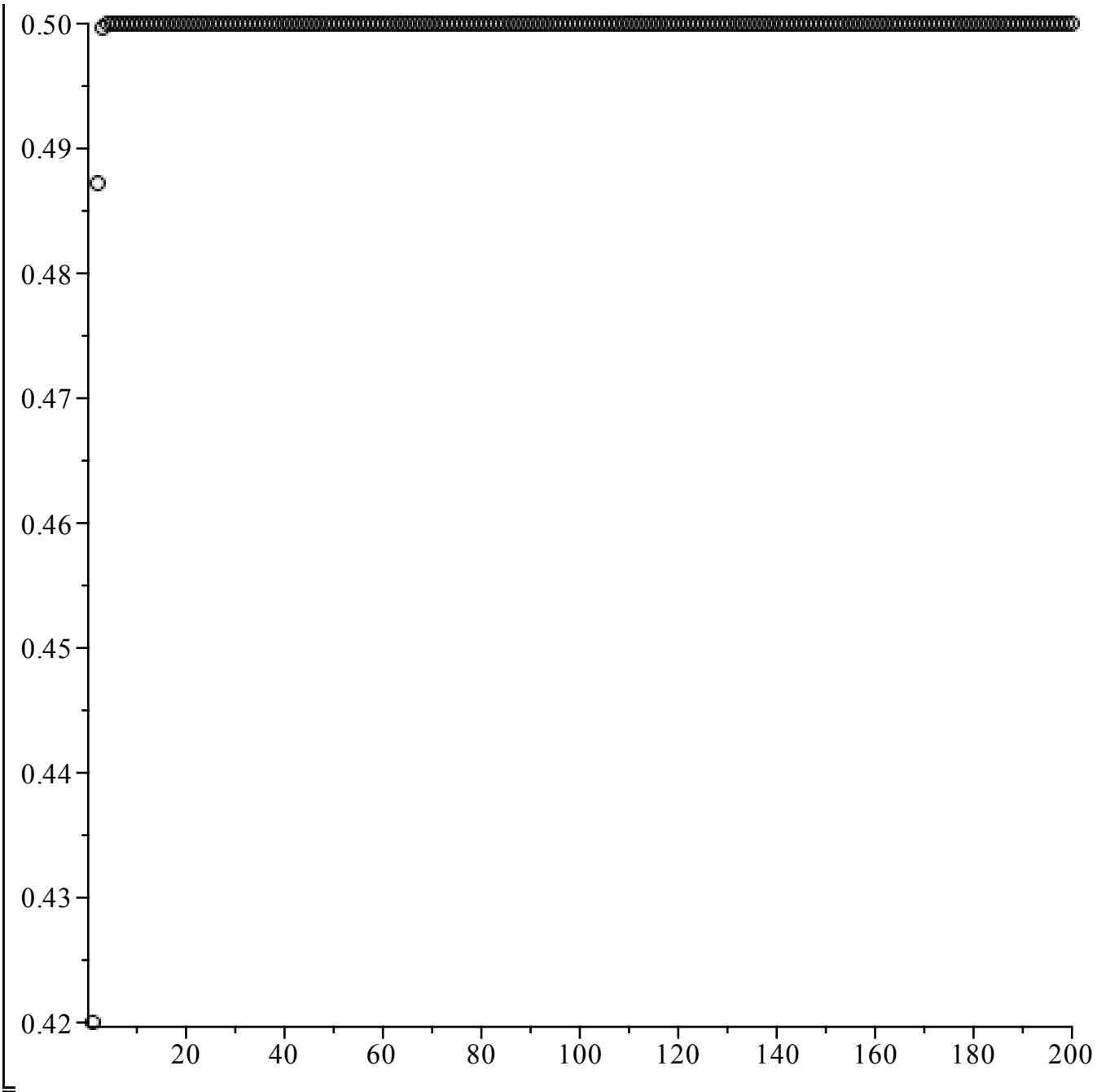
```
> for i from 1 to 200 do x := fl(x) : psi(i) := x; od:
> points := [[n, psi(n)]$n = 1..200] : pointplot(points, symbol = circle)
```




```
> x := 0.7
```

$$x := 0.7 \qquad \textbf{(14)}$$

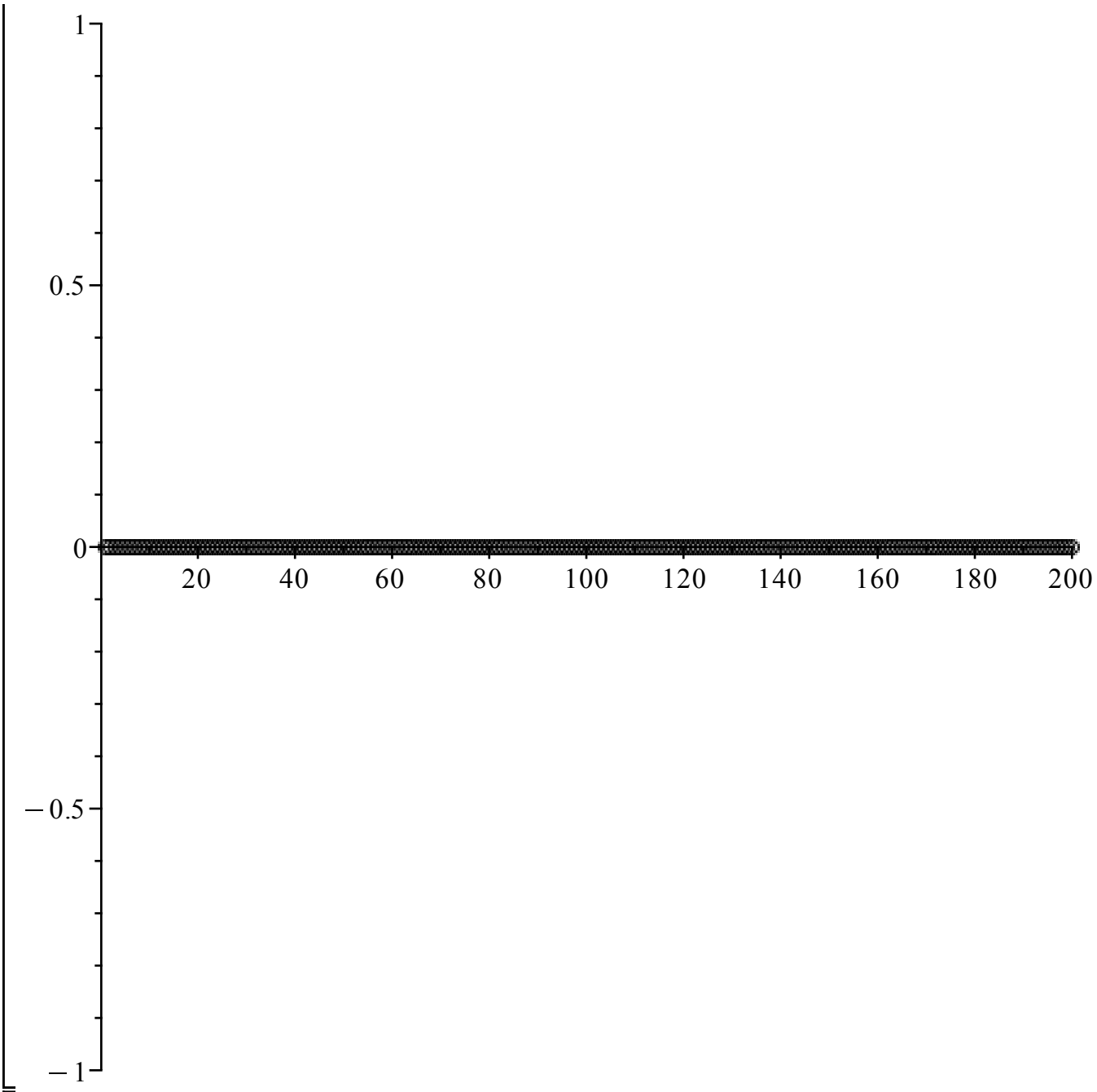
```
> for i from 1 to 200 do x := fl(x) : psi(i) := x;od:
> points := [[n, psi(n)]$n = 1..200] : pointplot(points, symbol = circle)
```




```
> x := 0
```

$$x := 0 \qquad \textbf{(15)}$$

```
> for i from 1 to 200 do x := fl(x) : psi(i) := x;od:
> points := [[n, psi(n)]$n = 1..200] : pointplot(points, symbol = circle)
```



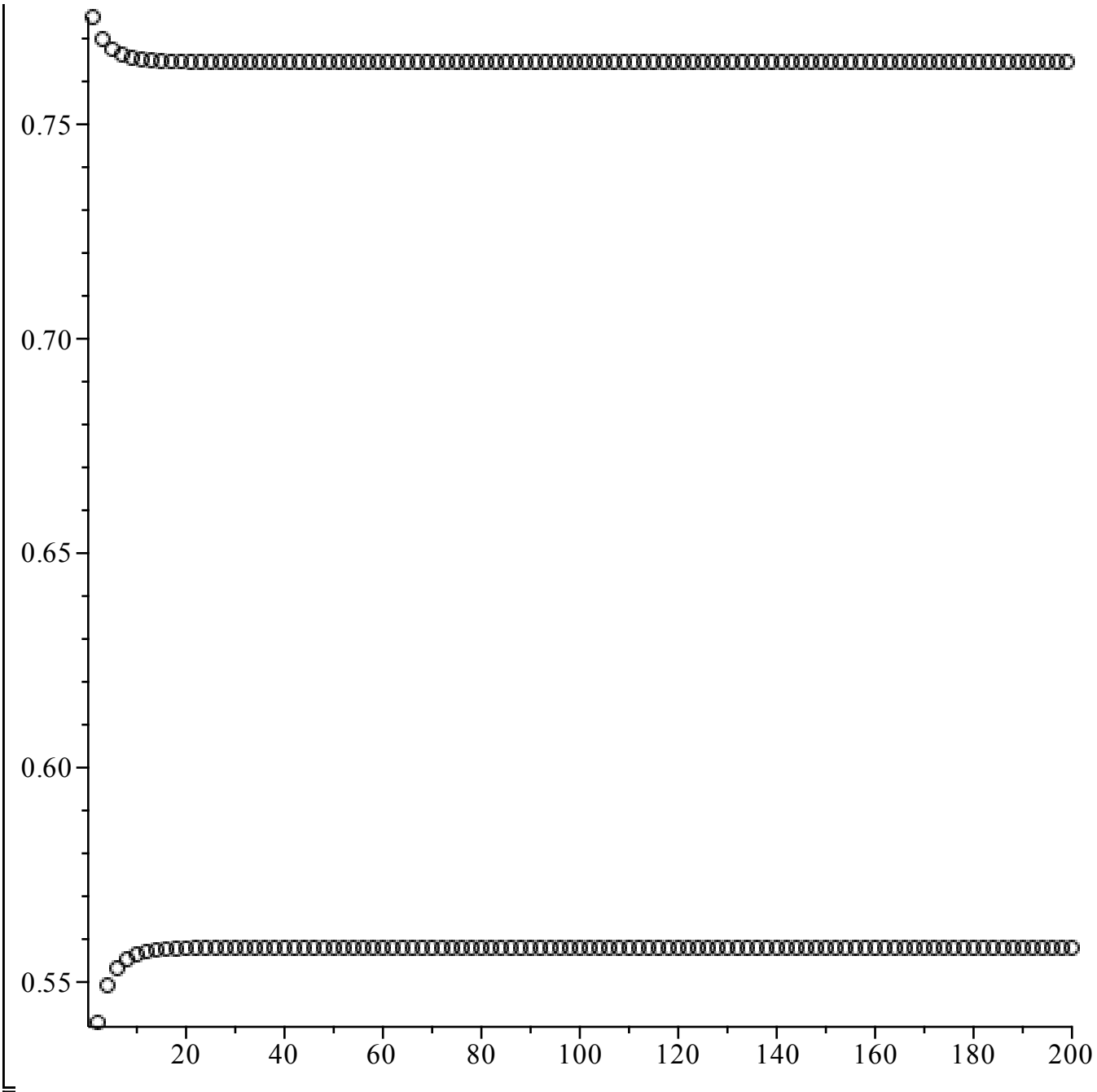

```
> lambda := 3.1
```

$$\lambda := 3.1 \quad \textbf{(16)}$$

```
> x := 0.5
```

$$x := 0.5 \quad \textbf{(17)}$$

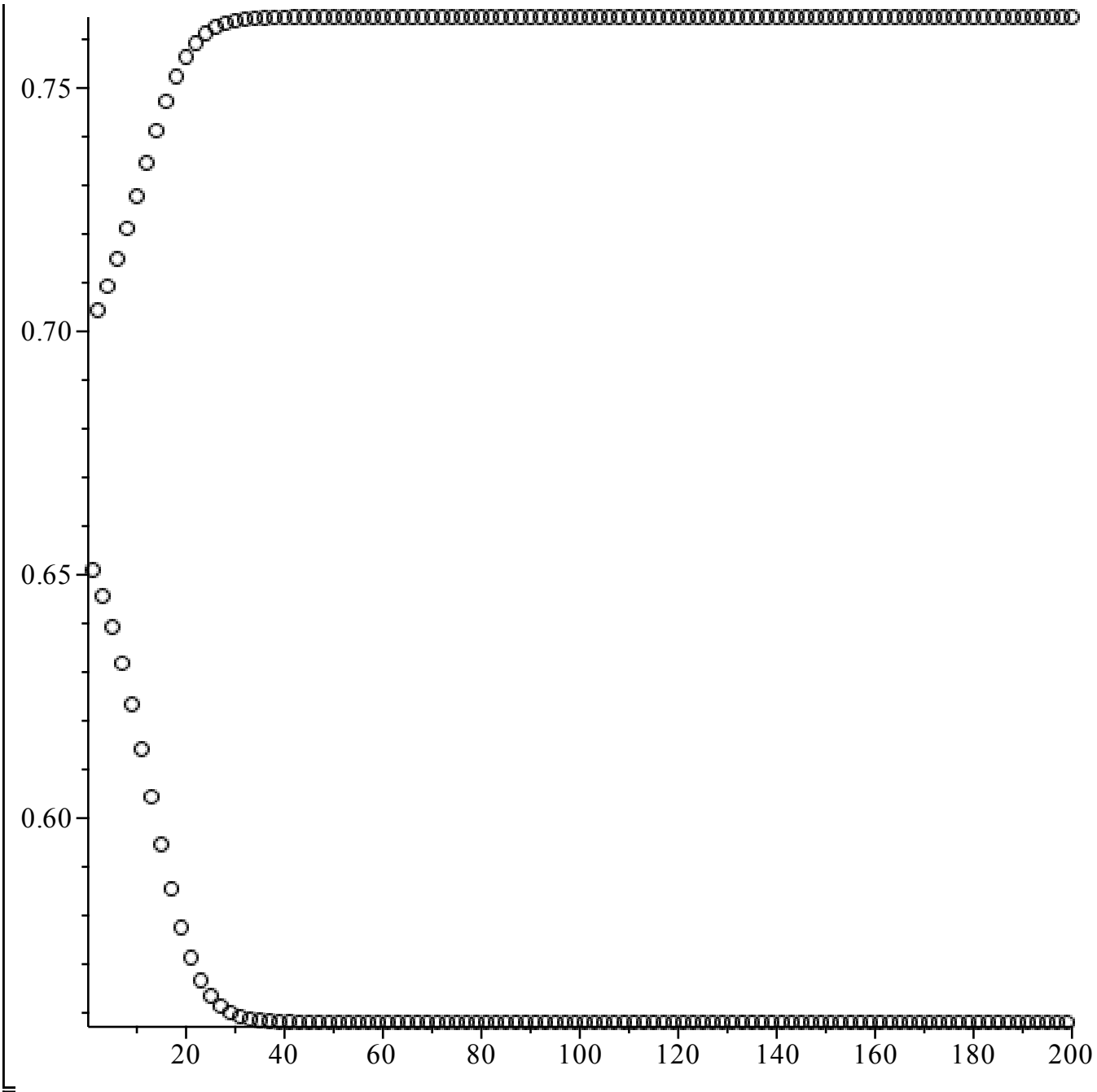
```
>   for i from 1 to 200 do x := fl(x) : psi(i) := x;od:
>     points := [[n, psi(n)]$n = 1..200] : pointplot(points, symbol = circle)
```




```
> x := 0.7
```

