> $eq1 := 2\,x - x\hat{}2 - x\cdot y = 0$

$$eq1 := -x^2 - x\,y + 2\,x = 0 \quad (1)$$

> $eq2 := -y + x\cdot y = 0$

$$eq2 := x\,y - y = 0 \quad (2)$$

> $dsolve(\{eq1, eq2\}, \{x, y\})$

$$\{x = 1, y = 1\}, \{x = 0, y = 0\}, \{x = 2, y = 0\} \quad (3)$$

> $with(LinearAlgebra) : with(VectorCalculus)$

[&x, `*`, `+`, `-`, `.`, <,>, <|>, About, AddCoordinates, ArcLength, BasisFormat, Binormal, ConvertVector, CrossProduct, Curl, Curvature, D, Del, DirectionalDiff, Divergence, DotProduct, Flux, GetCoordinateParameters, GetCoordinates, GetNames, GetPVDescription, GetRootPoint, GetSpace, Gradient, Hessian, IsPositionVector, IsRootedVector, IsVectorField, Jacobian, Laplacian, LineInt, MapToBasis, ∇, Norm, Normalize, PathInt, PlotPositionVector, PlotVector, PositionVector, PrincipalNormal, RadiusOfCurvature, RootedVector, ScalarPotential, SetCoordinateParameters, SetCoordinates, SpaceCurve, SurfaceInt, TNBFrame, TangentLine, TangentPlane, TangentVector, Torsion, Vector, VectorField, VectorPotential, VectorSpace, Wronskian, diff, eval, evalVF, int, limit, series] (4)

> $f1 := (x, y) \to 2\,x - x\hat{}2 - x\cdot y$

$$f1 := (x, y) \mapsto 2\cdot x + \left(-x^2\right) + \left(-x\cdot y\right) \quad (5)$$

> $f2 := (x, y) \to -y + x\cdot y$

$$f2 := (x, y) \mapsto -y + x\cdot y \quad (6)$$

> $Jm := Jacobian([f1(x, y), f2(x, y)], [x, y])$

$$Jm := \begin{bmatrix} -2\,x - y + 2 & -x \\ y & x - 1 \end{bmatrix} \quad (7)$$

> $p1 := 1$

$$p1 := 1 \quad (8)$$

> $p2 := 1$

$$p2 := 1 \quad (9)$$

> $A := subs([x = p1, y = p2], Jm)$

$$A := \begin{bmatrix} -1 & -1 \\ 1 & 0 \end{bmatrix} \quad (10)$$

> $Eigenvalues(A)$

$$\begin{bmatrix} -\frac{1}{2} + \frac{I\sqrt{3}}{2} \\ -\frac{1}{2} - \frac{I\sqrt{3}}{2} \end{bmatrix} \quad (11)$$

> $p1 := 0$

$$p1 := 0 \quad (12)$$

```
> p2 := 0
```

$$p2 := 0 \tag{13}$$

```
> A := subs([x = p1, y = p2], Jm)
```

$$A := \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix} \tag{14}$$

```
> Eigenvalues(A)
```

$$\begin{bmatrix} 2 \\ -1 \end{bmatrix} \tag{15}$$

```
> p1 := 2
```

$$p1 := 2 \tag{16}$$

```
> p2 := 0
```

$$p2 := 0 \tag{17}$$

```
> A := subs([x = p1, y = p2], Jm)
```

$$A := \begin{bmatrix} -2 & -2 \\ 0 & 1 \end{bmatrix} \tag{18}$$

```
> Eigenvalues(A)
```

$$\begin{bmatrix} 1 \\ -2 \end{bmatrix} \tag{19}$$

```
>
> with(DEtools)
```

[*AreSimilar*, *Closure*, *DEnormal*, *DEplot*, *DEplot3d*, *DEplot_polygon*, *DFactor*, *DFactorLCLM*, *DFactorsols*, *Dchangevar*, *Desingularize*, *FindODE*, *FunctionDecomposition*, *GCRD*, *Gosper*, *Heunsols*, *Homomorphisms*, *IVPsol*, *IsHyperexponential*, *LCLM*, *MeijerGsols*, *MultiplicativeDecomposition*, *ODEInvariants*, *PDEchangecoords*, *PolynomialNormalForm*, *RationalCanonicalForm*, *ReduceHyperexp*, *RiemannPsols*, *Xchange*, *Xcommutator*, *Xgauge*, *Zeilberger*, *abelsol*, *adjoint*, *autonomous*, *bernoullisol*, *buildsol*, *buildsym*, *canoni*, *caseplot*, *casesplit*, *checkrank*, *chinisol*, *clairautsol*, *constcoeffsols*, *convertAlg*, *convertsys*, *dalembertsol*, *dcoeffs*, *de2diffop*, *dfieldplot*, *diff_table*, *diffop2de*, *dperiodic_sols*, *dpolyform*, *dsubs*, *eigenring*, *endomorphism_charpoly*, *equinv*, *eta_k*, *eulersols*, *exactsol*, *expsols*, *exterior_power*, *firint*, *firtest*, *formal_sol*, *gen_exp*, *generate_ic*, *genhomosol*, *gensys*, *hamilton_eqs*, *hypergeometricsols*, *hypergeomsols*, *hyperode*, *indicialeq*, *infgen*, *initialdata*, *integrate_sols*, *intfactor*, *invariants*, *kovacicsols*, *leftdivision*, *liesol*, *line_int*, *linearsol*, *matrixDE*, *matrix_riccati*, *maxdimsystems*, *moser_reduce*, *muchange*, *mult*, *mutest*, *newton_polygon*, *normalG2*, *ode_int_y*, *ode_y1*, *odeadvisor*, *odepde*, *parametricsol*, *particularsol*, *phaseportrait*, *poincare*, *polysols*, *power_equivalent*, *rational_equivalent*, *ratsols*, *redode*, *reduceOrder*, *reduce_order*, *regular_parts*, *regularsp*, *remove_RootOf*, *riccati_system*, *riccatisol*, *rifread*, *rifsimp*, *rightdivision*, *rtaylor*, *separablesol*, *singularities*, *solve_group*, *super_reduce*, *symgen*, *symmetric_power*, *symmetric_product*, *symtest*, *transinv*, *translate*, **(20)**

untranslate, varparam, zoom]

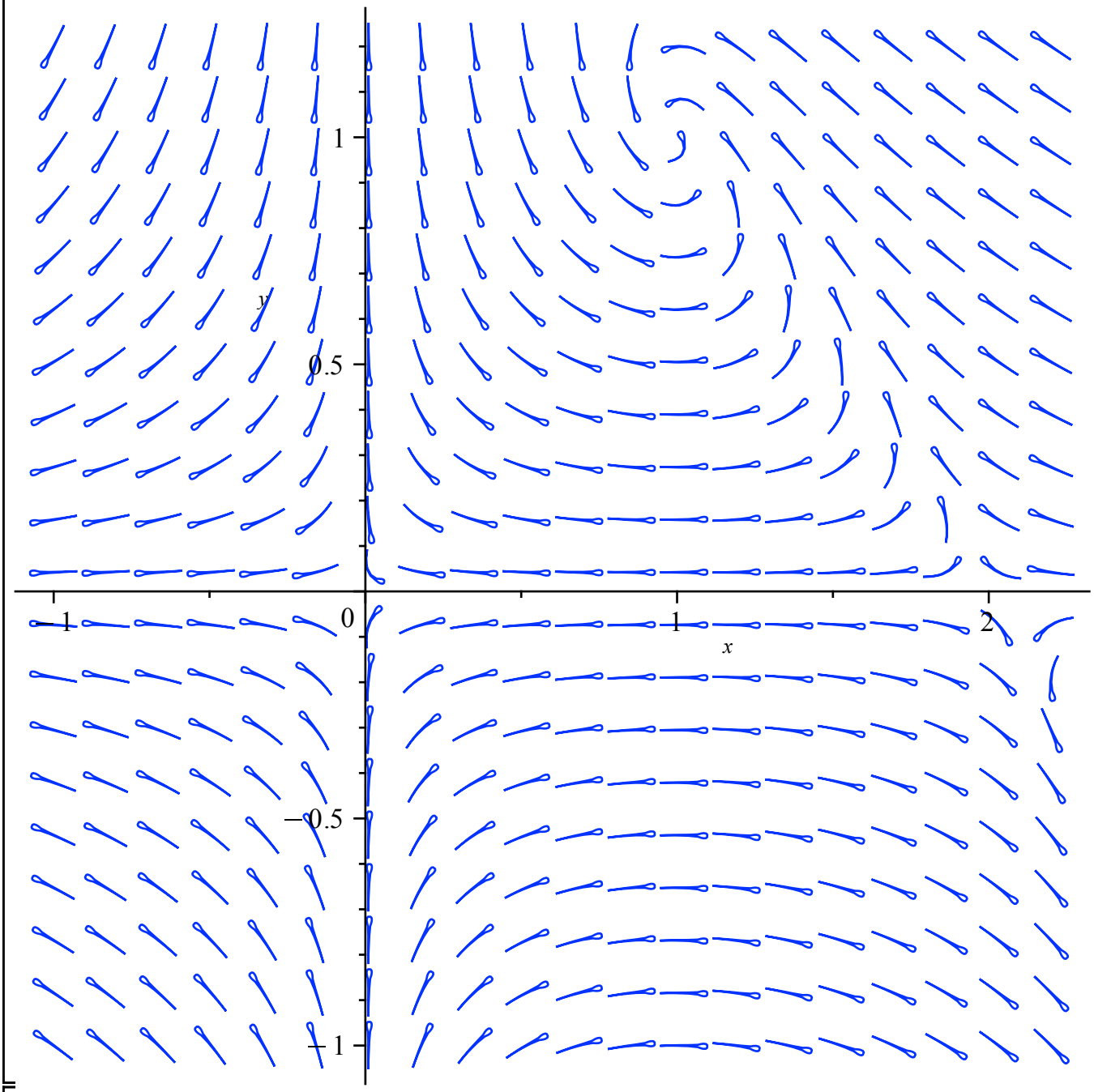
```
> eq1 := diff(x(t), t) = 2·x(t) − x(t)^2 − x(t)·y(t)
```

