Universidad del Valle de Guatemala
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Trabajo Guiado No. 4

1)
$$\int \frac{dx}{\sqrt{x}+9} = \qquad u = \sqrt{x}+9$$

$$= \int 2\frac{(u-9)}{u}\,du \qquad = 2\int \frac{u-9}{u}\,du$$

$$\frac{u-9}{u} : 1 - \frac{9}{u}$$

$$= 2\int 1 - \frac{9}{u}\,du \qquad = 2\left(\int 1du - \int \frac{9}{u}\,du\right.$$

$$1du = u \qquad \int \frac{9}{u}\,du = 9\ln|u| \qquad u = \sqrt{x}+9$$

$$= 2\left(\sqrt{x}+9 - 9\ln|\sqrt{x}+9|\right)$$
$$= 2\left(\sqrt{x}+9 - 9\ln|\sqrt{x}+9|\right)+c$$

2)
$$\int e^{\sqrt{x+1}}\,dx \qquad u = \sqrt{x+1}$$

$$= \int e^{u}\cdot 2u\,du \qquad = 2\cdot\int e^{u}u\,du \qquad u=u,$$
$$v' = e^{u}$$
$$= 2\left(e^{u}u - \int e^{u}du\right)$$

$$\int e^{u}du = e^{u}$$
$$= 2\left(e^{u}u - e^{u}\right) \qquad = 2\left(e^{\sqrt{x+1}}\sqrt{x+1} - e^{\sqrt{x+1}}\right)$$
$$= 2\left(e^{\sqrt{x+1}}\sqrt{x+1} - e^{\sqrt{x+1}}\right)+c$$

3) $\int \frac{3x-1}{x(x^2-4)}\,dx \qquad \frac{3x-1}{x(x^2-4)} : \frac{1}{4x} - \frac{7}{8(x+2)} + \frac{5}{8(x-2)}$

$= \int \frac{1}{4x} - \frac{7}{8(x+2)} + \frac{5}{8(x-2)}\,dx$

$= \int \frac{1}{4x}\,dx - \int \frac{7}{8(x+2)}\,dx + \int \frac{5}{8(x-2)}\,dx$

$\frac{1}{4x}\,dx = \frac{1}{4}\int \frac{1}{x}\,dx = \frac{1}{4}\ln|x|$

$u = x+2$

$\frac{7}{8(x+2)}\,dx = \frac{7}{8}\int \frac{1}{x+2}\,dx = \frac{7}{8}\ln|x+2|$

$\frac{5}{8(x-2)}\,dx = \frac{5}{8}\ln|x-2|$

$\frac{1}{4}\ln|x| - \frac{7}{8}\ln|x+2| + \frac{5}{8}\ln|x-2|$

$= \frac{1}{4}\ln|x| - \frac{7}{8}\ln|x+2| + \frac{5}{8}\ln|x-2| + c$

4) $\int \frac{\ln x}{(x-1)^2}\,dx \qquad u = \ln(x),\ v' = \frac{1}{(x-1)^2}$

$= \frac{\ln(x)}{x-1} - \int -\frac{1}{x(x-1)}\,dx$

$$\int -\frac{1}{x(x-1)}\,dx = -\ln|x-1| + \ln|x|$$

$$= -\frac{\ln|x|}{x-1} - \left(-\ln|x-1| + \ln|x|\right)$$

$$= -\frac{\ln|x|}{x-1} + \ln|x-1| - \ln|x|$$

$$= -\frac{\ln(x)}{x-1} + \ln|x-1| - \ln|x| + C$$

5) $\int \ln(x^2+4)\,dx \qquad u = \ln(x^2+4),\ v' = 1$

$$= x\ln(x^2+4) - \int \frac{2x^2}{x^2+4}\,dx$$

$$\int \frac{2x^2}{x^2+4}\,dx = 2\cdot\int \frac{x^2}{x^2+4}\,dx = \frac{-4}{x^2+4} + 1$$