$$x := 0.7 \qquad \textbf{(18)}$$

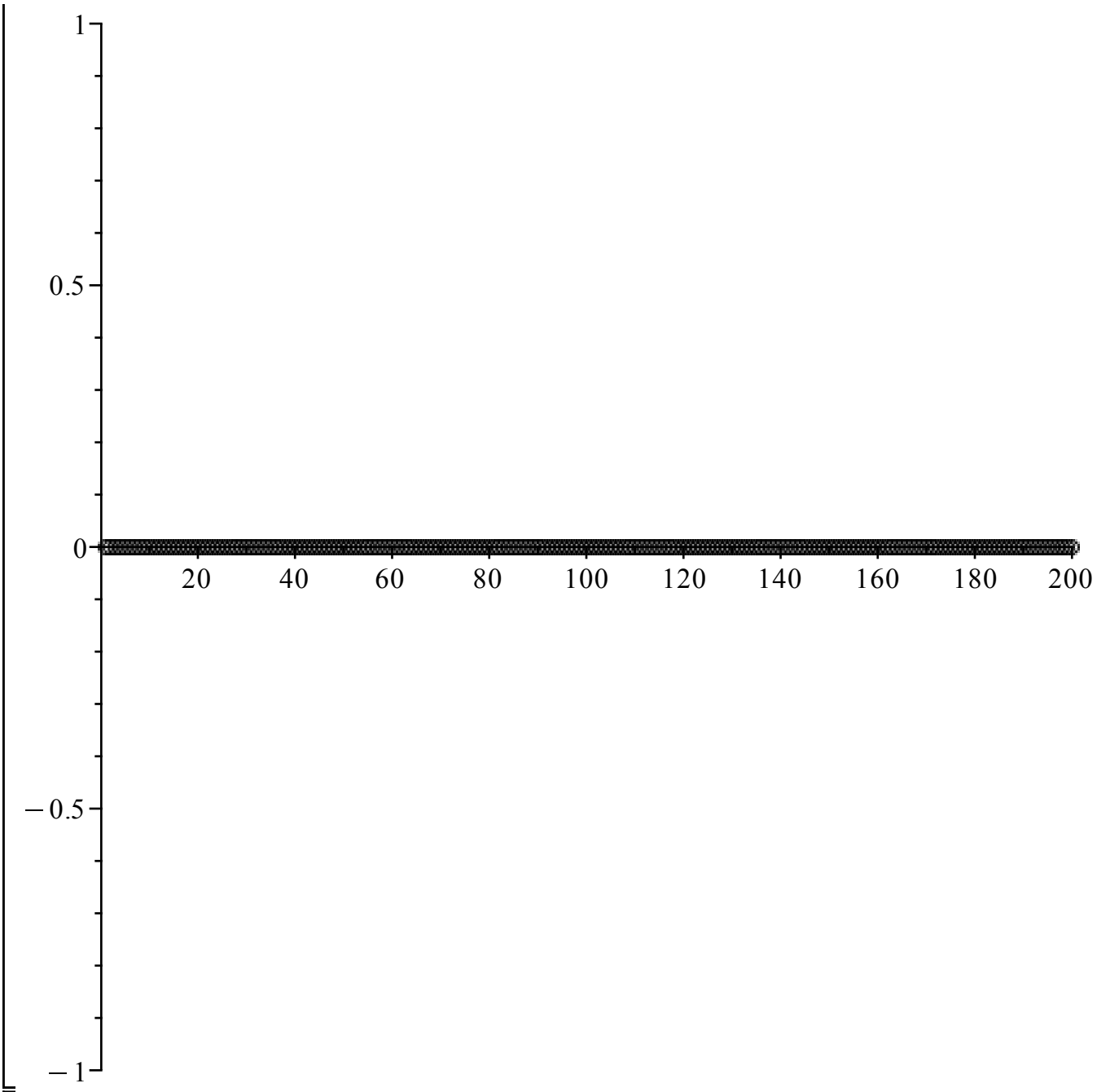
```
> for i from 1 to 200 do x := fl(x) : psi(i) := x;od:
> points := [[n, psi(n)]$n = 1..200] : pointplot(points, symbol = circle)
```




```
> x := 0
```

$$x := 0 \qquad \textbf{(19)}$$

```
> for i from 1 to 200 do x := fl(x) : psi(i) := x;od:
> points := [[n, psi(n)]$n = 1..200] : pointplot(points, symbol = circle)
```



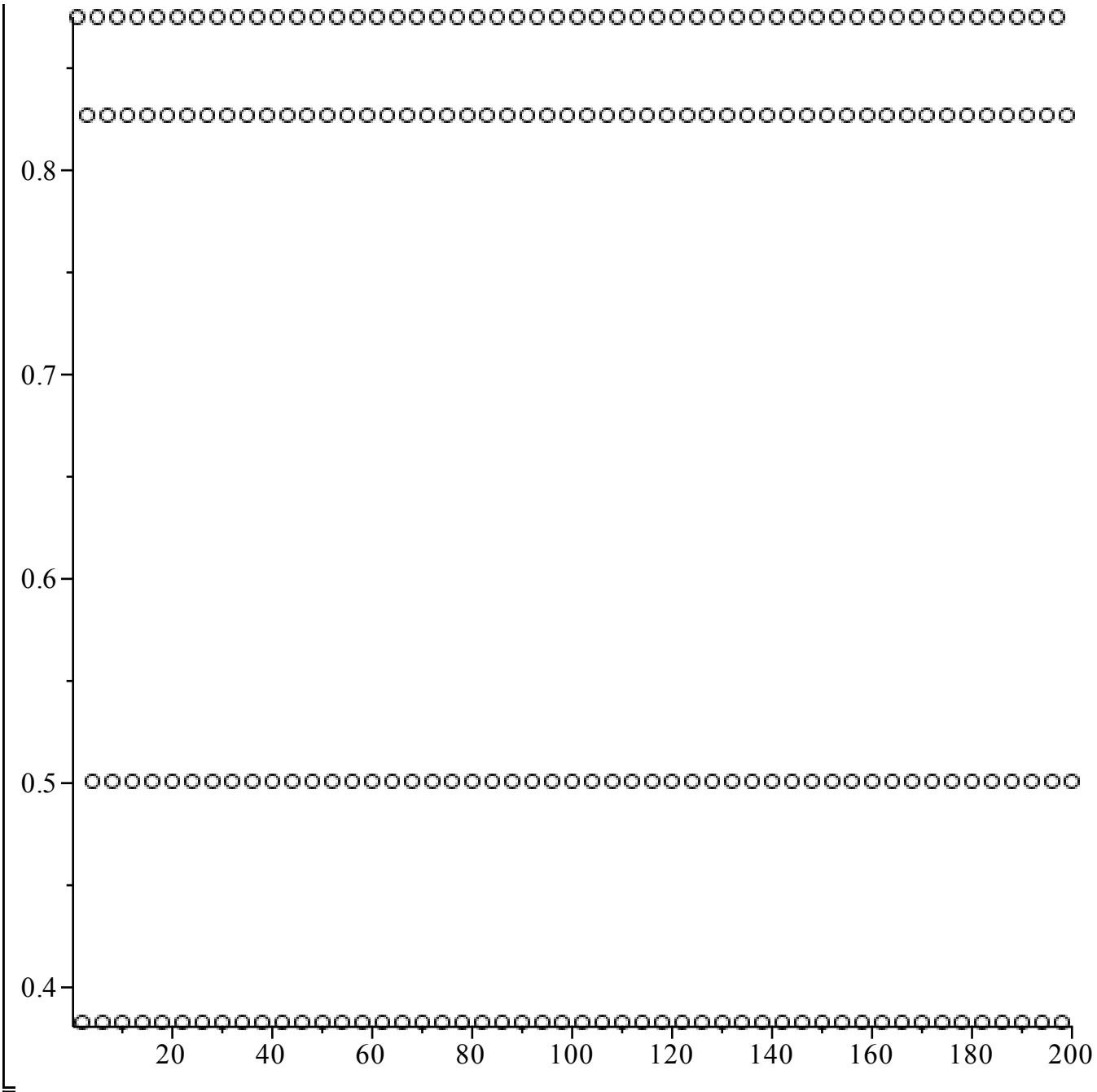

```
> lambda := 3.5
```

$$\lambda := 3.5 \quad \textbf{(20)}$$

```
> x := 0.5
```

$$x := 0.5 \quad \textbf{(21)}$$

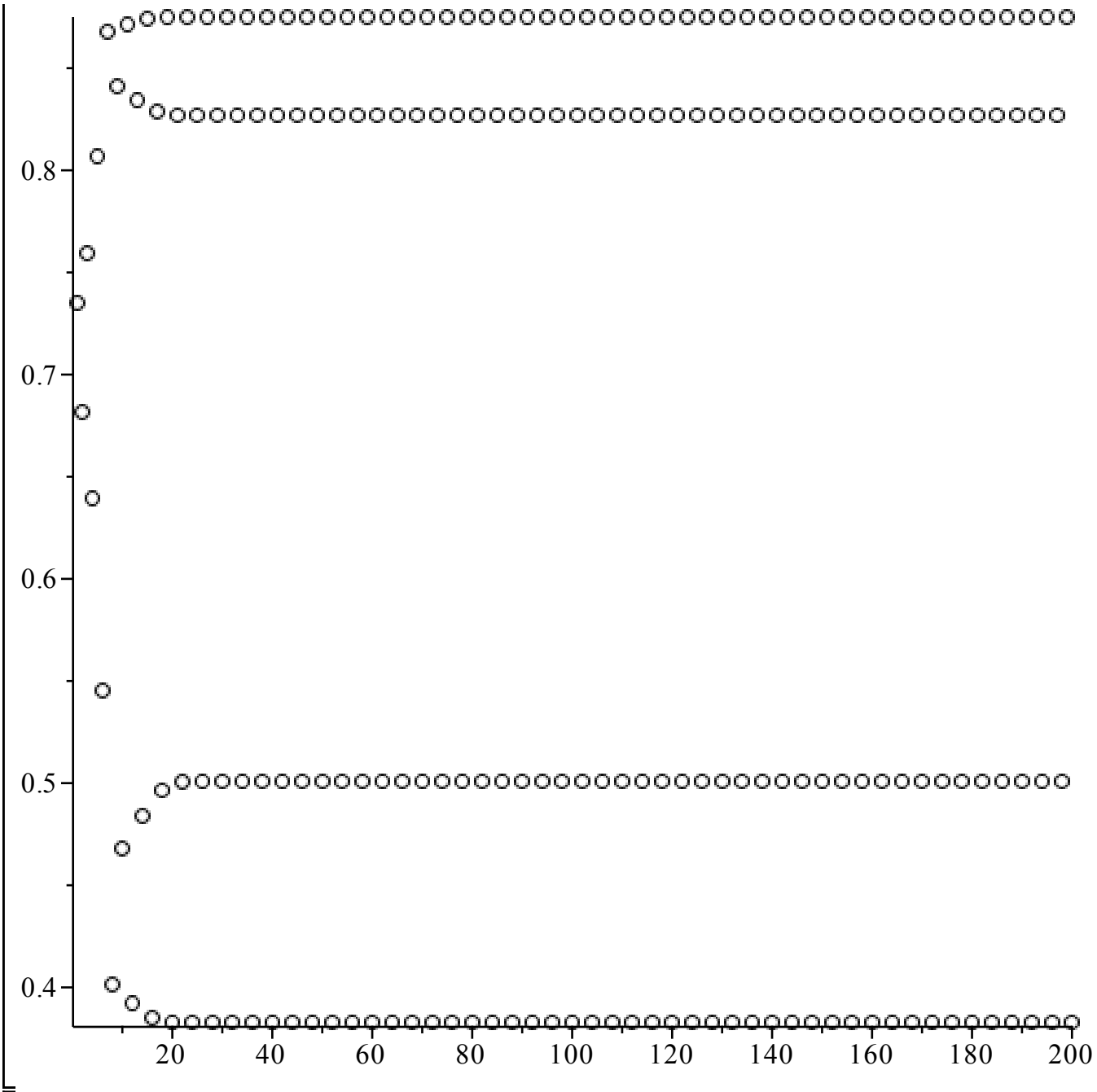
```
>     for i from 1 to 200 do x := fl(x) : psi(i) := x;od:
>       points := [[n, psi(n)]$n = 1..200] : pointplot(points, symbol = circle)
```




```
> x := 0.7
```

$$x := 0.7 \qquad \textbf{(22)}$$

```
> for i from 1 to 200 do x := fl(x) : psi(i) := x;od:
> points := [[n, psi(n)]$n = 1..200] : pointplot(points, symbol = circle)
```