$$eq1 := \frac{d}{dt} x(t) = 2\,x(t) - x(t)^2 - x(t)\,y(t) \tag{21}$$

```
> eq2 := diff(y(t), t) = x(t)·y(t) − y(t)
```

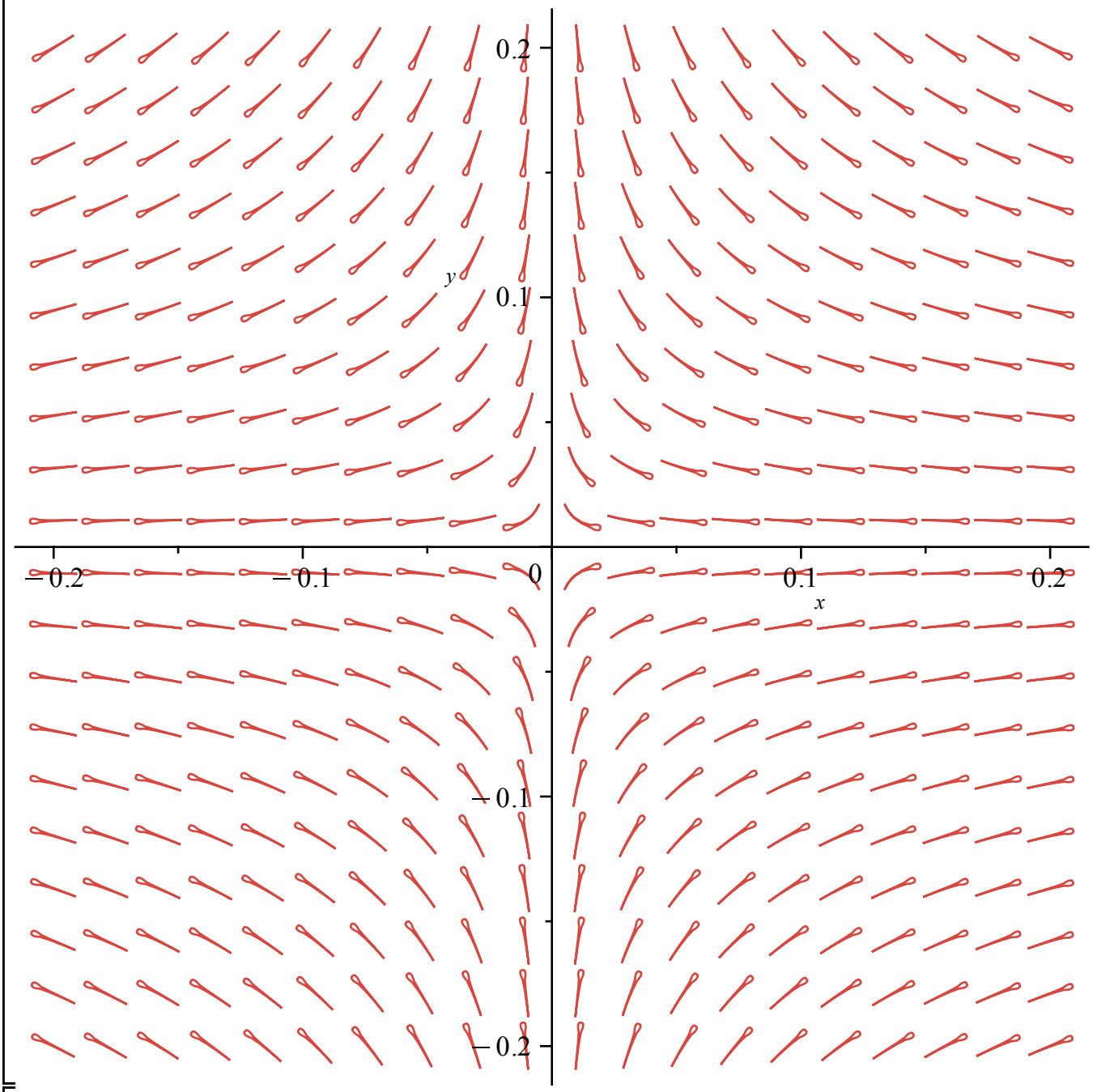
$$eq2 := \frac{d}{dt} y(t) = x(t)\,y(t) - y(t) \tag{22}$$

```
> dfieldplot([eq1, eq2], [x(t), y(t)], t = −100..100, x = −1..2.2, y = −1..1.2, arrows = comet,
      color = blue)
```

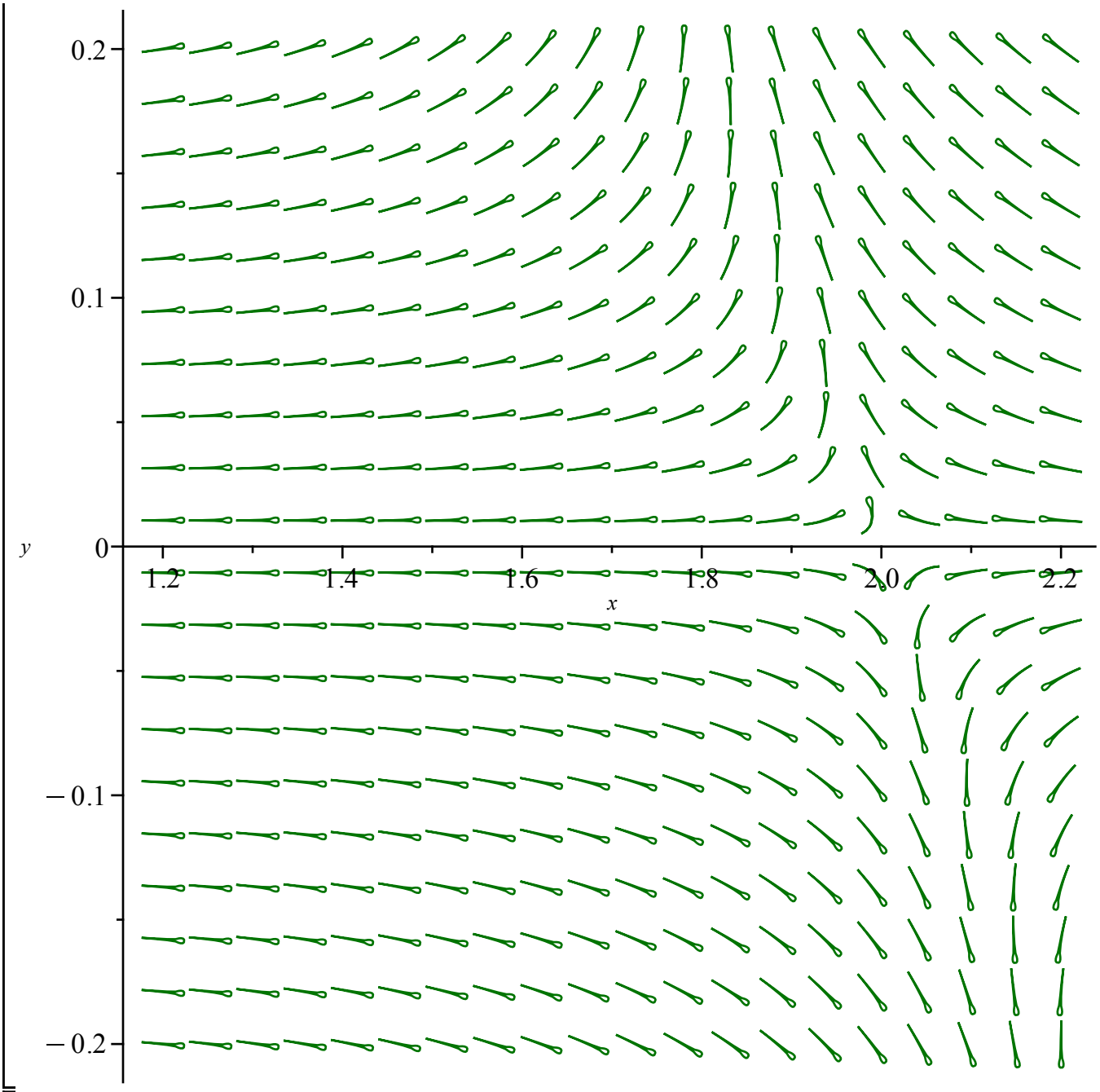



```
> dfieldplot([eq1, eq2], [x(t), y(t)], t = −1..1, x = −0.2..0.2, y = −0.2..0.2, arrows = comet,
```

