Math Tools

Note: Probability and measure

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Contents

References: STAT5005 and *Probability: Theory and Examples*, 4th edition, by Richard Durrett, published by Cambridge University Press.

1 Measure Theory

1.1 Expectation

Lemma 1.1. *Let $X \geq 0$, $p > 0$, we have $\mathbb{E}X^p = \int_0^\infty px^{p-1}\mathrm{P}(X > x)\mathrm{d}x$.*

2 Law of Large Numbers

2.1 Almost Surely Convergence

This lemma gives an equivalent relation between expectation and sum of tail probability.

Lemma 2.1. *Let X_i iid and $\varepsilon > 0$, then $\sum_{n=1}^\infty \mathrm{P}(|X_n| > n\varepsilon) \leq \varepsilon^{-1}\mathbb{E}\,|X_i| \leq \sum_{n=0}^\infty \mathrm{P}(|X_n| > n\varepsilon)$.*

References