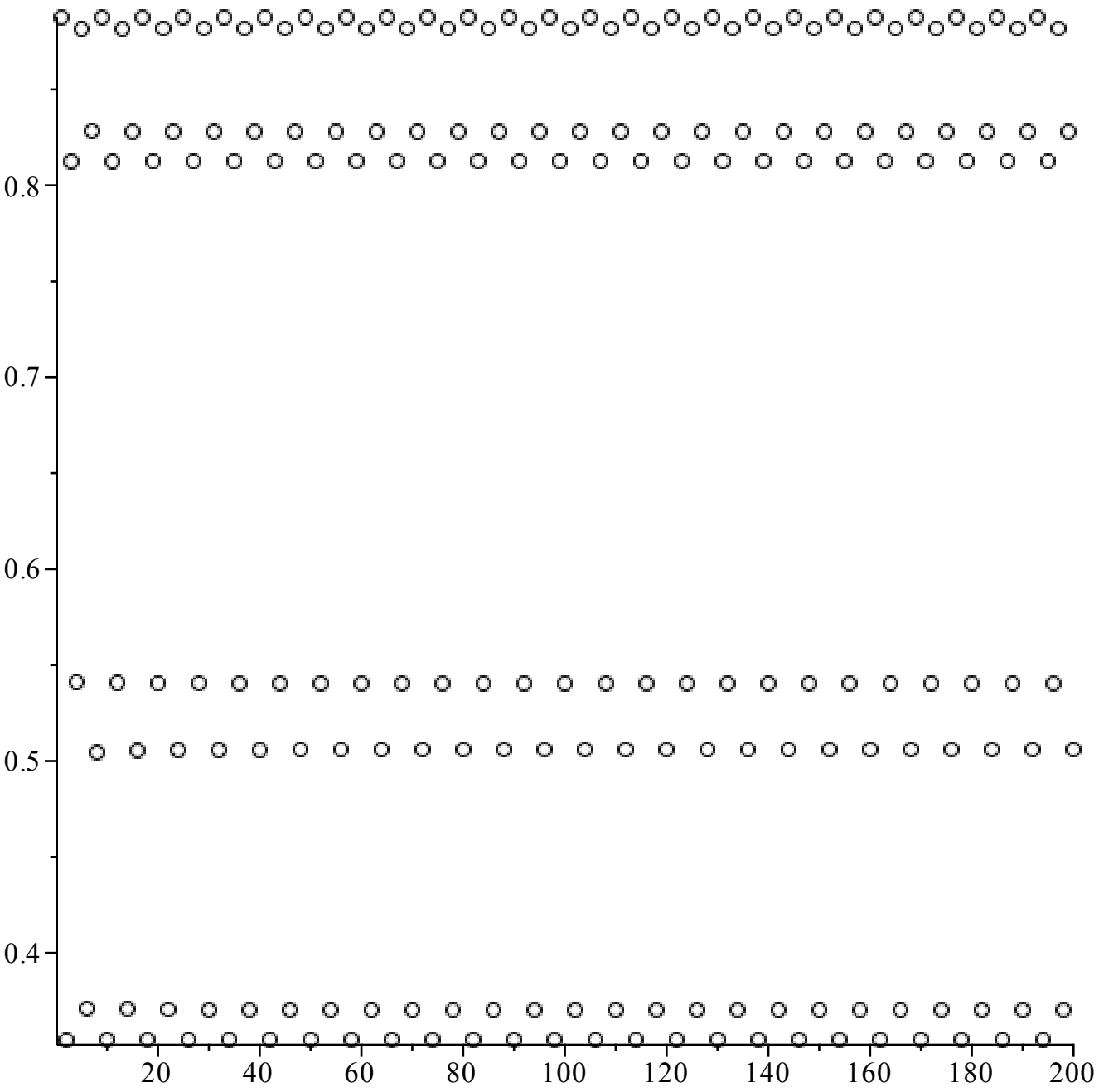

```
> lambda := 3.55
```

$$\lambda := 3.55 \quad \textbf{(23)}$$

```
> x := 0.5
```

$$x := 0.5 \quad \textbf{(24)}$$

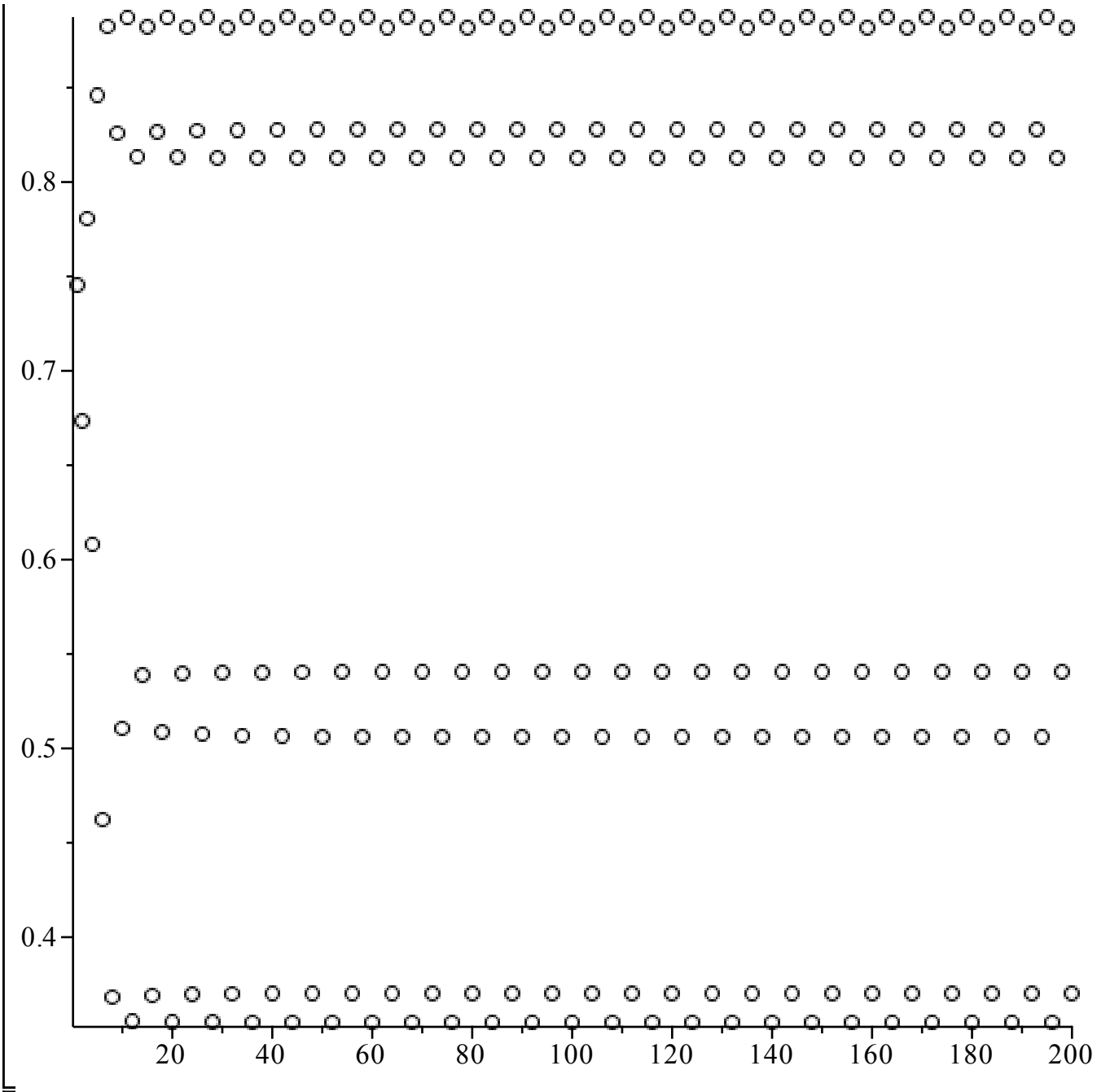
```
>     for i from 1 to 200 do x := fl(x) : psi(i) := x;od:
>       points := [[n, psi(n)]$n = 1..200] : pointplot(points, symbol = circle)
```




```
> x := 0.7
```

$$x := 0.7 \qquad \textbf{(25)}$$

```
>     for i from 1 to 200 do x := fl(x) : psi(i) := x;od:
> points := [[n, psi(n)]$n = 1..200] : pointplot(points, symbol = circle)
```



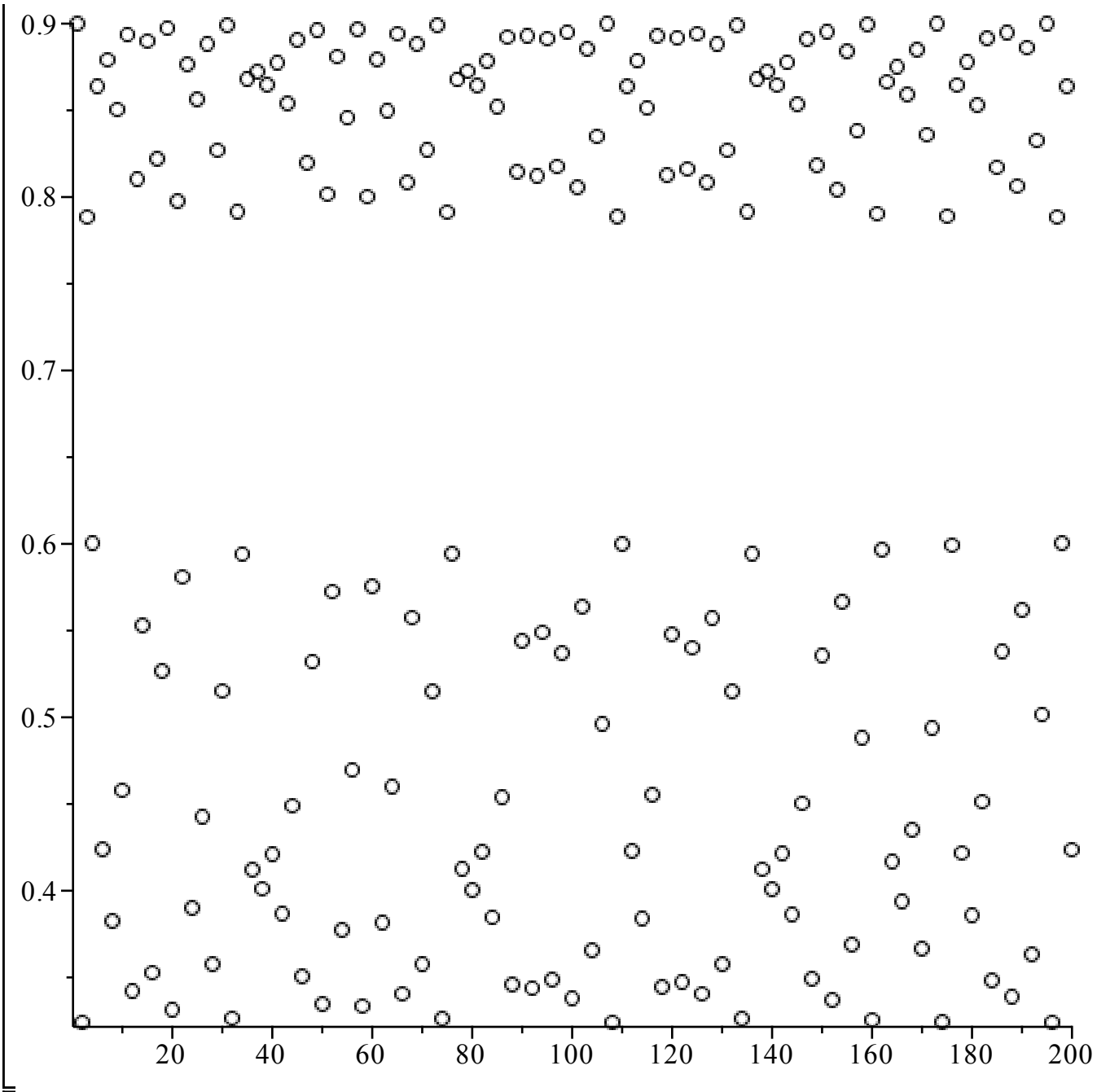

```
> lambda := 3.6
```

$$\lambda := 3.6 \quad \textbf{(26)}$$

```
> x := 0.5
```

$$x := 0.5 \quad \textbf{(27)}$$

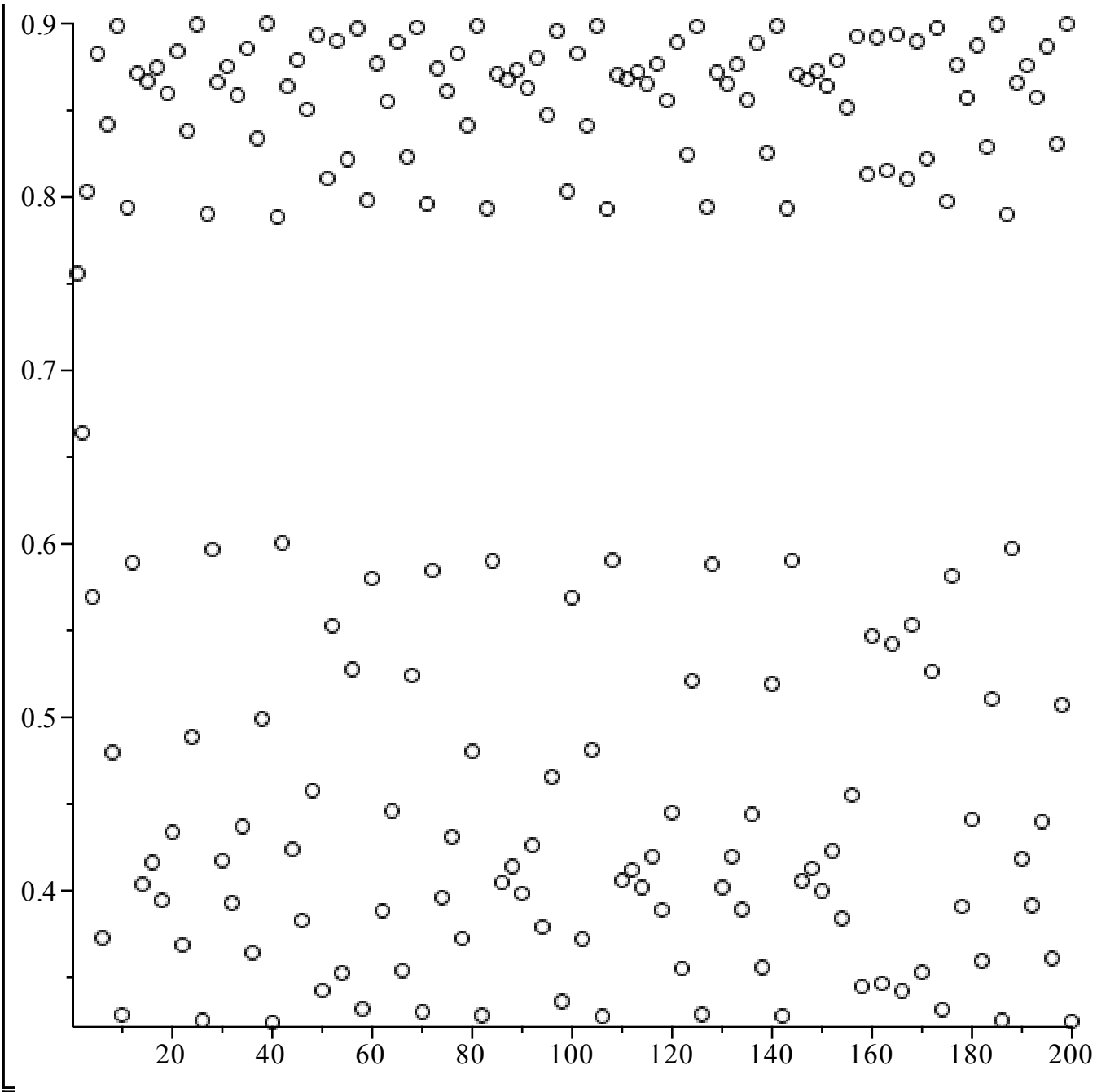
```
>     for i from 1 to 200 do x := fl(x) : psi(i) := x;od:
> points := [[n, psi(n)]$n = 1..200] : pointplot(points, symbol = circle)
```




```
> x := 0.7
```

$$x := 0.7 \qquad \textbf{(28)}$$

```
>     for i from 1 to 200 do x := fl(x) : psi(i) := x;od:
> points := [[n, psi(n)]$n = 1..200] : pointplot(points, symbol = circle)
```




> lambda := *3.8*

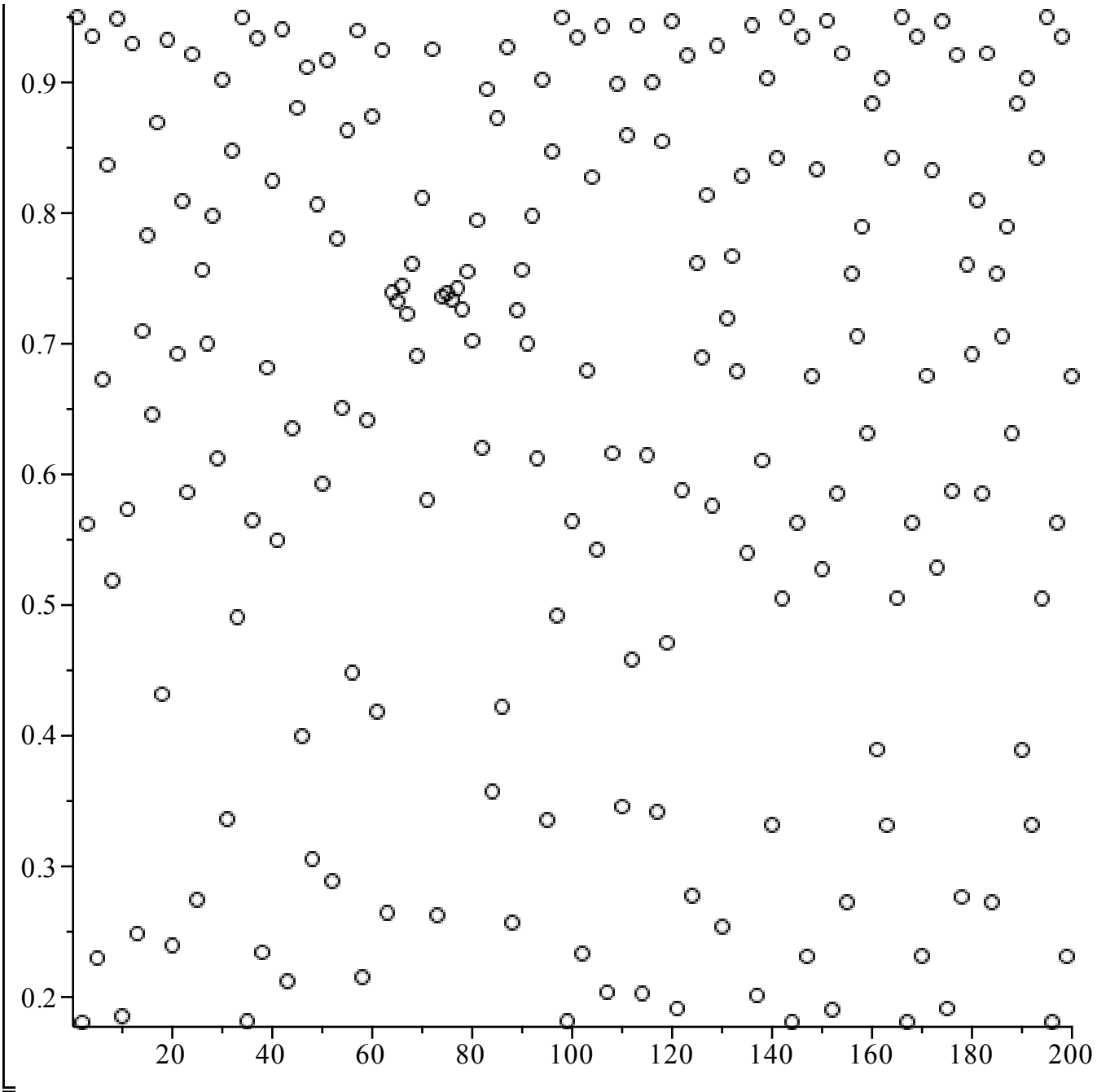
$$\lambda := 3.8 \qquad \textbf{(29)}$$

> $x := 0.5$

$$x := 0.5 \qquad \textbf{(30)}$$

> **for** *i* **from** 1 **to** 200 **do** $x := fl(x)$ : psi(*i*) := *x*;**od**:

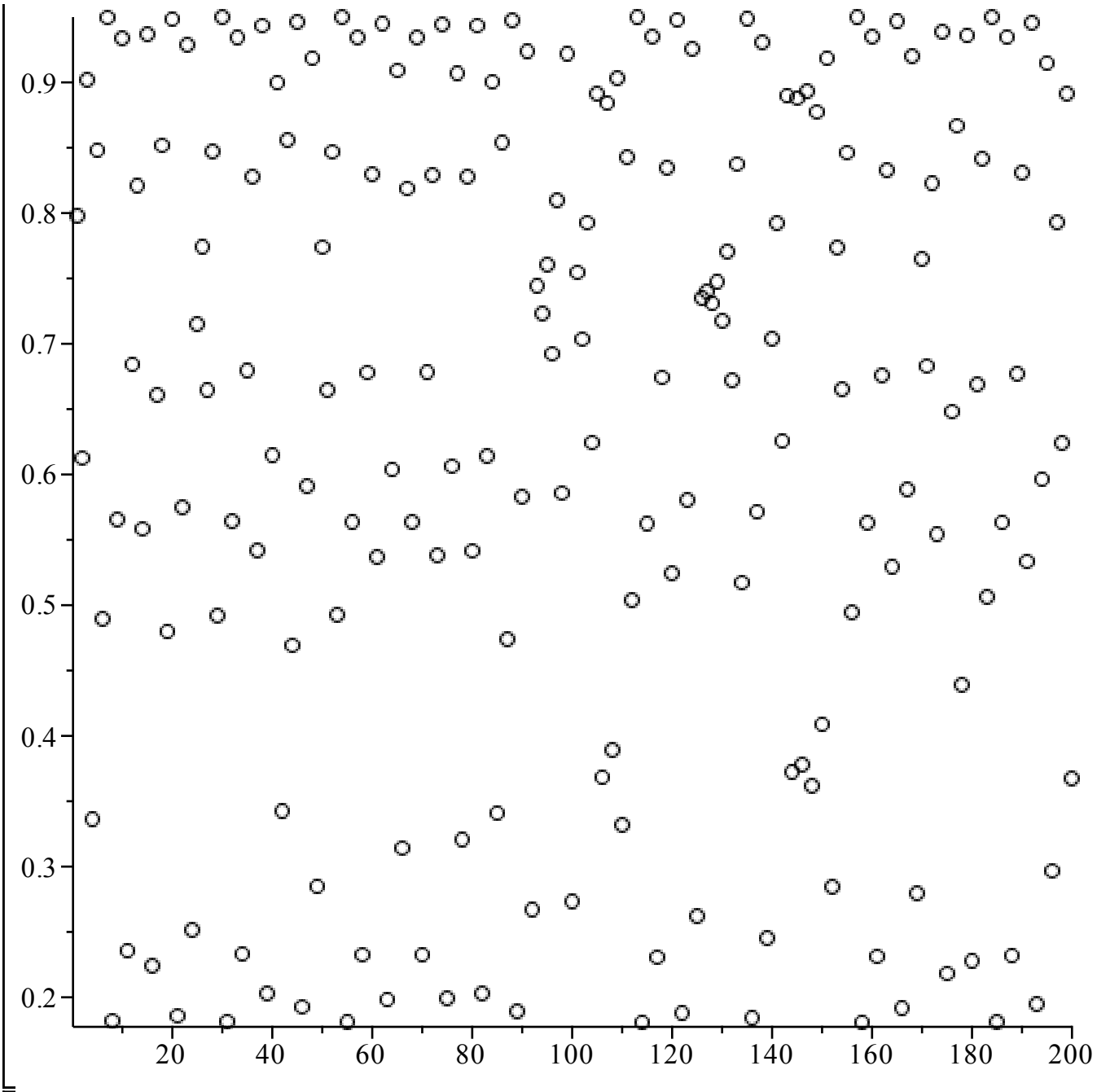
> *points* := [[*n*, psi(*n*)]$*n* = 1..200] : *pointplot*(*points*, *symbol* = *circle*)




```
> x := 0.7
```

$$x := 0.7 \qquad (31)$$

```
> for i from 1 to 200 do x := fl(x) : psi(i) := x;od:
> points := [[n, psi(n)]$n = 1..200] : pointplot(points, symbol = circle)
```




```
> f4 := x→4·x·(1 − x)
```

$$f4 := x \mapsto 4 \cdot x \cdot (1 - x) \quad \textbf{(32)}$$

```
> x := 0.67
```

$$x := 0.67 \quad \textbf{(33)}$$

```
> for i from 1 to 40 do x := f4(x) : psi(i) := x; print(x);od:
```

0.8844

0.40894656

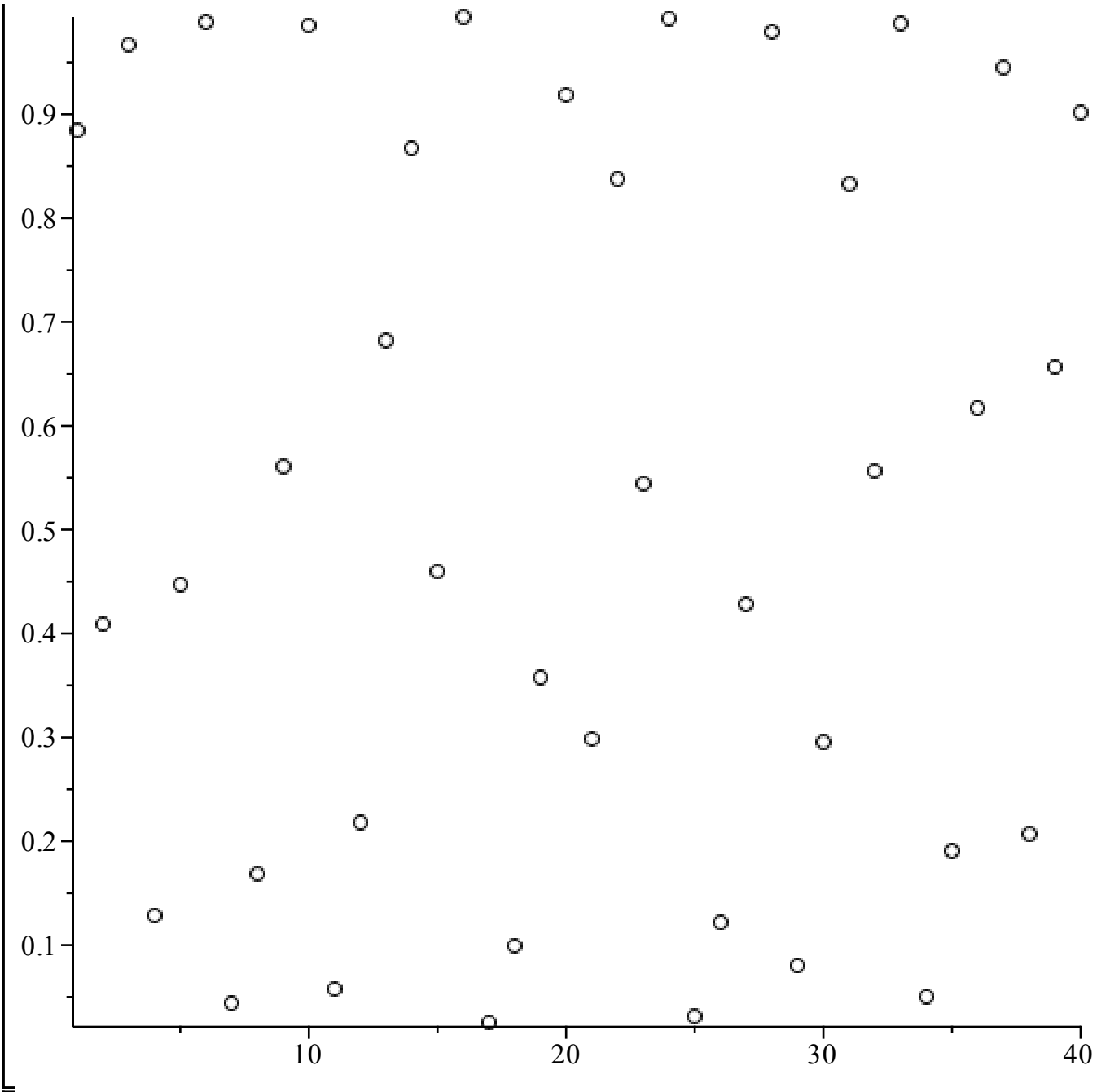
0.9668370844

0.1282525465

0.4472153232

0.9888551116
0.04408271944
0.1685577332
0.5605840952
0.9853182696
0.05786470876
0.2180655370
0.6820518344
0.8674285184
0.4599851356
0.9935952424
0.02545494672
0.09922796964
0.3575271188
0.9188059124
0.2984064310
0.8374401316
0.5445366304
0.9920659544
0.03148438608
0.1219724780
0.4283807704
0.9794827440
0.08038519284
0.2956936545
0.8330356688
0.5563489732
0.9872991728
0.05015806476
0.1905689332
0.6170096596
0.9452349584
0.2070633273
0.6567524232
0.9017147112 **(34)**

> $points := [[n, \mathrm{psi}(n)]\$n = 1..40] : pointplot(points, symbol = circle)$




> $x := 0.67001$

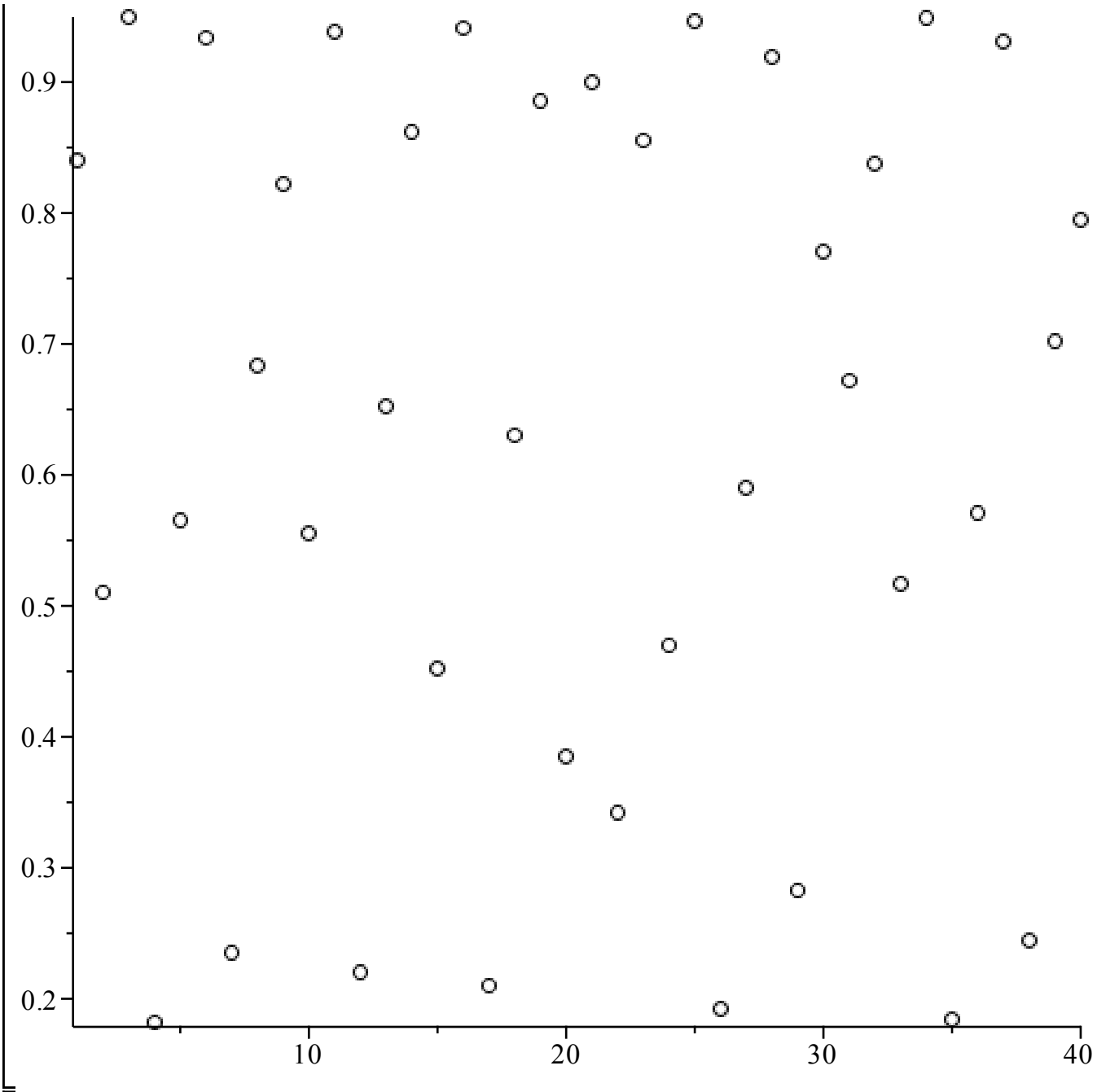
$$x := 0.67001 \quad \textbf{(35)}$$

> **for** $i$ **from** 1 **to** 40 **do** $x := fl(x) : \mathrm{psi}(i) := x; print(x);$**od**:

0.8401670796
0.5102881602
0.9495977844
0.1818749626
0.5654265502
0.9337335929
0.2351256475

0.6833979941
0.8221876680
0.5555414050
0.9382775789
0.2200685026
0.6522257557
0.8619438135
0.4521873683
0.9413130188
0.2099227138
0.6302496385
0.8855331202
0.3851840103
0.8999056962
0.3422866498
0.8554806967
0.4698072022
0.9465359007
0.1923016197
0.5902224857
0.9190676318
0.2826528159
0.7704887656
0.6719761451
0.8376119808
0.5168689716
0.9489186636
0.1841937272
0.5710123127
0.9308375554
0.2446402033
0.7022072220
0.7946265097 **(36)**

> $points := [[n, \mathrm{psi}(n)]\$n = 1..40] : pointplot(points, symbol = circle)$




> $f41 := x \to 4 \cdot x - 4 \cdot x\hat{}2$

$$f41 := x \mapsto 4 \cdot x - 4 \cdot x^2 \quad \textbf{(37)}$$

> $x := 0.67$

$$x := 0.67 \quad \textbf{(38)}$$

> **for** $i$ **from** 1 **to** 40 **do** $x := f41(x) : \mathrm{psi}(i) := x; print(x);$**od**:

0.8844

0.40894656

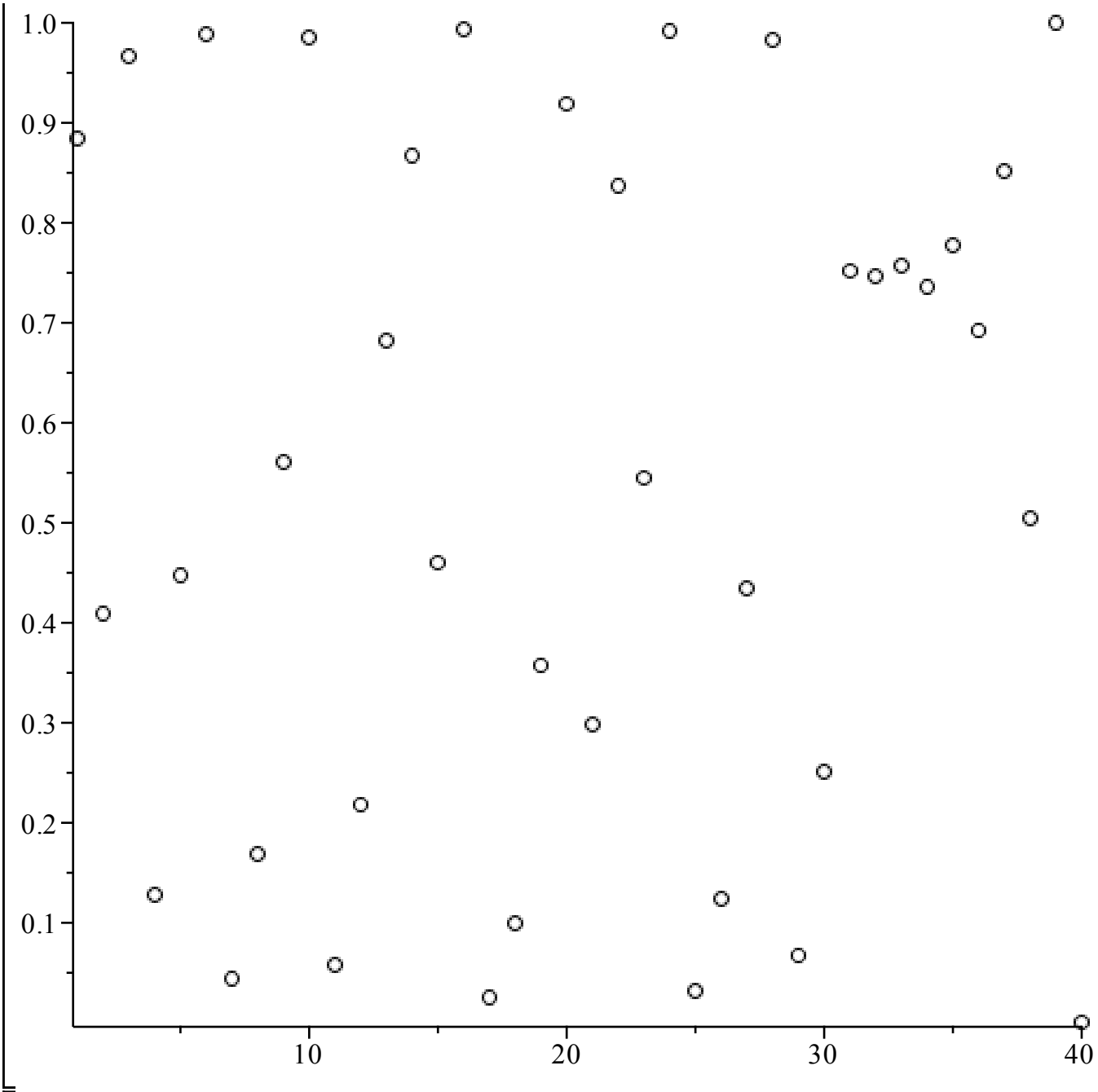
0.9668370844

0.128252547

0.4472153248

0.9888551122
0.044082717
0.1685577242
0.5605840712
0.985318281
0.057864664
0.2180653786
0.6820514770
0.867429039
0.459983605
0.9935947524
0.025456882
0.09923531664
0.3575506743
0.9188327582
0.298316483
0.8372950359
0.544928236
0.991925814
0.032035974
0.1240386815
0.4346123480
0.9828978200
0.067238782
0.2508709128
0.7517387914
0.746510324
0.756930641
0.735946583
0.777316840
0.692381481
0.851957463
0.504503777
0.999918864
0.000324518 **(39)**