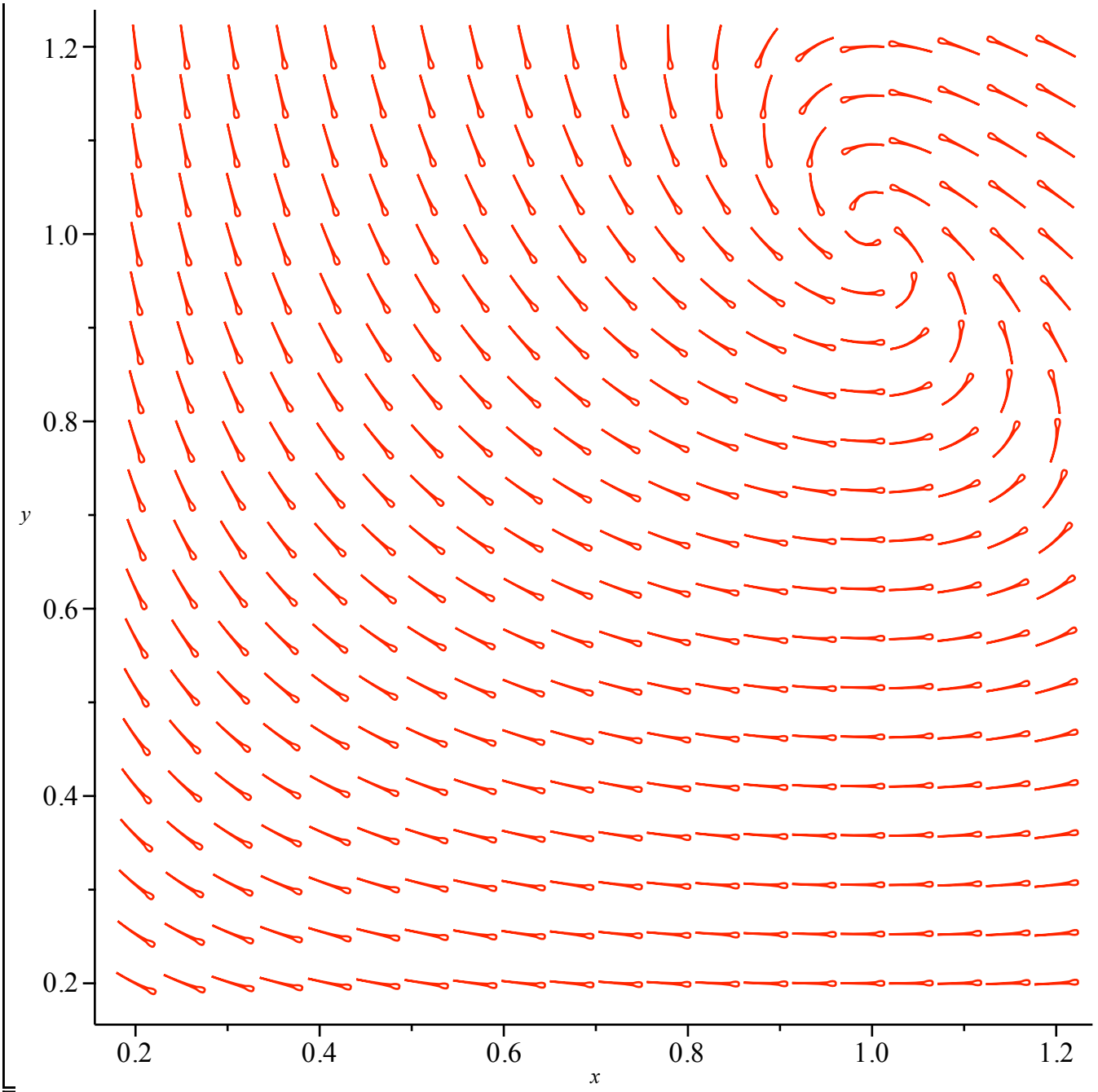
color = *orange*)



> *dfieldplot*([*eq1*, *eq2*], [*x*(*t*), *y*(*t*)], *t* = −10 ..10, *x* = 1.2 ..2.2, *y* = −0.2 ..0.2, *arrows* = *comet*, *color* = *darkgreen*)




```
> dfieldplot([eq1, eq2], [x(t), y(t)], t = −1 ..1, x = 0.2 ..1.2, y = 0.2 ..1.2, arrows = comet, color
  = red)
```