$$= -2\cdot\int -\frac{4}{x^2+4}\,dx \qquad x = 2v$$

$$2\cdot\int 2\left(-\frac{1}{v^2+1} + 1\right)dv = 2\cdot 2\left(-\int \frac{1}{v^2+1}\,dv + \int dv\right)$$

$$\int \frac{1}{v^2+1}\,dv = \arctan(v) \qquad \int 1\,dv = v$$

$$= 2\cdot 2\left(-\arctan(v) + v\right)$$

$$= 2\cdot 2\left(-\arctan\left(\frac{x}{2}\right) + \frac{x}{2}\right)$$

$= 4\left(-\arctan\left(\frac{x}{2}\right)+\frac{x}{2}\right)$

$= x\ln\left(x^2+4\right)-4\left(-\arctan\left(\frac{x}{2}\right)+\frac{x}{2}\right)$

$= x\ln(x^2+4)-4\left(-\arctan\left(\frac{x}{2}\right)+\frac{x}{2}\right)+c$

6) $\int 8te^{2t^2}dt \qquad u=2t^2$

$= 8\int \frac{e^u}{4}du \qquad = 8\cdot\frac{1}{4}\cdot\int e^u du$

$= 8\cdot\frac{1}{4}e^u \qquad = 8\cdot\frac{1}{4}e^{2t^2}$

$= 2e^{2t^2} \qquad \underline{= 2e^{2t^2}+c}$

7) $\int \frac{x\tan x}{\cos x}dx \qquad \frac{1}{\cos(x)} = \sec(x)$

$\int x\sec(x)\tan(x)dx \qquad u=x,\ v'=\sec(x)\tan(x)$

$= x\sec(x)-\int \sec(x)dx$

$\int \sec(x)dx = \ln\left|\tan(x)+\sec(x)\right|$

$= x\sec(x)-\ln\left|\tan(x)+\sec(x)\right|$

$= x\sec(x)-\ln\left|\tan(x)+\sec(x)\right|+c$

8) $\int \frac{t^5}{1+t^2} dt \qquad \frac{t^5}{1+t^2} : t^3 - t + \frac{t}{t^2+1}$

$= \int t^3 - t + \frac{t}{t^2+1}\, dt$

$= \int t^3 dt - \int t\, dt + \int \frac{t}{t^2+1}\, dt$

$\int t^3 dt = \frac{t^{3+1}}{3+1} = \frac{t^4}{4}$

$\int t\, dt = \frac{t^{1+1}}{1+1} = \frac{t^2}{2}$

$\int \frac{t}{t^2+1}\, dt = \frac{1}{2} \ln|t^2+1| \qquad u = t^2+1$

$= \int \frac{1}{2u}\, du \qquad = \frac{1}{2} \cdot \int \frac{1}{u}\, du$

$= \frac{1}{2} \ln|u| \qquad = \frac{1}{2} \ln|t^2+1|$

$= \frac{t^4}{4} - \frac{t^2}{2} + \frac{1}{2} \ln|t^2+1|$

$= \frac{t^4}{4} - \frac{t^2}{2} + \frac{1}{2} \ln|t^2+1| + c$

9) $\int \tan^{10} x \sec^4 x\, dx$

$$= \int \sec^2(x) \sec^2(x) \tan^{10}(x)\, dx$$

$$= \int (1+\tan^2(x)) \sec^2(x) \tan^{10}(x) dx$$

$u = \tan(x)$

$$= \int u^{10}(1+u^2)\, du \qquad = \int u^{10} + u^{12} du$$

$$u^{10} du = \frac{u^{10+1}}{10+1} = \frac{u^{11}}{11}$$

$$u^{12} du = \frac{u^{13}}{13}$$

$$= \frac{\tan^{11}(x)}{11} + \frac{\tan^{13}(x)}{13}$$

$$= \frac{\tan^{11}(x)}{11} + \frac{\tan^{13}(x)}{13} + C$$

10) $\displaystyle\int \frac{dx}{x^4+10x^3+25x^2}$:

$$\frac{1}{25x^2} + \frac{2}{125(x+5)} + \frac{1}{25(x+5)^2} - \frac{2}{125x}$$

$$\int \frac{1}{25x^2} dx = -\frac{1}{25x}$$

$$\int \frac{2}{125(x+5)} dx = \frac{2}{125} \ln|x+5|$$

$$\int \frac{1}{25(x+5)^2}\,dx = \frac{-1}{25(x+5)}$$

$$\int \frac{2}{125}\,dx = \frac{2}{125}\ln|x|$$

$$= -\frac{1}{25x} + \frac{2}{125}\ln|x+5| - \frac{1}{25(x+5)} - \frac{2}{125}\ln|$$

$$= -\frac{1}{25x} + \frac{2}{125}\ln|x+5| - \frac{1}{25(x+5)} - \frac{2}{125}\ln|x| + C$$

11) $\int \sin(mx)\cos(nx)\,dx$ para $m \neq n$

$$\cos(t)\sin(s) = \frac{\sin(s+t) + \sin(s-t)}{2}$$

$$= \int \frac{\sin(mx+nx) + \sin(mx-nx)}{2}\,dx$$

$$= \frac{1}{2}\int \sin(mx+nx) + \sin(mx-nx)\,dx$$

$$= \frac{1}{2}\left(\int \sin(mx+nx)\,dx + \int \sin(mx-nx)\,dx\right)$$

$$= 1/2\left(-1/_{m+n}\cos(mx+nx) - \frac{1}{m-n}\cos(mx-nx)\right) + C$$

12 $\int x^2 \operatorname{sen}(x^3)\,dx$

$$\int \cancel{x^2} \operatorname{sen}(x^3) \frac{1}{3\cancel{x^2}}\,dt$$

$$\int \operatorname{sen}(x^3)\,\frac{1}{3}\,dt$$

$$\int \frac{\operatorname{sen}(x^3)}{3}\,dt$$

$$\int \frac{\operatorname{sen}(t)}{3}$$

$$\frac{1}{3}\int \operatorname{sen}(t) = \frac{1}{3}\left(-\cos(t)\right)$$

$$\frac{1}{3}\left(-\cos(x^3)\right) = \frac{-\cos(x^3)}{3}$$

$$-\frac{\cos(x^3)}{3} + C$$

13 $\int e^{w}\left(1+e^{w}\right)^{5} dw$

$$\int \cancel{e^{w}}\left(1+e^{w}\right)^{5} \frac{1}{\cancel{e^{w}}}\, dp$$

$$\int \left(1+e^{w}\right)^{5} dp$$

$$\int p^{5}\, dp \qquad \frac{p^{5+1}}{5+1} = \frac{p^{6}}{6}$$

$$\frac{\left(1+e^{w}\right)^{6}}{6} = \frac{\left(1+e^{w}\right)^{6}}{6} + C$$

14) $\dfrac{dx}{(81-20x-x^2)^{3/2}}$

$u = x+10$

$du = dx$

$u = \sin^{-1}\left(\dfrac{u}{2\cdot 3^{3/2}}\right)$

$du = 2\cdot 3^{3/2}\cos$

$$\int \frac{dx}{[108-(x+10)^2]^{3/2}} = \frac{1}{108} = \int \frac{du}{(\omega^2)^{3/2}}$$

$$\frac{1}{108}\int \frac{1}{\cos^2 u}\,du = \frac{1}{108}\int \sec^2 u\,du$$

$$\frac{1}{108}\tan u + C = \frac{1}{108}\tan\left(\sin^{-1}\frac{u}{2\cdot 3^{3/2}}\right)$$