> $points := [[n, \mathrm{psi}(n)]\$n = 1..40] : pointplot(points, symbol = circle)$




```
> x := 0.67001
```

$$x := 0.67001 \quad \textbf{(40)}$$

```
> for i from 1 to 40 do x := f41(x) : psi(i) := x; print(x);od:
```

0.884386400

0.408988382

0.9668675416

0.128138794

0.4468769739

0.9887117768

0.044643197

0.1706007278

0.5659844779

0.982584195

0.068449979

0.2550583175

0.7600142887

0.729570279

0.789189948

0.665476696

0.890469852

0.390133179

0.9517171264

0.183806551

0.6000868112

0.959930521

0.153855664

0.5207363946

0.998280007

0.006868138

0.02728386672

0.1061578294

0.3795533786

0.9419704452

0.218648503

0.6833653406

0.865508607

0.465613833

0.9952703660

0.018829058

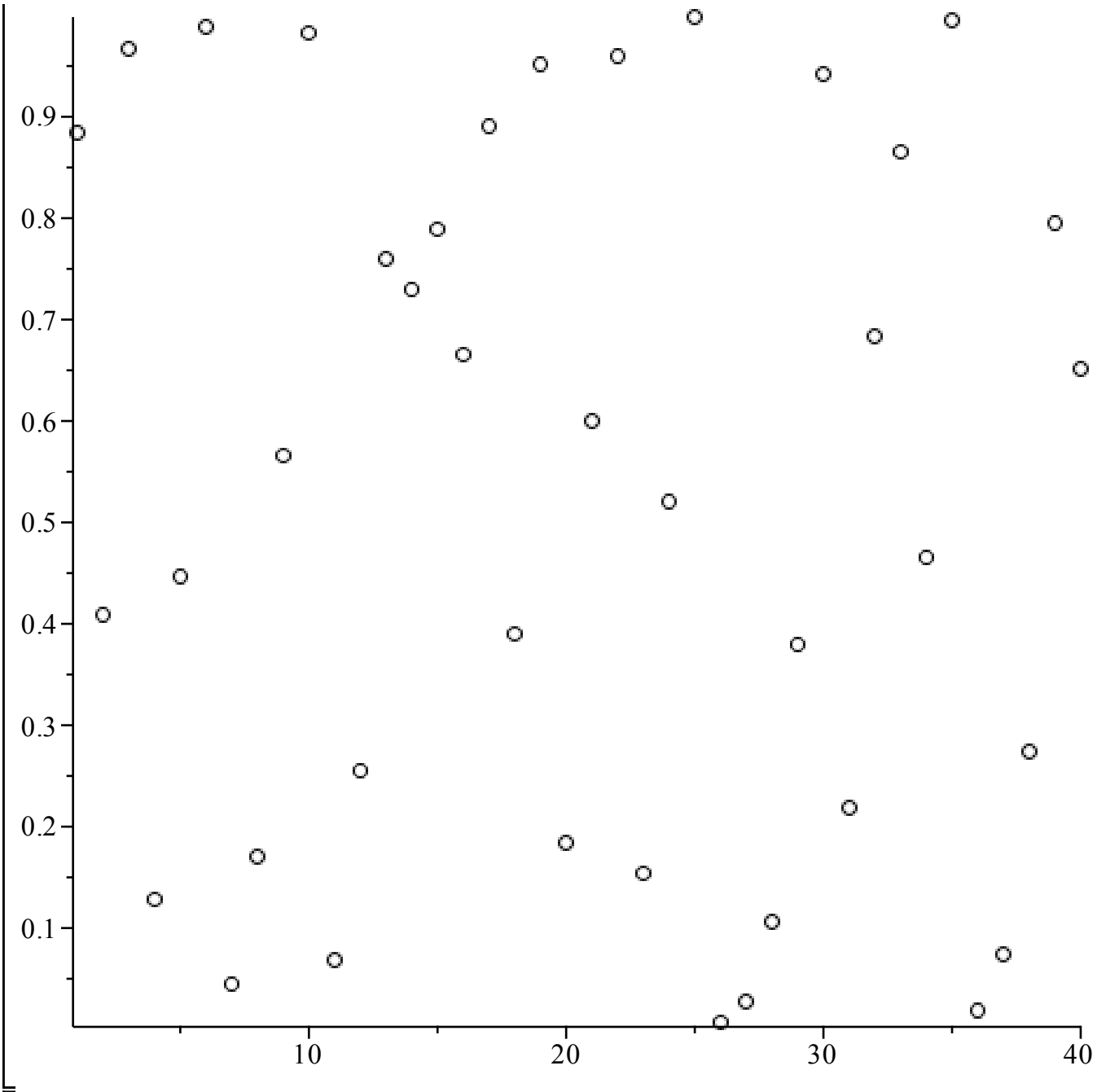
0.07389809830

0.2737486775

0.7952413563

0.651330166 **(41)**

```
> points := [ [n, psi(n) ]$n = 1 ..40 ] : pointplot( points, symbol = circle)
```




```
> restart :
> with(DEtools)
```

[*AreSimilar*, *Closure*, *DEnormal*, *DEplot*, *DEplot3d*, *DEplot_polygon*, *DFactor*, *DFactorLCLM*, *DFactorsols*, *Dchangevar*, *Desingularize*, *FindODE*, *FunctionDecomposition*, *GCRD*, *Gosper*, *Heunsols*, *Homomorphisms*, *IVPsol*, *IsHyperexponential*, *LCLM*, *MeijerGsols*, *MultiplicativeDecomposition*, *ODEInvariants*, *PDEchangecoords*, *PolynomialNormalForm*, *RationalCanonicalForm*, *ReduceHyperexp*, *RiemannPsols*, *Xchange*, *Xcommutator*, *Xgauge*, *Zeilberger*, *abelsol*, *adjoint*, *autonomous*, *bernoullisol*, *buildsol*, *buildsym*, *canoni*, *caseplot*, *casesplit*, *checkrank*, *chinisol*, *clairautsol*, *constcoeffsols*, *convertAlg*, *convertsys*, *dalembertsol*, *dcoeffs*, *de2diffop*, *dfieldplot*, *diff_table*, *diffop2de*, *dperiodic_sols*, *dpolyform*, *dsubs*, *eigenring*, **(42)**

*endomorphism_charpoly*, *equinv*, *eta_k*, *eulersols*, *exactsol*, *expsols*, *exterior_power*, *firint*, *firtest*, *formal_sol*, *gen_exp*, *generate_ic*, *genhomosol*, *gensys*, *hamilton_eqs*, *hypergeometricsols*, *hypergeomsols*, *hyperode*, *indicialeq*, *infgen*, *initialdata*, *integrate_sols*, *intfactor*, *invariants*, *kovacicsols*, *leftdivision*, *liesol*, *line_int*, *linearsol*, *matrixDE*, *matrix_riccati*, *maxdimsystems*, *moser_reduce*, *muchange*, *mult*, *mutest*, *newton_polygon*, *normalG2*, *ode_int_y*, *ode_y1*, *odeadvisor*, *odepde*, *parametricsol*, *particularsol*, *phaseportrait*, *poincare*, *polysols*, *power_equivalent*, *rational_equivalent*, *ratsols*, *redode*, *reduceOrder*, *reduce_order*, *regular_parts*, *regularsp*, *remove_RootOf*, *riccati_system*, *riccatisol*, *rifread*, *rifsimp*, *rightdivision*, *rtaylor*, *separablesol*, *singularities*, *solve_group*, *super_reduce*, *symgen*, *symmetric_power*, *symmetric_product*, *symtest*, *transinv*, *translate*, *untranslate*, *varparam*, *zoom* ]

> $f := (x, y) \to 2 \cdot x \cdot y$

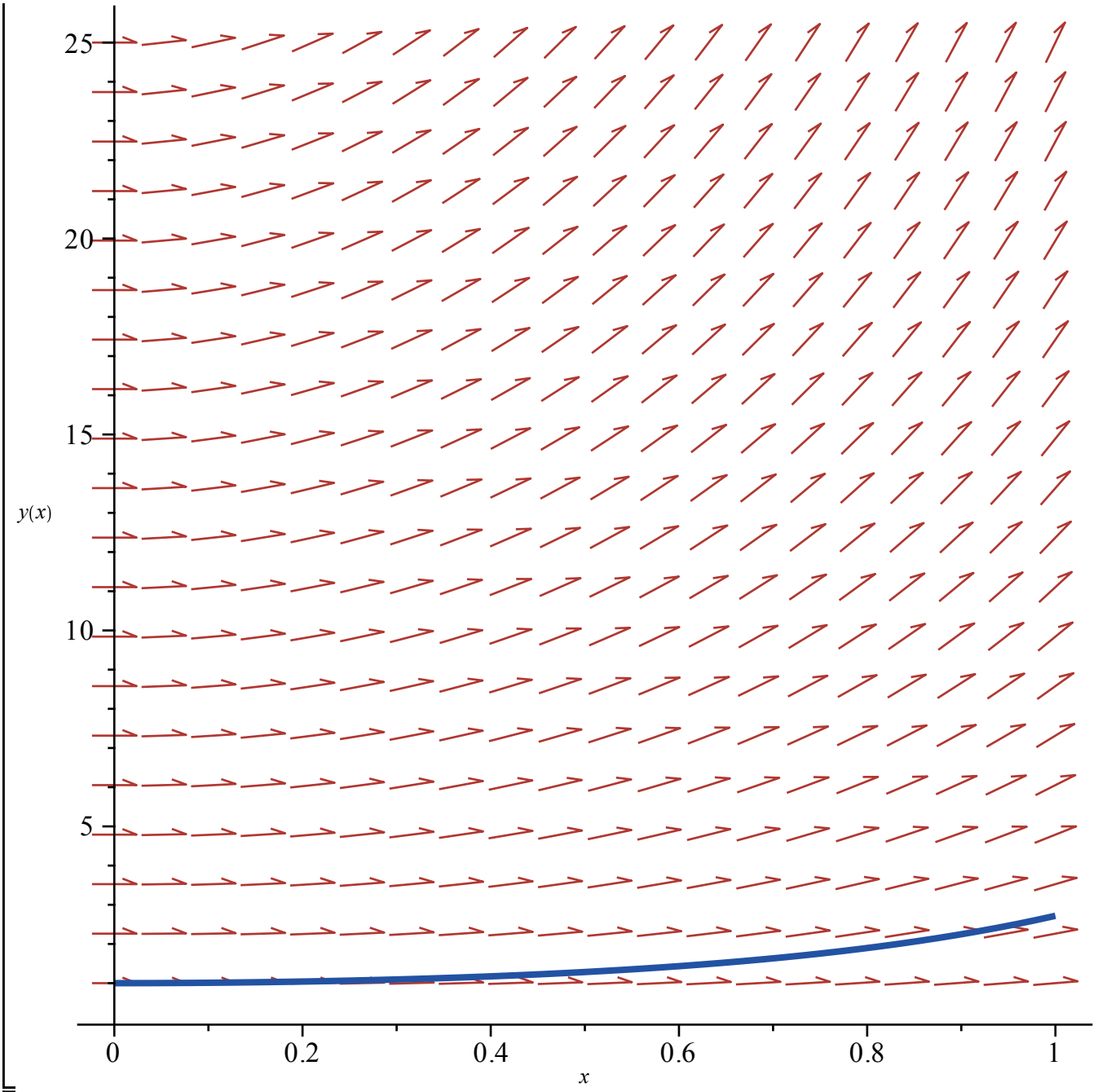
$$f := (x, y) \mapsto 2 \cdot y \cdot x \tag{43}$$

> *dsolve*( { *diff* ( $y(x), x$ ) = $f(x, y(x))$, $y(0) = 1$ }, $y(x)$ ); phi := *unapply*( *rhs* ( % ), $x$ )

$$y(x) = e^{x^2}$$

$$\phi := x \mapsto e^{x^2} \tag{44}$$

> *DEplot*( *diff* ( $y(x), x$ ) = $f(x, y(x))$, $y(x)$, $x = 0..1$, [ [ $y(0) = 1$ ] ], $y = 1..25$ )




> $h := 0.1$

$$h := 0.1 \quad \textbf{(45)}$$

> $x := 0; y := 1$

$$x := 0$$
$$y := 1 \quad \textbf{(46)}$$

> **for** $i$ **from** 1 **to** 10 **do** $y := y + h \cdot f(x, y)$; psi$(i) := y : x := x + h : print(x, y,$ phi$(x),$ abs$(y$ − phi$(x)))$; **od**;

$$y := 1.$$
$$\psi(1) := 1.$$
$$x := 0.1$$

0.1, 1., 1.010050167, 0.010050167

$y := 1.02$

$\psi(2) := 1.02$

$x := 0.2$

0.2, 1.02, 1.040810774, 0.020810774

$y := 1.0608$

$\psi(3) := 1.0608$

$x := 0.3$

0.3, 1.0608, 1.094174284, 0.033374284

$y := 1.124448$

$\psi(4) := 1.124448$

$x := 0.4$

0.4, 1.124448, 1.173510871, 0.049062871

$y := 1.21440384$

$\psi(5) := 1.21440384$

$x := 0.5$

0.5, 1.21440384, 1.284025417, 0.069621577

$y := 1.335844224$

$\psi(6) := 1.335844224$

$x := 0.6$

0.6, 1.335844224, 1.433329415, 0.097485191

$y := 1.496145531$

$\psi(7) := 1.496145531$

$x := 0.7$

0.7, 1.496145531, 1.632316220, 0.136170689

$y := 1.705605905$

$\psi(8) := 1.705605905$

$x := 0.8$

0.8, 1.705605905, 1.896480879, 0.190874974

$y := 1.978502850$

$\psi(9) := 1.978502850$

$x := 0.9$

0.9, 1.978502850, 2.247907987, 0.269405137

$y := 2.334633363$

$\psi(10) := 2.334633363$

$x := 1.0$

1.0, 2.334633363, 2.718281828, 0.383648465 **(47)**

```
> with(plots)
```

[animate, animate3d, animatecurve, arrow, changecoords, complexplot, complexplot3d, conformal, conformal3d, contourplot, contourplot3d, coordplot, coordplot3d, densityplot, display, dualaxisplot, fieldplot, fieldplot3d, gradplot, gradplot3d, implicitplot, implicitplot3d, inequal, interactive, interactiveparams, intersectplot, listcontplot, listcontplot3d, listdensityplot, listplot, listplot3d, loglogplot, logplot, matrixplot, multiple, odeplot, pareto, plotcompare, pointplot, pointplot3d, polarplot, polygonplot, polygonplot3d, polyhedra_supported, polyhedraplot, rootlocus, semilogplot, setcolors, setoptions, setoptions3d, shadebetween, spacecurve, sparsematrixplot, surfdata, textplot, textplot3d, tubeplot] **(48)**