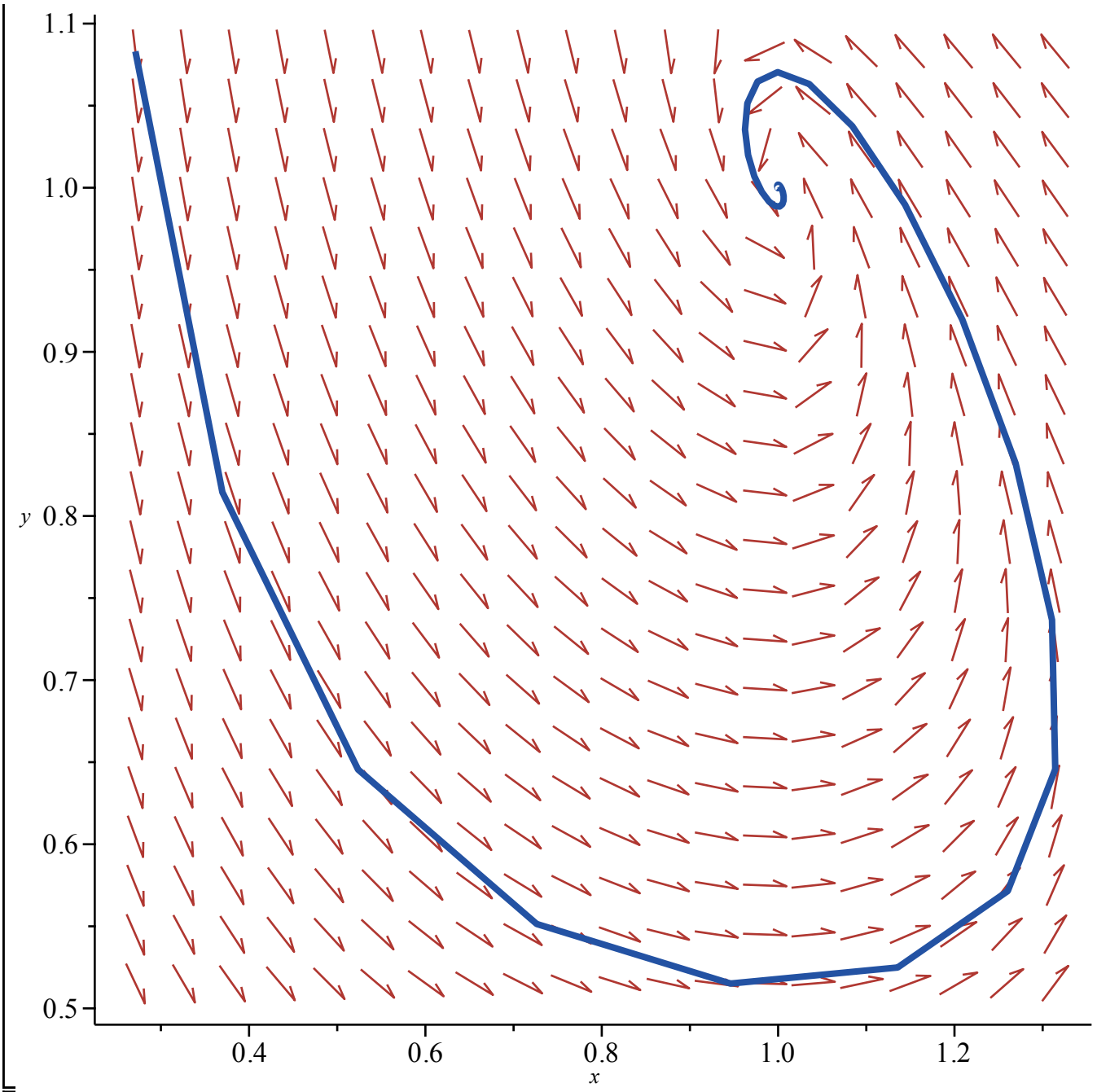
> $ic1 := x(1) = 0.6$

$$ic1 := x(1) = 0.6 \quad \textbf{(23)}$$

> $ic2 := y(1) = 0.6$

$$ic2 := y(1) = 0.6 \quad \textbf{(24)}$$

> $DEplot([eq1, eq2], [x(t), y(t)], t = 0..20, [[ic1, ic2]])$



> *eq1*

$$\frac{d}{dt}x(t) = 2\,x(t) - x(t)^2 - x(t)\,y(t) \quad \textbf{(25)}$$

> *eq2*

$$\frac{d}{dt}y(t) = x(t)\,y(t) - y(t) \quad \textbf{(26)}$$

> *ic1*

$$x(1) = 0.6 \quad \textbf{(27)}$$

> *ic2*

$$y(1) = 0.6 \quad \textbf{(28)}$$

> *eq1* := $x - 2\cdot x\cdot y = 0$

$$eq1 := -2xy + x = 0 \quad (29)$$

> $eq2 := \frac{(x\hat{}2)}{2} - y = 0$

$$eq2 := \frac{x^2}{2} - y = 0 \quad (30)$$

> $dsolve(\{eq1, eq2\}, \{x, y\})$

$$\{x = 0, y = 0\}, \left\{x = 1, y = \frac{1}{2}\right\}, \left\{x = -1, y = \frac{1}{2}\right\} \quad (31)$$

> $f1 := (x, y) \to x - 2 \cdot x \cdot y$

$$f1 := (x, y) \mapsto x + (-2 \cdot x \cdot y) \quad (32)$$

> $f2 := (x, y) \to \frac{(x\hat{}2)}{2} - y$

$$f2 := (x, y) \mapsto x^2 \cdot 2^{-1} + (-y) \quad (33)$$

> $Jm := Jacobian([f1(x, y), f2(x, y)], [x, y])$

$$Jm := \begin{bmatrix} -2y + 1 & -2x \\ x & -1 \end{bmatrix} \quad (34)$$

> $p1 := 0$

$$p1 := 0 \quad (35)$$

> $p2 := 0$

$$p2 := 0 \quad (36)$$

> $A := subs([x = p1, y = p2], Jm)$

$$A := \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad (37)$$

> $Eigenvalues(A)$

$$\begin{bmatrix} 1 \\ -1 \end{bmatrix} \quad (38)$$

> $p1 := 1$

$$p1 := 1 \quad (39)$$

> $p2 := \frac{1}{2}$

$$p2 := \frac{1}{2} \quad (40)$$

> $A := subs([x = p1, y = p2], Jm)$

$$A := \begin{bmatrix} 0 & -2 \\ 1 & -1 \end{bmatrix} \quad (41)$$

> $Eigenvalues(A)$

$$\begin{bmatrix} -\frac{1}{2}+\frac{I\sqrt{7}}{2} \\ -\frac{1}{2}-\frac{I\sqrt{7}}{2} \end{bmatrix} \quad (42)$$

> $p1 := -1$

$$p1 := -1 \quad (43)$$

> $A := subs([x = p1, y = p2], Jm)$

$$A := \begin{bmatrix} 0 & 2 \\ -1 & -1 \end{bmatrix} \quad (44)$$

> $Eigenvalues(A)$

$$\begin{bmatrix} -\frac{1}{2}+\frac{I\sqrt{7}}{2} \\ -\frac{1}{2}-\frac{I\sqrt{7}}{2} \end{bmatrix} \quad (45)$$

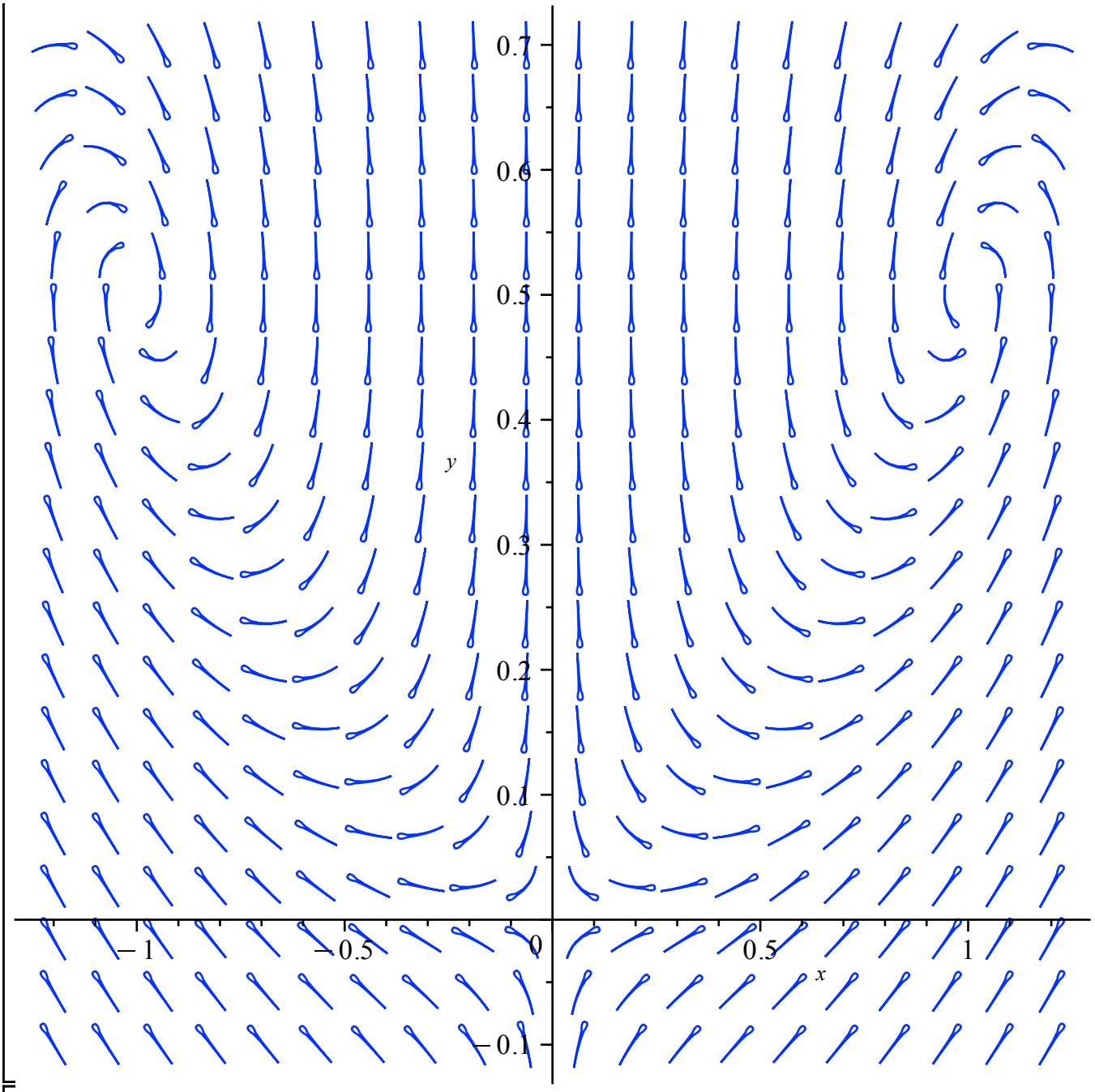
> $eq1 := diff(x(t), t) = x(t) - 2 \cdot x(t) \cdot y(t)$