$$\frac{x+10}{3\cdot 3^{3/2}\sqrt{1-\frac{(x+10)^2}{108}}} + C$$

$$\frac{x+10}{108\sqrt{-x-20x+8}} + C$$

15 $\int e^x \tan^2 e^x \, dx$

$\int \left(-1 + \sec^2(e^x)\right) e^x \, dx$

$\int \tan^2(u) \, du$

$\int -1 + \sec^2(u) \, du$

$-\int 1 \, du + \int \sec^2(u) \, du$

$1 \, du = u$

$\int \sec^2(u) \, du = \tan(u)$

$-u + \tan(e^x)$

$-e^x + \tan(e^x) + c$ //

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$$\int \sec^3(x)\, dx$$

$$\frac{\sec(x)\tan(x)}{2} + \frac{1}{2}\int \sec(x)\, dx$$

$$\frac{\ln(\tan(x)+\sec(x))}{2} + \frac{\sec(x)\tan(x)}{2}$$

$$\frac{\ln(|\tan(x)+\sec(x)|)}{2} + \frac{\sec(x)\tan(x)}{2} + C$$

$$\frac{\ln(||\tan(x)+\sec(x)||)}{2} + \frac{\sec(x)\tan(x)}{2} + C$$

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$$\int \cos^2\left(\frac{x}{2}\right) dx \qquad u = \frac{x}{2}$$

$$2\int \cos^2(u)\, du$$

$$\frac{\cos(u)\,\mathrm{sen}(u)}{2} + \frac{1}{2}\int 1\, du =$$

$$\frac{\cos(u)\,\mathrm{sen}(u)}{2} + \frac{u}{2}$$

$$2\int \cos^2(u)\, du = \cos(u)\,\mathrm{sen}(u) + u$$

$$\frac{x}{2} + \cos\left(\frac{x}{2}\right)\mathrm{sen}\left(\frac{x}{2}\right)$$

$$\frac{x}{2} + \cos\left(\frac{x}{2}\right)\mathrm{sen}\left(\frac{x}{2}\right) + C$$

$$\frac{\mathrm{sen}(x) + x}{2} + C$$

18 $\int \frac{\operatorname{sen}(x)}{1+\operatorname{sen}(x)}\,dx$

$$\frac{\operatorname{sen}(x)}{1+\operatorname{sen}(x)} = -\frac{1}{1+\operatorname{sen}(x)} + 1$$

$$\frac{\operatorname{sen}(x) - (1+\operatorname{sen}(x))}{1+\operatorname{sen}(x)} + 1$$

$$\frac{\operatorname{sen}(x) - (1+\operatorname{sen}(x))}{1+\operatorname{sen}(x)} = \frac{-1}{1+\operatorname{sen}(x)}$$

$$\frac{\operatorname{sen}(x) - (1+\operatorname{sen}(x))}{1+\operatorname{sen}(x)} = \operatorname{sen}(x) - (1+\operatorname{sen}(x))$$

$$\int \frac{-1}{1+\operatorname{sen}(x)} + 1\,dx = -\int \frac{1}{1+\operatorname{sen}(x)}\,dx + \int 1\,dx$$

$$\int \frac{1}{1+\operatorname{sen}(x)}\,dx = -\frac{2}{\tan\left(\frac{x}{2}\right)+1} = 2\cdot\int \frac{1}{1+u^2+2u}\,du$$

$$2\cdot\int \frac{1}{(u+1)^2}\,du = 2\cdot\frac{\left(\tan\left(\frac{x}{2}\right)+1\right)^{-2+1}}{-2+1}$$

$$\frac{2}{\tan\left(\frac{x}{2}\right)+1} + x + c$$

19 $$\int \frac{\sqrt{x^2+9}}{x^2}\,dx$$

$$\frac{\sqrt{x^2+9}}{x} - \int -\frac{1}{\sqrt{x^2+9}}\,dx$$

$$\int -\frac{1}{\sqrt{x^2+9}}\,dx = -\ln\left|\frac{1}{3}\left(x+\sqrt{9+x^2}\right)\right|$$