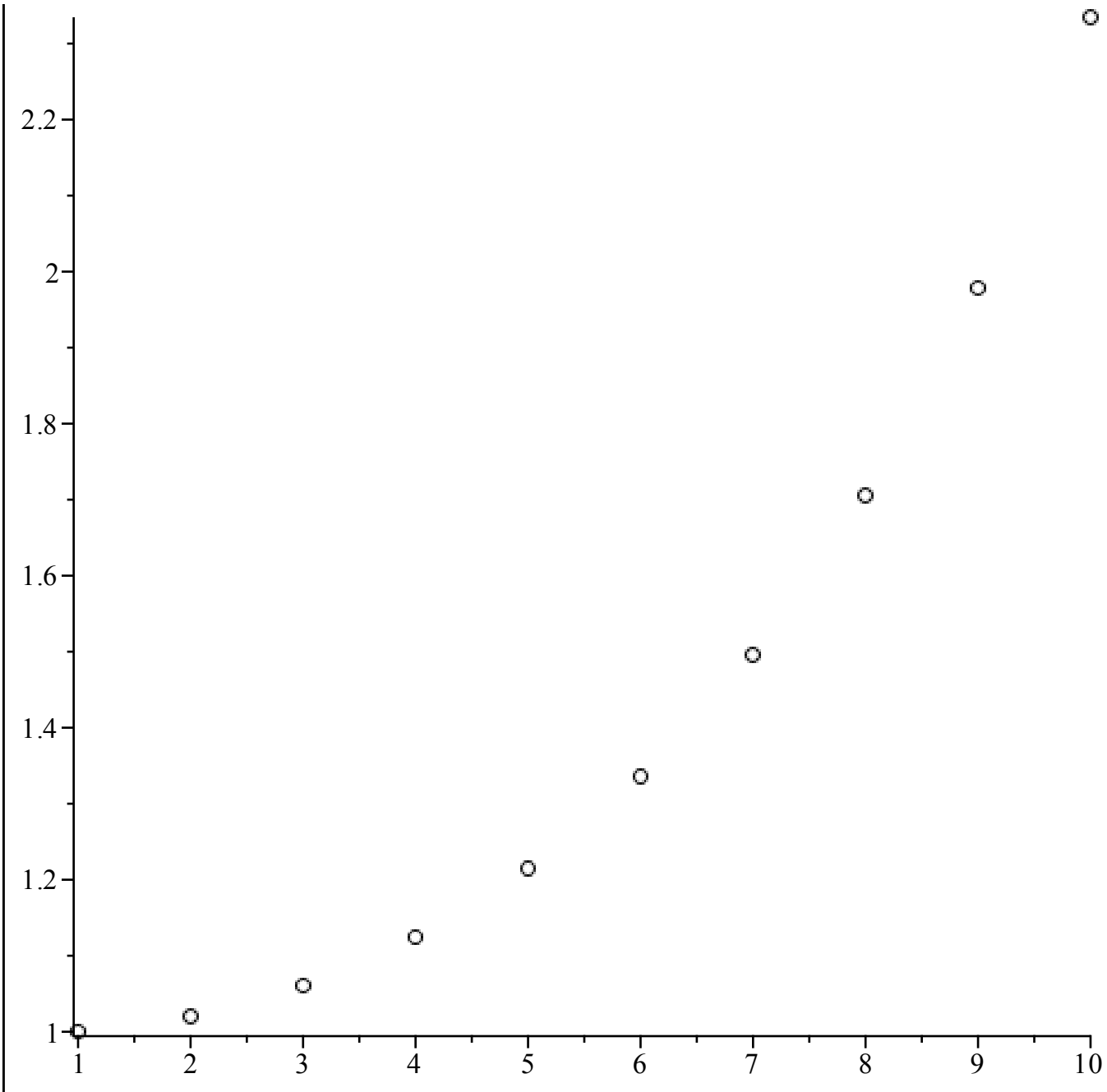
```
> y := 'y'
```

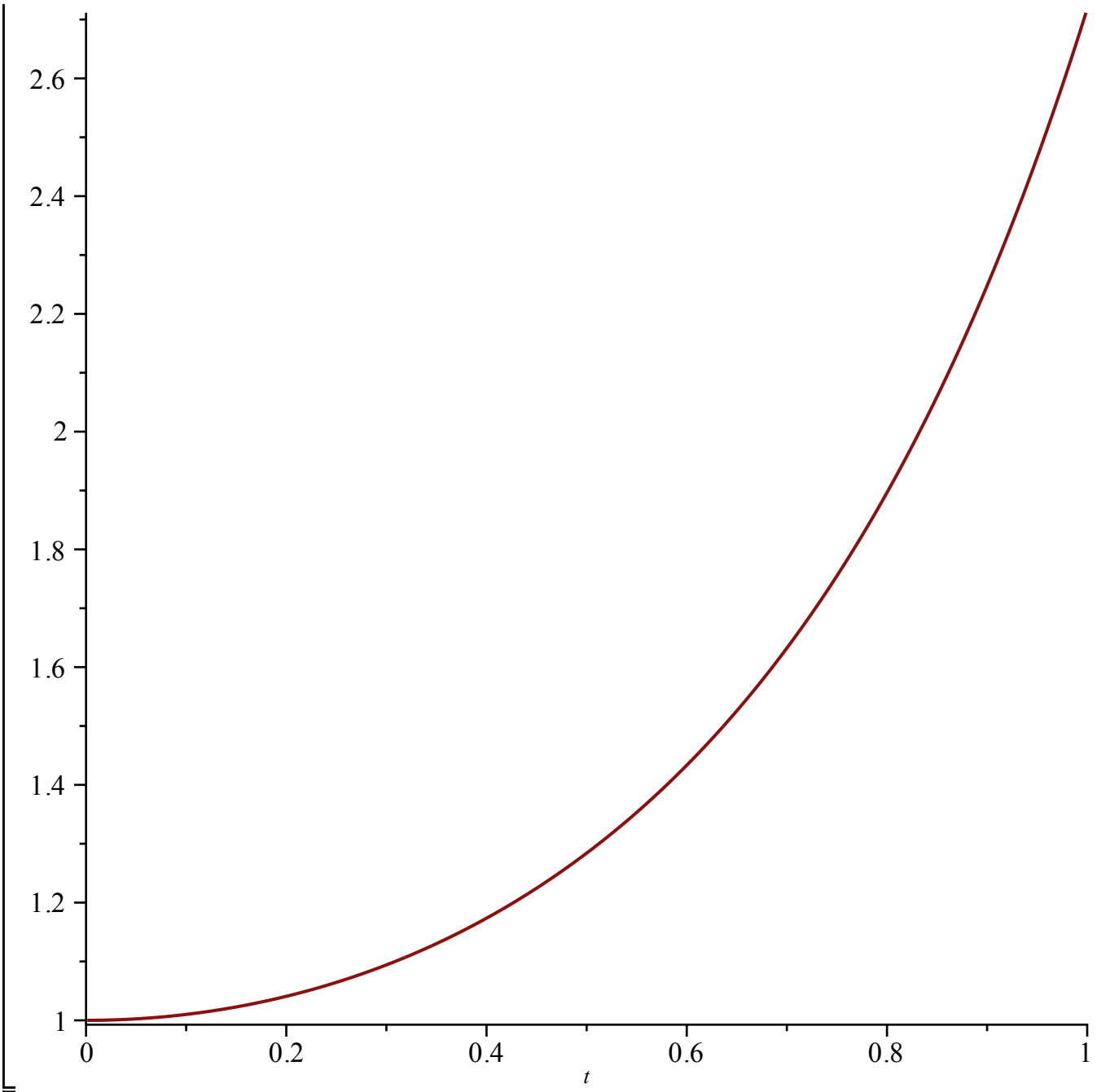
$$y := y \quad \textbf{(49)}$$

```
> y
```

$$y \quad \textbf{(50)}$$

```
> points := [[n, psi(n)]$n = 1..10] : with(plots) : pointplot(points, symbol = circle);
      plot(phi(t), t = 0..1)
```


2.2
2
1.8
1.6
1.4
1.2
1
1
2
3
4
5
6
7
8
9
10




```
> restart
> phi := x→exp(x^2)
```

$$\phi := x \mapsto e^{x^2} \quad \textbf{(51)}$$

```
> h := 0.1
```

$$h := 0.1 \quad \textbf{(52)}$$

```
> x := 0
```

$$x := 0 \quad \textbf{(53)}$$

```
> y := 1
```

$$y := 1 \quad \textbf{(54)}$$

```
> f := (x, y)→2·x·y
```

**(55)**

$$f := (x, y) \mapsto 2 \cdot y \cdot x \tag{55}$$

```
> for i from 1 to 10 do y := y + h/2 ·f(x, y) + h/2 ·f(x + h, y + h·f(x, y)) : psi(i) := y : x := x
  + h : print(x, y, phi(x), abs(y − phi(x))) : od
```

$$y := 1.010000000$$

$$\psi(1) := 1.010000000$$

$$x := 0.1$$

$$0.1,\ 1.010000000,\ 1.010050167,\ 0.000050167$$

$$y := 1.040704000$$

$$\psi(2) := 1.040704000$$

$$x := 0.2$$

$$0.2,\ 1.040704000,\ 1.040810774,\ 0.000106774$$

$$y := 1.093988045$$

$$\psi(3) := 1.093988045$$

$$x := 0.3$$

$$0.3,\ 1.093988045,\ 1.094174284,\ 0.000186239$$

$$y := 1.173192779$$

$$\psi(4) := 1.173192779$$

$$x := 0.4$$

$$0.4,\ 1.173192779,\ 1.173510871,\ 0.000318092$$

$$y := 1.283472900$$

$$\psi(5) := 1.283472900$$

$$x := 0.5$$

$$0.5,\ 1.283472900,\ 1.284025417,\ 0.000552517$$

$$y := 1.432355756$$

$$\psi(6) := 1.432355756$$

$$x := 0.6$$

$$0.6,\ 1.432355756,\ 1.433329415,\ 0.000973659$$

$$y := 1.630593792$$

$$\psi(7) := 1.630593792$$

$$x := 0.7$$

$$0.7,\ 1.630593792,\ 1.632316220,\ 0.001722428$$

$$y := 1.893445511$$

$$\psi(8) := 1.893445511$$

$$x := 0.8$$

$$0.8,\ 1.893445511,\ 1.896480879,\ 0.003035368$$

$$y := 2.242596863$$

$$\psi(9) := 2.242596863$$

$$x := 0.9$$

$$0.9, 2.242596863, 2.247907987, 0.005311124$$

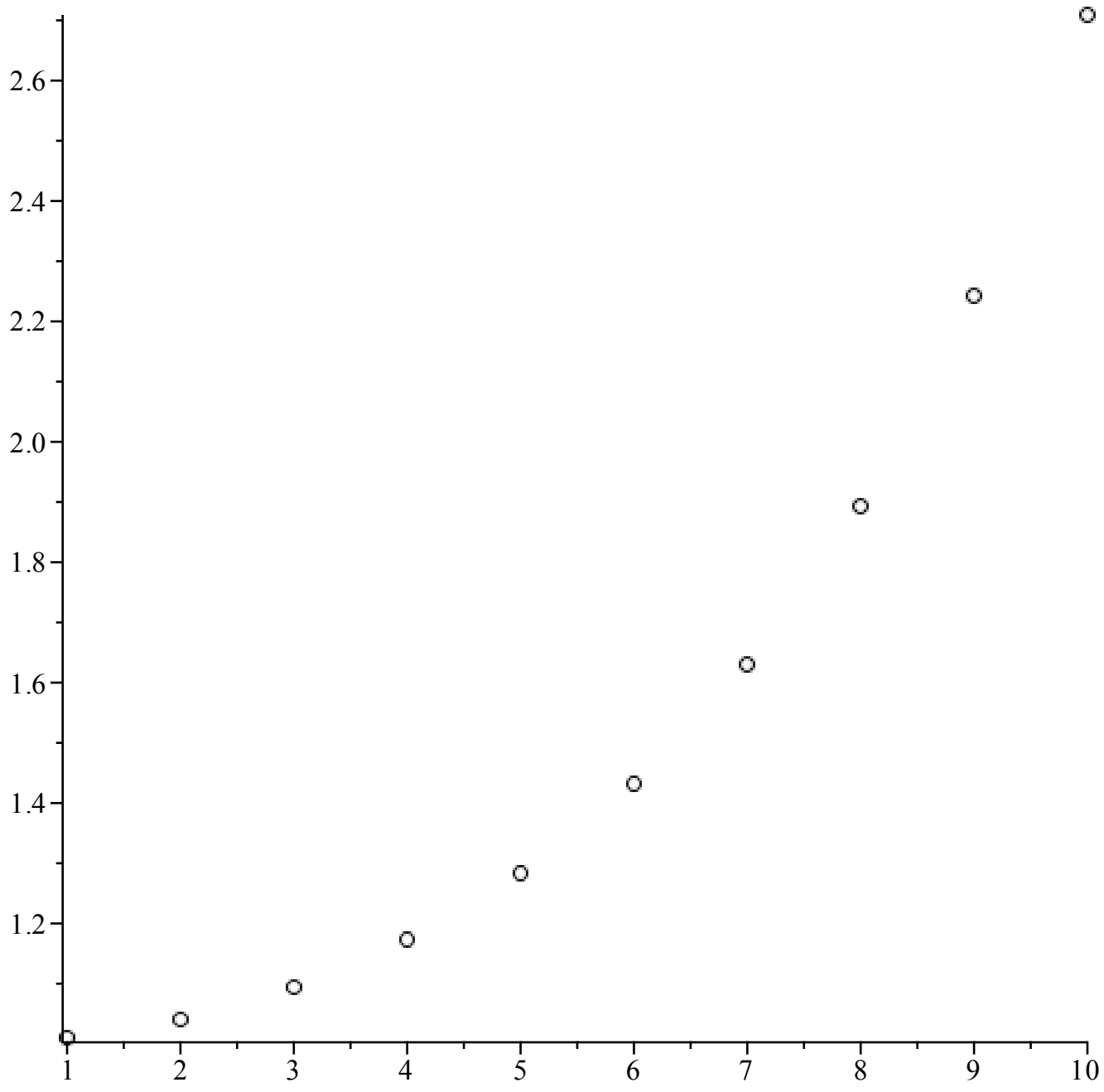
$$y := 2.709057011$$

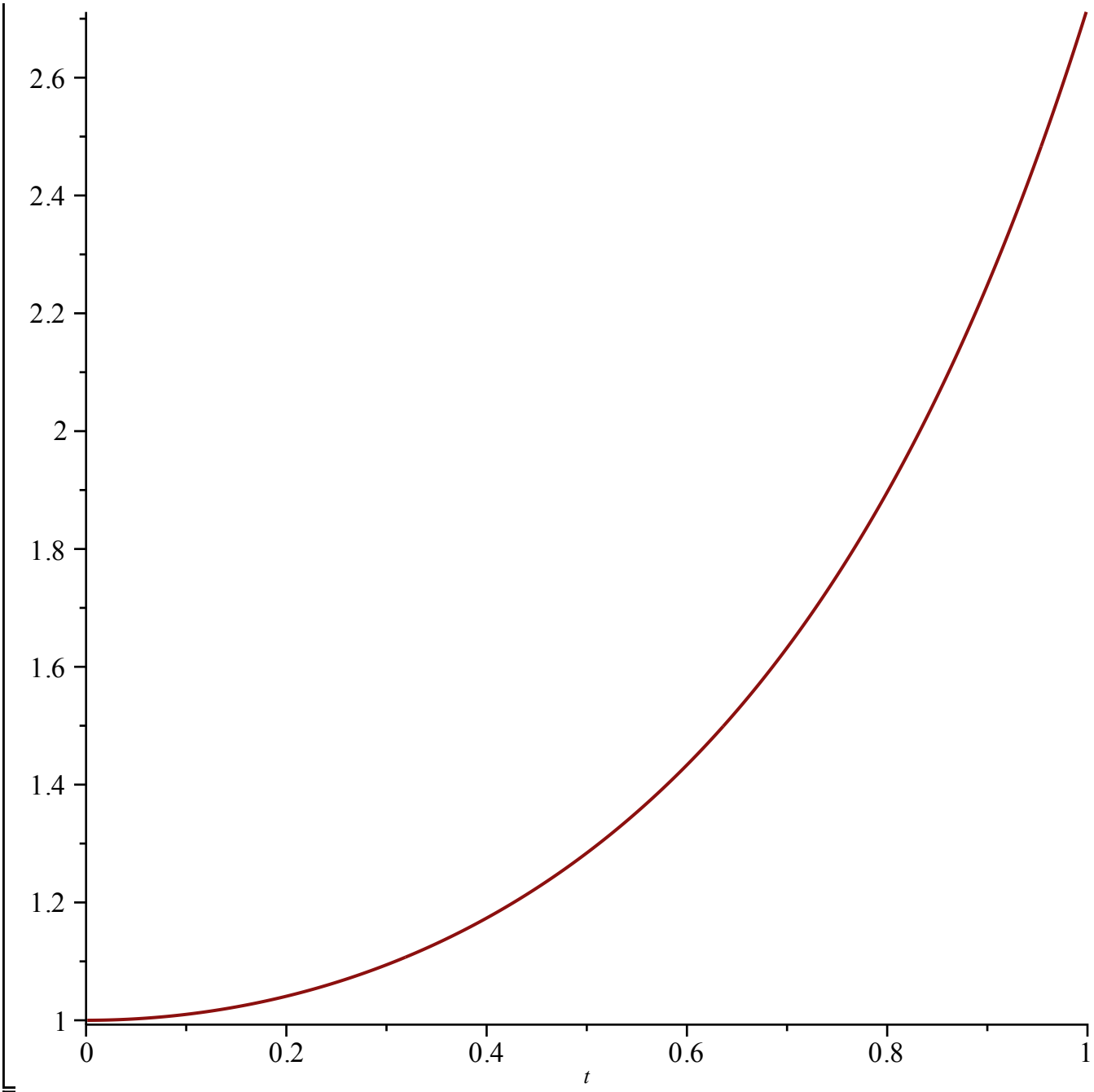
$$\psi(10) := 2.709057011$$

$$x := 1.0$$

$$1.0, 2.709057011, 2.718281828, 0.009224817 \quad \textbf{(56)}$$

```
> points := [[n, psi(n)]$n = 1..10] : with(plots) : pointplot(points, symbol = circle);
      plot(phi(t), t = 0..1)
```






```
> restart :
> with(DEtools)
```