$$eq1 := \frac{d}{dt} x(t) = x(t) - 2\, x(t)\, y(t) \quad (46)$$

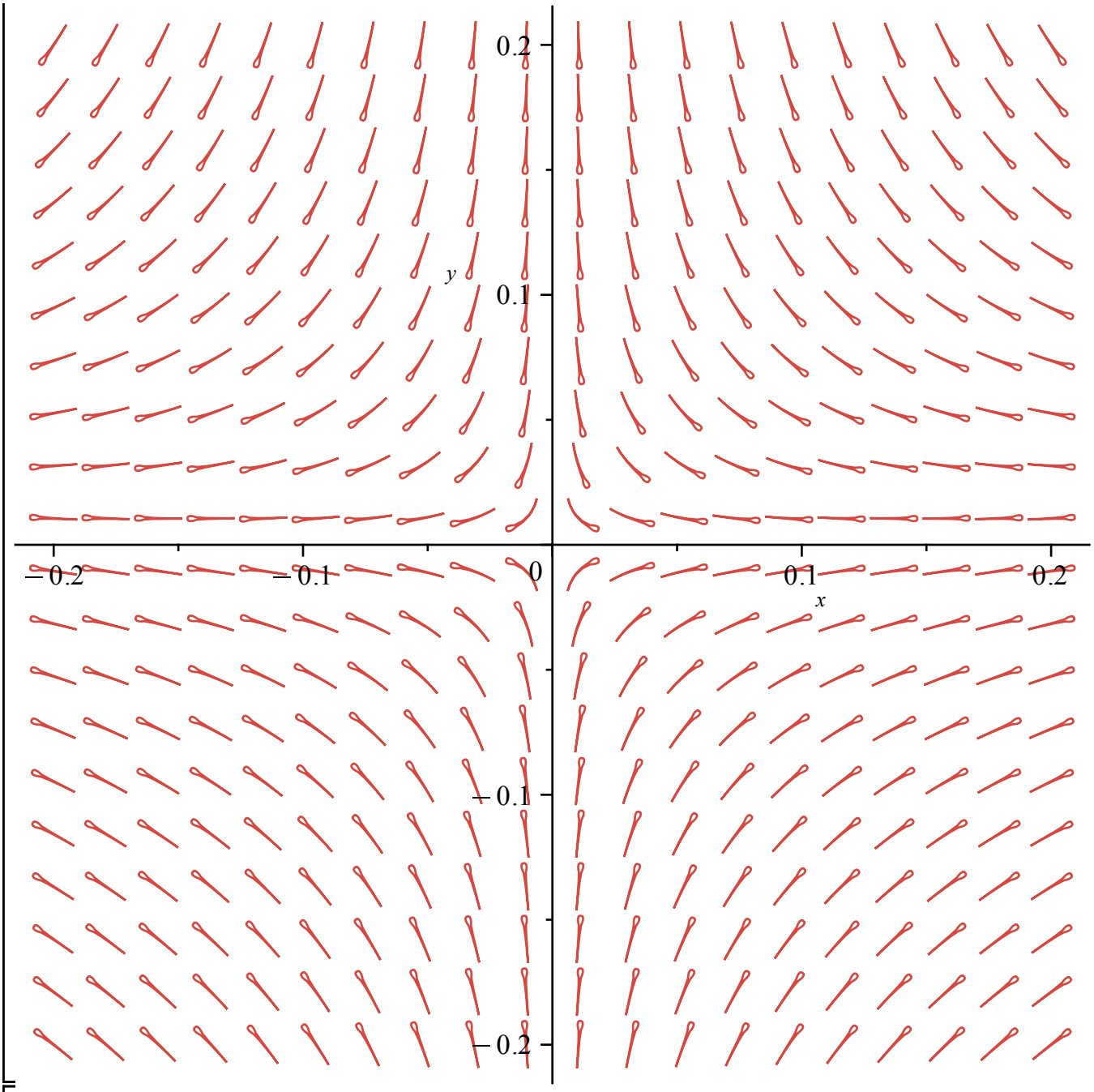
> $eq2 := diff(y(t), t) = \frac{(x(t)\hat{}2)}{2} - y(t)$

$$eq2 := \frac{d}{dt} y(t) = \frac{x(t)^2}{2} - y(t) \quad (47)$$

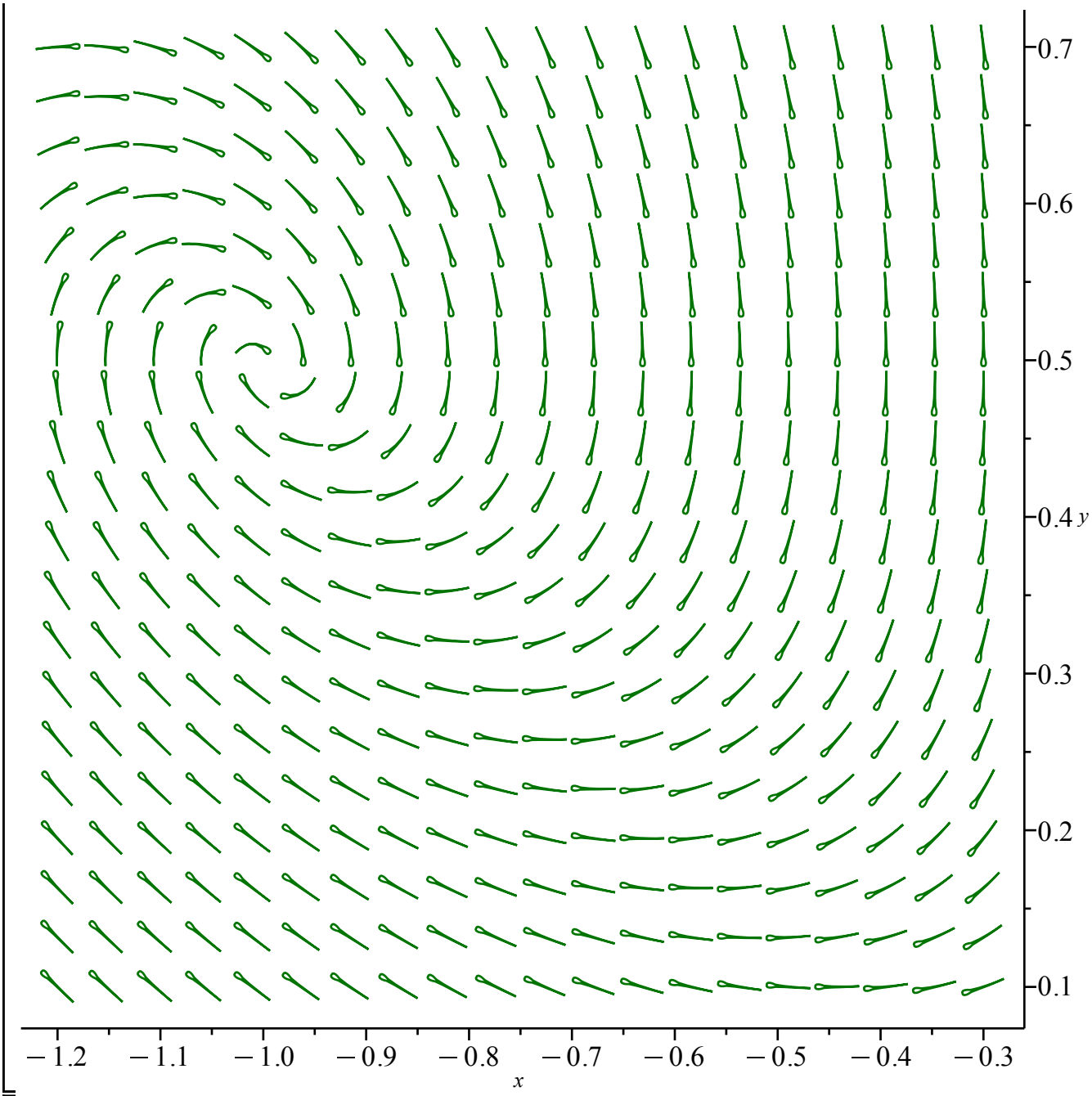
> $dfieldplot([eq1, eq2], [x(t), y(t)], t = -100..100, x = -1.2..1.2, y = -0.1..0.7, arrows = comet, color = blue)$