$$\int -\frac{1}{\sqrt{x^2+9}}\,dx$$

$$-\ln\left|\tan\left(\arctan\left(\tfrac{1}{3}x\right)\right) + \sec\left(\arctan\left(\tfrac{1}{3}x\right)\right)\right|$$

$$-\ln\left|\frac{1}{3}\left(x+\sqrt{9+x^2}\right)\right|$$

$$-\frac{\sqrt{x^2+9}}{x} - \left(-\ln\left|\frac{1}{3}\left(x+\sqrt{9+x^2}\right)\right|\right)$$

$$-\frac{\sqrt{x^2+9}}{x} + \ln\left|\frac{1}{3}\left(x+\sqrt{9+x^2}\right)\right| + C$$

20) $\int \sec^2(x) \ln(\tan(x))\, dx$

$u = \tan(x)$

$dx = \frac{1}{\sec^2(x)} du$

$\int \ln(u)\, du$

$u \ln(u) - \int 1\, du$

$\tan \ln(\tan) - u$

$\tan x \ln(\tan(x)) - \tan(x)$

$\tan(x)(\ln(\tan(x)) - 1) + C$

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$$\int \operatorname{sen}(2x)\, e^{\operatorname{sen} x}\, dx$$

$\operatorname{sen}(2x) = 2 \operatorname{sen} x \cos x$ $\qquad$ $u = \operatorname{sen} x$

$du = \cos x\, dx$

$$2\int \operatorname{sen} x \cos x\, e^{\operatorname{sen} x}\, dx$$

$$2\int u e^{u}\, du$$

$w = u$ $\qquad$ $dv = e^{u}$

$dw = du$ $\qquad$ $v = e^{u}$

$$2\left(we - \int e^{u} du\right) =$$

$$2we^{u} - 2e^{u} =$$

$$2 \operatorname{sen} x\, e^{\operatorname{sen} x} - 2e^{\operatorname{sen} x}$$

22) $\int \frac{1}{x^4+1} \, dx$

$$\frac{1}{x^2 - \sqrt{2x}+1)\ (x^2+\sqrt{2x}+1} \, dx$$

$$\int \frac{x+\sqrt{2}}{2^{3/2}(x^2+\sqrt{2}x+1)} - \frac{x-\sqrt{2}}{2^{3/2}(x^2-\sqrt{2}+x+1}) \, dx$$

$$\int \left(\left(\frac{2x+\sqrt{2}}{2(x^2+\sqrt{2}x+1}\right) + \left(\frac{1}{\sqrt{2}(x^2+\sqrt{2}+x+1)}\right)\right) dx$$

$$\frac{1}{2}\int \frac{2x+\sqrt{2}}{x^2+\sqrt{2}x+1} \, dx \quad \frac{1}{\sqrt{2}}\int \frac{1}{x^2+\sqrt{2}+x+1} \, dx$$

$U = x^2+\sqrt{2}x+2$ $\quad \int \frac{1}{U} \, du \rightarrow \ln(U)$

$dx = \frac{1}{2x+\sqrt{2}} \, du$

$\ln(x^2+\sqrt{2}x+1) \quad = \int \frac{1}{x^2+\sqrt{2}x+1} =$

$\int \frac{1}{\left(x+\frac{4}{\sqrt{2}}\right)+\frac{1}{2}}$ $\quad U = \sqrt{2}x+1$

$dx = \frac{1}{U} \, du$

$$\frac{\ln(U^2+\sqrt{2}x+1)}{2^{5/2}} - \frac{\ln(x^2-\sqrt{2}x+1)}{2^{5/2}} + \frac{\tan^{-1}(\sqrt{2}x+1)}{2^{3/2}}$$

$$+\frac{\tan^{-1}(\sqrt{2}x-1)}{2^{3/2}} + C$$