[*AreSimilar*, *Closure*, *DEnormal*, *DEplot*, *DEplot3d*, *DEplot_polygon*, *DFactor*, *DFactorLCLM*, *DFactorsols*, *Dchangevar*, *Desingularize*, *FindODE*, *FunctionDecomposition*, *GCRD*, *Gosper*, *Heunsols*, *Homomorphisms*, *IVPsol*, *IsHyperexponential*, *LCLM*, *MeijerGsols*, *MultiplicativeDecomposition*, *ODEInvariants*, *PDEchangecoords*, *PolynomialNormalForm*, *RationalCanonicalForm*, *ReduceHyperexp*, *RiemannPsols*, *Xchange*, *Xcommutator*, *Xgauge*, *Zeilberger*, *abelsol*, *adjoint*, *autonomous*, *bernoullisol*, *buildsol*, *buildsym*, *canoni*, *caseplot*, *casesplit*, *checkrank*, *chinisol*, *clairautsol*, *constcoeffsols*, *convertAlg*, *convertsys*, *dalembertsol*, *dcoeffs*, *de2diffop*, *dfieldplot*, *diff_table*, *diffop2de*, *dperiodic_sols*, *dpolyform*, *dsubs*, *eigenring*, **(57)**

*endomorphism_charpoly, equinv, eta_k, eulersols, exactsol, expsols, exterior_power, firint, firtest, formal_sol, gen_exp, generate_ic, genhomosol, gensys, hamilton_eqs, hypergeometricsols, hypergeomsols, hyperode, indicialeq, infgen, initialdata, integrate_sols, intfactor, invariants, kovacicsols, leftdivision, liesol, line_int, linearsol, matrixDE, matrix_riccati, maxdimsystems, moser_reduce, muchange, mult, mutest, newton_polygon, normalG2, ode_int_y, ode_y1, odeadvisor, odepde, parametricsol, particularsol, phaseportrait, poincare, polysols, power_equivalent, rational_equivalent, ratsols, redode, reduceOrder, reduce_order, regular_parts, regularsp, remove_RootOf, riccati_system, riccatisol, rifread, rifsimp, rightdivision, rtaylor, separablesol, singularities, solve_group, super_reduce, symgen, symmetric_power, symmetric_product, symtest, transinv, translate, untranslate, varparam, zoom* ]

> $f := (x,y) \to y\hat{}2 + x\hat{}2$

$$f := (x,y) \mapsto y^2 + x^2 \tag{58}$$

> $dsolve(\{diff(y(x),x) = f(x,y(x)), y(0)=0\}, y(x)); \text{phi} := unapply(rhs(\%), x)$

$$y(x) = -\frac{\left(-\text{BesselJ}\left(-\frac{3}{4}, \frac{x^2}{2}\right) + \text{BesselY}\left(-\frac{3}{4}, \frac{x^2}{2}\right)\right) x}{-\text{BesselJ}\left(\frac{1}{4}, \frac{x^2}{2}\right) + \text{BesselY}\left(\frac{1}{4}, \frac{x^2}{2}\right)}$$

$$\phi := x \mapsto -\frac{\left(-\text{BesselJ}\left(-\frac{3}{4}, \frac{x^2}{2}\right) + \text{BesselY}\left(-\frac{3}{4}, \frac{x^2}{2}\right)\right) \cdot x}{-\text{BesselJ}\left(\frac{1}{4}, \frac{x^2}{2}\right) + \text{BesselY}\left(\frac{1}{4}, \frac{x^2}{2}\right)} \tag{59}$$

> $dsolve(\{diff(y(x),x) = f(x,y(x)), y(0)=0\}, y(x)); \text{phi} := unapply(rhs(\%), x)$

$$y(x) = -\frac{\left(-\text{BesselJ}\left(-\frac{3}{4}, \frac{x^2}{2}\right) + \text{BesselY}\left(-\frac{3}{4}, \frac{x^2}{2}\right)\right) x}{-\text{BesselJ}\left(\frac{1}{4}, \frac{x^2}{2}\right) + \text{BesselY}\left(\frac{1}{4}, \frac{x^2}{2}\right)}$$

$$\phi := x \mapsto -\frac{\left(-\text{BesselJ}\left(-\frac{3}{4}, \frac{x^2}{2}\right) + \text{BesselY}\left(-\frac{3}{4}, \frac{x^2}{2}\right)\right) \cdot x}{-\text{BesselJ}\left(\frac{1}{4}, \frac{x^2}{2}\right) + \text{BesselY}\left(\frac{1}{4}, \frac{x^2}{2}\right)} \tag{60}$$

> $h := 0.1;\ x := 0; y := 0$

$$h := 0.1$$
$$x := 0$$
$$y := 0 \tag{61}$$

> **for** $i$ **from** 1 **to** 20 **do** $y := y + h \cdot f(x,y); \text{psi}(i) := y : x := x + h : print(x, y, \text{phi}(x), \text{abs}(y - \text{phi}(x))); \textbf{od};$

$$y := 0.$$

$$\psi(1) := 0.$$

$$x := 0.1$$

$$0.1,\ 0.,\ 0.0003333349060,\ 0.0003333349060$$

$$y := 0.001$$

$$\psi(2) := 0.001$$

$$x := 0.2$$

$$0.2,\ 0.001,\ 0.002666869814,\ 0.001666869814$$

$$y := 0.0050001$$

$$\psi(3) := 0.0050001$$

$$x := 0.3$$

$$0.3,\ 0.0050001,\ 0.009003473189,\ 0.004003373189$$

$$y := 0.01400260010$$

$$\psi(4) := 0.01400260010$$

$$x := 0.4$$

$$0.4,\ 0.01400260010,\ 0.02135938017,\ 0.00735678007$$

$$y := 0.03002220738$$

$$\psi(5) := 0.03002220738$$

$$x := 0.5$$

$$0.5,\ 0.03002220738,\ 0.04179114620,\ 0.01176893882$$

$$y := 0.05511234067$$

$$\psi(6) := 0.05511234067$$

$$x := 0.6$$

$$0.6,\ 0.05511234067,\ 0.07244786117,\ 0.01733552050$$

$$y := 0.09141607768$$

$$\psi(7) := 0.09141607768$$

$$x := 0.7$$

$$0.7,\ 0.09141607768,\ 0.1156598536,\ 0.02424377592$$

$$y := 0.1412517676$$

$$\psi(8) := 0.1412517676$$

$$x := 0.8$$

$$0.8,\ 0.1412517676,\ 0.1740802646,\ 0.0328284970$$

$$y := 0.2072469738$$

$$\psi(9) := 0.2072469738$$

$$x := 0.9$$

$$0.9,\ 0.2072469738,\ 0.2509066825,\ 0.0436597087$$

$$y := 0.2925421046$$

$$\psi(10) := 0.2925421046$$

$$x := 1.0$$

1.0, 0.2925421046, 0.3502318440, 0.0576897394

$$y := 0.4011001929$$

$$\psi(11) := 0.4011001929$$

$$x := 1.1$$

1.1, 0.4011001929, 0.4776170219, 0.0765168290

$$y := 0.5381883294$$

$$\psi(12) := 0.5381883294$$

$$x := 1.2$$

1.2, 0.5381883294, 0.6410767262, 0.1028883968

$$y := 0.7111529972$$

$$\psi(13) := 0.7111529972$$

$$x := 1.3$$

1.3, 0.7111529972, 0.8528799930, 0.1417269958

$$y := 0.9307268557$$

$$\psi(14) := 0.9307268557$$

$$x := 1.4$$

1.4, 0.9307268557, 1.133112675, 0.2023858193

$$y := 1.213352104$$

$$\psi(15) := 1.213352104$$

$$x := 1.5$$

1.5, 1.213352104, 1.517447543, 0.304095439

$$y := 1.585574437$$

$$\psi(16) := 1.585574437$$

$$x := 1.6$$

1.6, 1.585574437, 2.076423381, 0.490848944

$$y := 2.092979066$$

$$\psi(17) := 2.092979066$$

$$x := 1.7$$

1.7, 2.092979066, 2.972797219, 0.879818153

$$y := 2.820035203$$

$$\psi(18) := 2.820035203$$

$$x := 1.8$$

1.8, 2.820035203, 4.688130710, 1.868095507

$$y := 3.939295058$$

$$\psi(19) := 3.939295058$$

$$x := 1.9$$

1.9, 3.939295058, 9.566995342, 5.627700284

$$y := 5.852099613$$

$$\psi(20) := 5.852099613$$

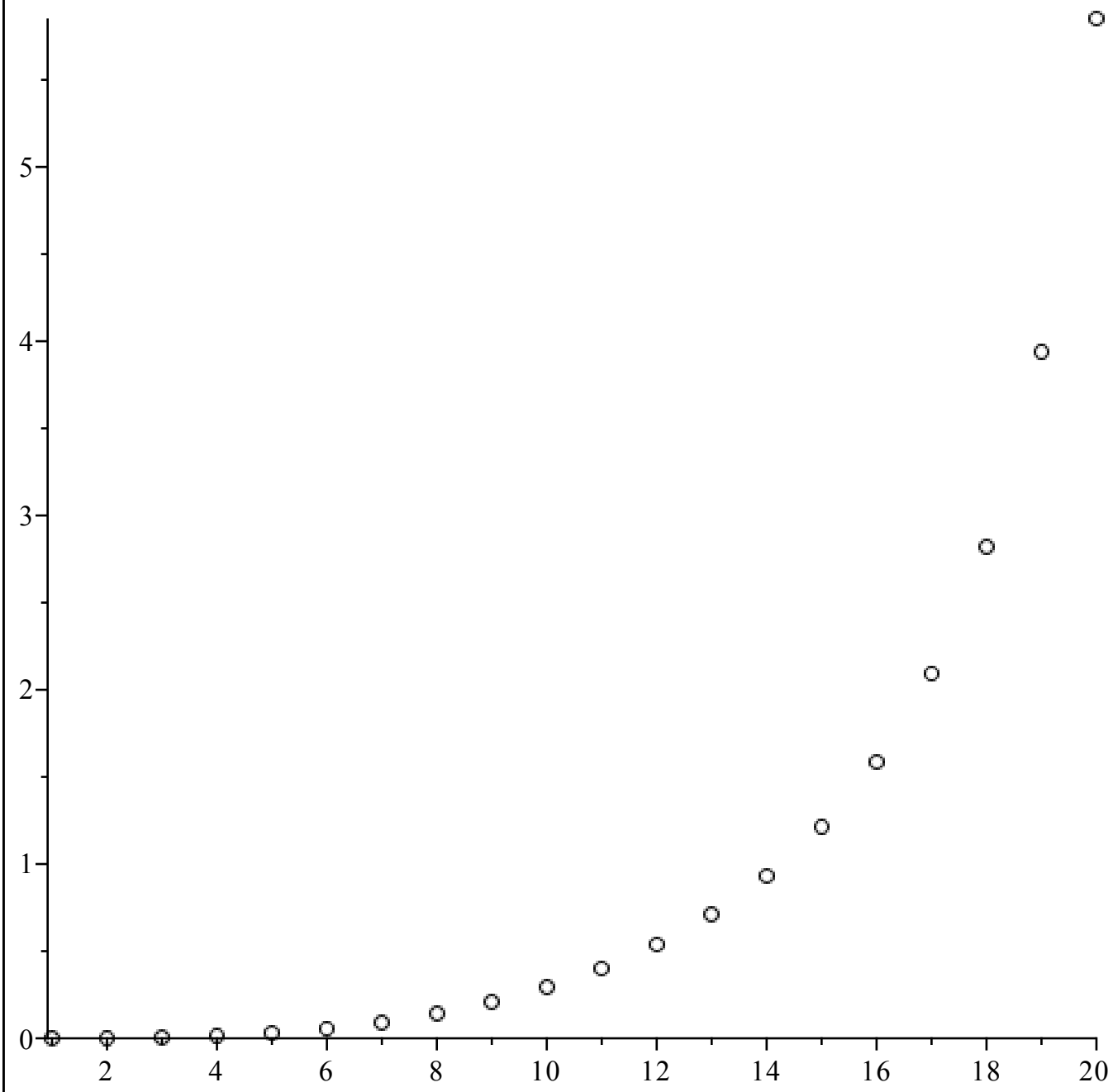
$$x := 2.0$$

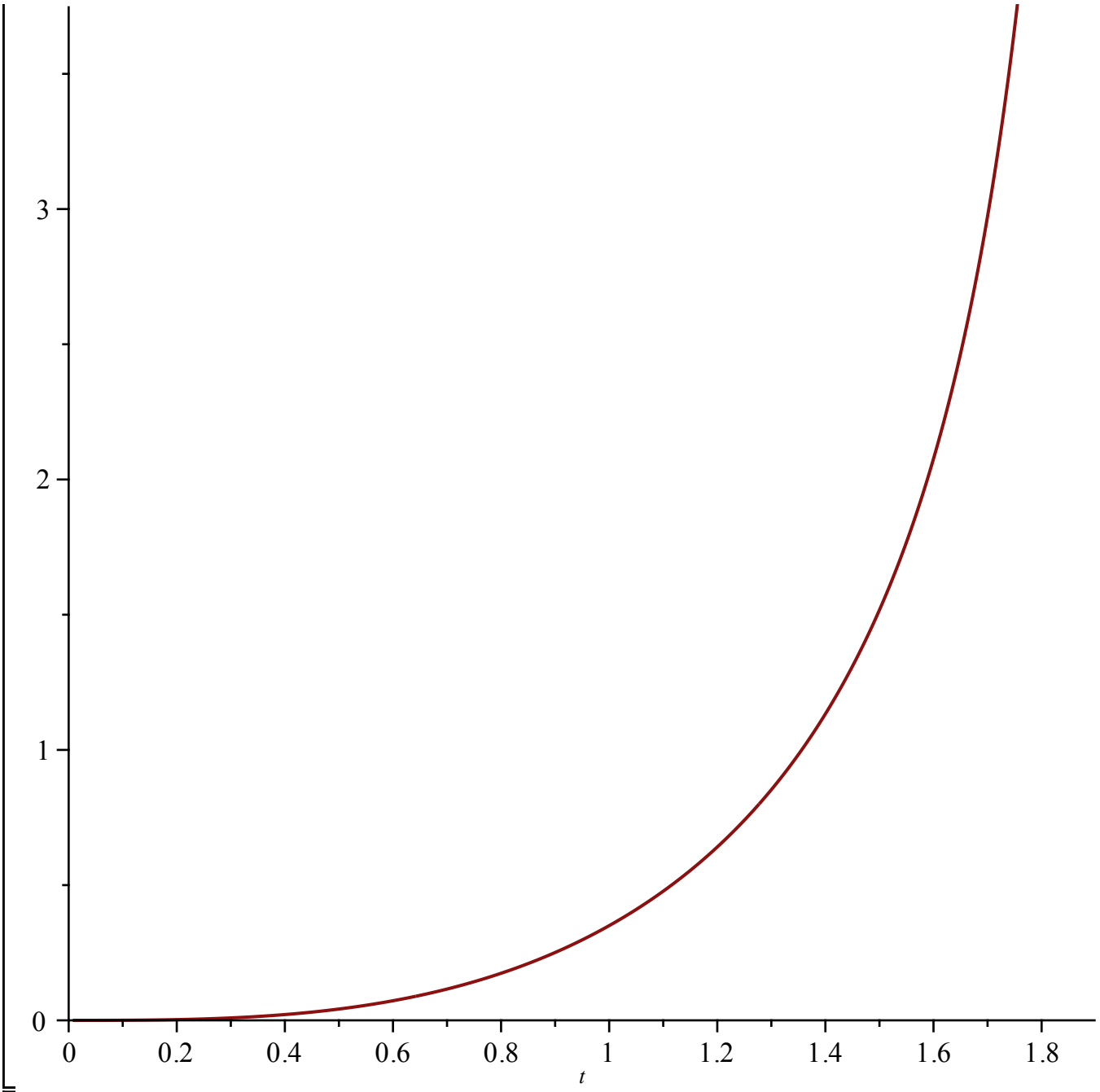
2.0, 5.852099613, 317.7224621, 311.8703625 **(62)**

```
> y := 'y'
```

$$y := y \quad \textbf{(63)}$$

```
> points := [[n, psi(n)]$n = 1..20] : with(plots) : pointplot(points, symbol = circle);
  plot(phi(t), t = 0..2)
```






```
> restart
> with(DEtools)
```