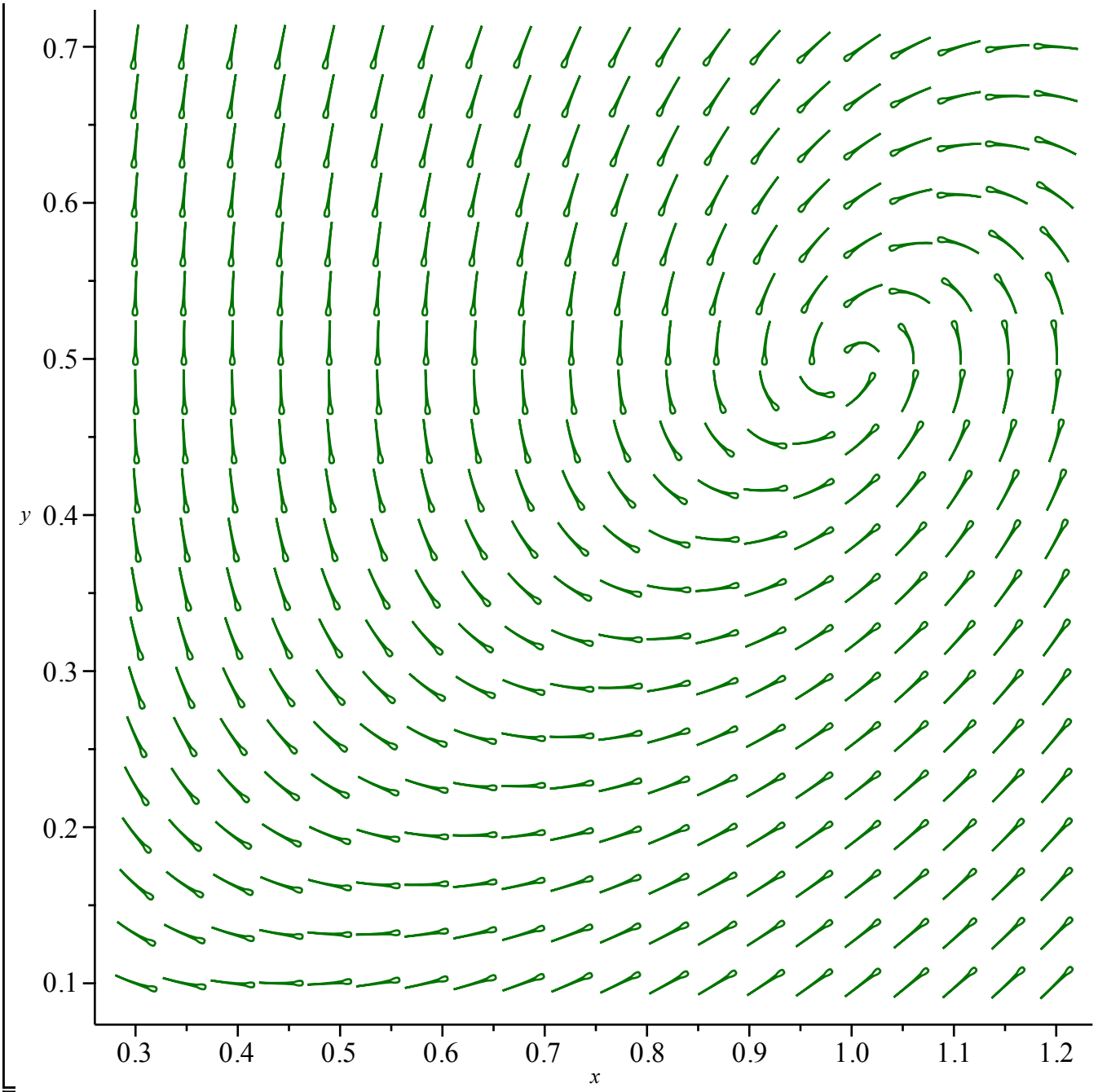

```
> dfieldplot([eq1, eq2], [x(t), y(t)], t = −1 ..1, x = −0.2 ..0.2, y = −0.2 ..0.2, arrows = comet,
  color = orange)
```




```
> dfieldplot([eq1, eq2], [x(t), y(t)], t = −10..10, x = −1.2..−0.3, y = 0.1..0.7, arrows = comet,
    color = darkgreen)
```




```
> dfieldplot( [eq1, eq2], [x(t), y(t) ], t = −10 ..10, x = 0.3 ..1.2, y = 0.1 ..0.7, arrows = comet,
    color = darkgreen)
```



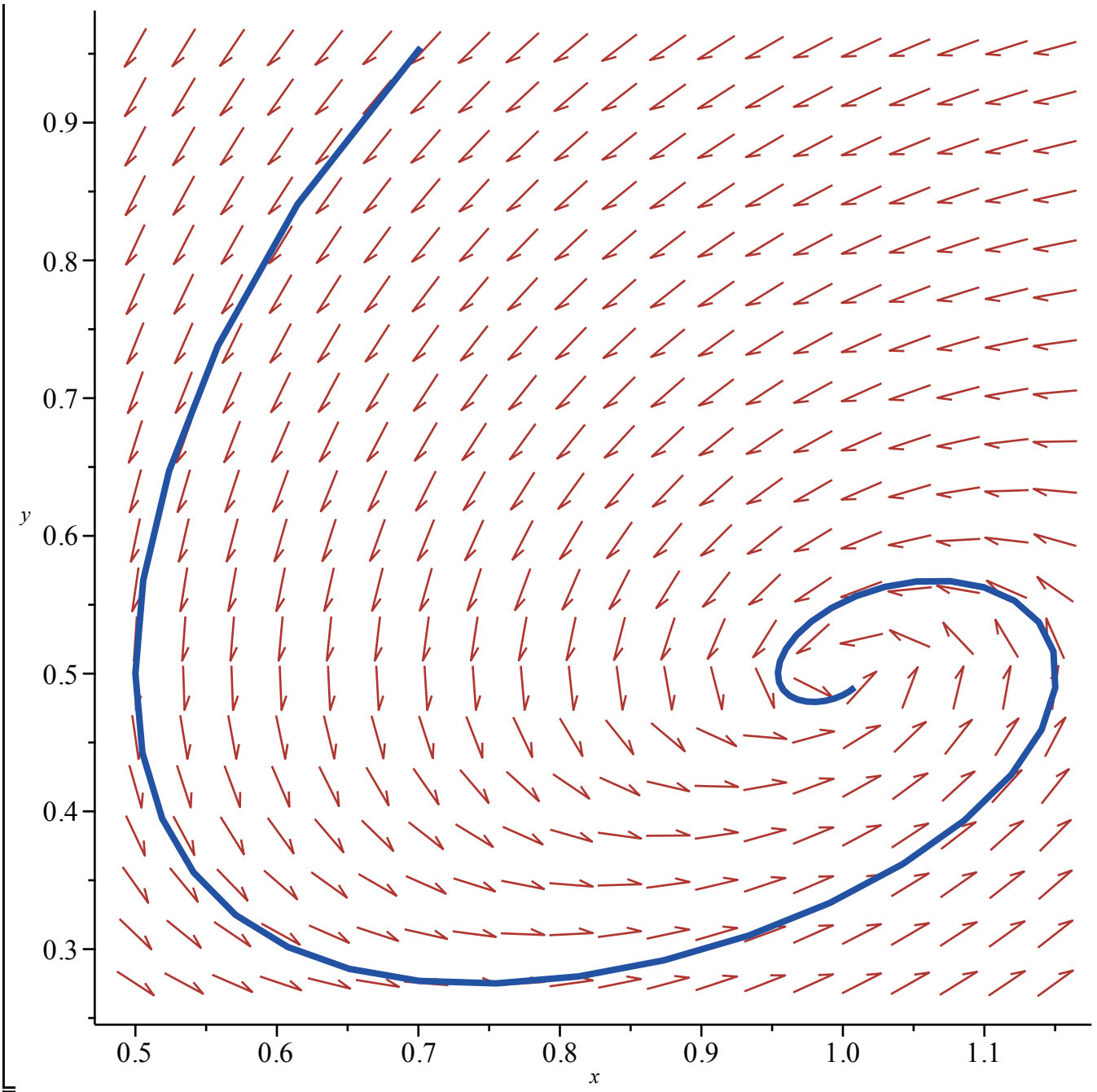
> $ic1 := x(1) = 0.5$

$$ic1 := x(1) = 0.5 \quad \textbf{(48)}$$

> $ic2 := y(1) = 0.5$

$$ic2 := y(1) = 0.5 \quad \textbf{(49)}$$

> $DEplot([eq1, eq2], [x(t), y(t)], t = 0..8, [[ic1, ic2]])$



> $f1 := (x, y) \to y$

$$f1 := (x, y) \mapsto y \tag{50}$$

> $f2 := (x, y) \to -4 \cdot \sin(x)$

$$f2 := (x, y) \mapsto -4 \cdot \sin(x) \tag{51}$$

> $Jm := Jacobian([f1(x, y), f2(x, y)], [x, y])$

$$Jm := \begin{bmatrix} 0 & 1 \\ -4\cos(x) & 0 \end{bmatrix} \tag{52}$$

> $A := subs([x = 0, y = 0], Jm)$

(53)

$$A := \begin{bmatrix} 0 & 1 \\ -4\cos(0) & 0 \end{bmatrix} \quad (53)$$

> A

$$\begin{bmatrix} 0 & 1 \\ -4\cos(0) & 0 \end{bmatrix} \quad (54)$$

> $A[2]$

$$\begin{bmatrix} -4\cos(0) & 0 \end{bmatrix} \quad (55)$$

> $A[2][1]$

$$-4 \quad (56)$$

> A

$$\begin{bmatrix} 0 & 1 \\ -4\cos(0) & 0 \end{bmatrix} \quad (57)$$

> $Eigenvalues(A)$

$$\begin{bmatrix} 2\,I \\ -2\,I \end{bmatrix} \quad (58)$$

> $dsolve\left(diff(y(x),x) = -\frac{4\sin(x)}{y(x)}\right)$

$$y(x) = \sqrt{8\cos(x) + c_1}, y(x) = -\sqrt{8\cos(x) + c_1} \quad (59)$$

> $H := (x,y) \to y\hat{}2 - 8\cdot\cos(x)$

$$H := (x,y) \mapsto y^2 + (-8\cdot\cos(x)) \quad (60)$$

> $diff(H(x,y),x)\cdot y$

$$8\sin(x)\,y \quad (61)$$

> $-4\cdot\sin(x)\cdot diff(H(x,y),y)$

$$-8\sin(x)\,y \quad (62)$$

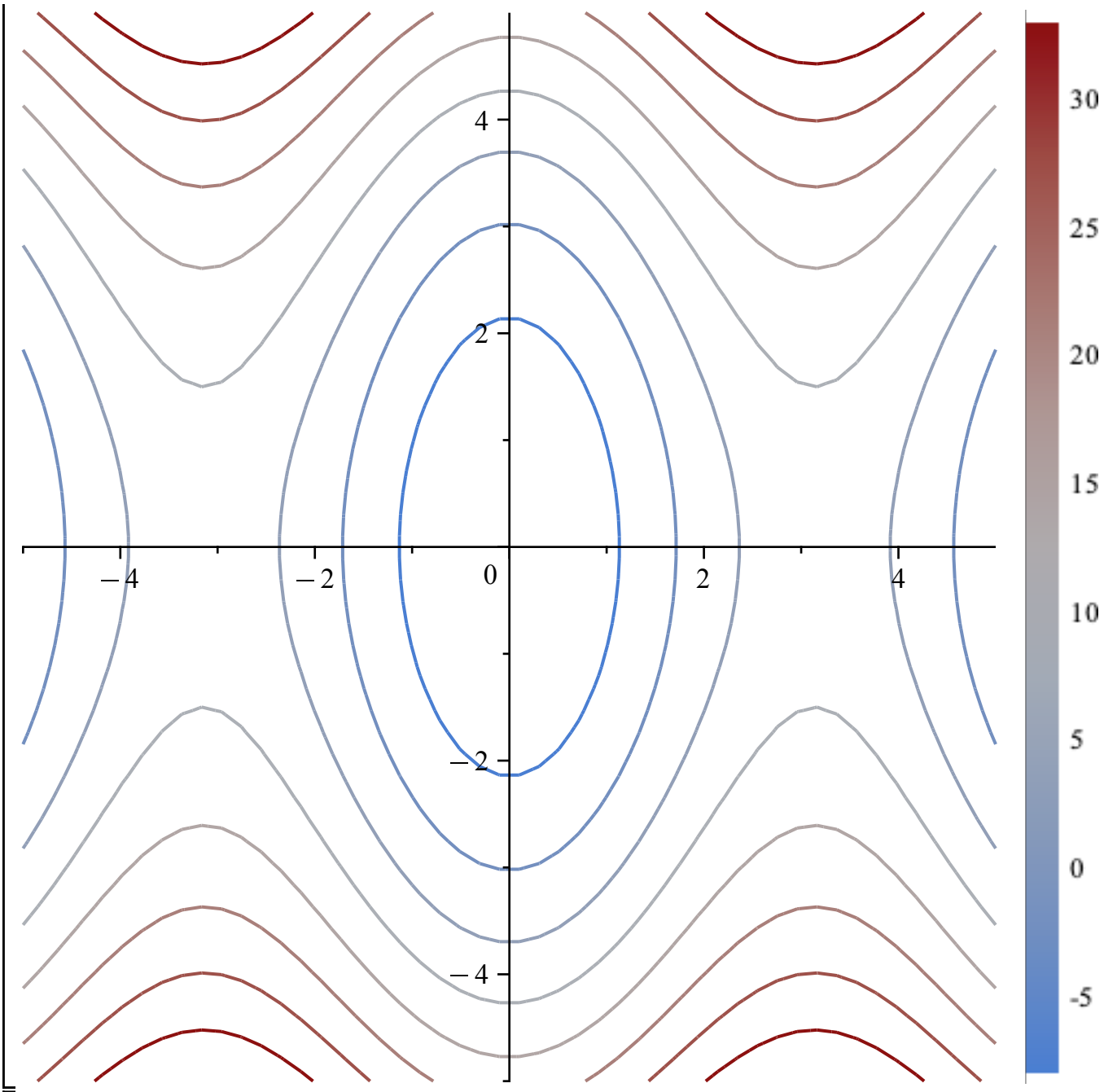
> $diff(H(x,y),x)\cdot y - 4\cdot\sin(x)\cdot diff(H(x,y),y)$