[*AreSimilar*, *Closure*, *DEnormal*, *DEplot*, *DEplot3d*, *DEplot_polygon*, *DFactor*, *DFactorLCLM*, *DFactorsols*, *Dchangevar*, *Desingularize*, *FindODE*, *FunctionDecomposition*, *GCRD*, *Gosper*, *Heunsols*, *Homomorphisms*, *IVPsol*, *IsHyperexponential*, *LCLM*, *MeijerGsols*, *MultiplicativeDecomposition*, *ODEInvariants*, *PDEchangecoords*, *PolynomialNormalForm*, *RationalCanonicalForm*, *ReduceHyperexp*, *RiemannPsols*, *Xchange*, *Xcommutator*, *Xgauge*, *Zeilberger*, *abelsol*, *adjoint*, *autonomous*, *bernoullisol*, *buildsol*, *buildsym*, *canoni*, *caseplot*, *casesplit*, *checkrank*, *chinisol*, *clairautsol*, *constcoeffsols*, *convertAlg*, *convertsys*, *dalembertsol*, *dcoeffs*, *de2diffop*, *dfieldplot*, *diff_table*, *diffop2de*, *dperiodic_sols*, *dpolyform*, *dsubs*, *eigenring*, **(64)**

*endomorphism_charpoly, equinv, eta_k, eulersols, exactsol, expsols, exterior_power, firint, firtest, formal_sol, gen_exp, generate_ic, genhomosol, gensys, hamilton_eqs, hypergeometricsols, hypergeomsols, hyperode, indicialeq, infgen, initialdata, integrate_sols, intfactor, invariants, kovacicsols, leftdivision, liesol, line_int, linearsol, matrixDE, matrix_riccati, maxdimsystems, moser_reduce, muchange, mult, mutest, newton_polygon, normalG2, ode_int_y, ode_y1, odeadvisor, odepde, parametricsol, particularsol, phaseportrait, poincare, polysols, power_equivalent, rational_equivalent, ratsols, redode, reduceOrder, reduce_order, regular_parts, regularsp, remove_RootOf, riccati_system, riccatisol, rifread, rifsimp, rightdivision, rtaylor, separablesol, singularities, solve_group, super_reduce, symgen, symmetric_power, symmetric_product, symtest, transinv, translate, untranslate, varparam, zoom* ]

> $f := (x, y) \to y\hat{}2 + x\hat{}2$

$$f := (x, y) \mapsto y^2 + x^2 \tag{65}$$

> $dsolve(\{diff(y(x), x) = f(x, y(x)), y(0) = 0\}, y(x)); \text{phi} := unapply(rhs(\%), x)$

$$y(x) = -\left\{ \begin{array}{ll} 0 & x = 0 \\ \dfrac{x\left(\text{BesselJ}\left(-\frac{3}{4}, \frac{x^2}{2}\right) - \text{BesselY}\left(-\frac{3}{4}, \frac{x^2}{2}\right)\right)}{\text{BesselJ}\left(\frac{1}{4}, \frac{x^2}{2}\right) - \text{BesselY}\left(\frac{1}{4}, \frac{x^2}{2}\right)} & otherwise \end{array} \right.$$

$$\phi := x \mapsto -\left\{ \begin{array}{ll} 0 & x = 0 \\ \dfrac{x \cdot \left(\text{BesselJ}\left(-\frac{3}{4}, \frac{x^2}{2}\right) - \text{BesselY}\left(-\frac{3}{4}, \frac{x^2}{2}\right)\right)}{\text{BesselJ}\left(\frac{1}{4}, \frac{x^2}{2}\right) - \text{BesselY}\left(\frac{1}{4}, \frac{x^2}{2}\right)} & otherwise \end{array} \right. \tag{66}$$

> $h := 0.1; x := 0; y := 0$

$$h := 0.1$$
$$x := 0$$
$$y := 0 \tag{67}$$

> **for** $i$ **from** 1 **to** 20 **do** $y := y + \frac{h}{2} \cdot f(x, y) + \frac{h}{2} \cdot f(x + h, y + h \cdot f(x, y)) : \text{psi}(i) := y : x := x + h : print(x, y, \text{phi}(x), \text{abs}(y - \text{phi}(x))) :$ **od**

$$y := 0.0005000000000$$
$$\psi(1) := 0.0005000000000$$
$$x := 0.1$$
$$0.1,\ 0.0005000000000,\ 0.0003333349060,\ 0.0001666650940$$
$$y := 0.003000125004$$
$$\psi(2) := 0.003000125004$$

$$x := 0.2$$

$$0.2,\ 0.003000125004,\ 0.002666869814,\ 0.000333255190$$

$$y := 0.009503025760$$

$$\psi(3) := 0.009503025760$$

$$x := 0.3$$

$$0.3,\ 0.009503025760,\ 0.009003473189,\ 0.000499552571$$

$$y := 0.02202467595$$

$$\psi(4) := 0.02202467595$$

$$x := 0.4$$

$$0.4,\ 0.02202467595,\ 0.02135938017,\ 0.00066529578$$

$$y := 0.04262140864$$

$$\psi(5) := 0.04262140864$$

$$x := 0.5$$

$$0.5,\ 0.04262140864,\ 0.04179114620,\ 0.00083026244$$

$$y := 0.07344210066$$

$$\psi(6) := 0.07344210066$$

$$x := 0.6$$

$$0.6,\ 0.07344210066,\ 0.07244786117,\ 0.00099423949$$

$$y := 0.1168165840$$

$$\psi(7) := 0.1168165840$$

$$x := 0.7$$

$$0.7,\ 0.1168165840,\ 0.1156598536,\ 0.0011567304$$

$$y := 0.1753963673$$

$$\psi(8) := 0.1753963673$$

$$x := 0.8$$

$$0.8,\ 0.1753963673,\ 0.1740802646,\ 0.0013161027$$

$$y := 0.2523742135$$

$$\psi(9) := 0.2523742135$$

$$x := 0.9$$

$$0.9,\ 0.2523742135,\ 0.2509066825,\ 0.0014675310$$

$$y := 0.3518301326$$

$$\psi(10) := 0.3518301326$$

$$x := 1.0$$

$$1.0,\ 0.3518301326,\ 0.3502318440,\ 0.0015982886$$

$$y := 0.4792938348$$

$$\psi(11) := 0.4792938348$$

$$x := 1.1$$

1.1, 0.4792938348, 0.4776170219, 0.0016768129

$$y := 0.6427029949$$

$$\psi(12) := 0.6427029949$$

$$x := 1.2$$

1.2, 0.6427029949, 0.6410767262, 0.0016262687

$$y := 0.8541363558$$

$$\psi(13) := 0.8541363558$$

$$x := 1.3$$

1.3, 0.8541363558, 0.8528799930, 0.0012563628

$$y := 1.133184603$$

$$\psi(14) := 1.133184603$$

$$x := 1.4$$

1.4, 1.133184603, 1.133112675, 0.000071928

$$y := 1.514119178$$

$$\psi(15) := 1.514119178$$

$$x := 1.5$$

1.5, 1.514119178, 1.517447543, 0.003328365

$$y := 2.062972003$$

$$\psi(16) := 2.062972003$$

$$x := 1.6$$

1.6, 2.062972003, 2.076423381, 0.013451378

$$y := 2.924894430$$

$$\psi(17) := 2.924894430$$

$$x := 1.7$$

1.7, 2.924894430, 2.972797219, 0.047902789

$$y := 4.487143656$$

$$\psi(18) := 4.487143656$$

$$x := 1.8$$

1.8, 4.487143656, 4.688130710, 0.200987054

$$y := 8.165117641$$

$$\psi(19) := 8.165117641$$

$$x := 1.9$$

1.9, 8.165117641, 9.566995342, 1.401877701

$$y := 23.42048639$$

$$\psi(20) := 23.42048639$$

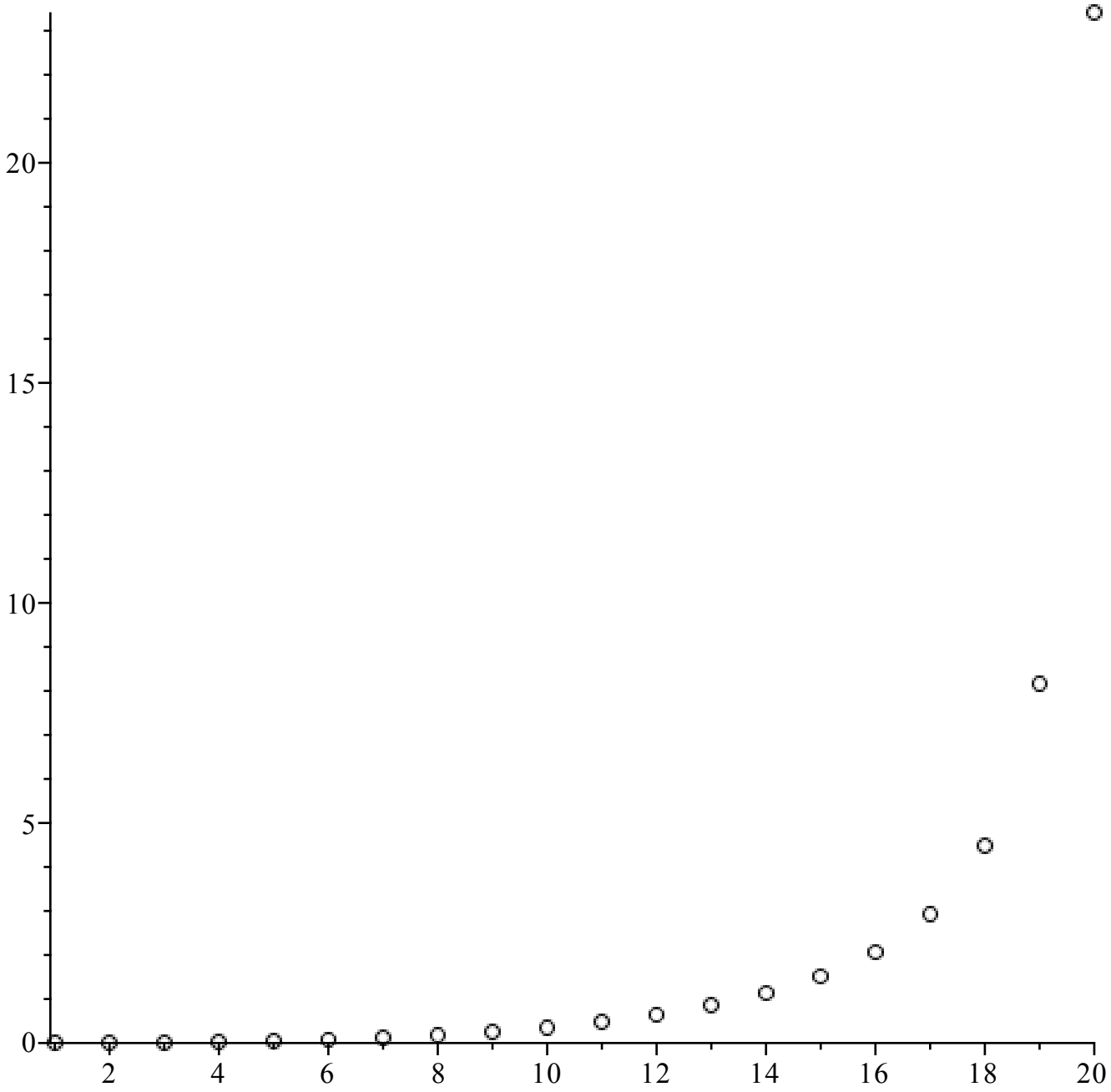
$$x := 2.0$$

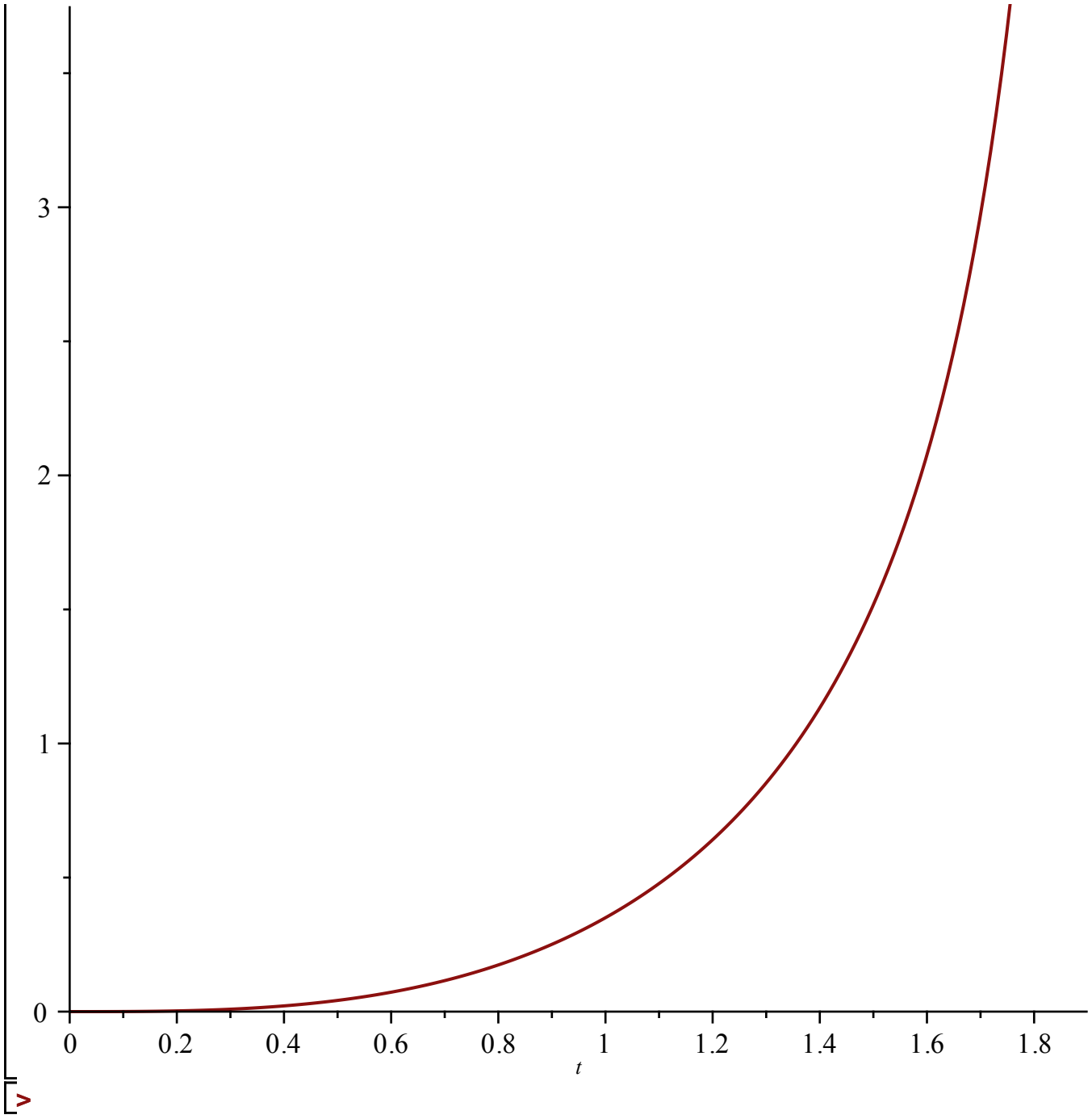
2.0, 23.42048639, 317.7224621, 294.3019757

```
> y := 'y'
```

$$y := y \tag{69}$$

```
> points := [[n, psi(n)]$n = 1..20] : with(plots) : pointplot(points, symbol = circle);
      plot(phi(t), t = 0..2)
```




3
2
1
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
t
v