$$0 \quad (63)$$

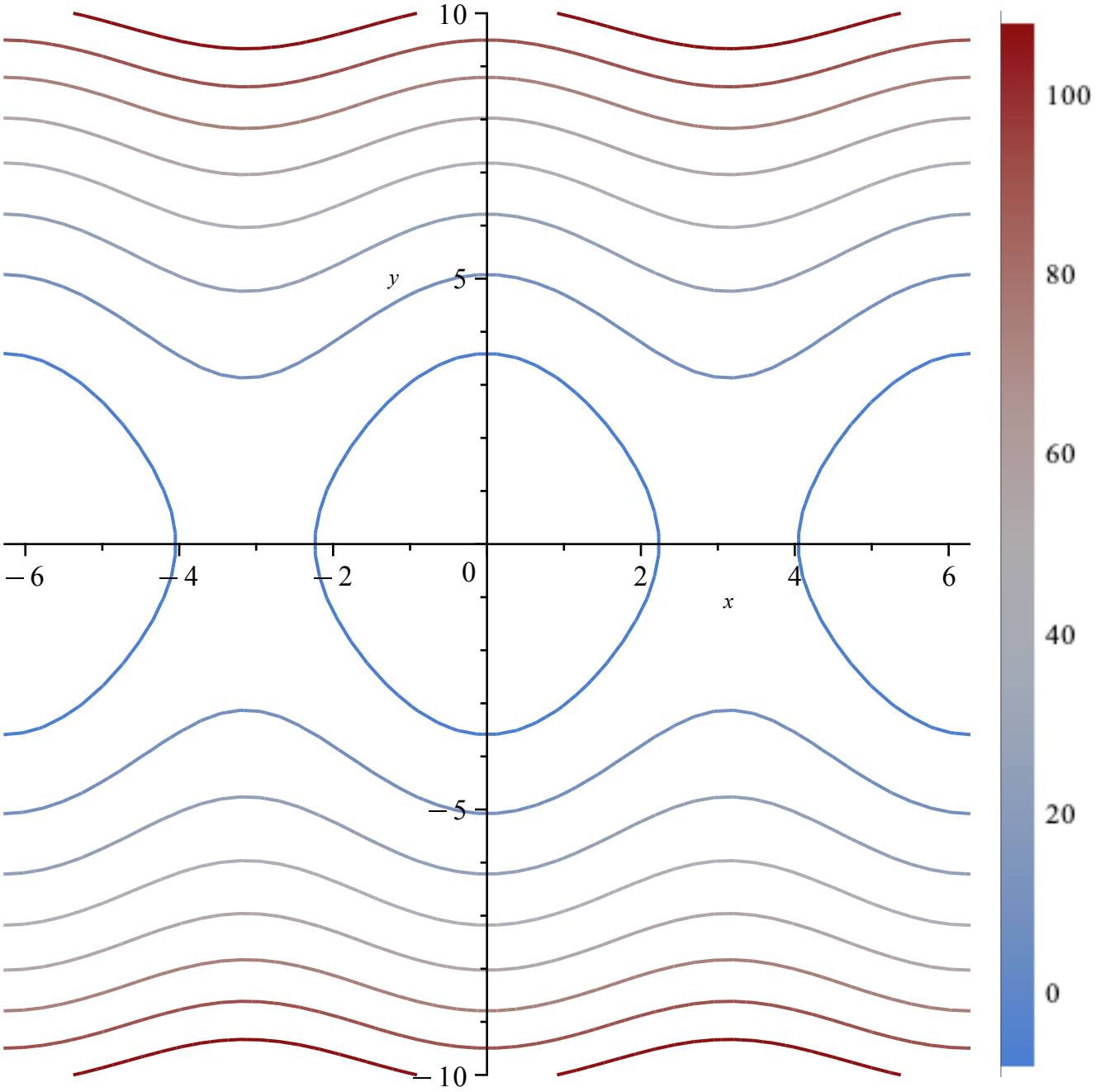
> $with(plots)$

[*animate*, *animate3d*, *animatecurve*, *arrow*, *changecoords*, *complexplot*, *complexplot3d*, *conformal*, *conformal3d*, *contourplot*, *contourplot3d*, *coordplot*, *coordplot3d*, *densityplot*, *display*, *dualaxisplot*, *fieldplot*, *fieldplot3d*, *gradplot*, *gradplot3d*, *implicitplot*, *implicitplot3d*, *inequal*, *interactive*, *interactiveparams*, *intersectplot*, *listcontplot*, *listcontplot3d*, *listdensityplot*, *listplot*, *listplot3d*, *loglogplot*, *logplot*, *matrixplot*, *multiple*, *odeplot*, *pareto*, *plotcompare*, *pointplot*, *pointplot3d*, *polarplot*, *polygonplot*, *polygonplot3d*, *polyhedra_supported*, *polyhedraplot*, *rootlocus*, *semilogplot*, *setcolors*, *setoptions*, *setoptions3d*, *shadebetween*, *spacecurve*, *sparsematrixplot*, *surfdata*, *textplot*, *textplot3d*, *tubeplot*] (64)

> $contourplot(H(x,y), x = -5..5, y = -5..5)$




```
> contourplot(H(x, y))
```



> $f1 := (x, y) \to x - x \cdot y$

$$f1 := (x, y) \mapsto x + (-x \cdot y) \quad \textbf{(65)}$$

> $f2 := (x, y) \to -0.3 \cdot y + 0.3 \cdot x \cdot y$

$$f2 := (x, y) \mapsto -0.3 \cdot y + 0.3 \cdot x \cdot y \quad \textbf{(66)}$$

> $Jm := Jacobian([f1(x, y), f2(x, y)], [x, y])$

$$Jm := \begin{bmatrix} -y + 1 & -x \\ 0.3\,y & -0.3 + 0.3\,x \end{bmatrix} \quad \textbf{(67)}$$

> $A := subs([x = 1, y = 1], Jm)$

(68)

$$A := \begin{bmatrix} 0 & -1 \\ 0.3 & 0. \end{bmatrix} \quad (68)$$

> $Eigenvalues(A)$

$$\begin{bmatrix} 0. + 0.547722557505166\,\mathrm{I} \\ 0. - 0.547722557505166\,\mathrm{I} \end{bmatrix} \quad (69)$$

> $f1(x, y)$

$$-x\,y + x \quad (70)$$

> $H := (x, y) \to y - \ln(y) + 0.3(x - \ln(x))$

$$H := (x, y) \mapsto y + (-\ln(y)) + 0.3 \cdot (x + (-\ln(x))) \quad (71)$$

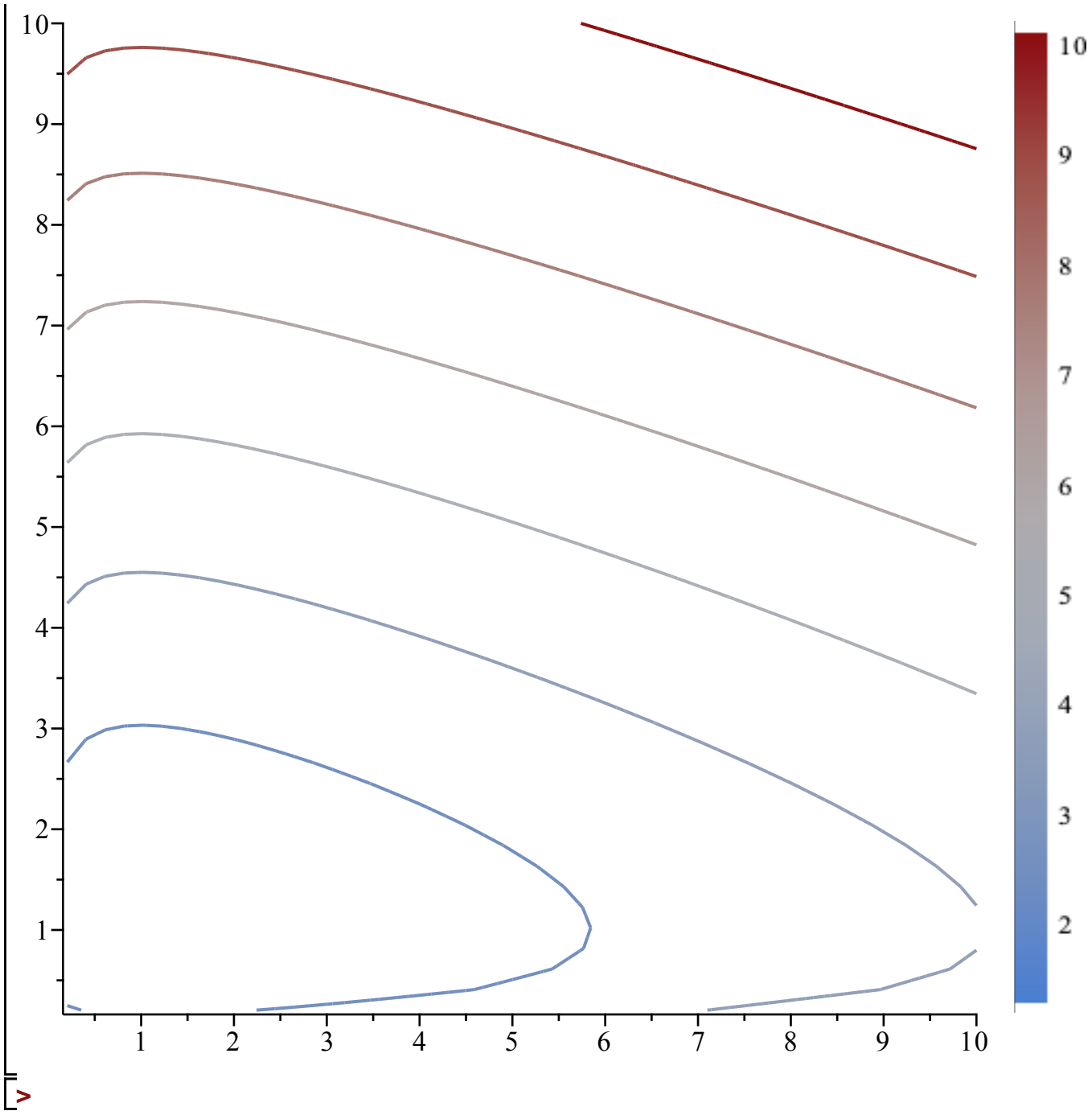
> $f1(x, y)$

$$-x\,y + x \quad (72)$$

> $expand(diff(H(x, y), x) \cdot f1(x, y)) + expand(diff(H(x, y), y) \cdot f2(x, y))$

$$0. \quad (73)$$

> $contourplot(H(x, y))$


10
9
8
7
6
5
4
3
2
1
1
2
3
4
5
6
7
8
9
10
v
10
9
8
7
6
5
4
3
2